Robust Control Configured Design Method for Systems with Multi-objective Specifications

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Abstract

In this paper we propose a new design method for robust control configured systems with multiple design specifications. Because this design problem is formulated as the multi-objective minimax optimization problem, we use the Genetic Algorithm (GA) based technique to obtain the optimal solution. This design method is based on the input deviation, the minimax design approach and pareto-partitioning GA. And we apply this design method for designing the 4WS(4 wheeled steering) car system. Some design examples are included to show the applicability and the effectiveness of the proposed method.

1. Introduction

This paper is concerned with a design method for systems with multiple design specifications, for example, robust stability, robust performance, and so on.

Generally, the process of developing an engineering system involves many steps. These steps falls into three categories roughly. Those are "structure design ", "control design" and "implementation". And it is important that the control engineer assists a mechanical engineer on designing structure for developing a system with good performance[1, 2]. In view of this point, a study and development on Control Configured Vehicle (CCV)[3] has been made over the last few decades. The basic concept of CCV is configuring the control design on designing the structure [4, 5, 6].

On the other hand, various kinds of uncertainties exist in a system structure. Thus, controllers for such a system structure are required to have robustness property in that it can attain acceptable control performance and closed-loop stability in the presence of parametric perturbations.

This paper, therefore, proposes a new design method for robust control configured systems with multiple design specifications. Because this design problem is formulated as the multi-objective minimax optimization problem, we use the Genetic Algorithm (GA) based technique to obtain the optimal solution. This design method is based on the input deviation[1], the minimax design approach[2, 7] and pareto-partitioning genetic algorithm (GA) based technique[8].

First, the input deviation is the criterion for evaluating structure design. This criterion is to evaluate structure from the viewpoint of the control designing. If this input deviation for all actual inputs in all domains becomes less, the input-output relation of actual system is made closer to the ideal one. We can thus evaluate the structure design and obtain the nominally optimal values of physical parameters used in actual system using this input deviation criterion.

Secondly, the minimax design approach is one of the worst case design methods, namely, parameters of controller are adjusted so as to minimize the performance measure maximized by parameters representing uncertainties of the structure. Hence the designed controller by this approach has robust property against the structural uncertainties.

Thirdly, the reason why we use GA is that GA can give good design results for many practical situations even when the standard gradient based technique fails. GA has multiple individuals that can search in parallel and simultaneously evaluates many points in the search region. GA based technique, therefore, is more likely to converge toward the global optimum. And, in this design method, to prevent a partial convergence of non-dominated solutions in the trade-off surface on the multi-objective optimization problem, the pareto partitioning GA uniformly controls a distribution of non-dominated solutions. This GA assigns all non-dominated individuals the different fitness on the basis of the distribution in pre-specified regions.

Furthermore, some design examples are included to show the applicability and the effectiveness of the proposed method.

2. Control configured design method

The proposed design method, control configured design method, basically consists of two design procedures. First one is the phisical parameter design and second one is robust minimax control design. We explain these two design procedures in following sections.

2.1 Physical parameter design

Generally, the process of making a system involves many steps. The scenario is as follows typically:

Step 1. Design and make the physical system without controller.

Step 2. Identify the system for resulting a model to be controlled.

Step 3. Analyze the model.

Step 4. Decide on performance specification.

Step 5. Design a controller to meet specs, if impossible, modify the specs or go to *step 1* and remake the system and repeat.

Step 6. Simulate the resulting control system.

Step 7. Implement the controller.

These steps falls into three categories roughly. Those are "structure design " (Step 1), "control design"(Steps $2 \sim 5$) and "implementation" (Steps 6 and 7). Then, as a rule, a control engineer's role is considered merely as designing controller for structures. But, it is important that the control engineer assists a mechanical engineer in the designing of structure system (on Step 1), for the better control performance system made [1, 4].

From this viewpoint, let's consider how to evaluate *structure*. At first, we assume the following conditions:

1. We can define the ideal output of system
2. We can classify the actual input of system into the finite typical patterns
3. We can define the inverse system

Next, we define the ideal and actual input-output relations of system as

- u_i : actual inputs of system
  $\{ u_i \in U \,;\, U$: a group of function $\}$
- y_m : ideal outputs of actual system
  $\{ y_m \in Y \,;\, Y$: a group of function $\}$
- f : the transfer function of actual system.
  (From above assumptions, f^{-1} always exists.)
- u_o : optimal input.
  (if and only if u_o is inputted into actual system, then output is y_m. $\{ u_o = f^{-1}(y_m) \}$)

From above notations, we can define the following performance function for evaluating *structure*.

$$J = \sum_{i=1}^{n} p_i \parallel u_o - u_i \parallel = \sum_{i=1}^{n} p_i \left[\int_{a_i}^{b_i} \mid u_o - u_i \mid^2 dt \right]^{1/2} \tag{1}$$

where

- n : the number of classified inputs
- (a_i, b_i) : time domains of classified inputs
- p_i : weights for classified inputs

Now, $\mid u_o - u_i \mid$ is named " input deviation ". If this input deviation for all actual inputs in all domains becomes less, the input-output relation of actual system is made closer to the ideal one. We can thus evaluate the structure design and obtain the nominally optimal values of physical parameters used in actual system using this input deviation criterion. This problem is evaluating the structure and getting the nominal optimal values of physical parameters. So we can construct a first problem for physical parameter design as following;

First problem :

$$\min_{\theta \in \Theta_1} J \tag{2}$$

where $\theta \in \Theta_1$ is the set of phisical parameters of structure. It is assumed that Θ_1 is a prior given bounded set.

Solving this problem, we can get the nominal optimal values of phisical parameters.

2.2 Robust minimax control design

Although the nominal optimal values of phisical parameters are obtained by solving first problem, perturbations of physical parameters are not able to take acconut in first problem. As I have mentiond before, structure of systems have various kind of uncertainties described parametric perturbations. Therefore, the system is required to have robustness property against these uncertainties of structures. Thus the robust against parameter perturbation control design problem is formulated as the following minimax optimization problem:

Second problem :

$$\min_{q \in Q} \max_{\theta \in \Theta_2} J \tag{3}$$

s.t. closed-loop system is stable of all $\theta \in \Theta_2$

where Θ_2 is the set of perturbations of the optimal nominal values of physical parameters obtaind in the first problem and $q \in Q$ is the parameter vector of the controller. These perturbation ranges are able to estimate by using some set-membership identification based method

[9, 10]. We also assume that Q and Θ_2 are prior given.

Then, this minimax optimization problem is one of the worst case design, namely, parameters of controller are adjusted so as to minimize the J maximized by parameters of structure. We can check the robust stability by Edge Theorem [11]. But, it should be noted that the plant models do not satisfy the premis of Edge's Theorem that every parameter of characteristic equation is linearly affine. Hence, the results are a little conservative.

And it should be noted that a detail of whole design algorithm is depend on the design object. In next section, therefore, let's apply this design method to design of 4 wheeled Steering (4WS) system of car.

3. Application to 4WS system of car

3.1 Model of 4WS system

4WS car is the system which has steer angle not only the front wheels but rear wheels like shown in fig. 1.

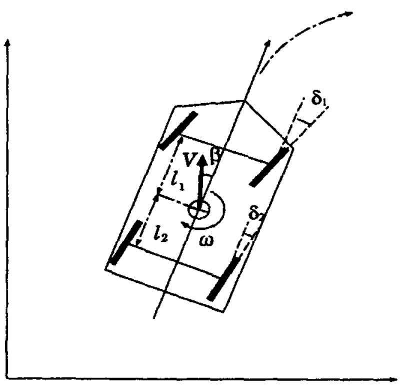


Figure 1: Car with 4WS system

The state space equation of this model as shown in fig. 1 is obtained as:

$$\frac{d}{dt}\begin{bmatrix}\beta\\ \omega\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\beta\\ \omega\end{bmatrix}+\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}\delta_1\\ \delta_2\end{bmatrix} \quad (4)$$

$$a_{11}=\frac{-(k_1+k_2)}{mV} \qquad b_{11}=\frac{k_1}{mV}$$

$$a_{12}=\frac{-mV^2-l_1k_1+l_2k_2}{mV^2} \qquad b_{12}=\frac{k_2}{mV}$$

$$a_{21}=\frac{-(l_1k_1-l_2k_2)}{I} \qquad b_{21}=\frac{k_1l_1}{I}$$

$$a_{22}=\frac{-(l_1^2k_1+l_2^2k_2)}{IV} \qquad b_{22}=\frac{-k_2l_2}{I}$$

where the outputs of this system are slip angle β of center of gravity and yawing angular velocity ω. The definition of parameters are shown in table 1.

Table 1: Parameters of a car model

β	slip angle of center of gravity
ω	yawing angular velocity
δ_1	steering angle of front wheel
δ_2	steering angle of rear wheel
V	longitudinal velocity of center of gravity
m	mass
I	yawing moment of inertia
k_1	cornering power of front wheel
k_2	cornering power of rear wheel
l_1	length between center of gravity and front wheel
l_2	length between center of gravity and rear wheel
L	wheel base ($L=l_1+l_2$)

And the inputs of this system are steering angle of front wheel (δ_1) and rear wheel (δ_2). The inputs of δ_1 is thought to be given by human who drive the car.

On the other hand, we employ following contol law for δ_2.

$$\delta_2=K\omega \quad (5)$$

where K is controller.

3.2 Preparation for design

We use the following performance function for evaluating the structure and controller of car.

$$J=\sum_{i=1}^{n}p_i\|\delta_0-\delta_i\|=\sum_{i=1}^{n}p_i\left[\int_0^\infty|\delta_0-\delta_i|^2\,dt\right]^{\frac{1}{2}} \quad (6)$$

where δ_0 is optimal input of front steering angle and δ_i is actual input of front steering angle.

The output is defined the slip angle of center of gravity (β) and the ideal output is assumed $\beta=\dot{\beta}=0$, namely

$$\forall\delta_i\rightarrow\beta=\dot{\beta}=0 \quad (7)$$

Then the optimal inputs δ_{0*} are derived as[1]:

$$\delta^o_{2ws}(t) \;=\; c\exp(\Phi) \quad (8)$$

$$\delta^o_{4ws}(t) \;=\; c'\exp(\Phi' t) \quad (9)$$

where

$$\Phi \;=\; \frac{mV^2l_1-k_2l_2L}{IV}$$

$$\Phi' \;=\; \frac{mV^2l_1-k_2L(KV+l_2)}{IV}$$

where c and c' are constants. Eqs. (8) and (9) show optimal inputs for 2WS car and for 4WS car respectively.

Actual inputs are defined as shown in figs. 3 and 4 with eqs. (10) and (11).

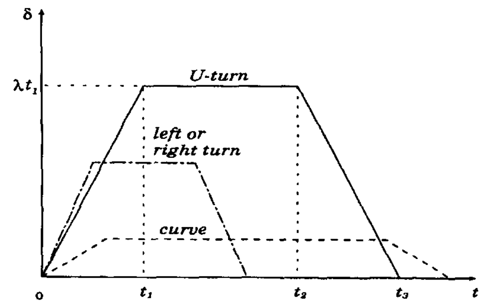


Figure 2: Actual inputs by driver (a)

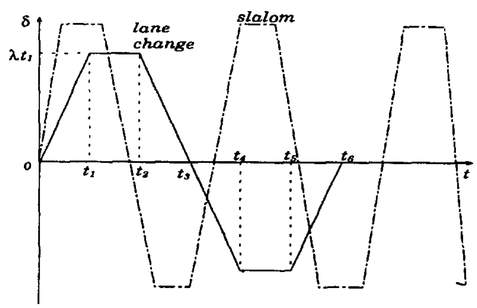


Figure 3: Actual inputs by driver (b)

$$\delta_i(\text{ for (a)}) = \begin{cases} \lambda t & : 0 \le t \le t_1 \\ \lambda t_1 & : t_1 \le t \le t_2 \\ -\lambda(t - t_3) & : t_2 \le t \le t_3 \end{cases} \quad (10)$$

$$\delta_i(\text{ for (b)}) = \begin{cases} \lambda t & : 0 \le t \le t_1 \\ \lambda t_1 & : t_1 \le t \le t_2 \\ -\lambda(t - t_3) & : t_2 \le t \le t_4 \\ -\lambda(t_4 - t_3) & : t_4 \le t \le t_5 \\ \lambda(t - t_6) & : t_5 \le t \le t_6 \end{cases} \quad (11)$$

3.3 Design algorithm

The parameters of structure are m(mass), I(yawing moment of inertia), k_2(cornering power of rear wheel), l_1(length between center of gravity and front wheel), l_2(length between center of gravity and rear wheel) and K(controller). On first problem, δ^o_{2ws} is used as optimal input. This process means to find the nominal optimal parameters of 2WS car system as structure design. Next, in second problem, we design the robust controller K. Here δ^o_{4ws} is used as optimal input for second problem.

The design algorithm is as follows in brief.

Step 1. Solve the first problem and get nominal optimal values. The first problem is

$$\min_{\theta \in \Theta_1} J \quad (12)$$

where $\theta \in \Theta_1$ is the set of physical parameters of structure of car(m, I, l_1, l_2, k_2). And where Θ_1 is a prior given bounded set.

Step 2. Set the perturbation $\pm N\%$ of nominal optimal values obtained on *Step 1*.

Step 3. Solve the second problem and get the robust controller K. The second problem is

$$\min_{K} \max_{\theta \in \Theta_2} J \quad (13)$$

where $\theta \in \Theta_2$ is (m, I, l_1, l_2, k_2). And where Θ_2 is $\pm N\%$ perturbed bounded set of nominal optimal values.

In this minimax design problem, we can see that there does not generally exist saddle point, namely minmax $J \neq$ maxmin J, from eq. (6). And we also recognize that this problem has multiple specifications. For solving this design problem, therefore, we developed the GA with pareto partitioning method[8] as mentioned below.

3.4 GA with pareto patitioning method

To prevent a partial convergence of non-dominated solutions in the trade-off surface, we propose the Pareto partitioning method which uniformly controls a distribution of non-dominated solutions. The proposed method assigns all non-dominated individuals the differ fitness on the basis of the distribution in pre-specified regions.

The proposed method consists of following procedure. First, the objective space is divided pre-specified regions. The edge points of the whole region correspond the best solutions for each objective function. Then, the fitness f_i of the individual p_i is defined as $f_i = 1/n_i$. The value of n_i denotes the number of non-dominated solutions in the identical region with the individual p_i.

The continuous generation model with a pareto-optimal preservation strategy is employed. In this model, a pareto-optimal set is always forced to appear in the following generation.

The proposed procedure consists of the following steps:

Step 1 Set a generation number $t = 0$. Randomly generate an initial population $P(t)$ of M individuals.

Step 2 Calculate the fitness of each solution in the current population according to the multiobjective ranking. If the non-dominated solutions occupy the current population, apply the pareto partitioning method for the fitness assignment of all non-dominated individuals.

Step 3 Generate a new population $P'(t)$ from $P(t)$ by using a crossover operator.

Step 4 Apply a mutation operator to the newly generated population $P'(t)$ a ccording to mutation rate.

Step 5 Calculate the fitness both of $P(t)$ and $P'(t)$.

Step 6 Select M individuals from all population member on the basis of the fitness. If the non-dominated solutions is over M, select M individuals from whole population according to the proposed fitness assignment.

Step 7 If a terminal condition is satisfied, stop and return the best individuals. Otherwise set $t = t + 1$ and go to *Step 2*.

In this procedure, update of the current population size is always constant M. There is no question that selection of individuals for the next generation should be proportional to their fitness values. However, when a pure proportional approach is adopted, in practice the population is rapidly dominated by a few super-individuals, resulting in a dramatic decrease in genetic diversity. Here, to avoid the rapid loss of genetic diversity, multiple equivalent individuals are eliminated from the current population.

3.4.1 GA formulation

To design GA for the design problem mentiond above, certain problem-dependent algorithm elements need to be defined. They influence both the efficiency of the algorithm and the quality of its results.

A. Representation of individuals

Gray-code string representation is employed for candidate solutions. Each of the parameters x_1 and x_2 is subject to interval constraints. Two such strings are concatenated into a binary string, representing a point in the parameter space to be searched by the algorithm.

B. Fitness function

The linear scaling method based on the multiobjective ranking is employed as a fitness function. The fitness f_i of the individual p_i is computed through two steps. First, the rank R_i $(i = 1, \ldots, M)$ of each individual in the current population $P(t)$ is calculated by the multiobjective ranking. Then, the fitness f_i of the individual p_i is defined as follows:

$$\begin{aligned} f_i &= (F_{\max} - F_s \times (R_i - 1)) \\ F_s &= \frac{1}{M-1}(F_{\max} - F_{\min}) \end{aligned} \tag{14}$$

where M denotes the population size, $F_{\max}$ and $F_{\min}$ denote the maximum and the minimum fitness values. In computer simulations, the values of $(F_{\max}, F_{\min})$ are held constants $(10.0, 1.0)$, namely, the fitness of best individuals is 10.0, on the contrary, the fitness of worst one is 1.0.

C. Genetic operators

The genetic operators applied in the algorithm are uniform crossover, bit mutation and roulette wheel selection. Uniform crossover generates two offspring by exchanging a predefined number of alternate subsections between two parent strings. The recombination operator, mutation, is implemented as altering bit values at randomly selected string positions.

3.5 Design result

We assumed the following range for Θ_1 at the first problem.

Table 2: Search ranges of Θ_1

1000.0	$\leq$	m	$\leq$	1500.0
1200.0	$\leq$	I	$\leq$	2000.0
1.0	$\leq$	l_1	$\leq$	1.5
1.0	$\leq$	l_2	$\leq$	1.5
30000.0	$\leq$	k_2	$\leq$	45000.0

And we also asuumed Θ_2 is $\pm 50\%$ bounded set of nominal optimal values.(This means $N = 50$.) Then we can get the nominal optimal parameters of 4WS car system by the proposed design method.

Furthermore, in this design example, GA parameters are defined as follows. Each parameter x_1 and x_2 of representation has 20 bits precision, namely, each population member consists of a 40-bit string. And the population size is 50, the mutation rate is 0.1 and the stopping condition is 100 generations. Furthermore, in the Pareto partitioning method, the number of partitioning region is $50^2 = 2500$.

The values of parameters are obtained as shown in table 3.

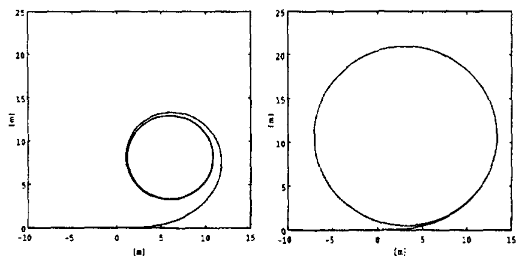
Table 3: Design results

	nominal optimal values
m	1272
I	1208
l_1	0.908
l_2	1.077
k_2	35340

The value of controller is obtained by the second problem is $K = 0.012$. We used all inputs which are classified in eqs. (10) and (11).

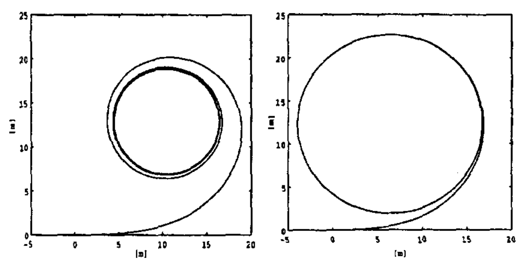
Figs. 4 and 5 show the loci of center of gravity of car with values in table 3 in the rotation movement in the case of $V = 80[km/h]$ and $V = 150[km/h]$

respectively. The input is a step for front steering angle which is given as $\delta_1 = 0.5[rad]$ on coordinates $(0,0)$ at time $t = 0$. The simulation time is 5 $[sec.]$. And the value of k_1 is set $32000[N/deg]$. Both of (a) show results in the case of car with nominal optiomal values of physical parameters in figs. 4 and 5. On the other hand, both of (b) show results in the case of car with worst case values of phisical parameters with bounded sets given in tabel 2.



(a). Nominal case (b). Worst case

Figure 4: Loci of car movement ($V = 80km/h$)



(a). Nominal case (b). Worst case

Figure 5: Loci of car movement ($V = 150km/h$)

From these figures, we observe that the car can turn in stable even in the worst case. Hence we can see that good robust performance of this control system designed by proposed method.

On the other hand, we get the fact that if we use the general values of structure of the conventional 4WS car, we can not find the realistic solution of the second problem with the constraint conditions involved robust stability of closed-loop system against 50% parameter perturbations in the situation of $V \geq 110.5km/h$. This means that if the speed of car (V) is greater than $110.5km/h$, the behavior of the car is unstable and dangerous with 50% parameter perturbations.

5. Conclusion

In this paper, we propose a new design method for robust control configured systems with multiple design specifications. Because this design problem is formulated as the multi-objective minimax optimization problem, we use the GA based technique to obtain the optimal solution. This design method is based on the input deviation, the minimax design approach and GA with the pareto partitioning method. And applying this method to design 4WS car system, we confirm the effectiveness of the proposed method.

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