CHAPTER 2

Cultural Algorithms for Optimization

EVOLUTIONARY ALORITHMS have been successfully applied in a wide variety of optimization problems [26, 30, 100]. However, when used as optimizers, evolutionary algorithms are "blind" techniques in the sense that they do not require specific information about the problem but only a way of estimating how good is a solution with respect to the others (the so-called *fitness function*, which is, in general, a variation of a normalized version of the objective function to be minimized or maximized).

The use of domain knowledge within an evolutionary algorithm with the aim of improving its performance has been a research topic during several years [53, 63]. However, the incorporation of domain-specific knowledge evidently removes generality to an evolutionary algorithm, since such knowledge is specific for a particular problem (or class of problems). Additionally, the incorporation of domain-specific knowledge also replaces some of the stochastic nature of the evolutionary algorithm by deterministic information. Such extra information will certainly increase the selection pressure and will normally speed up convergence, although the risk of having premature convergence will also increase.

Nevertheless, it is worth noticing that the incorporation of domain-specific knowledge into an evolutionary algorithm is one of the choices that have been suggested [38] to circumvent the limitations imposed by the famous "No Free Lunch" theorem, which roughly states that all heuristics are equally efficient when assessing their performance over all possible problems [116].

Cultural algorithms are evolutionary computation techniques that extract domain knowledge (which is normally stored) during the evolutionary process with the aim of improving performance (normally, by providing a biased behavior for the evolutionary operators). In this chapter, we provide a review of the use of cultural algorithms for both single- and multi-objective optimization.

The remainder of this chapter is organized as follows. Section 2.1 provides an introduction to cultural algorithms. Section 2.2 reviews the most relevant work done on the use of cultural algorithms for single-objective optimization. The most relevant work on the use of cultural algorithms in multi-objective optimization is described in

Section 2.3. Some sample applications of cultural algorithms in real-world problems are provided in Section 2.4. Some possible paths for future research from the authors' perspective are briefly described in Section 2.5. Finally, our conclusions are provided in Section 2.6.

2.1 CULTURAL ALGORITHMS

Cultural algorithms were originally proposed by Robert Reynolds in the mid-1990s [88], as an approach that tries to add domain knowledge to an evolutionary algorithm during the search process, avoiding the need to add it *a priori*.

According to Reynolds [89] cultural algorithms were developed as a complement to the metaphor which inspired evolutionary algorithms (natural selection and genetic concepts). Thus, cultural algorithms are based on some sociological and anthropological theories, which have tried to model the phenomenon called *cultural evolution*. Such theories propose that the evolution of societies where culture exists, is slightly more complicated than only genetic evolution, and it can be seen as a process of inheritance at two levels: the *micro-evolutionary level*, which consist on the genetic material inherited by parents to their descendants; and the *macro-evolutionary level*, which is conformed by the knowledge acquired by the individuals through their experiences, that once encoded and stored, is useful for guiding the behavior of new individuals in a population (not only descendants in a genetic line) [86, 37].

Culture, then, can be seen as a set of ideological phenomena shared by a population, that influences the way in which an individual interprets its experiences and decides its behavior (i.e., how to act). Culture affects the success and survival of individuals and groups, leading to evolutionary processes that are every bit as real and important as those that shape genetic variation [95]. In these models, it is easy to appreciate the component of the system that is shared by the population: the knowledge, collected by the members of a society, but encoded in a way that is potentially accessible for all the population. Similarly, the individual components of the system are the experiences, and the way they can contribute to the shared knowledge, for the other individuals to learn them indirectly.

Reynolds adopted this phenomenon of double inheritance, as inspiration to create *cultural algorithms* [88]. The aim is to increase the convergence or learning rates, and therefore, that the system responds better to a variety of problems [42].

The components of cultural algorithms are the following:

- **The population space:** This space maintains a set of individuals (potential solutions to the problem). Each individual possesses characteristics independently from other individuals, and these characteristics define its fitness in the environment (the problem to solve). Throughout generations, individuals can be replaced by their descendants, obtained by means of the application of operators that somehow affect the population.

- **The belief space:** In this space is where the knowledge, acquired by the individuals through the generations, will be stored. This knowledge must be accessible to any individual in the population, and can be used to influence its

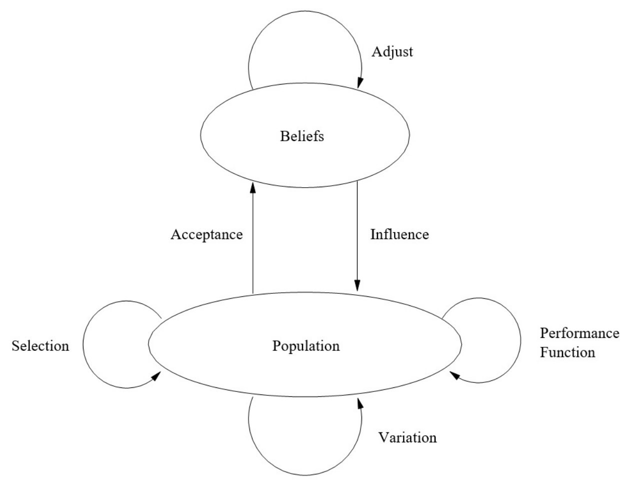

Figure 2.1 Spaces of a cultural algorithm

behavior (modify its characteristics and then modify its fitness). It is worth noting that the belief space is normally designed for a specific problem or class of problems to be solved.

- **A communication protocol:** which is necessary to link both spaces, defining rules about the type of information that the spaces will interchange (i.e., which information will pass from the population space to the belief space and vice versa).

At each generation, a cultural algorithm selects some individuals from the population, in order to extract information from them that can be useful during the search.

Such an information is used to update the belief space. The belief space will then influence the operators of the evolutionary algorithm, to transform them in informed operators and enhance the search process. These interactions between the spaces of a cultural algorithm are depicted in Figure 2.1.

In his original proposal, Reynolds [88] adopted the population of a genetic algorithm (and its associated operators) as the population space and the version spaces [75] were used as the belief space. This cultural algorithm was called *Version Space guided Genetic Algorithm*, (VGA), and was applied to solve some instances of the *Boole problem*[1] [115], with encouraging results. In this early application, a graph (the

[1] This problem consists of infering the characteristic function for an unknown Boolean multipliexer.

belief space) of the solutions in the population was built and classified based on the fitness of each particular instance. This was a very illustrative approach, because the graph's dynamics reflects the discovering of good and bad solutions. Based on those findings, Reynolds argued about the usefulness of cultural algorithms with an adaptation of the schema theorem, taking advantage of the genetic algorithm-based approach previously introduced. The *schema theorem* is an expression that bounds the propagation of the best solutions within the population of a genetic algorithm [48]. This modification indicates that a genetic algorithm, with the addition of a belief space, can improve its performance by increasing its convergence rates.

Cultural algorithms have had a limited use as optimizers in the specialized literature, but hey have been adopted for both single- and multi-objective optimization as reviewed in the next two sections.

2.2 CULTURAL ALGORITHMS FOR SINGLE-OBJECTIVE OPTIMIZATION

The general (single-objective) global optimization problem is defined as follows[2]:

$$\begin{array}{rl} \text{Minimize} & f(\vec{x}) \\ \text{Subject to} & lb_j \leq x_j \leq ub_j, \quad j = 1, 2, ..., D \end{array} \tag{2.1}$$

where $\vec{lb}, \vec{ub} \in^D$ are the lower bound and the upper bound of the decision variables $\vec{x}$, respectively. $f :^D \rightarrow$ is the objective function. The feasible solution space is defined as: $\Omega = \{\vec{x} \in^D | lb_j \leq x_j \leq ub_j, \forall j \in \{1, 2, 3, ..., D\}\}$.

Single-objective optimization problems may also be subject to constraints:

$$\begin{array}{ll} g_i(\vec{x}) \geq 0 & i = 1, 2, \ldots, m \\ h_i(\vec{x}) = 0 & i = 1, 2, \ldots, p \end{array} \tag{2.2}$$

where $g_i(\vec{x})$ denote inequality constraints and $h_i(\vec{x})$ denote equality constraints. Constraints are said to be *active* when in the global optimum $g_i(\vec{x}) = 0$. By definition, all equality constraints are active.

2.2.1 Static Optimization

Reynolds et al. [93] and Chung & Reynolds [17] explored the use of cultural algorithms for global optimization with very encouraging results. Chung and Reynolds used a hybrid of evolutionary programming [41] and GENOCOP [71] in which they incorporated an interval constraint-network to represent the constraints of the problem at hand.

Chung and Reynolds [17] used evolutionary programming with a mutation operator influenced by the best individual found so far, and the intervals where good solutions had been found. They called this approach the CAEP (*Cultural Algorithms with Evolutionary Programming*) for global optimization. CAEP provided a very rich model in the belief space, and produced very encouraging results.

[2]Without loss of generality, we will assume minimization.

Table 2.1 Knowledge sources for real-parameter optimization in cultural algorithms

Knowledge source	**Description**
Situational knowledge	Consists of the best exemplars found in the population, which represent leaders to follow. The individuals generated through this source, will tend to be closer to the leaders.
Normative knowledge	Consists of a set of intervals for each decision variable where good solutions have been found. The individuals generated through this source are more likely to be within the intervals, so they exploit good regions.
Topographical knowledge	Consists of a set of cells that represent a region of the search space. Each cell stores a characteristic of the region it represents; for example, the feasability of that region. The individuals generated through this source will be closer to the best cells.
History knowledge	Consists of a set of previous local optima, and its function is to extract patterns about their position. The individuals generated through this source will try to find in advance the location of the next local optimum. This knowledge source can also be used to add diversity to the algorithm, since it attempts to explore new regions.
Domain knowledge	It has no defined structure, because it depends of the problem which is to be solved. Its function is to exploit some knowledge about the problem, if available.

Reynolds and Chung [91, 16] proposed a formal model of self-adaptation in cultural algorithms, that supports the three main levels of self-adaptation in evolutionary algorithms (population, individual and component level). The royal road functions [74] were adopted as a case of study in this work.

The CAEP was tested on a number of global optimization problems [17], showing its improvement when compared to the standard evolutionary programming algorithm. In this case, the belief space was divided in two parts, called knowledge sources, specifically designed for real-valued problems: the *situational knowledge* and the *normative knowledge*. A general description of these knowledge sources, and those designed and added later, is provided in Table 2.1.

Jin and Reynolds [52] proposed an extension of CAEP for nonlinear constrained optimization. In order to handle constraints, an additional knowledge source, called *topographical knowledge*, was added to the belief space. It consists of a set of cells, which store some characteristic of the region of the search space they represent. In this case, they store a map of the feasible region (i.e., a discrete representation of the search space), based on the points that have been explored so far.

Coello Coello and Landa Becerra [22, 21] extended Jin and Reynolds' approach, improving its computational efficiency and overcoming its scalability problems. In the original approach the topographical knowledge was stored as an n-dimentional grid of the search space. This was replaced by a spatial data structure, that requires a controlled amount of memory even when the number of dimensions grows. Additionally, the authors presented an empirical study in which this approach was validated using a well-known benchmark adopted in evolutionary constrained optimization, and results were compared with respect to constraint-handling techniques that were representative of the state-of-the-art in the area at that time. This approach was able to find competitive results, while performing only about 15% of the total number of fitness function evaluations required by the other approaches with respect to which it was compared.

Reynolds and Peng [94] provide a study of how the knowledge sources associated with cultural knowledge control the search process in a cultural algorithm used for solving engineering optimization problems. In this case, evolutionary programming is adopted for the population space. The authors observed that the meta-level interaction of the knowledge sources of a cultural algorithm allow the generation of feasible solutions from a fully infeasible initial population and also helps to speed up convergence. Although the study is indeed interesting, only one (relatively simple) engineering optimization problem was used to validate this approach.

Gao et al. [43] used a genetic algorithm with real numbers encoding for the population space of a genetic algorithm. However, in this case, the only modification made to the genetic algorithm consists of introducing the mutation scheme adopted in the regional-based sliding cultural algorithm proposed by Jin and Reynolds [52]. The validation is also very poor in this case, since this proposal is only tested in one constrained problem having only one decision variable and one nonlinear constraint.

Landa Becerra and Coello Coello [60, 61] have also adopted differential evolution for the population space of a cultural algorithm designed for constrained optimization. In this case, all the knowledge sources were adapted for their use with the differential evolution operator, providing some adaptation of its components. The approach was tested on a well-known benchmark commonly adopted to validate new constraint-handling techniques and also on some engineering optimization problems, providing very competitive results.

Tang and Li [109] made an interesting proposal in which an Anti-Culture Population was adopted with the purpose of having individuals that disobey the guidance of the knowledge provided by the cultural algorithm. Such individuals aim to avoid local minima and to speed up convergence. This approach was called *Triple Spaces Cultural Algorithm* (TSCA) and it was implemented using a genetic algorithm in the population space. Mutation operations are used to move individuals away from the biased introduced by the knowledge obtained during the search. The authors validated their TSCA using the same test problems from [60] and compared results with respect to that approach as well.

Nguyen and Yao [77] proposed a hybrid of a cultural algorithm with iterated local search for multimodal optimization. In this proposal, a shared knowledge space is adopted for integrating the knowledge produced from pre-defined multi-populations and knowledge migration is used to bias the search towards different directions. This approach was validated using the test problems from the special session on real-parameter optimization held at the *2005 IEEE Congress on Evolutionary Computation* (CEC'2005) [106], and presented competitive results.

Yang et al. [118] proposed a cultural algorithm that integrates a hybdrid of the so-called quantum-behaved particle swarm optimization (QPSO) [107] with differential evolution. In QPSO each particle has a quantum behavior, and we can only learn the probability that a particle has of appearing in a certain position from a probability density function, the form of which depends on the potential field in which the particle lies in. In this proposal, the population is divided into two sub-swarms: (1) common particles and (2) elite particles. *Elite* particles have better fitness values than *common* particles and are meant to represent the mainstream *culture* of the entire swarm. Elite particles are evolved using differential evolution and common particles are evolved using QPSO. Common particles are influenced both by the other common particles and by the elite particles. This approach was validated using the test problems from the special session on real-parameter optimization held at the *2005 IEEE Congress on Evolutionary Computation* (CEC'2005) [106], comparing results with respect to QPSO, a simple PSO and a very completitive PSO called *Comprehensive Learning PSO* (CLPSO) [65]. The proposed approach was able to outperform QPSO and was competitive with respect to PSO and CLPSO.

Ali et al [2, 6] introduced the use of a *social fabric influence function* in a cultural algorithm designed to solve nonlinear constrained optimization problems. This social fabric is seen as the authors as some sort of computational tool that influences the action and interactions of the different knowledge sources adopted by the cultural algorithm. Particle swarm optimization was adopted for the population memory in this case. In this proposal, at each time step every individual is influence by one of five possible knowledge sources. Individuals are connected using a certain topology and after the first one is influenced by a particular knowledge source, it passes the signal to adjacent individuals. Different particle swarm topologies were adopted by the authors for interconnecting the individuals in the population. Several benchmark problems taken from [72] were adopted to validate this proposal.

Awad et al. [7] proposed a cultural algorithm with an improved local search mechanism for global optimization. In this proposal the initial solution to which the local search is applied is selected using a niching technique called *clearing* [82]. Once a solution is selected, several neighbors are generated using the five knowledge sources of the cultural algorithm which give rise to five different sub-local search methods. A set of test problems with low dimensionality (10 to 30 decision variables) was adopted to validate this approach.

Ali and Reynolds [5] proposed a cultural algorithm that incorporated an embedded local tabu search [44] component for solving constrained optimization problems. Evolutionary programming is used in this case for the population space. The main goal of using tabu search is to improve diversity in the population, as to avoid premature convergence and stagnation. Since tabu search was originally designed for discrete search spaces, the authors had to adopt a version developed by Siarry and Berthiau [99] for continuous search spaces. The core idea of this approach is to use tabu search to encapsulate the History Knowledge so that it can be adopted to select appropriate search paths. This algorithm was only tested with three engineering optimization problems.

Omran [78] proposed the intellect-masses optimizer (IMO) which is a variation of the cultural algorithm from Yang et al. [118]. In this case, the population is divided into two sub-populations: (1) the *intellects*, which are the fittest individuals and (2) the *masses* which refers to the rest of the population. The intellects are meant to learn from each other and to focus on exploitation whereas the masses are meant to learn from the intellects and from themselves and focuse on exploration. The author uses differential evolution for evolving the intellects and a modified artificial bee colony [1] to evolve the masses. This approach is validated using the test problems from the special session on real-parameter optimization held at the *2005 IEEE Congress on Evolutionary Computation* (CEC'2005) [106], and its results are compared with respect those from Yang et al. [118] and with respect to several other metaheuristics, including the artificial bee colony and CLPSO [65].

Ali et al. [3] proposed a hybrid of a cultural algorithm with multiple trajectory search (MTS) [111] for multimodal optimization. The core idea of MTS is to move in decision variable space based on different step sizes. Each step size s applied according to a certain local search method. The original MTS algorithm uses simulated orthogonal arrays to generate the initial solutions of the basic multiple trajectory model. In this case, the authors use instead the knowledge sources of the cultural algorithm for the same purpose. This approach was validated using the test problems from the special session on real-parameter optimization held at the *2005 IEEE Congress on Evolutionary Computation* (CEC'2005) [106], and its results were compared with respect to a wide variety of global optimizers, obtaining very promising results in problems with up to 100 decision variables.

Ali et al. [4] proposed a framework for developing cultural algorithms based on differential evolution in which the main emphasis is provide a proper balance between exploration and exploitation. This approach, called b-hCA-DE incorporates four knowledge sources (topographical, situational, normative and temporal) and uses a population that is shared by all the knowledge sources. This approach is validated using the 28 test problems from the special session and competition held at the *2013 IEEE Congress on Evolutionary* [64] with 10, 30 and 50 decision variables. The results obtained by this approach were found to be competitive with respect to those of a high variety of state-of-the-art global optimizers.

Ali et al. [4] proposed re-structuring the social fabric (social network) [90] of the connections that link the individuals in the population space of a cultural algorithm with the aim of enhancing its performance. For this re-structuring, the authors adopt a dynamic neighborhood topology. The methaphore behind this proposal is that the knowledge sources of the cultural algorithm can weave a networked "fabric" of individuals that are performing the search. Thus, the population space consists of a set of subgroups called "tribes", which are meant to represent the building blocks of the population of search engines. Tribal subroups can be merged with each other using different regrouping schemes. The re-structuring of the social fabric is based on the success of each of the knowledge sources incorporated in the cultural algorithm. This approach was validated using the problems from the *2011 IEEE Congress on Evolutionary Computation (CEC'2011)* Competition on Testing Evolutionary Algorithms on Real-World Numerical Optimization Problems [29]. The approach was compared with respect to other cultural algorithms and with respect to state-of-the-art global optimizers producing very promising results.

Awad et al. [8] proposed CADE which hybridizes a cultural algorithm and differential evolution. The core idea in this case is to select the best individuals in the population and use them to update the knowledge sources in the belief space (the authors adopt topographical, situational, normative and domain knowledge). Then, the knowledge sources that will influence the evolutionary process are selected. At this point, differential evolution is used to improve the exploratory capabilities of the cultural algorithm. In this approach, both the cultural algorithm and differential evolution are executed in parallel sharing the same population and a success-based quality function is used to guide the search. A set of 50 test problems having up to 100 decision variables each was adopted to validate this approach and results were compared with respect to six other metaheuristics. CADE had a very competitive performance.

Ravichandran [85] proposed a cultural algorithm based on decomposition (CA/D) to decompose a dynamic multi-objective optimization problem into several subproblems that are then optimized using information shared by neighboring problems. This approach consists of a culturized version of MOEA/D-DP [13]. In this case, the historical knowledge is used to track the environmental changes, the situational knowledge is used to preserve the best solutions and the normative knowledge is used to distribute solutions along the Pareto front. The author experimented with both Tchebycheff decomposition and reference points. Although this is clearly a multi-objective optimization algorithm the author used it to solve single-objective optimization problems. In fact, the test problems from the special session on real-parameter optimization held at the *2005 IEEE Congress on Evolutionary Computation* (CEC'2005) [106] were adopted to validate it.

2.2.2 Dynamic Optimization

Saleem and Reynolds [97] added two more knowledge sources to cultural algorithms, in order to deal with dynamic environments: *history knowledge* and *domain knowledge*. The first of these sources was designed to extract patterns about the changes of position of optimal points at each environmental change. The second source was designed to exploit the known characteristics of the function generator. Even when these knowledge sources were designed for dynamic problems, they have also been used in static environments [60].

Peng and Reynolds [81] adopted particle swarm optimization [56] for the population space of a cultural algorithm. In this case, the authors used all of the previously designed knowledge sources, and they investigated the role of the belief space in the different stages of a dynamic optimization process. The authors argued in this case that a cultural algorithm provides an additional degree of adaptability to the one provided by evolutionary algorithms. This work is extended in [80] in which evolutionary programming is adopted for the population space and a dynamic problems generator is used to simulate the changes of the fitness landscape that the cultural algorithm is meant to emulate. The authors show the emergence of swarms of solutions in both the population space and the belief space which are able to detect the changes in the location of the optimum. Situational and domain knowledge are found to play a key role in this case.

Jiang et al. [51] proposed a cultural-based particle swarm optimizer for dynamic environments. The authors see the belief space as a knowledge repository that stores information about the environmental changes. Thus, the core idea of this proposal is to use this information to predict the location of the new optimum. For this sake, the authors need to learn to identify the types of belief knowledge structures required for finding and tracking a moving optimum. This approach is only validated with a single problem (a dynamic version of the Rastrigin function).

Daneshyari and Yen [28] proposed a cultural based particle swarm optimizer in which five knowledge sources are adopted (situational, temporal, domain, normative and space). These knowledge sources store information from the used particle swarm optimizer and such information is then used to detect changes in the environment. The knowledge sources also assists the optimizer to respond to these changes through a diversity maintenance mechanism (called repulsion) and a migration operator that acts among swarms in the population space.

Chen et al. [15] proposed a cultural algorithm for the path planning of an unmanned aerial vehicle (UAV) in real time. In this case, the belief space incorporates both situational and normative knowledge. This is a dynamic problem, since the environment is changing over time. When a change occurs, it is not necessary to regenerate the full path to avoid an obstacle. Only a portion of the path needs to be readjusted. This approach was compared with respect to the D* algorithm [83]. The cultural algorithm was found to have a better real-time performance, a lower path planning cost and produced paths of a shorter length.

Kinnaird-Heether and Reynolds [57] analyzed different knowledge distribution mechanisms for a cultural algorithm. The knowledge distribution mechanism is used to handle the conflict resolution between the competing knowledge sources and the belief space. Researchers have adopted a variety of knowlegde distribution mechanisms including voting [14], auctions [92] and game-theory [114]. In this work, the authors also proposed a new (sub-cultured) approach that allows the cultural algorithm to learn to use a combination of different mechanisms. This scheme was adopted for solving dynamic optimization problems. The authors reported that the use of their proposed sub-cultured mechanism was better than any of the mechanisms that it combined when used separately.

2.3 CULTURAL ALGORITHMS FOR MULTI-OBJECTIVE OPTIMIZATION

In multi-objective optimization, the aim is to solve problems of the type[3]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{2.3}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2.4}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{2.5}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

A few additional definitions are required to introduce the notion of optimality used in multi-objective optimization:

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.
w **Definition 3.** We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

Therefore, our aim is to obtain the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2.4) and (2.5).

Apparently, the first attempt to use a Pareto-based approach for solving multi-objective problems adopting a cultural algorithm is the proposal of Coello Coello and Landa Becerra [23]. In this case, evolutionary programming is used for the population space and Pareto ranking is adopted for selecting nondominated solutions. This approach also uses the approximation of the dimensions of the Pareto front in the belief space, which works as a guide for the individuals to reach regions in which new nondominated solutions can be found. The belief space also includes a mechanism to produce a good distribution of solutions along the Pareto front (i.e., a density estimator [20]). An algorithm based on this approach was used by Gu and Wu [47] for solving a water resources problem.

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[3] Without loss of generality, we will assume only minimization problems.

Landa Becerra and Coello Coello [62] proposed to combine the cultural algorithm based on differential evolution that they proposed in [60] with the ϵ-constraint method for solving complex multi-objective optimization problems. This approach is computationally expensive because a high number of single objective optimizations need to be performed to solve the multi-objective problem. Thus, for justifying the use of this approach, they solved the most difficult instances from the Deb-Thiele-Laumanns-Zitzler (DTLZ) [32] and the Walking-Fish-Group test suites [49] which could not be properly solved by any of the multi-objective evolutionary algorithms available at that time (including the NSGA-II).

Best [9] and Best et al. [10] proposed a framework for designing multi-objective cultural algorithms called MOCA in which four knowledge sources are considered: (1) situational knowledge, which influences individuals to produce children near to known good solutions, (2) domain knowledge, which searches in the four cardinal solutions with the aim of moving towards the true Pareto front of the problem, (3) normative knowledge which tries to capture the regions in decision variable space that contain the fittest solutions, (4) historical knowledge, which is used to spread solutions along the Pareto front, (5) topographical knowledge, which avoids getting trapped in a local Pareto front. The acceptance function is based on the use of Pareto ranking, but for selecting the best individual, a random scheme (from among the nondominated solutions generated so far) is adopted. The knowledge sources are selected in order to influence members of the population by sampling a dynamic probability distribution. The DTLZ test problems are adopted to validate this approach and, in general, the results of this proposal are not better than those generated by NSGA-II.

Daneshyari and Yen [27] proposed a Cultural-Based Multi-Objective Particle Swarm Optimizer (MOPSO). The core idea in this case is to adapt the parameters of the MOPSO using the knowledge sources of the cultural algorithm. In this approach, Pareto dominance is used as the acceptance function and the authors adopt three knowledge sources: (1) situational knowledge, which is used to adapt the local acceleration of the MOPSO and to select the personal best, (2) normative knowledge, which is used to adapt the global acceleration and to find the global best and (3) topographical knowledge, which is also used to adapt the global acceleration and to find the global best. This approach also incorporates the same sort of bounded global archive adopted in [25].

Reynolds and Liu [87] proposed an extension of MOCA [10] in which they allow all knowledge sources to contribute to the optimization process. What the authors actually do in this case is to improve the implementation of MOCA by introducing several changes to the previous version. For example, for the topographical knowledge the original had severe scalability problems because grid cells were used to represent decision variable space. In this new version, hypercubes in objective space are adopted instead. In this paper, a single problem is adopted to validate the proposal (from the old Zitzler-Deb-Thiele (ZDT) test suite [123]) and only a visual comparison of results is done with respect to old multi-objective evolutionary algorithms (e.g., the original Nondominated Sorting Genetic Algorithm (NSGA) [102]).

Stanley et al. [103] proposed CAPSO, which is a parallelized hybrid optimization system designed for solving multi-objective optimization problems. This approach combines cultural algorithms with particle swarm optimization and the vector evaluated genetic algorithm (VEGA) [98]. Five knowledge sources are adopted in this case: (1) situational, (2) normative, (3) historical, (4) topographical and (5) domain knowledge. For determining the best solutions, one objective is considered each time (as in VEGA). Although the authors indicate that VEGA was selected because it's easy to parallelize, it's unclear if a parallel implementation was actually developed. This approach is validated with very old (and rather simple) multi-objective test problems. Nevertheless, CAPSO has been used to analyze trends in a study of the impact of "El Niño" and climate change on artisanal fishermen behavior from the early 1980s in Peru [54] (CAPSO was adopted to perform nonlinear regressions).

Mao et al. [70] proposed a multi-objective cultural algorithm called MOFECO in which the five-elements-cycle-optimization algorithm [66] is used in the population space. This approach adopts normative, situational, topographical and historical knowledge. This approach was validated using several problems from the ZDT [123] and the DTLZ [32] test suites and results were compared with respect to several multi-objectve evolutionary algorithms (including MOEA/D [120]) using two performance measures: hypervolume [122] and inverted generational distance [18]).

2.4 SOME APPLICATIONS

Cultural algorithms have been used in a wide variety of applications in which they have been used to solve both single- and multi-objective optimization problems [96, 39]. A sample of these applications is provided next:

Electrical Engineering: Goudarzi et al. [46] proposed four different versions of a cultural algorithm (each one adopting a different knowledge source) for solving the combined environmental economic dispatch problem. This is actually a multi-objective problem in which the aim is to simultaneously minimize fuel cost and emission, while satisfying several power systems constraints. However, in this case, the authors combine the two objective into an aggregating function. Three systems having 5, 20 and 50 generating units were adopted to validate this approach and results were compared with respect to a variety of metaheuristics showing promising results.

Other authors have solved economic dispatch problems using cultural algorithms combined with differential evolution [36], evolutionary programming [11], quantum-behaved particle swarm optimization [67], an artificial immune system [45] and the self-migrating algorithm [35].

Additionally, this problem has also been tackled using multi-objective approaches. For example, Zhang et al. [121] proposed the *Enhanced Multi-Objective Cultural Algorithm* (EMOCA) to solve this problem. EMOCA combines the framework of cultural algorithms with particle swarm optimization and adopts two knowledge sources tailored for the specific problem to be solved. The authors also proposed a constraint-handling technique as part of their approach.

Lu et al. [68] proposed the Hybrid Multi-Objective Cultural Algorithm for short-term environmental/economic hydrothermal scheduling. This approach adopts a Pareto-based version of differential evolution for the population space. This approach is validated using two case studies in which its results were compared with respect to NSGA-II [31] and two other multi-objective optimization algorithms based on evolutionary programming and simulated annealing.

Other related applications include the optimization of the operation of a hydropower station [69] and the optimization of a doubly-fed induction generator to attain an efficient and improved dynamic response of a wind energy conversion system [112].

Mechanical Engineering: Coelho and Mariani [34] proposed two Gaussian PSO approaches combined with a cultural algorithm, which are called cultural PSO (PSO-CA) and GPSO-CA. Evidently, the main difference with respect to a traditional PSO lies on the use of a Gaussian distribution for computing the velocities of the particles. This is meant to improve the local exploration of the algorithm. This approach was used to solve mechanical engineering design problems. Similar engineering optimization problems to those adopted in [34] have been adopted by some other authors as well (see for example [24, 61, 3, 4]).

Jalili and Hosseinzadeh [50] proposed a cultural algorithm combined with evolutionary programming (following the proposal from [91]) for the optimal design of trusses. Results are compared with respect to those of several other metaheuristics using 4 test problems which include a 120-bar dome truss. The cultural algorithm is able to produce competitive results while performing a lower number of objective function evaluations than the other algorithms.

Image Processing: Wang et al. [113] proposed an adaptive cultural algorithm with improved quantum-behaved particle swarm optimization (ACA-IQPSO) for detecting underwater sonar images. The authors adopt situational, normative and domain knowledge and introduce a new communication protocol that considers not only the acceptance function, but also an influence function which is used to guide the evolution of the particles with a poor performance using the knowledge stored in the belief space. This approach is found to be very effective for both floating and underwater object detection.

Yan et al. [117] proposed a cultural algorithm with isolated niching (as a mechanism to improve diversity) for image matching problems. Tan and Yang [108] proposed a cultural algorithm to maximize the entropy function used to do multi-threshold image segmentation (for infrared images) in order to reduce the computational time as well as to improve the segmentation efficiency.

Cai [12] proposed a scheme for increasing the detectability of sea-surface floating weak targets which consists of using a cultural algorithm aided time-frequency distribution fusion strategy without any prior information. The authors adopted several sets of experimental data collected by an instrument-quality radar system to verify the accuracy and efficiency of this proposal. Six representative time-frequency distributions of experimental signals were obtained and their performance were quantitatively analyzed in terms of effective resolution and entropy. Additionally, the authors adopted the Volterra-series weighted averaging model as their fusion rule and a cultural algorithm for the optimization. The authors reported that their proposed approach was able to outperform other detectability techniques.

Scheduling: Soza et al. [101] proposed a cultural algorithm for solving timetabling problems. Three knowledge sources are considered in this case: (1) situational, (2) normative and (3) domain knowledge. These knowledge sources are combined with three specialized variation operators: interchange, sequencing and simple mutation. Only one exploration operator is applied to each individual at a time. This approach was validated using a benchmark with 20 instances, and results were compared with respect to an evolutionary algorithm with specialized crossover operators, a memetic algorithm, and a simulated annealing approach that won a timetabling competition organized by the Metaheuristic Network. The results obtained indicated that the cultural algorithm was a viable alternative for solving timetabling problems in an efficient manner

Mojab et al. [76] proposed a cultural algorithm for workshop scheduling in cloud computing. The authors considered the situation in which there is a deadline for the workflow and the goal is to minimize the monetary cost of running in the cloud while satisfying the given deadline. Three knowledge sources were adopted in this case: normative, situational and domain knowledge. The proposed approach was validated using four synthetic workflow applications based on real scientific workflows and results were compared with respect to those of a random scheme, a genetic algorithm, a particle swarm optimizer and a traditional cultural algorithm. The authors reported a better performance of their proposal.

Finance: Sternberg and Reynolds [104] proposed to embed a fraud detection expert system called DETECT into a cultural algorithm. In order to simulate a dynamic environment in this application, the authors considered four objectives: characterizing fraudulent claims, nonfraudulent claims, false positive claims (i.e., nonfraudulent claims predicted as fraudulent) and false negative claims (i.e., fraudulent claims predicted as nonfraudulent). The authors reported that the use of a cultural algorithm allowed to respond to changing objectives in an effective way.

Ostrowski et al. [79] proposed a cultural algorithm for optimizing strategies in agent-based models and demonstrated its use in an application used to model pricing strategies. For a more effective evaluation of parameter configurations, the authors adopted white and black box testing, which are well-known software engineering techniques. The authors indicated that their proposed approach was able to derive a near-optimal pricing strategy in less periods than traditional evolutionary approaches.

2.5 FUTURE PERSPECTIVES

There are several paths for future research in this area. For example:

- **Use of other cultural paradigms:** Kuo and Lin [59] developed a *Cultural Evolutionary Algorithm* based on Steward's socio-cultural integration theory [105]. It would be interesting to see the development of new cultural algorithms based on other theories in the years to come.

- **Parallelism:** The use of cultural algorithms has been somehow limited due to their potentially high computational cost (depending on the particular application and the knowledge sources adopted). One way to deal with this limitation could be to use parallelism, but so far, very few proposals of cultural algorithms seem to involve parallel implementations (see for example [73, 33, 55]).

- **Multi-Objective Optimization:** Although there are several proposals to adopt cultural algorithms for multi-objective optimization, there is no multi-objective cultural optimizer currently available which had been validated with state-of-the-art test problems and compared with respect to state-of-the-art multi-objective evolutionary algorithms. Particularly, to the authors' best knowledge, no multi-objective cultural algorithm has been succesfully applied to many-objective problems (i.e., multi-objective problems having 4 or more objectives) nor to inverted test problems [119].

  Furthermore, most of the current paradigms used in multi-objective evolutionary algorithms have not been culturized [19]. Particularly, no multi-objective cultural algorithm has been developed based on performance indicators [40] or on decomposition [110] (the approach reported in [85] was used for single-objective optimization). In other areas, such as dynamic multi-objective optimization [84], cultural algorithms seem to be an obvious choice, but to the authors' best knowledge, they have not been applied in such problems yet.

- **Culturizing other Evolutionary Algorithms:** Most of the current cultural algorithms are based on traditional evolutionary algorithms (i.e., evolutionary programming, genetic algorithms and differential evolution). However, the use of other algorithms such as genetic programming [58] has been fairly limited (see for example [89]). The use of genetic programming would allow to extend the range of applications of cultural algorithms to areas such as symbolic regression and classification.

2.6 CONCLUSIONS

This chapter has provided an overview of cultural algorithms and their use on optimization. The aim was not to be comprehensive, but to cover most of the areas (within optimization) in which they have been used. The aim is to provide a general overview of the field both to students and researchers who are interested on doing research in this area.

The topics covered in this chapter include static and dynamic single-objective optimization as well as multi-objective optimization. Additionally, a few application areas of cultural algorithms have been also provided. In the final part of the chapter, some research paths that are worth exploring in the future (from the authors' perspective) are also delineated.

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