

The Evolution of Optimality: De Novo Programming

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Abstract. Evolutionary algorithms have been quite effective in dealing with single-objective “optimization” while the area of Evolutionary Multiobjective Optimization (EMOO) has extended its efficiency to Multiple Criteria Decision Making (MCDM) as well. The number of technical publications in EMOO is impressive and indicative of a rather explosive growth in recent years. It is fair to say however that most of the progress has been in applying and evolving algorithms and their convergence properties, not in evolving the optimality concept itself, nor in expanding the notions of true optimization. Yet, the conceptual constructs based on evolution and Darwinian selection have probably most to contribute – at least in theory – to the evolution of optimality. They should be least dependent on a priori fixation of anything in problem formulation: constraints, objectives or alternatives. Modern systems and problems are typical for their *flexibility*, not for their fixation. In this paper we draw attention to the impossibility of optimization when crucial variables are given and present *Eight basic concepts of optimality*. In the second part of this contribution we choose a more realistic problem of linear programming where constraints are not “given” but flexible and to be optimized and objective functions are multiple: *De novo programming*.

1 Introduction

Evolutionary algorithms have been quite effective in dealing with single-objective “optimization” while the area of Evolutionary Multiobjective Optimization (EMOO) has extended its efficiency to Multiple Criteria Decision Making (MCDM) [10] as well. The number of technical publications in EMOO is impressive and indicative of a rather explosive growth in recent years [1]. It is fair to say however that most of the progress has been in applying and evolving algorithms and their convergence properties, not in evolving the optimality concept itself, nor in expanding the notions of true optimization. Yet, the conceptual constructs based on evolution and Darwinian selection have probably most to contribute – at least in theory – to the evolution of optimality. They should be least dependent on a priori fixation of anything in problem formulation: constraints, objectives or alternatives.

The notion of optimality and the process of optimization are pivotal to the areas of economics, engineering, as well as management and business. What does it mean to state that something is ‘optimal’? If optimal means ‘the best’, then asking ‘What is the best?’ remains a legitimate and still mostly unanswered question.

Any maxima or minima could be declared optimal under specific circumstances, but optima are not necessarily maxima or minima. The two concepts are different: maximizing (or minimizing) is not optimizing.

Although dictionaries commonly use optimization as a synonym for maximization, we shall develop the concept of optimality in the *sense of balance* among multiple criteria or objectives.

When there is only a single dimension or attribute chosen to describe reality, then maximization or minimization with respect to constraints is sufficient. When there are multiple criteria (measures or yardsticks), as is true in most situations, then optimality and optimization (in the sense of balancing) need to be developed.

Optimization applies to an *economic problem* only: when scarce means (constraints) are used to satisfy alternative ends (multiple objectives). If the means are scarce, but there is only a single end, then the problem of how to use the means is a *technical problem*: no value judgments enter into its solution, no balancing is needed, and no optimization can take place. Only knowledge of physical and technical relationships is needed.

In other words, if all my constraints are “given” (fixed) and if my objective function is single, then the solution is fully defined and determined by mathematical problem formulation. The solution just needs to be revealed, explicated or computed by an algorithm. No optimization is possible: all is given and fully determined.

The technical problem is not what we wish to address when dealing with optimality and optimization.

2 The evolution of optimality

Balancing of multiple criteria is about optimization, not about “satisficing”. Simon acknowledged this quite simply by saying: ‘No one in his right mind will satisfice if he can just as well optimize.’

Surprisingly, multiple criteria or multiple objective functions - the necessary prerequisites for optimization - were not recognized and acknowledged by the optimization sciences until the early 1970s. Optimization in the sense of balancing multi-dimensionality is not compatible with the traditional concepts of “optimality” characterized by scalar or scalarized schemes, based on unique solutions under complete information. These are rather limited in capturing the richness and complexity of human problem solving, decision making and optimization.

We must strive to understand decision making not merely as computation of the given, already-constructed world, but as a way of constructing our lo-

cal world, ordering of individual and collective experience. It is necessary to acknowledge multiple concepts of optimality.

3 Multiple concepts of optimality

There are some axiomatic prerequisites that must be at the base of any optimization scheme.

For example, what is determined or given a priori cannot be subject to subsequent optimization and thus, clearly, does not need to be optimized: it is given.

What is not given must be selected, chosen or identified and is therefore, by definition, subject to optimization.

Consequently, different optimality concepts can be derived from different distinctions between what is given and what is yet to be determined in problem-solving or decision-making formulations.

For example, if I determine the value of the objective function a priori, set it at a predetermined value, then I cannot optimize it (nor maximize or minimize). If I set a value of the constraint a priori, then I cannot optimize that constraint. Constraints have to become objectives in order to be optimized, see also [2]. Even if I do not determine the value of the objective a priori, but the constraints are fixed, I still cannot optimize it – it is strictly implied (given) by the constraints.

Traditionally, by optimal solution or optimization we implicitly understand maximizing (or minimizing) a single, pre-specified objective function (or criterion) with respect to a given, fixed set of decision alternatives (or situation constraints). Both criterion and decision alternatives are given, only the (optimal) solution itself remains to be calculated.

There are at least *eight distinct optimality concepts*, all mutually irreducible, all characterized by different applications, interpretations and mathematical formalisms. For details see [13, 14, and 15].

Single-objective optimality

This is not really optimization but refers to the conventional maximization (or ‘optimization’) problem. It should be included for the sake of completeness, out of respect for tradition and as a potential special case of bona fide optimization.

To maximize a single criterion it is fully sufficient to perform technical measurement and algorithmic search processes. Once X and f are formulated or specified, the ‘optimum’ (that is, maximum) is found by computation, not by decision processes or balancing. Search for optimality is reduced to ‘scalarization’: assigning each alternative a number (scalar) and then identifying the largest-numbered alternative.

Numerical example

Consider the following linear-programming problem with two variables and five constraints:

$$\begin{aligned} \text{Max } f &= 400x + 300y \\ \text{subj. to } 4x &\leq 20 \\ 2x + 6y &\leq 24 \\ 12x + 4y &\leq 60 \\ 3y &\leq 10.5 \\ 4x + 4y &\leq 26 \end{aligned}$$

The maximal solution to the above problem is $x^* = 4.25$, $y^* = 2.25$, and $f^* = 2375$. Observe that all is given here and considering market prices of resources is unnecessary. However, if $p_1 = 30$, $p_2 = 40$, $p_3 = 9.5$, $p_4 = 20$ and $p_5 = 10$ were respective market prices (\$/unit) of the five respective resources, the total cost of current resource portfolio (20, 24, 60, 10.5, 26) would be $B = \$2600$.

Multi-objective optimality

More generally, optimality, to be distinct from maximizing, should involve balancing and harmonizing multiple criteria. In the real world, people continually resolve conflicts among multiple criteria which are competing for their attention and assignments of importance. This corresponds to the vector optimization problem. This maximization of individual functions should be non-scalarized, separate and independent, that is, not subject to superfunctional aggregation which would effectively reduce multi-objective optimality to single-objective maximization: there would be no reason to consider multiple criteria other than for constructing the superfunction. Multiple criteria, if they are to be meaningful and functional, should be optimized (or balanced) in the non-scalarized vector sense, in mutual competition with each other.

Numerical example

$$\begin{aligned} \text{Max } f_1 &= 400x + 300y \\ \text{and } f_2 &= 300x + 400y \\ \text{subj. to } 4x &\leq 20 \\ 2x + 6y &\leq 24 \\ 12x + 4y &\leq 60 \\ 3y &\leq 10.5 \\ 4x + 4y &\leq 26 \end{aligned}$$

The maximal solution with respect to f_1 is $x^* = 4.25$ and $y^* = 2.25$, $f_1(4.25, 2.25) = 2375$. The maximal solution with respect to f_2 is $x^* = 3.75$ and $y^* = 2.75$, $f_2(3.75, 2.75) = 2225$. The set of optimal (non-dominated) solutions X^* includes the two maximal solutions (extreme points) and their connecting (feasible) line defined by $4x + 4y = 26$. For example, $0.5(4.25, 2.25) + 0.5(3.75, 2.75) = (4.0,$

2.5) is another non-dominated point in the middle of the line. Total cost of resource portfolio remains $B = \$2600$.

Optimal system design: single criterion

Instead of optimizing a given system with respect to selected criteria, humans often seek to form or construct an optimal system of decision alternatives (optimal feasible set), designed with respect to such criteria. Single-criterion design is the simplest of such concepts: it is analogous to single-criterion 'optimization', producing the best (optimal) set of alternatives X at which a given, single objective function f is maximized subject to the cost of design (affordability).

Numerical example

$$\begin{aligned} \text{Max } f &= 400x + 300y \\ \text{subj. to } 4x &\leq 29.4 \\ 2x + 6y &\leq 14.7 \\ 12x + 4y &\leq 88.0 \\ 3y &\leq 0 \\ 4x + 4y &\leq 29.4 \end{aligned}$$

where the right-hand sides (resource portfolio) have been optimally designed. Solving the above optimally designed system will yield $x^* = 7.3446$, $y^* = 0$ and $f(x^*) = 2937.84$. If market prices of the five resources ($p_1 = 30$, $p_2 = 40$, $p_3 = 9.5$, $p_4 = 20$ and $p_5 = 10$) remain unchanged, then the total cost of the resource portfolio (29.4, 14.7, 88, 0, 29.4) is again $B = \$2600$.

Optimal system design: multiple criteria

As before, multiple criteria cannot be scalarized into a superfunction. Rather, all criteria compete independently or there would be no need for their separate treatment.

Numerical example

$$\begin{aligned} \text{Max } f_1 &= 400x + 300y \\ \text{and } f_2 &= 300x + 400y \\ \text{subj. to } 4x &\leq 16.12 \\ 2x + 6y &\leq 23.3 \\ 12x + 4y &\leq 58.52 \\ 3y &\leq 7.62 \\ 4x + 4y &\leq 26.28 \end{aligned}$$

The above represents an optimally designed portfolio of resources: maximal solution with respect to both f_1 and f_2 is $x^* = 4.03$ and $y^* = 2.54$, $f_1(4.03, 2.54) = 2375$ and $f_2(4.03, 2.54) = 2225$. This can be compared (for reference only)

with the f_1 and f_2 performances in the earlier case of given right-hand sides. Assuming the same prices of resources, the total cost of this resource portfolio is $B = \$2386.74 \leq \2600 . One could therefore design even better performing portfolios by spending the entire budget of 2600 (or the additional \$213.26).

Optimal valuation: single criterion

All previously considered optimization forms assume that decision criteria are given *a priori*. However, in human decision making, different criteria are continually being tried and applied, some are discarded, new ones added, until an optimal (properly balanced) mix of both quantitative and qualitative criteria is identified. There is nothing more suboptimal than engaging perfectly good set of alternatives X towards unworthy, ineffective or arbitrarily determined criteria (goals or objectives).

If the set of alternatives X is given and fixed *a priori*, we face a problem of optimal *valuation*: According to what measures should the alternatives be evaluated or ordered? According to criterion f_1 , f_2 or f_3 ? Which of the criteria captures best our values and purposes? What specific criterion engages the available means (X) in the most effective way?

Numerical example

In order to evaluate X, should we maximize f_1 or f_2 ? How do we select a criterion if only one is allowed (possible) or feasible?

$$\text{Max } f_1 = 400x + 300y$$

$$\text{or } f_2 = 300x + 400y$$

$$\text{subj. to } 4x \leq 20$$

$$2x + 6y \leq 24$$

$$12x + 4y \leq 60$$

$$3y \leq 10.5$$

$$4x + 4y \leq 26$$

The maximal solution with respect to f_1 is $x^* = 4.25$, $y^* = 2.25$, $f_1 (4.25, 2.25) = 2375$.

Maximal solution with respect to f_2 is $x^* = 3.75$, $y^* = 2.75$, $f_2 (3.75, 2.75) = 2225$. Is 2375 of f_1 better than 2225 of f_2 ? Only one of these valuation schemes can be selected.

Optimal valuation: multiple criteria

If the set of alternatives X is given and fixed *a priori*, but a set of multiple criteria is still to be selected for the evaluation and ordering of X, we have a problem of multiple-criteria valuation:

Which set of criteria best captures our value complex? Is it (f_1 and f_2)? Or (f_2 and f_3)? Or perhaps (f_1 and f_2 and f_3)? Or some other combination?

Numerical example

How do we select a set of criteria f_1 or f_2 or (f_1 and f_2) that would best express a given value complex?

$$\begin{aligned} \text{Max } f_1 &= 400x + 300y \\ \text{or/and Max } f_2 &= 300x + 400y \\ \text{subj. to } 4x &\leq 20 \\ 2x + 6y &\leq 24 \\ 12x + 4y &\leq 60 \\ 3y &\leq 10.5 \\ 4x + 4y &\leq 26 \end{aligned}$$

The maximal solution with respect to f_1 is $x^* = 4.25$, $y^* = 2.25$, $f_1(4.25, 2.25) = 2375$. Maximal solution with respect to f_2 is $x^* = 3.75$, $y^* = 2.75$, $f_2(3.75, 2.75) = 2225$. Should we use f_1 or f_2 or should we use both f_1 and f_2 to achieve the best valuation of X ? Only one of possible (single and multiple criteria) valuation schemes is to be selected.

Optimal pattern matching: single criterion

All previously considered optimization concepts assume that relevant decision criteria are given and determined a priori. Yet, that is not how human decision-making processes are carried out: different criteria are being tried and applied, some are discarded, new ones added, until a proper balanced mix (or portfolio) of both quantitative and qualitative criteria is derived.

Like any other decision-problem factors, criteria should be determined and designed in an optimal fashion. There is nothing more wasteful than engaging perfectly good means and processes towards unworthy, ineffective or only arbitrarily determined criteria.

There is a problem formulation representing an 'optimal pattern' of interaction between alternatives and criteria. It is this optimal, ideal or balanced problem formulation or pattern that is to be approximated or matched by decision makers. Single-objective matching of such cognitive equilibrium [18] is once more the simplest special case.

Numerical example

Should we maximize f_1 or f_2 ? How do we select a single criterion if only one is allowed, possible or feasible?

$$\text{Max } f_1 = 400x + 300y$$

or $\text{Max } f_2 = 300x + 400y$
 subj. to $4x \leq 29.4$ or 0
 $2x + 6y \leq 14.7$ 41.27
 $12x + 4y \leq 88$ 27.52
 $3y \leq 0$ 20.63
 $4x + 4y \leq 29.4$ 27.52

The above presents two optimally designed portfolios of resources with respect to f_1 and f_2 respectively. Among the possible patterns are ($x^* = 7.3446$, $y^* = 0$, $f_1(x^*) = 2937.84$, $B = \$2600$) and ($x^* = 0$, $y^* = 6.8783$, $f_2(y^*) = 2751.32$, $B = \$2600$).

Suppose that the value complex requires that the chosen criterion should minimize the opportunity cost of the unchosen criteria, other things being equal. Choosing f_1 would make f_2 drop only to 80.08 per cent of the opportunity performance, whereas choosing f_2 would make f_1 drop to 70.24 per cent. So, f_1 has preferable opportunity impact, and the first pattern and its resource portfolio would be selected.

A value complex indicating that deployed resource quantities should be as small as possible would require choosing f_2 and thus the second pattern.

Optimal pattern matching: multiple criteria

Pattern matching with multiple criteria is more involved and the most complex optimality concept examined so far. In all/matching' optimality concepts there is a need to evaluate the closeness (resemblance or match) of a proposed problem formulation (single or multi-criterion) to the optimal problem formulation.

Numerical example

How do we select a set of criteria f_1 , f_2 or (f_1, f_2) that would best express our current value complex?

$\text{Max } f_1 = 400x + 300y$
 or/and $\text{Max } f_2 = 300x + 400y$
 subj. to $4x \leq 29.4$ or 0 or 19.98
 $2x + 6y \leq 14.7$ 41.27 28.78
 $12x + 4y \leq 88$ 27.52 72.48
 $3y \leq 0$ 20.63 9.39
 $4x + 4y \leq 29.4$ 27.52 32.50

The above describes three optimally designed portfolios of resources with respect f_1 , f_2 and (f_1, f_2) respectively. So, among the possible patterns are ($x^* = 7.3446$, $y^* = 0$, $f_1(x^*) = 2937.84$, $B = \$2600$), ($x^* = 0$, $y^* = 6.8783$, $f_2(y^*) = 2751.32$, $B = \$2600$) and ($x^* = 4.996$, $y^* = 3.131$, $f_1(x^*) = 2937.84$, $f_2(y^*) =$

2751.32, B = \$2951.96).

If the value complex requires that B = 2600 is not to be exceeded, we may ‘match’ the third optimal pattern to that level by scaling it down by the optimum-path ratio $r = 2600/2951.96 = 0.88$. The new pattern is ($x^* = 4.396$, $y^* = 2.755$, $f_1(x^*) = 2585.30$, $f_2(x^*) = 2421.16$, B = \$2600). If producing both products is of value, then the choice could be maximization of both f_1 and f_2 .

4 Summary of Eight Concepts

In Figure 1 we summarize the eight major optimality concepts according to a dual classification: single versus multiple criteria versus the extent of the ‘given’, ranging from ‘all-but’ to ‘none except’. The traditional concept of optimality, characterized by too many ‘givens’ and a single criterion, naturally appears to be the most remote from any sort of optimal conditions or circumstances for problem solving as represented by cognitive equilibrium (optimum) with multiple criteria.

The third row of Figure 1 can be solved by De novo programming in linear cases. In the next section we summarize the basic formalism of De Novo programming, as it applies to linear systems [11, 12, 16, and 17]. It is only with multiple objectives that optimal system design becomes fully useful, even though a single-objective formulation can also lead to performance improvements.

5 Formal Summary of De Novo Programming

Formulate linear programming problem:

$$\text{Max } Z = Cx \text{ s. t. } Ax - b \leq 0, pb \leq B, x \geq 0, (1)$$

where $C \in \mathbb{R}^{q \times n}$ and $A \in \mathbb{R}^{m \times n}$ are matrices of dimensions $q \times n$ and $m \times n$, respectively, and $b \in \mathbb{R}^m$ is m -dimensional *unknown* resource vector, $x \in \mathbb{R}^n$ is n -dimensional vector of decision variables, $p \in \mathbb{R}^m$ is the vector of the unit prices of m resources, and B is the given total available budget.

Solving problem (1) means finding the optimal allocation of B so that the corresponding resource portfolio b maximizes simultaneously the values $Z = Cx$ of the product mix x .

Obviously, we can transform problem (1) into:

$$\text{Max } Z = Cx \text{ s. t. } Vx \leq B, x \geq 0, (2)$$

$$\text{where } Z = (z_1, \dots, z_q) \in \mathbb{R}^q \text{ and } V = (V_1, \dots, V_n) = pA \in \mathbb{R}^n.$$

Let $z_{k*} = \max z_k$, $k = 1, \dots, q$, be the optimal value for k th objective of Problem (2) subject to $Vx \leq B$, $x \geq 0$. Let $Z^* = (z_{1*}, \dots, z_{q*})$ be the q -objective value for the ideal system with respect to B . Then, a *metaoptimum problem* can

Number of Criteria Given	Single	Multiple
Criteria & Alternatives	Traditional "Optimality"	MCDM
Criteria Only	Optimal Design (De Novo Programming)	Optimal Design (De Novo Programming)
Alternatives Only	Optimal Valuation (Limited Equilibrium)	Optimal Valuation (Limited Equilibrium)
"Value Complex" Only	Cognitive Equilibrium (Matching)	Cognitive Equilibrium (Matching)

Fig. 1. Eight concepts of optimality

be constructed as follows:

$$\text{Min } Vx \text{ s. t. } Cx \geq Z^*, x \geq 0. \quad (3)$$

Solving Problem (3) yields x^* , B^* ($= Vx^*$) and b^* ($= Ax^*$). The value B^* identifies the minimum budget to achieve Z^* through x^* and b^* .

Since $B^* \geq B$, the optimum-path ratio for achieving the ideal performance Z^* for a given budget level B is defined as:

$$r^* = B/B^* \quad (4)$$

and establish the optimal system design as (x, b, Z) , where $x = r^*x^*$, $b = r^*b^*$ and $Z = r^*Z^*$. The optimum-path ratio r^* provides an effective and fast tool for efficient optimal redesign of large-scale linear systems.

Shi [8] observed that two additional types of budget (other than B and B^*) can be usefully introduced. One is B_j^k , the budget level for producing optimal x_j^k with respect to the k th objective, referring to a single-objective De Novo programming problem.

The other, B^{**} , refers to the case $q \leq n$ (the number of objectives smaller than the number of variables). If x^{**} is the degenerate optimal solution, then $B^{**} = Vx^{**}$ (See Shi [8]). It can be shown that $B^{**} \geq B^* \geq B \geq B_j^k$, for $k = 1, \dots, q$.

Shi defines six types of optimum-path ratios:

$$\begin{aligned} r_1 &= B^*/B^{**}; r_2 = B/B^{**}; r_3 = \sum \lambda_k B_j^k / B^{**}; \\ r_4 &= r^* = B/B^*; r_5 = \sum \lambda_k B_j^k / B^*; r_6 = \sum \lambda_k B_j^k / B, \end{aligned}$$

leading to six different optimal system designs. Comparative economic interpretations of all optimum-path ratios are still to be fully researched.

The following numerical example is adapted from Zeleny [11]:

$$\begin{aligned} \text{Max } z_1 &= 50 x_1 + 100 x_2 + 17.5 x_3 \\ z_2 &= 92 x_1 + 75 x_2 + 50 x_3 \\ z_3 &= 25 x_1 + 100 x_2 + 75 x_3 \end{aligned}$$

subject to

$$\begin{aligned} 12 x_1 + 17 x_2 &\leq b_1 \\ 3 x_1 + 9 x_2 + 8 x_3 &\leq b_2 \\ 10 x_1 + 13 x_2 + 15 x_3 &\leq b_3 \quad (5) \\ 6 x_1 + 16 x_3 &\leq b_4 \\ 12 x_2 + 7 x_3 &\leq b_5 \\ 9.5 x_1 + 9.5 x_2 + 4 x_3 &\leq b_6 \end{aligned}$$

We assume that the objective functions z_1 , z_2 , and z_3 are equally important. We are to identify the optimal resource levels of b_1 through b_6 when the current unit prices of resources are $p_1 = 0.75$, $p_2 = 0.60$, $p_3 = 0.35$, $p_4 = 0.50$, $p_5 = 1.15$ and $p_6 = 0.65$. The initial budget $B = 4658.75$.

We calculate $Z^* = (10916.813; 18257.933; 12174.433)$ with respect to given $B (= 4658.75)$. The feasibility of Z^* can be only assured by the metaoptimum solution $x^* = (131.341, 29.683, 78.976)$ at the cost of $B^* = 6616.5631$.

Because the optimal-path ratio $r^* = 4658.75/6616.5631 = 70.41\%$, the resulting $x = (92.48, 20.90, 55.61)$ and $Z = (7686.87; 12855.89; 8572.40)$. It follows that the optimal portfolio b , with respect to $B = 4658.75$, can be calculated by substituting x into the constraints (5). We obtain:

$$\begin{aligned} b_1 &= 1465.06 \\ b_2 &= 910.42 \\ b_3 &= 2030.65 \\ b_4 &= 1444.64 \text{ (6)} \\ b_5 &= 640.07 \\ b_6 &= 1299.55 \end{aligned}$$

If we spend precisely $B = 4658.8825$ (approx. 4658.75) the optimum portfolio of resources to be purchased at current market prices is displayed in (6), allowing us to produce x and realize Z in criteria performance. The analysis with respect to the additional Shi ratios would proceed along similar lines.

6 Extended De Novo Formulation

There are many interesting extensions of De Novo programming [3-8]. But the main extension always concerns the objective function [12]. The multiobjective form of $\text{Max } (cx - pb)$ appears to be the right function to be maximized in a globally competitive economy. This is compatible with achieving long-term maximum sustainable yields from deployed resources. Another realistic feature would be multiple pricing and quantity discounts in both resources and products markets.

Searching for a better portfolio of resources (redefining the b_i s of right-hand sides) is tantamount to the continuous reconfiguration and “reshaping” of systems boundaries. Such practical considerations lead to a more general programming formulation, starting to approximate the real concerns of free-market producers.

For example, the following optimal-design formulation of the production problem, although still quite incomplete, takes a full advantage of the De novo programming computational efficiency while delivering the necessary decision inputs:

$$\begin{aligned} \text{Max } z &= \sum_j c_j(x_j)x_j - [\sum_{i \in I1} p_i b_i] \pi_1 - \dots - [\sum_{i \in Ir} p_i b_i] \pi_r \\ \text{s.t } \sum_j a_{ij} x_j - b_i &\leq 0 \quad i \in I \end{aligned}$$

$$[\sum_{i \in I} p_i b_i] \beta \leq B,$$

where $I = I_1 \cup \dots \cup I_r$, $I_s \cap I_{s+1} = \emptyset$, $0 < \pi_s < 1$, $s = 1, \dots, r$, $\beta \geq 1$

and

$$\begin{aligned} c_{j1} x_j &\leq x_{j1} \\ c_{j2} x_{j1} &< x_j \leq x_{j2} \\ &\vdots \\ c_j(x_j) &= \cdot \\ &\vdots \\ c_{jkj} x_{jkj-1} &< x_j, \\ \text{where } c_{jh} &\geq c_{h+1}, h = 1, \dots, k_j. \end{aligned}$$

The formulation above is more practical than the traditional LP- systems, but perhaps still quite far away from the useful formulation of the real world-class management systems.

7 Conclusion

The challenge for EMOO is obvious: how to evolve its conceptual and technical capabilities to transform itself into a truly evolutionary vehicle that would encompass all practical concepts of optimality, including at least the eight basic ones, as outlined in this paper.

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