

# Multi-Objective Airfoil Shape Optimization Using a Multiple-Surrogate Approach

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**Abstract**—In this paper, we present a surrogate-based multi-objective evolutionary optimization approach to optimize airfoil aerodynamic designs. Our approach makes use of multiple surrogate models which operate in parallel with the aim of combining their features when solving a costly multi-objective optimization problem. The proposed approach is used to solve five multiobjective airfoil aerodynamic optimization problems. We compare the performance of a multi-objective evolutionary algorithm with surrogates with respect to the same approach without using surrogates. Our preliminary results indicate that our proposal can achieve a substantial reduction in the number of objective function evaluations, which has obvious advantages for dealing with expensive objective functions such as those involved in aeronautical optimization problems.

## I. INTRODUCTION

Multi-Objective Optimization (MOO) provides designers with more trade-off solutions to choose from, in situations in which we aim to fulfill several (conflicting) objectives. This contrasts with traditional design, in which a single solution is obtained. The trade-off solutions that are obtained by multi-objective optimization techniques are referred to as the Pareto optimal set (in decision variable space) and their corresponding objective function values form the so-called Pareto front [1]. This sort of approach contrasts with traditional design optimization techniques, which only produce one (the best possible) solution without providing alternative choices to the designer.

In aeronautical systems design as well as in the design of propulsion system components, such as turbine engines, aerodynamics plays a key role. Thus, aerodynamic shape optimization (ASO) is a crucial task, which has been extensively studied and developed. This discipline has recently benefitted from the use of multi-objective evolutionary algorithms (MOEAs), which have gained an increasing popularity in the last few years [2]. However, MOEAs have to face several challenges when applied to ASO problems:

- 1) The flow field for some ASO applications, can be extremely complex. Therefore, complex Computational Fluid Dynamics (CFD) Navier-Stokes computations (which are very expensive, computationally speaking) are required.

- 2) The performance of aerodynamic shapes such as wing's airfoils or turbine airfoils blades, is very sensitive to the shape itself. Thus, an airfoil must be modeled with a large number of decision variables. In addition, the objective function landscape of an ASO problem is often multimodal and nonlinear because the flow field is governed by a system of nonlinear partial differential equations.
- 3) ASO problems are usually subject to several constraints and in some cases, such constraints can be evaluated only after performing a CFD simulation, turning it into a very expensive process (computationally speaking).
- 4) MOEAs require a considerable number of fitness function calls to the CFD simulation code in order to conduct an appropriate search. This may turn them impractical if the objective functions are too costly.

Thus, there is an evident need to have mechanisms that allow the solution of computationally expensive problems in reasonably short periods of time. A common approach is the use of parallel processing techniques, which, however, may not be sufficient in some cases [3]. Another alternative that has been widely adopted in the engineering optimization literature is the use of surrogates (also called metamodels), which use (computationally cheap) approximate models of the problem which are periodically adjusted (using real objective function evaluations).

In this paper, we present a surrogate-based multiobjective evolutionary approach to optimize airfoil aerodynamic designs. Our approach makes use of multiple surrogate models which operate in parallel, aiming to combine the features of different approximation models in order to produce the combination that reduces, as much as possible, the computational cost of the MOEA being used.

The remainder of this paper is organized as follows. Section II presents some basic concepts related to multi-objective optimization. In Section III, we present general concepts about surrogate modeling emphasizing some of the main points to consider when adopting them. Section IV describes our proposed approach. Section V describes the experimental setup used to validate our proposed approach. Section VI presents the results obtained and a discussion of them. Finally, in Section VII, we present our conclusions and some possible paths for future research.

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## II. MULTI-OBJECTIVE OPTIMIZATION

A multi-objective optimization problem (MOP) can be mathematically defined as<sup>1</sup>:

$$\min \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \quad (1)$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \dots, m \quad (2)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \quad (3)$$

where  $\vec{x} = [x_1, x_2, \dots, x_n]^T$  is the vector of decision variables,  $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $i = 1, \dots, k$  are the objective functions and  $g_i, h_j : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, p$  are the constraint functions of the problem.

Other relevant definitions are the following:

**Definition 1.** A vector of decision variables  $\vec{x} \in \mathbb{R}^n$  **dominates** another vector of decision variables  $\vec{y} \in \mathbb{R}^n$ , (denoted by  $\vec{x} \prec \vec{y}$ ) if and only if  $\vec{x}$  is partially less than  $\vec{y}$ , i.e.  $\forall i \in \{1, \dots, k\}, f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1, \dots, k\} : f_i(\vec{x}) < f_i(\vec{y})$ .

**Definition 2.** A vector of decision variables  $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$  is **nondominated** with respect to  $\mathcal{X}$ , if there does not exist another  $\vec{x}' \in \mathcal{X}$  such that  $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$ .

**Definition 3.** A vector of decision variables  $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$  ( $\mathcal{F}$  is the feasible region) is **Pareto optimal** if it is nondominated with respect to  $\mathcal{F}$ .

**Definition 4.** The **Pareto Optimal Set**  $\mathcal{P}^*$  is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimal}\}$$

**Definition 5.** The **Pareto Front**  $\mathcal{PF}^*$  is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

The goal when solving a MOP consists on determining the Pareto optimal set from the set  $\mathcal{F}$  of all the decision variable vectors that satisfy (2) and (3).

## III. SURROGATE MODELS

We will limit our discussion to the use of surrogate models in multi-objective ASO problems, since that is the scope of this paper. In this context, surrogate models [4] replace direct calls to any CFD simulation code. Figure 1 shows the general flow in any surrogate-based optimization approach.

When designing and/or using a surrogate modeling approach several issues need to be addressed [5]:

- 1) **Model to use:** Several options are available: Response Surface Methods (RSM) based on low-order polynomial functions, Gaussian processes or Kriging, Radial Basis Functions (RBFs), Artificial Neural Networks (ANNs), and Support Vector Machines (SVMs), among others [6].

<sup>1</sup>Without loss of generality, minimization is assumed in the following definitions, since any maximization problem can be transformed into a minimization one.

- 2) **Global/local surrogate model:** A global approximation model, associated with a reduced accuracy can be designed with a better ability to reflect general tendencies in the fitness landscape, allowing the designer to perform an explorative design search in the whole design space in an efficient and rapid manner. When a local surrogate model is adopted, the accuracy of the approximation can be increased with a better ability to capture local tendencies in the fitness landscape, but its region of validity is limited to a predefined neighborhood in the design space, and the designers are able to explore only small regions of it.
- 3) **Sample size and distribution for initial surrogate training:** The metamodel must first be trained using a number of initial simulations, whose evaluation is costly. These initial points are defined by a design of experiment (DoE) technique [7], and must be kept to a minimum. Several initial point distributions have been proposed. A relatively common approach is to use Latin Hypercube Sampling (LHS) [7].
- 4) **Infilling criterion:** Once the surrogate model is initialized, its operation will require a selection of (surrogate-obtained) optimal designs which will be evaluated with the real objective functions. These evaluations will be used to adjust the model (aiming to reduce its approximation error). Clearly, these so-called *infill points*, must be carefully selected (using a good *infilling criterion*). This is not an easy task, since we aim not only at reducing the approximation errors, but also at exploring as many different regions of the search space as possible. Thus, accuracy and diversity need somehow to be balanced within our infilling criterion.

### A. Surrogate modeling in ASO problems

Next we present a short review of some representative research work on the use of surrogate modeling for solving ASO problems. The way in which the issues indicated before are addressed in each paper is emphasized in our discussion.

From the different mathematical models available, RSM based on low order polynomials are probably the most popular choice in the literature. For example, Lian and Liou [8] presented the use of MOGA [9] coupled to a second order polynomial based RSM, for solving the bi-objective ASO of a turbine blade. This is probably a natural extension of methods that have been found to be effective for the single-objective case. The main advantage of using polynomial based RSM is probably its generalization abilities. However, its training cost is proportional to the number of sampling points, and a high number of them is required for getting a good accuracy of the model. More recently, RSM based on low order polynomials have become less common, probably due to its limitation in accurately representing fitness landscapes which are very rough and with high nonlinearities. For such cases, some reserachers have relied on more elaborated surrogate models as in Karakasis et al. [10], who presented the multi-objective optimization of a turbine engine compressor blade using RBF surrogate models. RBFs are very powerful functions

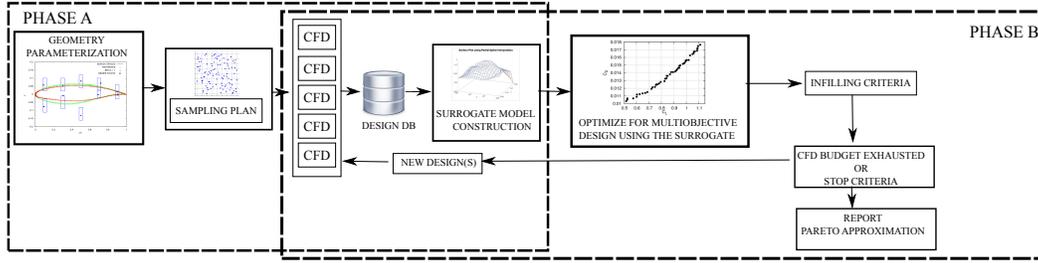


Fig. 1. Surrogate-based optimization framework

to represent complex fitness landscapes, and for some kernel functions, there exist some tuning parameters to control the accuracy of the approximation model. Emmerich et al. [11] and Keane [12] presented the application of a Kriging-based metamodel for multi-objective airfoil shape optimization in transonic flow conditions. Kriging has a strong mathematical basis, and is probably one of the most powerful interpolation methods currently available. Also, kriging is able to provide an estimate of its associated accuracy, and it allows the model to be tuned for an improved accuracy. However, its cost increases as the dimensionality and the number of training points in the problem increase.

Concerning the globality/locality of the model, the RSM presented by Lian and Liou [8] corresponds to a global one, while the model presented by Karakasis et al. [10] corresponds to multiple local models. In this case, the database containing the history of designs evaluated with the CFD tool, is subdivided into clusters, using a self-organizing map technique. This sort of technique aims at training local RBF surrogate models with small subpopulations, but also guaranteeing that the whole design space is covered by allowing the overlapping of the local RBF models. In the work of Emmerich et al. [11], the model is also local, i.e., local kriging models in the neighborhood of the solution are evaluated. In the work of Keane [12], the global kriging model is adopted.

Regarding the initial sampling technique, most of the research works that we reviewed use a DoE technique, based on either a particular LHS approach [8] (Improved Hypercube Sampling (IHS) algorithm [13]), or the  $LP\tau$  technique [12]. The size of the initial population is mainly a choice based both on the dimensionality of the problem, as well as on the number of CFD evaluations available.

Finally, regarding the infilling criterion, we identified the following options. Lian and Liou [8], do not adopt an infilling criterion. In this case, the trained model, which comprised 1,024 design solutions, is used to obtain the approximation of the Pareto front. From this set, some candidate solutions are selected to be evaluated with the CFD simulation code. This is probably the simplest possible technique, but designers must rely on very accurate models in this case. In the works of Karakasis et al. [10], and Emmerich et al. [11], the infilling criterion corresponds to a prescreening technique. In this technique, the offspring are evaluated with the local models and with this evaluation they are either ranked based on

Pareto dominance [10] or their Hypervolume contribution is estimated [11]. In both cases, the most promising individuals are selected to be evaluated with the CFD simulation code. Finally, in the work of Keane [12], the infilling criterion corresponds to a multi-objective extension of a commonly used technique for single-objective kriging models. This approach consists in adopting a metric defined in terms of the probability of improvement, and on the expected improvement which can be computed from the estimated accuracy of interpolation given by the model.

#### IV. OUR PROPOSED SURROGATE MODELING APPROACH

Our proposed surrogate-based multiobjective evolutionary approach has the following features:

(a).- **Model:** When a single surrogate is needed, it is common practice to train several surrogate models and pick one based on their accuracy or their cross-validation error [14]. Another option is to combine different surrogate models into a single one by weighting their contribution [15]. In our case, and inspired by the notion of “*blessing and curse of uncertainty*” in approximation models [16], we propose to use not only one surrogate model, but a set of them. So, the idea is to train, in parallel,  $N$  surrogate models ( $SM_1, SM_2, \dots, SM_N$ ). Thus,  $N$  new solutions will be selected for updating the database and to update the set of surrogate models itself. The motivation behind this choice is also that, by using different models, which are trained with the same data points, we can inherently balance the exploration/exploitation required in any surrogate-based optimization technique. Solutions selected from a surrogate model with high accuracy will emphasize exploitation, while solutions selected from a surrogate with low accuracy will emphasize exploration. Towards the end of the evolutionary process, it is expected that all surrogate models will have a high accuracy and will, therefore, contribute more to the exploitation. By performing the search in parallel, we can reduce the computational cost associated with the MOEA being used. In our approach, the surrogate models can be any combination of the options previously mentioned. Alternatively, it is also possible to use a single surrogate model but with different tuning parameters which will be tested in parallel.

(b).- **Globality/locality:** In our proposed approach we decided to adopt a global model, but this globality is defined in terms of the training points in the database. Thus, the models are trained in the design space implicitly defined by the database points. We propose to define an initial number of training points  $NTP^{init}$ , and a maximum allowable number of training points to hold in the database  $NTP^{max}$ . Once this upper limit is reached, we still allow the insertion of new points, but the database is pruned until reaching again the maximum allowable number of solutions. The pruning technique adopted here is based on Pareto ranking (i.e., individuals in the database with the highest Pareto ranks, which are the worst in terms of Pareto optimality, are removed, until reaching the upper limit allowed in the database). The motivation for defining a maximum number of points to be kept in the database, is to reduce the computational cost associated to the training process, and to adapt the model in the neighborhood of the Pareto front approximation, as the evolution progresses.

(c).- **Initial sampling:** In the experiments that we present here, we adopted a sampling procedure based on the Halton distribution of points. Regarding the initial database size  $NTP^{init}$ , this will depend on the budget of CFD evaluations available and on the number of dimensions of the problem. We will provide guidelines for this, later on.

(d).- **Infilling criterion:** In our approach, we adopt  $N$  parallel surrogate models, and we extract one solution from each of them. Therefore,  $N$  new solutions will be generated at each design cycle, which will also be evaluated in parallel. For performing this selection, we first define a set of weight vectors  $[\lambda^1, \lambda^2, \dots, \lambda^{NP}]$ , where  $NP$  is the population size used in the MOEA. Next, and from each surrogate model, we select a solution that minimizes a scalar function for a selected weight vector. For that sake, we adopt the Tchebycheff scalarization function given by:

$$g(\vec{x}|\lambda, z^*) = \max_{1 \leq i \leq k} \{\lambda_i^j |f_i(x) - z_i^*|\} \quad (4)$$

In the above equation,  $\lambda^j, j = 1, \dots, NP$  represents the set of weight vectors used to distribute the solutions along the Pareto front.  $z^*$  corresponds to a reference point, defined in objective space and determined with the minimum objective values of the population used in the MOEA. In order to cover the whole Pareto front, each surrogate model must choose a different weight vector from the set. Additionally, in each design cycle, a different weight vector must be selected. For doing this, we perform, in each surrogate model, a sweeping in the set of weight vectors, and start from a different weight vector. This process is illustrated in Figure 2. The indicated sweeping is done in a cyclic manner, i.e. once the last weight vector is selected, the next one is picked from the beginning. The aim of this technique is to guarantee the coverage of all regions of the Pareto front. Evidently, once the  $N$  solutions are selected, they are evaluated with the CFD code and added to the database, and the approximation of the Pareto front is

updated.

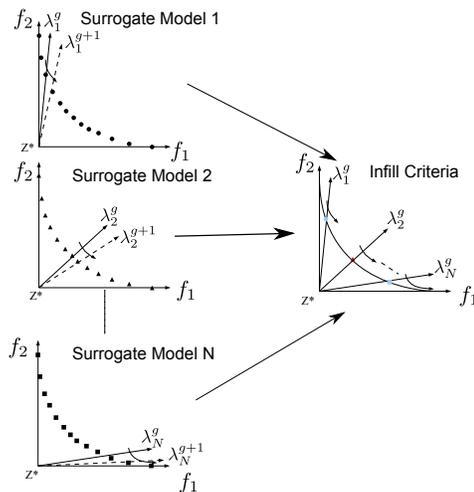


Fig. 2. Infill criterion adopted by our proposed approach

## V. EXPERIMENTAL SETUP

In order to validate our proposed approach, we defined five ASOs, based on problems found in the specialized literature. Three of these problems are bi-objective, and the other two have three objectives each.

We are interested in designing airfoil shapes (*AirfGeom*), with optimal values in their aerodynamics forces and moments, and for different operating conditions. In airfoil design and analysis, it is common to define these forces and moments as scalar coefficients. It follows that for an airfoil shape (*AirfGeom*) at a given flow incidence angle ( $\alpha$ ), the lift ( $C_l$ ), drag ( $C_d$ ), and pitching moment ( $C_m$ ) are a function of:

$$[C_l, C_d, C_m]_{AirfGeom} = f(\alpha, Re, M) \quad (5)$$

where the Reynolds number ( $Re$ ) is the dimensionless ratio of the inertial forces to viscous forces and quantifies their respective relevance for a given operating condition. The Mach number ( $M$ ) is a measure of the air velocity against the speed of sound. The CFD solver adopted in this benchmark corresponds to XFOIL [17]. These coefficients, as well as their ratio in some cases, have different effects on aircraft performance. Thus, for the benchmark we defined the following ASO MOPs, aiming at presenting different Pareto front geometries, as well as different fitness landscapes.

**ASO-MOP1:**

$\min(C_d)$  @  $\alpha = 0.0^\circ, Re = 4.0 \times 10^6, M = 0.2$   
 $\min(2.0 - C_l)$  @ same flow conditions.

**ASO-MOP2:**

$\min(C_d/C_l)$  @  $\alpha = 4.0^\circ, Re = 2.0 \times 10^6, M = 0.1$   
 $\min(C_m^2)$  @ same flow conditions.

ASO-MOP3:

$\min(C_d/C_l)$  @  $\alpha = 1.0^\circ$ ,  $Re = 3.0 \times 10^6$ ,  $M = 0.3$   
 $\min(C_d^2/C_l^3)$  @  $\alpha = 5.0^\circ$ ,  $Re = 1.5 \times 10^6$ ,  $M = 0.15$

ASO-MOP4:

$\min(C_d)$  @  $\alpha = 4.0^\circ$ ,  $Re = 3.0 \times 10^6$ ,  $M = 0.3$   
 $\min(2.0 - C_l)$  @ same flow conditions.  
 $\min(C_m^2)$  @ same flow conditions.

ASO-MOP5:

$\min(C_d/C_l)$  @  $\alpha = 1.0^\circ$ ,  $Re = 4.0 \times 10^6$ ,  $M = 0.3$   
 $\min(C_d^2/C_l^3)$  @  $\alpha = 3.0^\circ$ ,  $Re = 3.0 \times 10^6$ ,  $M = 0.3$   
 $\min(C_d^2/C_l^3)$  @  $\alpha = 5.0^\circ$ ,  $Re = 2.0 \times 10^6$ ,  $M = 0.3$

### A. Geometry Parameterization

We adopt here the PARSEC airfoil representation [18]. Figure 3 illustrates the 11 basic parameters used for this representation. In our case, a modified PARSEC geometry representation was adopted, allowing us to define independently the leading edge radius, both for upper and lower surfaces. Thus, 12 variables in total were used. Their allowable ranges are defined in Table I.

	$r_{leup}$	$r_{lelo}$	$\alpha_{te}$	$\beta_{te}$	$Z_{te}$	$\Delta Z_{te}$
min	0.0085	0.002	7.0	10.0	-0.006	0.0025
max	0.0126	0.004	10.0	14.0	-0.003	0.0050

	$X_{up}$	$Z_{up}$	$Z_{xxup}$	$X_{lo}$	$Z_{lo}$	$Z_{xxlo}$
min	0.41	0.11	-0.9	0.20	-0.023	0.05
max	0.46	0.13	-0.7	0.26	-0.015	0.20

TABLE I  
PARAMETER RANGES FOR THE MODIFIED PARSEC AIRFOIL REPRESENTATION ADOPTED HERE

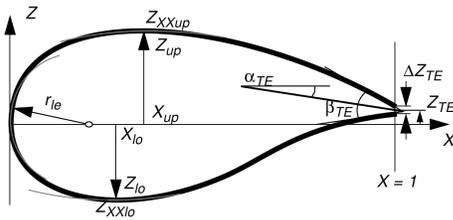


Fig. 3. PARSEC airfoil parameterization

The PARSEC airfoil geometry representation adopted here uses a linear combination of shape functions for defining the upper and lower surfaces. These linear combinations are given by:

$$Z_{upper} = \sum_{n=1}^6 a_n x^{(n-\frac{1}{2})}, Z_{lower} = \sum_{n=1}^6 b_n x^{(n-\frac{1}{2})} \quad (6)$$

In the above equations, the coefficients  $a_n$ , and  $b_n$  are determined as a function of the 12 described geometric parameters, by solving two systems of linear equations.

For solving the above ASO MOPs, we adopted a MOEA called MODE-LD+SS [19] as our search engine. This MOEA makes use of the differential evolution operators and incorporates the concept of local dominance and scalar selection

mechanisms for improving, on the one hand, its convergence rate, and, on the other hand, to distribute solutions along the Pareto front. The details of this MOEA, which has been compared to state-of-the-art MOEAs, can be found in [19].<sup>2</sup> Since we were only interested in evaluating the role of our performance modelling scheme, we only compared the results obtained by our original MODE-LD+SS with respect to those of the version that incorporates surrogate modelling.

### B. Performance Measures

In the context of MOEAs, it is common to compare results on the basis of some performance measures [1]. Next, and for performance assessment purposes, we report the hypervolume ( $Hv$ ) values attained by each of the two MOEAs compared (with and without surrogate modelling). However, we present first the definition of the  $Hv$  measure:

**Hypervolume (Hv):** Given a Pareto approximation set  $PF_{known}$ , and a reference point in objective space  $z_{ref}$ , this performance measure estimates the non-overlapping volume of all the hypercubes formed by the reference point and every vector in the Pareto set approximation. This is mathematically defined as:

$$HV = \{\cup_i vol_i | vec_i \in PF_{known}\} \quad (7)$$

$vec_i$  is a nondominated vector from the Pareto set approximation, and  $vol_i$  is the volume for the hypercube formed by the reference point and the nondominated vector  $vec_i$ . This performance measure is Pareto compliant [20], [21], and is used to assess both convergence and maximum spread of the solutions from the approximation of the Pareto front obtained with a MOEA. High values of this measure indicate that the solutions are closer to the true Pareto front and that they cover a wider extension of it.

### C. Parameters Settings

**Surrogate Model:** As previously indicated, we use a set of surrogate models. For this benchmark, we adopted five RBF models defined by the following kernels:

- Cubic:  $\varphi(r) = r^3$
- Thin Plate Spline:  $\varphi(r) = r^2 \ln(r)$
- Gaussian:  $\varphi(r) = e^{-r^2/(2\sigma^2)}$
- Multiquadratic:  $\varphi(r) = \sqrt{r^2 + \sigma^2}$
- Inverse Multiquadratic:  $\varphi(r) = 1/\sqrt{r^2 + \sigma^2}$

Above,  $r = ||x - c_i||$ ,  $c_i, i = 1, 2, \dots, h$ , is the center for the RBF, and  $h$  is the number of hidden layers. The first two RBF models contain no tuning parameters, while in the other three the  $\sigma$  parameter can be adjusted to improve the model accuracy. All the models are trained with the points stored in the actual database. For each model, the approximated function is defined by:

<sup>2</sup>Is interesting to note that, since this MOEA already uses a weight vector set in its selection mechanism, coupling it to a surrogate model is straightforward by adopting the infilling criterion described in Section IV.

$$\hat{y}_{SM} = \sum_{j=1}^h \omega_j \varphi_j(x) \quad (8)$$

We used a value of  $h = 20$  for the number of hidden layers.  $\varphi(r)$  is the kernel of the hidden layer, and  $\omega_j$  is the weighting coefficient. Since  $h$  is less than the number of training points in the database, we adopted a  $K - means$  clustering technique to obtain the respective center for each hidden layer. The training process for each RBF model, required the determination of the weighting parameters  $\omega_j$  by means of:

$$[\omega_1, \omega_2, \dots, \omega_p]^T = (H^T H)^{-1} H^T Y_S \quad (9)$$

where  $Y_S$  corresponds to the vector of the objective functions for the sample points, and

$$H = \begin{bmatrix} \varphi_1(X_1) & \varphi_2(X_1) & \cdots & \varphi_p(X_1) \\ \varphi_1(X_2) & \varphi_2(X_2) & \cdots & \varphi_p(X_2) \\ \dots & \dots & \vdots & \dots \\ \varphi_1(X_{NTP}) & \varphi_2(X_{NTP}) & \cdots & \varphi_p(X_{NTP}) \end{bmatrix} \quad (10)$$

*MOEA parameters:* We used the following set of parameters.

*MODE-LD+SS without surrogate modelling:*

$F = 0.5$ ,  $CR = 0.5$ ,  $NB = 5$ ,  $GMAX = 20$  and  $NP = 100$  for the bi-objective problems. We only changed  $NP = 120$  for the problems with three objectives. We defined a budget of 2000 objective function evaluations (OFEs). This was based on the OFEs commonly reported in the specialized literature for the problems of our interest, which range from 1000 [11] to 2000 [8].

*MODE-LD+SS with surrogate modelling:*

$F = 0.5$ ,  $CR = 0.5$ ,  $GMAX = 100$  and  $NP = 300$ <sup>3</sup> for both, the bi-objective and the three objective cases. The number of cycles in the surrogate approach was adjusted for performing a total of 2000 OFEs.

*Weight vector index for the infilling criterion:* The index was defined in terms of the current iteration or generation ( $gen$ ) using the following expression:

$$Weight\_Index = (SM_i - 1) \times \frac{NP}{N} + Shift \times (gen - 1) \quad (11)$$

From this expression, we can observe that each surrogate model starts the infilling criterion at a different weight vector, and then the whole set of vectors is swept during the evolutionary process. In this equation,  $NP$  is the population size used for the MOEA when the surrogate model is searched for,  $N$  is the number of surrogate models adopted,  $gen$  is the current generation number and  $Shift$  is a constant used for the sweeping process defined in the infilling criterion. In our experiments, this constant was

<sup>3</sup>A higher population size is adopted, because the evolutionary process is performed on the surrogate model, and, therefore, has a low computational cost.

set to 13 in order to minimize the number of times that a weight vector is selected during the evolutionary process.

*Initial and maximum training points in the database:* We adopted:  $NTP^{init} = 200$  and  $NTP^{max} = 300$  for both, the bi-objective and the three objective problems. Here we propose to set this parameter to approximately twice the number of points corresponding to the population size of the MOEA, when no surrogate model is used. The upper limit aims at reducing the training cost associated to the RBF models, specially for those where the tuning parameters are adjusted for improving their accuracy.

## VI. RESULTS AND DISCUSSION

Table II summarizes the results obtained for the five ASO MOPs adopted and for each of the two MOEAs compared. In this table, the average HV measure, and its standard deviation are obtained from 32 independent runs for each MOP. The Hv measures shown here correspond to a total of 2000 real objective function evaluations. From this table, we can observe that the surrogate model approach consistently obtained better values than the approach not using it, both for the HV mean value and for their standard deviation. According to a Wilcoxon rank-sum statistical test [22] with a significance level of 0.05, for all the ASO-MOPs, the surrogate approach were significantly better.

MOP	MODE-LD+SS		MODE-LD+SS w/s	
	Hv Mean	Std Dev	Hv Mean	Std Dev
ASO-MOP1	5.6593E-04	5.3074E-06	<b>5.8790E-04</b>	4.3753E-06
ASO-MOP2	6.3550E-04	6.2108E-06	<b>6.4942E-04</b>	2.1426E-06
ASO-MOP3	1.6747E-06	7.4601E-08	<b>1.9076E-06</b>	1.7524E-08
ASO-MOP4	4.3639E-04	3.4658E-06	<b>4.4202E-04</b>	2.5353E-06
ASO-MOP5	5.8814E-09	3.0200E-10	<b>7.0975E-09</b>	6.0863E-11

TABLE II  
SUMMARY OF RESULTS

In order to better analyze the impact of the proposed surrogate model approach, in Figures 4 through 8, we present, for all the test cases adopted, the Hv measure convergence plots, and the Pareto fronts approximations obtained after 1000 OFEs. From these convergence plots we can observe that, in general, at the beginning of the evolutionary process, the proposed surrogate model approach, has very good convergence properties. Considering as a first stage that defined up to reaching 500 OFEs, a high improvement of the Hv measure is achieved by the surrogate model approach. In fact, in all cases, except for the ASO-MOP4 problem, the surrogate model approach attains a Hv measure similar to that obtained after performing 2000 OFEs with the MOEA that does not incorporate a surrogate approach. For the case of the ASO-MOP4 test problem, about 50 additional OFEs are required for attaining the same state. Thus, if we consider the Hv measure improvement, we can estimate that our proposed surrogate-based optimization approach can produce savings of about 75% in the number of OFEs performed. This sort of savings can be considered a significant one for the type

of application being analyzed, because it translates into very important CPU time reductions.

Taking a closer look at the convergence plots for ASO-MOP1 and ASO-MOP3, after approximately 350 and 400 OFEs, respectively, we can observe that, prior to these points, the convergence rate clearly shows a tendency to be substantially reduced and probably even to stagnate. However, after these points, the convergence rate suddenly increases. This behavior can be explained in part by the combined action of the exploration/exploitation abilities of the different models incorporated in our approach. A more detailed analysis is, however, required, to confirm our hypothesis. After performing 500 OFEs, our proposed approach continues to show an improvement in the HV measure, but the rate has considerably reduced. Nevertheless, our approach is still able to consistently attain higher values than those achieved by the MOEA without surrogate modelling.

Looking at the Pareto front approximations, compared at an intermediate stage of 1000 OFEs, we can also observe that, in general, our proposed approach is able to, consistently, improve convergence towards the true Pareto front, and to cover a wider area along it. The first condition is clearly exemplified in the ASO-MOP3 problem, while the second condition is clearly seen on the ASO-MOP1 and the ASO-MOP2 problems. These same conditions apply to the problems with three objectives.

## VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented a surrogate-based multiobjective evolutionary optimization approach. The main characteristic of our proposed approach is that it uses not only one surrogate model, but a set of them. Unlike other approaches in which only one model is picked up from a set of trained models, or several models are combined by means of a weighting approach, here, each model is searched for Pareto optimal solutions using a MOEA. From the solutions obtained, some of them are selected to solve the MOP in parallel and in a collaborative manner.

Our proposal was tested on five ASO MOPs. Our results indicated that our proposal helps to speed up convergence and that it can produce a substantial reduction in the number of objective function evaluations performed (reaching savings of up to 75% with respect to the same MOEA not using surrogates).

As part of our future work, we plan to add more combinations of surrogate models to the ones adopted here. It would also be interesting to couple our surrogate-based approach to other MOEAs and, more interestingly, to combinations of them. This would allow a higher degree of variability during the search which could probably lead to further reductions on the number of objective function evaluations performed.

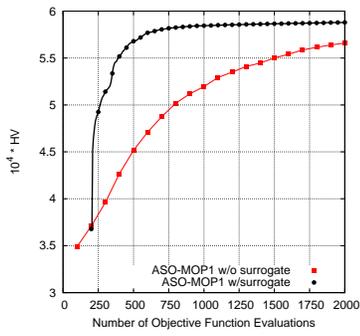
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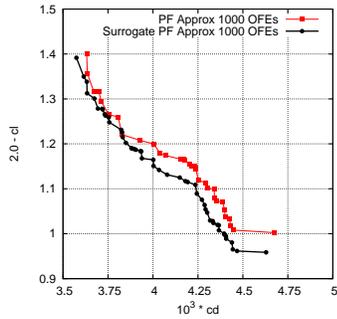
The third author acknowledges support from CONACyT project no. 79809.

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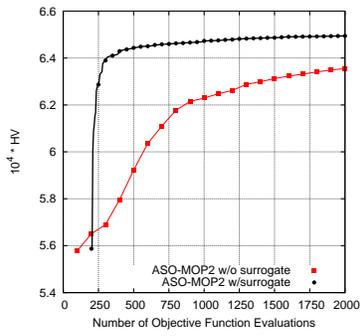


(a).- HV Convergence history

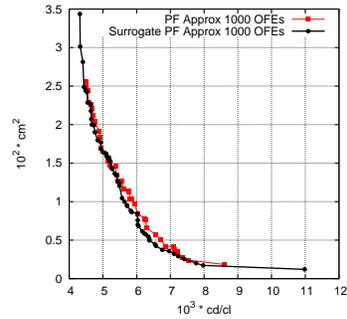


(b).- Pareto front approximation

Fig. 4. ASO-MOP1

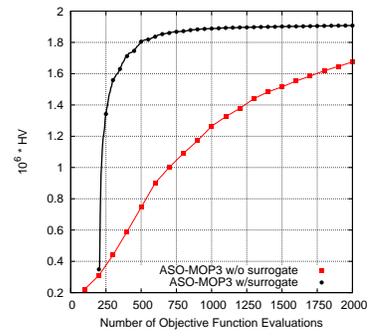


(a).- HV Convergence history

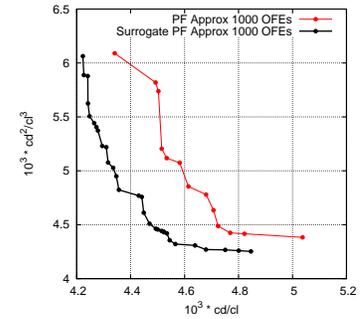


(b).- Pareto front approximation

Fig. 5. ASO-MOP2

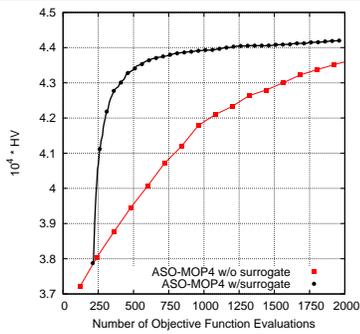


(a).- HV Convergence history

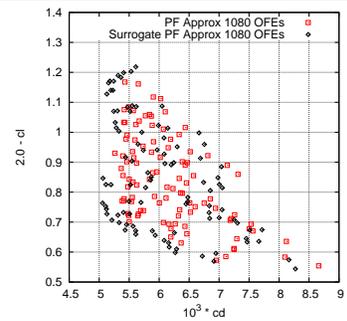


(b).- Pareto front approximation

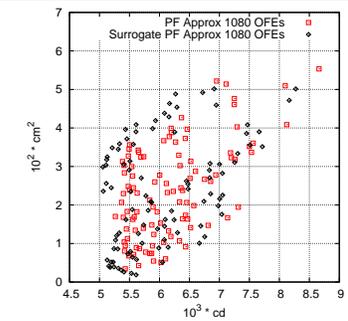
Fig. 6. ASO-MOP3



(a).- HV Convergence history

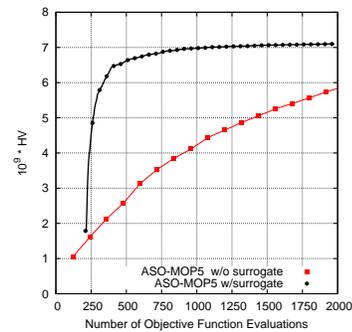


(b).- Pareto front approximation  $f_1$  vs.  $f_2$

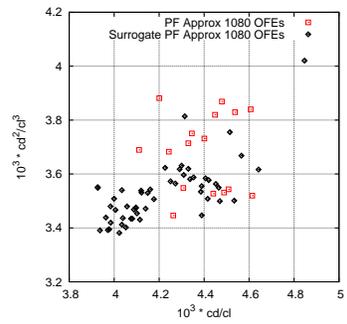


(c).- Pareto front approximation  $f_1$  vs.  $f_3$

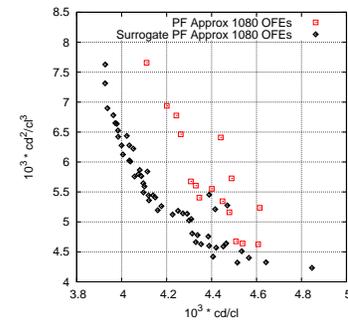
Fig. 7. ASO-MOP4



(a).- HV Convergence history



(b).- Pareto front approximation  $f_1$  vs.  $f_2$



(c).- Pareto front approximation  $f_1$  vs.  $f_3$

Fig. 8. ASO-MOP5