Multi-Objective Evolutionary Algorithms: Past, Present and Future

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1 Introduction

The idea of using techniques based on the emulation of the mechanism of natural selection to solve optimization problems can be traced back to the 1960s when the three main techniques based on this notion were developed: genetic algorithms [61], evolution strategies [136] and evolutionary programming [41]. These approaches, which are now collectively denominated "evolutionary algorithms", have been very effective for solving single-objective optimization problems [49, 137, 42].

The solution of problems having two or more objectives (which are normally in conflict with each other) has attracted a considerable interest in recent years. The solution of the so-called *multi-objective optimization problems* (MOPs) consists of a set of solutions representing the best possible trade-offs among the objectives. Such solutions, defined in decision variable space constitute the so-called *Pareto optimal set*, and their corresponding objective function values form the so-called *Pareto front*.

Although a variety of mathematical programming techniques to solve MOPs have been developed since the 1970s [104], such techniques present several limitations, from which two of the most important are that these algorithms are normally very susceptible to the shape or continuity of the Pareto front and that they tend to generate a single element of the Pareto

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optimal set per run. Additionally, in some real-world MOPs, the objective functions are not provided in algebraic form, but are the output of a black-box software (which, for example, runs a simulation to obtain an objective function value). This severely limits the applicability of mathematical programming techniques.

Evolutionary algorithms seem particularly suitable for solving multi-objective optimization problems because they deal simultaneously with a set of possible solutions (the so-called population) which allows them to obtain several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front (e.g., they can easily deal with discontinuous and concave Pareto fronts), whereas these two issues are a real concern for mathematical programming techniques.

The potential of evolutionary algorithms for solving MOPs was first pointed out by Rosenberg in the 1960s [125], but the first actual implementation of a multi-objective evolutionary algorithm (MOEA) was produced until the mid-1980s by David Schaffer [133, 134]. Nevertheless, it was until the mid-1990s that MOEAs started to attract serious attention from researchers. Nowadays, it is possible to find applications of MOEAs in practically all areas of knowledge.[1]

The contents of this chapter is organized as follows. Some basic concepts required to make of this a self-contained chapter are provided in Section 2. Section 3 describes the main algorithmic paradigms (as well as some representative MOEAs belonging to each of them) developed from 1984 up to the early 2000s. In Section 4, the most popular MOEAs developed from the mid-2000s to date are briefly described. Some representative applications of these MOEAs are also provided in this section. Then, in Section 5, some possible paths for future research in this area are briefly described. Finally, our conclusions are provided in Section 6.

2 Basic Concepts

We are interested in solving problems of the type[2]:

$$\text{minimize } \mathbf{f}(\mathbf{x}) := [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})] \tag{1}$$

subject to:

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[1] The first author maintains the EMOO repository, which currently contains over 12,400 bibliographic references related to evolutionary multi-objective optimization. The EMOO repository is located at: `https://emoo.cs.cinvestav.mx`.

[2] Without loss of generality, we will assume only minimization problems.

$$g_i(\mathbf{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\mathbf{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

Definition 1. Given two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$, we say that $\mathbf{x} \leq \mathbf{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\mathbf{x}$ **dominates** $\mathbf{y}$ (denoted by $\mathbf{x} \prec \mathbf{y}$) if $\mathbf{x} \leq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

Definition 2. We say that a vector of decision variables $\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\mathbf{x}' \in \mathcal{X}$ such that $\mathbf{f}(\mathbf{x}') \prec \mathbf{f}(\mathbf{x})$.

Definition 3. We say that a vector of decision variables $\mathbf{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\mathbf{x} \in \mathcal{F} | \mathbf{x} \text{ is Pareto optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\mathbf{f}(\mathbf{x}) \in \mathbb{R}^k | \mathbf{x} \in \mathcal{P}^*\}$$

We thus wish to determine the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). Note however that in practice, not all the Pareto optimal set is normally desirable (e.g., it may not be desirable to have different solutions that map to the same values in objective function space) or achievable.

3 The Past

As indicated before, the earliest attempt to use evolutionary algorithms for solving multi-objective optimization problems dates back to Richard S. Rosenberg's PhD thesis [125] in which he suggested to use multiple properties (i.e., nearness to certain specified chemical composition) in his simulation of the genetics and chemistry of a population of single-celled organisms. Although his model considered two properties (i.e., two objectives), he transformed one of them into a constraint and dealt with a constrained single-

objective optimization problem. The first actual implementation of a multi-objective evolutionary algorithm (MOEA) was developed by David Schaffer in his PhD thesis [133]. His approach, called *Vector Evaluated Genetic Algorithm* (VEGA) [134] will be briefly described next.

In this section, we will review the approaches proposed in the period from 1984 to the early 2000s. This period is divided in three parts each of which corresponds to a different algorithmic paradigm: (1) Non-Elitist Non-Pareto Approaches, (2) Non-Elitist Pareto-based Approaches and (3) Elitist Pareto-based Approaches. Some representative algorithms within each of these three groups are briefly described next.

3.1 Non-Elitist Non-Pareto Approaches

These are the oldest MOEAs and are characterized for not incorporating elitism and for having selection mechanisms that do not incorporate the notion of Pareto optimality. Here, we will briefly review the following approaches:

- Linear aggregating functions
- Vector Evaluated Genetic Algorithm (VEGA)
- Lexicographic ordering
- Target-vector approaches

3.1.1 Linear aggregating functions

The most straightforward way of transforming a vector optimization problem into a scalar optimization problem is through the use of a linear combination of all the objectives (e.g., using addition). These techniques are called "aggregating functions" because they combine (or *aggregate*) all the objectives into a single one. This is indeed the oldest mathematical programming method developed for solving multi-objective problems, and it can be derived from the Kuhn-Tucker conditions for nondominated solutions [82].

The most typical linear aggregating function is the following:

$$\min \sum_{i=1}^{k} w_i f_i(\mathbf{x}) \tag{4}$$

where $w_i \geq 0$ are the weighting coefficients representing the relative importance of the k objective functions of our problem (the objectives need to be normalized). It is usually assumed that:

$$\sum_{i=1}^{k} w_i = 1 \tag{5}$$

In order to generate different elements of the Pareto optimal set, the weights must be varied and this is in fact the most common way in which linear aggregating functions have been incorporated into evolutionary algorithms for solving multi-objective problems (see for example [35]). The main problem of linear aggregating functions is that they cannot generate non-convex portions of the Pareto front, regardless of the weights that we adopt [29]. Nevertheless, some clever proposals were made in the early 2000s to overcome this limitation (see for example [74]).

3.1.2 Vector Evaluated Genetic Algorithm (VEGA)

This is the first actual implementation of an evolutionary multiobjective optimization technique, which, as indicated before, was developed by David Schaffer [133, 134] in the mid-1980s. The *Vector Evaluated Genetic Algorithm* (VEGA) basically consisted of a simple genetic algorithm (GA) with a modified selection mechanism. At each generation, a number of sub-populations (as many as the number of objectives of the problem) were generated by performing proportional selection according to each objective function in turn. Thus, for a problem with k objectives, k sub-populations of size N/k each would be generated (assuming a total population size of N). These sub-populations would then be shuffled together to obtain a new population of size N, on which the GA would apply the crossover and mutation operators in the usual way. VEGA has several limitations. For example, Schaffer realized that the solutions generated by his system were locally nondominated but not necessarily globally non-dominated. Also, he noted that producing individuals which were the best in one objective is not a good idea in multi-objective optimization (in fact, this sort of selection mechanism opposes the notion of Pareto optimality). Nevertheless, the selection mechanism of VEGA has been adopted by some researchers (see for example [19]) and some other population-based selection schemes which combine VEGA with linear aggregating functions have been adopted by other researchers (see for example [70]).

3.1.3 Lexicographic ordering

In this method, the user is asked to rank the objectives in order of importance. The optimum solution is then obtained by minimizing the objective functions, starting with the most important one and proceeding according to the assigned order of importance of the objectives, but maintaining the best solutions previously produced.

Fourman [44] was the first to suggest a selection scheme based on lexicographic ordering for a MOEA. In a first version of his algorithm, objectives are assigned different priorities by the user and each pair of individuals are

compared according to the objective with the highest priority. If this resulted in a tie, the objective with the second highest priority was used, and so on. In another version of this algorithm (that apparently worked quite well), an objective is randomly selected at each run. Several other variations of lexicographic ordering have been adopted by other authors (see for example [45, 111]), but this sort of approach is clearly not suitable for complex multi-objective problems or even for (not so complex) problems having more than two objectives [23].

3.1.4 Target-vector approaches

This category encompasses methods in which we have to define a set of goals (or targets) that we wish to achieve for each objective function under consideration. The MOEA in this case will try to minimize the difference between the current solution generated and the vector of desirable goals (different metrics can be used for this purpose). Although target vector approaches can be considered as another aggregating approach, these techniques can generate (under certain conditions) concave portions of the Pareto front, whereas approaches based on simple linear aggregating functions cannot.

The most popular techniques included here are hybrids of MOEAs with: Goal Programming [30, 149, 129], Goal Attainment [150, 155], and the min-max algorithm [54, 21]. These techniques are relatively simple to implement and are very efficient (computationally speaker). However, their main disadvantage is the difficulty to define the desired goals. Additionally, some of them can generate nondominated solutions only under certain conditions [23].

3.2 Non-Elitist Pareto-based Approaches

Goldberg discussed the main drawbacks of VEGA in his seminal book on genetic algorithms [49] and proposed an approach to solve multi-objective optimization problems which incorporated the concept of Pareto optimality (this approach is now known as Pareto ranking) and also suggested the use of a mechanism to block the selection mechanism so that a diverse set of solutions could be generated in a single run of a MOEA (he suggested fitness sharing for this sake [50]). Such a mechanism is known today as *density estimator* and is a standard procedure in modern MOEAs [23].

Early Pareto-based MOEAs relied on variations of Goldberg's proposal and adopted relatively simple density estimators. Here, we will briefly review the following MOEAs:

- Multi-Objective Genetic Algorithm (MOGA)
- Nondominated Sorting Genetic Algorithm (NSGA)
- Niched-Pareto Genetic Algorithm (NPGA)

3.2.1 Multi-Objective Genetic Algorithm (MOGA)

It was proposed by Fonseca and Fleming in 1993 [43]. In MOGA, the rank of a certain individual corresponds to the number of individuals in the population by which it is dominated. However, it adopts a clever Pareto ranking scheme which classifies all individuals in a single pass and assigns fitness to each of them based on their ranks. All nondominated individuals are assigned the same fitness value and all dominated individuals are assigned a fitness value that decreases proportionally to the number of individuals that dominate them (as more individuals dominate a certain solution, its fitness value becomes lower).

MOGA was used by an important number of researchers, particularly in automatic control (see for example [113, 97]).

3.2.2 Nondominated Sorting Genetic Algorithm (NSGA)

This algorithm was proposed by Srinivas and Deb in the mid-1990s and it was the first MOEA published in a specialized journal [139]. NSGA is based on the creation of several layers of classifications of the individuals (this procedure is now called *nondominated sorting*) as suggested by Goldberg [49]. Before selection is performed, the population is ranked on the basis of Pareto optimality: all nondominated individuals are classified into one category or layer (using a dummy fitness value, which is proportional to the population size). The density estimator in this case is fitness sharing (which is applied on the dummy fitness values). Once a group of individuals has been classified, then such a group is ignored and another layer of nondominated individuals is considered. This process is repeated until all the individuals in the population had been classified.

Several applications of NSGA were developed in the 1990s and early 2000s (see for example [148, 96, 8]).

3.2.3 Niched-Pareto Genetic Algorithm (NPGA)

It was proposed by Horn et al. in the mid-1990s [62]. NPGA uses binary tournament selection based on Pareto dominance. Thus, two individuals are randomly chosen and compared against a subset from the entire population (typically, the number of individuals in this set is of around 10% of the total population size). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins the tournament. Otherwise, (i.e., when both competitors are either dominated or nondominated), the result of the tournament is decided through fitness sharing [50].

NPGA was not as popular as NSGA or MOGA, but there are some applications of this MOEA reported in the literature (see for example [156, 118]).

3.3 Elitist Pareto-based Approaches

MOEAs developed in the late 1990s started to incorporate the notion of elitism. In the context of evolutionary multi-objective optimization, elitism refers to retaining the nondominated solutions generated by a MOEA. The most popular mechanism for implementing elitist is through the use of an external archive (also called *secondary population*) which may or may not intervene in the selection process. This external archive stores the nondominated solutions generated by a MOEA and is normally bounded and pruned once it is full. This is done for two main reasons: (1) to facilitate direct comparisons among different elitist MOEAs and (2) to dilute the selection process (when the external archive participates in the selection process) and/or to avoid storing an excessively large number of solutions.

Elitism is a very important mechanism in MOEAs, because it is required to (theoretically) guarantee convergence [127].

It is worth noting that elitism can also be introduced through the use of a $(\mu+\lambda)$-selection in which parents compete with their children and those which are nondominated (and possibly comply with some additional criterion such as providing a better distribution of solutions) are selected for the following generation. This is the elitist mechanism adopted by NSGA-II [33].

The most representative elitist Pareto-based approaches developed in the later 1990s and early 2000s which will be briefly described here are the following:

- The Strength Pareto Evolutionary Algorithm (SPEA)
- The Pareto Archived Evolution Strategy (PAES)
- The Nondominated Sorting Genetic Algorithm-II (NSGA-II)

It is also worth indicating that alternative density estimators were proposed with these MOEAs as will be indicated next.

3.3.1 The Strength Pareto Evolutionary Algorithm (SPEA)

It was introduced by Zitzler & Thiele in the late 1990s [161]. This approach was conceived as a way of integrating different MOEAs. SPEA uses an external archive that contains the nondominated solutions previously generated. At each generation, nondominated individuals are copied to the external archive. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of MOGA, since it is

proportional to the number of solutions to which a certain individual dominates. The fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. Additionally, a clustering technique called "average linkage method" [110] is used as the density estimator.

SPEA has been used in a variety of applications (see for example [103, 3]).

In 2001, a revised version of this algorithm (called SPEA2) was proposed [163]. This approach has three main differences with respect to its original version: (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions. SPEA2 has also been widely applied (see for example [114, 147]).

3.3.2 The Pareto Archived Evolution Strategy (PAES)

This MOEA was introduced by Knowles and Corne [80] and it consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with an external archive that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is compared. An interesting aspect of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective function space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective function space). This sort of density estimator (i.e., the so-called adaptive grid) is a very nice idea, but unfortunately, it does not scale property when increasing the number of objectives [20].

There are a few applications of PAES reported in the specialized literature (see for example [1, 122]).

3.3.3 The Nondominated Sorting Genetic Algorithm-II (NSGA-II)

Deb et al. [31, 33] proposed a revised version of the NSGA [139], called NSGA-II, which is more efficient (computationally speaking), uses elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters (it is based on how close are the neighbors of a

solution). This algorithm is, in fact, quite different from the original NSGA, since even its nondominated sorting process is done in a more efficient way.

As indicated before, NSGA-II does not use an external memory but adopts instead an elitist mechanism that consists of combining the best parents with the best offspring obtained (i.e., a $(\mu + \lambda)$-selection).

NSGA-II has been the most popular MOEA developed so far. This is mainly because of its efficiency, efficacy and because of the availability of its source code in the public domain (in several versions). However, its crowded comparison operator does not scale properly as the number of objectives increases [83], which motivated a number of variations of this algorithm including the NSGA-III [32], which is discussed in a further section.

There are many applications of NSGA-II reported in the literature (see for example [90, 27, 56]).

4 The Present

In addition to the MOEAs briefly discussed in the previous section, many others were developed, but few of them were adopted by researchers different from their developers (see for example [26, 25, 20, 140]).

Nevertheless, for over 10 years, Pareto-based MOEAs remained as the most popular approaches in the specialized literature. In 2004, a different type of algorithmic design was proposed, although it remained underdeveloped for several years: indicator-based selection.[3] The core idea of this sort of MOEA was introduced in the **Indicator-Based Evolutionary Algorithm** (IBEA) [162], which consists of an algorithmic framework that allows the incorporation of any performance indicator into the selection mechanism of a MOEA. IBEA was originally tested with the hypervolume [160] and the binary ϵ indicator [162].

The limitations of Pareto-based selection for dealing with problems having 4 or more objectives (the so-called *many-objective optimization problems*) motivated researchers to look for alternative approaches. Indicator-based selection became an attractive option because these schemes can properly deal with any number of objectives. Much of the early interest in this area was motivated by the introduction of the **S Metric Selection Evolutionary Multiobjective Algorithm** (SMS-EMOA) [36]. SMS-EMOA randomly generates an initial population and then produces a single solution per iteration (i.e., it uses a steady state selection scheme) adopting the crossover and mutation operators from NSGA-II. Then, it applies nondominated sorting (as in NSGA-II). When the last nondominated front has more than one solution, SMS-EMOA uses hypervolume [160] to decide which solution should be removed.

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[3] It is worth indicating that indicator-based archiving was introduced earlier (see [79, 78]).

Beume et al. [7] proposed an improved version of SMS-EMOA in which the hypervolume contribution is not used when, in the nondominated sorting process, we obtain more than one front (i.e., the hypervolume is used as a density estimator in this case). When this happens, they use the number of solutions that dominate to a certain individual (i.e., the solution that is dominated by the largest number of solutions is removed). This makes SMS-EMOA a bit more efficient. However, since this MOEA relies on the use of exact hypervolume contributions, it becomes too computationally expensive as we increase the number of objectives [6]. SMS-EMOA started a trend for designing indicator-based MOEAs (several of which rely on the hypervolume indicator) although it is worth indicating that in such approaches, the performance indicator has been mostly used as a density estimator (see for example [66]). The use of "pure" indicator-based selection mechanisms has been very rare in the specialized literature (see for example [101]).

Researchers quickly realized that the efficacy and efficiency of indicator-based MOEAs relies on the adopted performance indicator. So far, the only performance indicator that is known to have the mathematical properties to guarantee convergence (from a theortical point of view) is the hypervolume (i.e., it is a Pareto compliant performance indicator [164]). The hypervolume (which is also known as the $\mathcal{S}$ metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively. As indicated before, one of its main advantages are its mathematical properties, since it has been proved that the maximization of this performance measure is equivalent to finding the Pareto optimal set [40]. Additionally, empirical studies have shown that (for a certain number of points previously determined) maximizing the hypervolume does produce subsets of the true Pareto front [78, 36]. As a performance indicator, the hypervolume assesses both convergence and, to a certain extent, also the spread of solutions along the Pareto front (although without necessarily enforcing a uniform distribution of solutions). Nevertheless, there are several practical issues regarding the use of the hypervolume. First, the computation of this performance indicator depends of a reference point, which can influence the results in a significant manner. Some people have proposed to use the worst objective function values in the current population, but this requires scaling the objectives. Nevertheless, the most serious limitation of the hypervolume is its high computational cost. The best algorithms known to compute hypervolume have a polynomial complexity on the number of points used, but such complexity grows exponentially on the number of objectives [6]. This has motivated a significant amount of research related to the development of sophisticated algorithms that can reduce the computational cost of computing the hypervolume and the hypervolume contributions, which is what we need for a hypervolume-based MOEA[4] (see for example [84, 72, 52]). Today, most researchers believe that it's not possible to overcome the high computational

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[4] See:
http://ls11-www.cs.uni-dortmund.de/rudolph/hypervolume/start

cost of computing exact hypervolume contributions. An obvious alternative to deal with this issue is to approximate the actual hypervolume contributions. This is the approach adopted by the **Hypervolume Estimation Algorithm for Multi-Objective Optimization** (HyPE) [4] in which Monte Carlo simulations are adopted to approximate exact hypervolume values. In spite of the fact that HyPE can efficiently solve multi-objective problems having a very large number of objectives, its results are not as competitive as when using exact hypervolume contributions.

Another alternative is to use a different performance indicator whose computation is relatively inexpensive. Unfortunately, the hypervolume is the only unary indicator which is known to be Pareto compliant [164], which makes less attractive the use of other performance indicators. Nevertheless, there are some other performance indicators which are weakly Pareto compliant, such as $R2$ [11] and the Inverted Generational Distance plus ($IGD+$) [69]. Although several efficient and effective indicator-based MOEAs have been proposed around these performance indicators (see for example [57, 12, 94, 86, 95]), their use has remained relatively scarce until now. Another interesting idea that has been only scarcely explored is the combination of performance indicators in order to take advantage of their strenghts and compensate for their limitations (see for example [37]).

In 2007, a different sort of approach was proposed, quickly attracting a lot of interest: the **Multi-Objective Evolutionary Algorithm based on Decomposition** (MOEA/D) [157]. The idea of using decomposition (or scalarization) methods was originally proposed in mathematical programming in the late 1990s [28] and it consists in transforming a multi-objective optimization problem into several single-objective optimization problems which are then solved to generate the nondominated solutions of the original problem. Unlike linear aggregating functions, the use of scalarization (or decomposition) methods allows the generation of non-convex portions of the Pareto front and works even in disconnected Pareto fronts. MOEA/D presents an important advantage with respect to methods proposed in the mathematical programming literature (such as Normal Boundary Intersection (NBI) [28]): it uses neighborhood search to solve simultaneously all the single-objective optimization problems generated from the transformation. Additionally, MOEA/D is not only effective and efficient, but can also be used for solving problems with more than 3 objectives although in such cases it will require higher population sizes (however, the population size needs to be increased linearly with respect to the number of objectives).

Decomposition-based MOEAs became fashionable at around 2010 and have remained as an active research area since then [131]. In fact, this sort of approach influenced the development of the **Nondominated Sorting Ge-**

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http://people.mpi-inf.mpg.de/~tfried/HYP/
http://iridia.ulb.ac.be/~manuel/hypervolume

netic Algorithm-III (NSGA-III[5]) [32] which adopts both decomposition and reference points to deal with many-objective optimization problems. However, it was recently found that decomposition-based MOEAs do not work properly with certain Pareto front geometries [71]. This has motivated a lot of research that aims to overcome this limitation.

4.1 Some Applications

In recent years, a significant number of applications of MOEAs have been reported in the literature [22].

Roughly, we can classify applications in three large groups: (1) engineering, (2) industrial and (3) scientific. Some specific areas within each of these groups are indicated next. We will start with the engineering applications, which are, by far, the most popular in the literature. This is not surprising because engineering disciplines normally have problems with better known and understood mathematical models which facilitates the use of MOEAs. A sample of engineering applications is the following:

- Electrical engineering [51, 88]
- Hydraulic engineering [146, 93]
- Structural engineering [67, 60]
- Aeronautical engineering [115, 152]
- Robotics [73, 39]
- Automatic Control [145, 63]
- Telecommunications [64, 9]
- Civil engineering [85, 75]
- Transport engineering [24, 76]

A sample of industrial applications of MOEAs is the following:

- Design and manufacture [92, 77]
- Scheduling [154, 126]
- Management [121, 138]

Finally, we have a variety of scientific applications:

- Chemistry [17, 38]
- Physics [109, 46]
- Medicine [128, 102]
- Bioinformatics [124, 34]
- Computer science [89, 55]

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[5] NSGA-III was designed to solve many-objective optimization problems and its use is relatively popular today.

5 The Future

A large number of MOEAs have been developed since 1984 (most of them, during this century), but few of them have become popular among practitioners. This raises a relevant question: where is the research on MOEAs leading to? In other words, can we design new MOEAs that can become popular? This is indeed an interesting question, although people working on the development of MOEAs evidently consider that there is room for new (sometimes highly specialized MOEAs).

If we look at specific domains, it is easier to justify the development of particular MOEAs to tackle them. Let's consider the following examples:

- **Large-Scale Multi-Objective Optimization:** It refers to solving multi-objective problems with more than 100 decision variables (something not unusual in real-world applications). Little work has been done in this area, and cooperative coevolutionary approaches (which are popular in single-objective large-scale optimization) have been the most popular choice (see for example [105, 91, 159]). However, there is still plenty of research to be done in this area. For example, appropriate test suites for large-scale multi-objective optimization are required (see for example [18]).
- **Expensive Objective Functions:** The design of parallel MOEAs seems as the most obvious choice for dealing with expensive objective functions (see for example [10]). However, basic research on parallel MOEAs has remained scarce and most of the current papers on this topic focus either on applications [123, 24] or on straightforward parallelizations of well-known MOEAs (see for example [153]). Many other topics remained to be explored, including the development of asynchronous parallel MOEAs [130], the study of theoretical aspects of parallel MOEAs [108] and the proper use of modern architectures such as Graphical Processing Units (GPUs) for designing MOEAs [116]. Another alternative to deal with expensive objective functions is the use of surrogate methods. When using surrogates, an empirical model that approximates the real problem is built through the use of information gathered from actual objective function evaluations [112]. Then, the empirical model (on which evaluating the fitness function is computationally inexpensive) is used to predict new (promising) solutions [2, 100]. Although frequently used in engineering applications, surrogate methods can normally be adopted only in problems of low dimensionality, which is an important limitation when dealing with real-world problems. Additionally, surrogate models tend to lack robustness which is also an important issue in optimization problems. Nevertheless, there has been recent research oriented towards overcoming the scalability and robustness limitations of surrogate methods (see for example [151, 117]).
- **Many-Objective Optimization:** Developing MOEAs for properly solving multi-objective problems having more than 3 objectives is indeed a hot research topic nowadays. In spite of the existence of a number of indicator-

based MOEAs and decomposition-based MOEAs that were explicitly designed for many-objective optimization, a number of other approaches are also possible. For example, we can use alternative ranking schemes (different from nondominated sorting) (see for example [47]), machine learning techniques (as in MONEDA [99]), or approaches such as the two-archive MOEA, which uses one archive for convergence and another for diversity [120]. It is also possible to use dimensionality reduction techniques which identify redundant objectives (i.e., objective that can be removed without changing the dominance relation induced by the original objective set) and remove them so that the actual dimensionality of the problem can be reduced (see for example [13, 132]). Additionally, several other topics related to many-objective optimization still require further research. Two good examples are visualization techniques [141] and density estimators [59] for problems having a large number of objectives. Another relevant topic is the solution of large-scale many-objective problems (see for example [16, 158]).

Nevertheless, a more profound and complex question is the folliwing: *is it possible to design MOEAs in a different way?* This is a question of great relevance because if it is no longer possible to produce new algorithmic proposals, this entire research area may stagnate and even disappear.

Clearly, it is not trivial to produce a selection mechanism that does not belong to any of the paradigms that we revised in this chapter (i.e., Pareto-based, decomposition-based or indicator-based), but this is indeed possible. For example, Molinet Berenguer and Coello Coello [5, 107], proposed an approach that transforms a multi-objective optimization problem into a linear assignment problem using a set of weight vectors uniformly scattered. Uniform design is adopted to obtain the set of weights, and the Kuhn-Munkres (Hungarian) algorithm [81] is used to solve the resulting assignment problem. This approach was found to perform quite well (and at a low computational cost) even in many-objective optimization problems. This approach constitutes an intriguing new family of MOEAs, since it does not belong to any of the three types of schemes previously described. But designing a new type of MOEA is not enough. It is perhaps more challenging (and certainly more difficult) that such an approach becomes popular.

In addition to the proposal of new algorithmic paradigms, many other approaches are possible. For example, it is possible to combine components of MOEAs under a single framework that allows to exploit their advantages. This is the basic idea of **Borg** [53], which adopts ϵ-dominance, a measure of convergence speed called ϵ progress, an adaptive population size, multiple recombination operators and a steady-state selection mechanism. Related to this sort of approach is the notion of being able to automatically design MOEAs for particular applications/domains, which is something that has been suggested by researchers from automated parameter tuning for single-objective evolutionary algorithms (see for example [65]).

A related idea is the design of hyper-heuristics for multi-objective optimization. A hyper-heuristic is a search method or learning mechanism for selecting or generating heuristics to solve complex search problems [14]. Hyper-heuristics are high-level approaches that operate on a search space of heuristics or on a pool of heuristics instead of the search space of the problem at hand. They constitute an interesting choice to provide more general search methodologies, since simple heuristics tend to work well on certain types of problems, but can perform very poorly on other classes of problems or even in slight variations of a certain class in which they perform well. Although the ideas behind hyper-heuristics can be traced back to the early 1960s in single-objective optimization, their potential hadn't been explored in multi-objective optimization until relatively recently. Several multi-objective hyper-heuristics have been proposed for combinatorial problems (see for example [15, 142, 98]) but they are still rare in continuous multi-objective optimization (see for example [143, 144, 58]).

Another interesting path for future research in this area is to gain a deeper understanding of the limitations of current MOEAs. For example, knowing that some scalarizing functions offer advantages over others is very useful to design good decomposition-based and even indicator-based MOEAs (see for example [119]). It is also important to design new mechanisms (e.g., operators, encodings, etc.) for MOEAs aimed for particular real-world problems (e.g., variable length encodings, expensive objective functions, uncertainty, etc.). See for example [87].

Other evolutionary computation areas can also be brought to this field to enrich the design of MOEAs. One example is coevolutionary approaches, which have been used so far mainly for large-scale multi-objective optimization, but could have more applications in this area (e.g., they could be used to solve dynamic multi-objective optimization problems [48]). Clearly, the potential of coevolutionary schemes has been only scarcely explored in multi-objective optimization (see [106]).

6 Conclusions

This chapter has provided a review of the research on the development of multi-objective evolutionary algorithms that has been conducted since their inception in 1984 to date. In addition to providing short descriptions of the main algorithmic proposals, several ideas for future research in the area have been provided.

This overview has shown that the design of MOEAs has been a very active research area, which still has a wide variety of topics to be explored. Clearly, evolutionary multi-objective optimization is still a very promising research area which should remain very active for several more years. However, it is important to work in a diverse set of topics in order to avoid focusing only on

the work by analogy (for example, producing more small variants of existing MOEAs).

Additionally, many fundamental topics still remain unexplored, thus offering great research opportunities for those interested in tackling them. For example, we are lacking theoretical studies related to the limitations of current MOEAs, which are fundamental for the development of the area. An interesting example of the importance of this topic is the study conducted by Schütze et al. [135] in which the actual source of difficulty in many-objective problems was analyzed. This study concluded that adding more objectives to a multi-objective problem does not necessarily makes it harder. According to this study (which has been largely ignored by several researchers working on many-objective optimization), the difficulty of many-objective problems is really associated to the intersection of the descent cones of the objectives (these descent cones are obtained with the combination of the gradients of each objective). This was somehow corroborated by an empirical study conducted by Ishibuchi et al. [68] in which it was shown that NSGA-II could properly solve many-objective knapsack problems in which the objectives were highly correlated. Clearly, the study of Schütze et al. [135] could had re-directed the research conducted in many-objective optimization, if researchers working in this area had taken it into account.

The main goal of this chapter is to serve as an introductory guide to those interested in tackling some of the many challenges that this research area still has to offer during the next few years. These days, such topics are not trivial to identify within the vast volume of references available on this topic and therefore the importance of providing a highly compressed overview of the research that has been conducted during 35 years around this topic. Hopefully, this chapter will serve to that purpose.

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