HANDLING CONSTRAINTS AS OBJECTIVES IN A MULTIOBJECTIVE GENETIC BASED ALGORITHM

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Abstract

In this paper, a novel approach based on handling constraints as objectives together with a modified Parks & Miller elitist technique, to solve constrained multiobjective optimization problems, is analyzed with Niched Pareto Genetic Algorithm. The performance of this approach is compared with the classical procedure of handling constraints that is the exterior penalty function method. Results are obtained applying both procedures of handling constraints, with and without elitism. Especially when using the modified elitist technique, simulation results suggest the effectiveness of the proposed technique.

I. INTRODUCTION

Most of the real world engineering optimization problems are multiobjective, constrained and hard computing, requiring an algorithm capable to handle the constraints and determine the Pareto-optimal (PO) front efficiently. When genetic based algorithms are used, the most common way of handling constraints is by using penalty techniques like the exterior penalty method. There are some weaknesses in this classical approach because good values for the penalty parameters are not known. Very high values for the penalty parameters do not highlight the objective functions and thus most effort is spent in finding feasible solutions. Low values can guide the search in the direction of infeasible points.

In this paper, constraints in constrained multiobjective optimization problems are handled as objectives and the resulting problem is solved by the Niched Pareto Genetic Algorithm - NPGA [1]. Handling constraints as objectives was recently presented for single-objective optimization [2]. The original NPGA was modified by incorporating the Parks & Miller elitist technique (P&M) [3], which needed some changes when constraints were treated as objectives. The required changes were essential to avoid convergence toward an infeasible space.

Two analytical test problems, TBU [4] and CPT7 [5], that were designed with special features to difficult the PO front search, are chosen to compare the approaches of handling constraints as objectives and using penalty function.

Comparing different procedures to solve multiobjective optimization problems is a hard task because these problems in general have a lot of nondominated solutions. So, just one metric is not

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enough and a set of them must be used to spot the differences among approaches under investigation. The methodology adopted here to compare the results from both new and classical approaches made use of three quantitative metrics: generational distance, coverage relationship and timing analyses [6], [7]. This choice enabled a realistic sight of the techniques discussed, pointed out the advantages of the new approach. After that, both classical and new procedures were applied to find the nondominated front by solving the constrained multiobjective optimization problem TEAM22 [8].

II. MATHEMATICAL FORMULATION

The multiobjective optimization involves a set of k decision variables, m objective functions and n constraints. In terms of minimization we can write this problem as:

minimize:

$$\bar{f} = \{f_1(\bar{x}), f_2(\bar{x}), \cdots, f_m(\bar{x})\}^T \tag{1}$$

subjected to:

$$\bar{g} = \{g_1(\bar{x}), g_2(\bar{x}), \cdots, g_n(\bar{x})\}^T \leq \{0, 0, \cdots, 0\}^T \tag{2}$$

where $\bar{x} = \{x_1, x_2, \cdots, x_k\}^T$.

As NPGA is not capable to deal directly with constrained problems, some way must be found to handle the constraints. In this paper, two approaches are considered. First, the constraints are incorporated to the fitness function by using penalty functions. This procedure will be denoted here as classical approach. Initially, the original optimization problem is rewritten as an unconstrained one. As an example, for the ith objective, a pseudo-objective function ffi can be written as:

$$ff_i(\bar{x}) = f_i(\bar{x}) + \sum_{j=1}^{n} \rho_j \left(g_j(\bar{x})\right)_+^2 \tag{3}$$

where ρj is the jth penalty parameter associated to the jth constraint and ()+ denotes that only violated constraints are considered. Usually, all n penalty parameters are taken with the same value, i.e., ρj = ρ for j = 1,.., n.

Second, the n constraints are transformed in n more objectives. To avoid confusion, this approach will be denoted here as new approach. Mathematically, the original problem is rewritten as:

minimize:

$$\bar{f} = \{f_1(\bar{x}), \cdots, f_m(\bar{x}), g_1(\bar{x}), \cdots, g_n(\bar{x})\}^T \tag{4}$$

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where $\bar{x} = \{x_1, x_2, \cdots, x_k\}^T$.

Using both approaches the constrained problem is transformed into an unconstrained one. Afterward, the ith fitness can be obtained by any procedure rewriting the minimization problem as maximization one.

III. TREATING CONSTRAINTS AS OBJECTIVES

A good question to be answered is ‘why is handling constraints as objectives interesting?’ As an example, imagine a hypothetical problem, in terms of multiobjective minimization, as illustrated in Fig. 1.

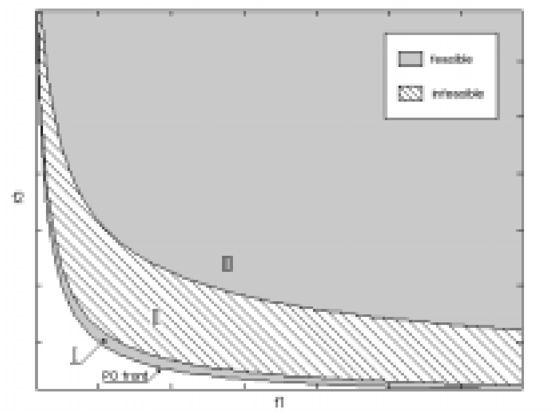

Fig. 1 – Hypothetical problem

The regions I and III are feasible ones but II is infeasible. As can be seen region III is the biggest one and probably more samples will be generated in this region in the first population. Using the classical approach the individuals of region II will be dominated by those of region III, because all objectives evaluated on points belonging to region II will be increased by a penalty value, guiding the search in direction of region III. Region II is as a ‘wall’ between regions I and III. Suppose that in the first population no point is sampled in region I, which is the most probable situation because this is the shortest region and it is very small. So finding a point belonging to this region would be a hard task. Therefore, one expects that the algorithm will converge most of times to the front defined in the region III.

On the other hand, when handling constraints as objectives region II is not viewed as a ‘wall’ because points of region III do not dominate those of II.

IV. MODIFIED PARKS & MILLER ELITISM

To improve the NPGA performance the Parks & Miller elitist technique [3] was used. It consists in incorporating the efficient individuals of the on-line population (Pon) to the off-line population (Poff), at each generation. When Poff size exceeds a threshold, the dominance criterion is applied, eliminating all dominated solutions. If Poff size continues bigger than the threshold, a distance criterion is applied. It is based on measuring the distance between the off-line individuals, taking

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two per turn, and if they are within some distance one of them is discarded, chosen randomly. This distance is measured in the objective space. The individuals of Poff are often reinserted in Pon to improve the convergence. This approach works well when the constraints are considered in the classical approach.

Although infeasible individuals (infeasible points) do not represent the wished PO front, keeping them in the on-line population seems to be a good idea because infeasible points near the PO front might be lost during the optimization process. When using constraints as objectives, i.e., the new approach, P&M elitism does not work properly because sometimes the Poff may be composed by a great deal of infeasible points, guiding the search in wrong way. To avoid this drawback, P&M technique was modified by two additional procedures to avoid convergence to the infeasible region: i) when eliminating individuals of Poff by the distance criterion, infeasible points are discarded if they are near to a feasible one and ii) when Poff size is bigger than a threshold, all infeasible points are discarded. This modified P&M will be denoted by M-P&M.

Without these procedures the nondominated set can partially converge toward an infeasible region due to reinserting many infeasible individuals of Poff in Pon. So, in the elitism process the constraints are not always viewed as objectives.

Eliminating individuals by distance criterion is done to preserve genetic diversity avoiding premature convergence. Distance measuring is done without considering the constraints, because diversity is needed only in the original objectives.

When using the classical approach (penalty functions) these procedures are not needed because just feasible individuals are placed in Poff.

V. PERFORMANCE MEASUREMENT

In general multiobjective optimization problems have uncountable solutions and it is not so easy to compare the quality of the solutions found using just one criterion. Usually, a unique criterion is not enough to compare the solutions for this kind of problems. To contrast the classical and new approaches with and without elitism, three metrics were used: generational distance (GD), coverage relationship (CR) and timing analyses (T) [6], [7].

A . Generational Distance (GD)

This metric represents the distance between the front wished and the front found. Mathematically:

$$GD \equiv \frac{\left(\sum_{i=1}^{n} d_i^2\right)^{1/2}}{n} \tag{5}$$

where n is the number of points in the nondominated set found and di is the Euclidean distance between each point and the nearest member of PO front.

B. Coverage Relationship (CR)

Given two sets of nondominated solutions, CR metric computes for each set the rate of solutions that is not covered (nondominated) by the other. The arithmetic mean of the number of solutions found per simulation is also presented.

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C. Timing analyses (T)

This metric is just the computing time that represents the computational effort. It was measured under the same computing conditions and the results are shown normalized.

These metrics give a good idea of the algorithm performance. At the end of the simulation process, just the feasible individuals were considered and all infeasible ones discarded.

VI. ANALYTICAL TEST FUNCTIONS

The first test function, TBU [4], has the following properties: i) the feasible region is non-convex and ii) some feasible PO solutions lie on boundaries between the feasible and infeasible regions. This problem is defined as:

minimize:

$$\bar{f} = \begin{Bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \end{Bmatrix} = \begin{Bmatrix} 4x_1^2 + 4x_2^2 \\ (x_1 - 5)^2 + (x_2 - 5)^2 \end{Bmatrix} \quad (6)$$

subjected to:

$$\bar{g} = \begin{Bmatrix} g_1(\bar{x}) \\ g_2(\bar{x}) \end{Bmatrix} = \begin{Bmatrix} (x_1 - 1)^2 + x_2^2 - 25 \\ -(x_1 - 8)^2 - (x_2 + 3)^2 + 7.7 \end{Bmatrix} \leq \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (7)$$

where $-15 \leq x_i \leq 30, \quad i = 1,2$.

This problem was simulated with population size and generation number equal to 80 and 100, respectively.

The main property of the second test function, CPT7 [5], is the disconnected PO front. This problem is mathematically defined as:

minimize:

$$\bar{f} = \begin{Bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \end{Bmatrix} = \begin{Bmatrix} x_1 \\ c(\bar{x})[1 - f_1(\bar{x})/c(\bar{x})] \end{Bmatrix} \quad (8)$$

subjected to:

$$\begin{aligned} g(\bar{x}) = \cos(\theta)[f_2(\bar{x}) - e] - sin(\theta)\, f_1(\bar{x}) \quad \geq \\ a \left| sin\left\{ b\pi \left[sin(\theta)\, (f_2(\bar{x}) - e) + \cos(\theta)\, f_1(\bar{x}) \right]^c \right\} \right|^d \end{aligned} \quad (9)$$

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where: $c(\bar{x}) = 41 + \sum_{i=2}^{5}\left[x_i^2 - 10\cos(2\pi x_i)\right],\ \theta = -0.05\pi,\quad a = 40,\quad b = 5,\quad c = 1,\quad d = 6,\quad e = 0$,
$0 \le x_1 \le 1, -5 \le x_i \le 5,\quad i = 2,3,4,5$.

This problem was simulated with population size and generation number equal to 40 and 400, respectively.

The NPGA was executed twenty times for each test problem. The arithmetic mean of these results using the metrics discussed above are shown in Table I and II, respectively, where OA denotes the approach of handling constraints as objectives, CA the classical penalty approach, P&M the Parks&Miller elitism and M-P&M the modified Parks&Miller elitism described before. The comparisons between OA and CA were done with and without elitism separately.

Table I - Results of TBU problem

	With Elitism		Without Elitism	
Metric	OA M-P&M	CA with P&M	OA	CA
CR (Mean)	100% (27.66)	99.33% (22.40)	100% (16)	57.01% (6.30)
GD	0.0155	0.0152	0.0190	0.0328
Time	1.72	1.28	1.78	1.16

Table II - Results of CPT7 problem

	With Elitism		Without Elitism	
Metric	OA M-P&M	CA with P&M	OA	CA
CR (Mean)	100% (12.5)	0.0% (0.0)	100% (0.1)	0.0% (0.0)
GD	0.0025	0.0615	0.2506	0.9315
Time	1.27	1.23	1.15	1.02

The results in Table I with elitism show that both approaches present comparable results. This can be seen from the CR and GD metric values. However, the front was better represented (see the mean of solutions found) when using the constraints as objectives. In the other situation, without elitism, the advantages of using constraints as objectives are clearer. It is observed that the computing time for OA increases proportionally with the number of constraints, but it is not significant for real world problems, because the cost in the optimization process to evaluate the

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original objectives is usually very high when compared with the additional cost to treat the constraints as objectives.

For the second test, CPT7 problem, an acceptable front was only found by using the new approach, that is, OA together with the modified elitist technique. This is clearly shown in Table II.

The fronts found for these two functions when using elitism are shown in Figs. 2 and 3, respectively. It is important to remember that just feasible individuals were saved, being all infeasible ones discarded at the end of simulation.

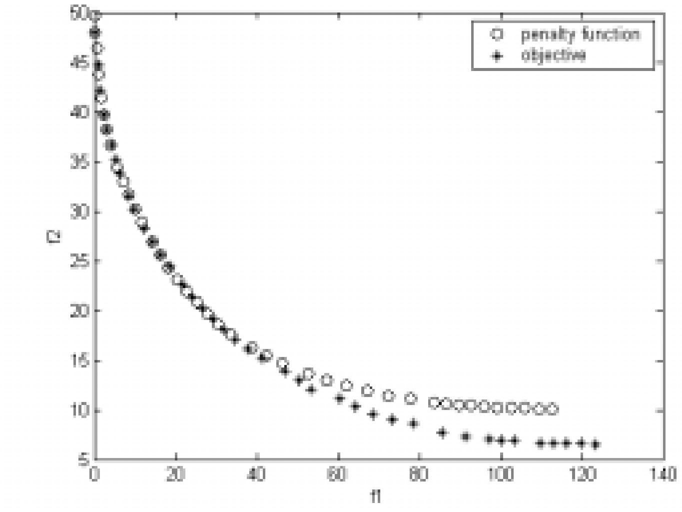

Fig. 2. Nondominated sets of TBU problem.

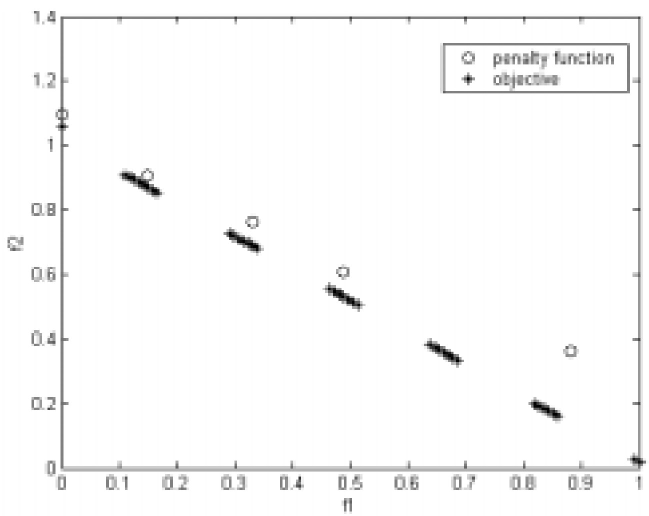

Fig. 3. Nondominated sets of CPT7 problem.

The front shown in Fig. 2 when using penalty function is partially dominated by that found using constraints as objectives. This behavior was expected because some feasible PO solutions lie on boundaries between the feasible and infeasible regions, as can be seen when plotting the complete graph of this function, in which case the penalty function approach does not work well as previously explained.

The front shown in Fig. 3 when using the classical penalty function is completely dominated by the one found using the new approach. As the constraint is extremely nonlinear the feasible

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front is disconnected, being each part of the feasible front separated by an infeasible one. This case confirms that is good keeping infeasible individuals that are near the PO front in the on-line population as is done in the new approach. This does not happen when using penalty function approach, causing convergence to an incorrect nondominated front.

VII. CONSTRAINED MULTIOBJECTIVE OPTIMIZATION IN ELECTROMAGNETICS

The TEAM'22 problem with three continuous variables was chosen to demonstrate the performance of the new approach in constrained multiobjective optimization problem in electromagnetics. This problem is well known in the literature so its complete description is omitted here. The aim of this problem is to optimize the Super-Conducting Energy Storage configuration with respect to two objectives and one constraint, to ensure minimal stray field (f1), 180MJ of stored energy (f2) and that physical quench condition is met [8].

This problem was solved using NPGA with both new and classical approaches to handle the physical quench condition. In both cases the modified and standard Parks&Miller elitisms were used and the population size and generation number were fixed to 30. The analysis of TEAM'22 problem was realized using a finite element code using triangular elements of first order. The results are presented in Fig. 4.

The front found using constraint as penalty function (classical approach) was completely dominated by that obtained using constraint as objective (new approach). The nondominated set of this problem was first presented neglecting the quench physical condition in [7].

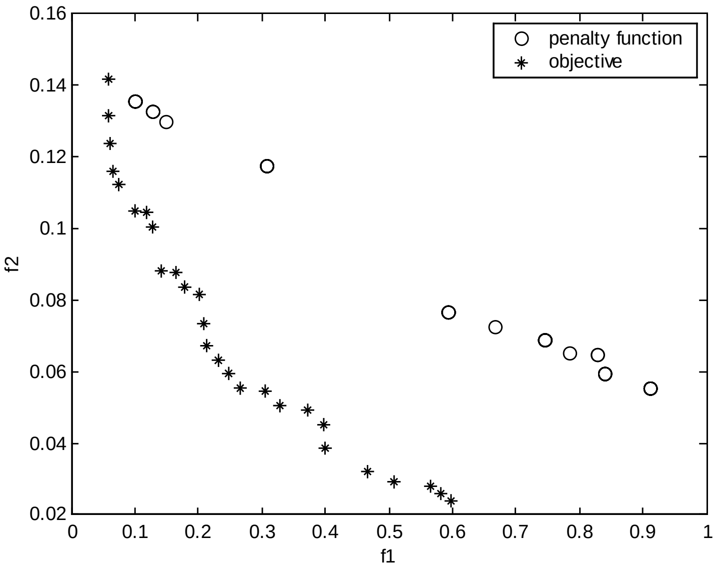

Fig. 4 – Nondominated sets of TEAM22 problem.

VIII. CONCLUSION

The results obtained by handling constraints as objectives when solving both test and TEAM'22 problems demonstrate that this approach works better than the classical method of handling constraints using penalty functions. As discussed earlier, the standard Parks and Miller elitism has

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been modified to consider the constraints transformed in objectives. The proposed modifications were important to avoid the convergence of NPGA toward infeasible regions and to represent better the nondominated front. These results pointed out the effectiveness of this new approach in multiobjective optimization problem.

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