An Empirical Study on the Use of the S-energy Performance Indicator in Mating Restriction Schemes for Multi-Objective Optimizers

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***Abstract*—Mating restrictions have been used to improve the performance of Multi-Objective Evolutionary Algorithms (MOEAs) by altering the way in which parents are selected in the recombination step. Originally proposed for single-objective optimization, mating restrictions have been implemented in different MOEAs obtaining mixed results. However, the role of mating restrictions in diversity management/maintenance and in the proper balance between exploration and exploitation within MOEAs has not been studied in sufficient detail in spite of its evident importance. In this paper, we present an empirical study on the impact of three new mating restrictions based on the s-energy performance indicator. When obtaining each individual's contribution to the total s-energy, we implicitly obtain vicinity information, since a high contribution means that an individual is relatively close to at least some other individual, i.e., it is in a crowded region. Conversely, an individual with a low contribution is in a non-crowded region. Using this information we explore different strategies aiming to improve the diversity of the population during its execution, as well as exploiting the least crowded regions of the objective space. One of the main advantages of our proposal are both its simplicity and its ability to scale up (in objective function space). We evaluate the impact of our proposals by implementing them in NSGA-III and comparing the obtained results with respect to those of the original algorithm. Our experimental results show that the use of mating restrictions does provide improvements in most of the test instances adopted for some of our proposed strategies.**

***Index Terms*—Multiobjective Optimization, Evolutionary Algorithms, Mating Restrictions.**

I. Introduction

The use of evolutionary algorithms to solve multi-objective optimization problems (MOPs) has become increasingly popular in the last 15 years. The so-called Multi-Objective Evolutionary Algorithms (MOEAs) present several advantages, from which the most remarkable are their population-based nature (which allows them to generate several elements of the Pareto optimal set in a single algorithmic execution) and their relatively low need of domain-specific information to operate.

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The role of diversity in MOEAs has been studied by a number of researchers (see for example [1]–[5]) over the years and the main outcome of such studies is the fact that the density estimator (responsible for blocking the selection mechanism of a MOEA with the aim of allowing it to generate different solutions in a single run) has become a standard mechanism in modern MOEAs.

Mating restrictions are discussed in Goldberg's seminal book on genetic algorithms [6] as a mechanism to prevent or minimize the propagation of the so-called "lethals" (offspring with low fitness values). In other words, mating restrictions were originally proposed as a mechanism to bias the way in which individuals mate during recombination. Their goal was to increase the effectiveness and efficiency of a genetic algorithm. Goldberg [6] provides a simple example of mating restrictions based on genotypic similarities and points out that, biologically, mating restrictions are equivalent to geographical isolation or to establishing a barrier that constrains the flow of genes. Thus, mating restrictions are closely related to speciation, which gives rise to new species. So, it should not be surprising that several niching techniques are based on mating restrictions.

Deb and Goldberg [7] proposed mating restrictions in single-objective genetic algorithms, as a way of biasing the selection of the individuals that were to be recombined. Their mechanism was based on the phenotypic distance between the individuals, and in order to find the mating companion of an individual, its mate was selected from individuals lying within a user-defined distance (defined with a parameter called σ_{mate}). By pairing relatively similar parents in objective space, their goal was to prevent, or decrease, the generation of *lethals*, hence improving the performance of the genetic algorithm. Ever since Deb and Goldberg's proposal, different mating restriction schemes have been proposed, exploring the effect of measuring the distance between individuals in both objective and decision space, as well as pairing similar or dissimilar individuals according to a user-defined metric [8]–[11]. There are also a few studies focused on the role of mating restrictions in MOEAs. For example, Zitzler and Thiele [12] as well as

Van Veldhuizen and Lamont [13] found that there was not enough empirical evidence to argue that the use of mating restrictions would either improve or worsen the performance of a MOEA. On the other hand, Ishibuchi studied the use of mating restrictions using either Euclidean or Hamming distances (as well as mating of either similar or dissimilar individuals) in MOPs with two and three objectives, finding that mating restrictions can indeed improve the performance of MOEAs but it is problem-dependent as well as algorithm-dependent [14], [15]. However, there are only a few works on the adoption of mating restrictions in modern MOEAs and/or in MOPs with four or more objectives (the so-called Many-objective Optimization Problems, or MaOPs).

Since mating restrictions determine which individuals are to be paired in the recombination step of a MOEA, they have a direct effect in both the exploration and the exploitation capacity of the algorithm. Hence, if the mating restriction biases the population towards the generation of new different individuals, it can be considered as a diversity maintenance technique.

In this paper, we are interested in this particular feature, and our main contribution is the proposal of new mating mating restriction schemes based on the s-energy performance indicator, as well as their experimental validation.

The remainder of this paper is organized as follows. In Section II, we present some basic concepts from multiobjective optimization. In Section III, a brief review of recent mating restriction schemes for multiobjective optimization is presented. In Section IV, we describe our proposed mating restriction schemes and in Section V we report the experimental results obtained. Finally, in Section VI, we present our conclusions and some possible paths for future work.

II. Basic Concepts

In multiobjective optimization, the aim is to solve problems of the type[1]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \quad (1)$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \quad (2)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \quad (3)$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

A few additional definitions are required to introduce the notion of optimality used in multiobjective optimization:

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

[1]Without loss of generality, we will assume only minimization problems.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

Therefore, our aim is to obtain the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). Note however that in practice, not all the Pareto optimal set is normally desirable or achievable, and decision makers tend to prefer certain types of solutions or regions of the Pareto front [16].

III. Previous Related Work

Mating restrictions were initially based on distance between individuals (either in objective or in decision space). However, there have been different proposals based on clustering in addition to distance, as well as others based on additional measures.

The mating restriction strategy based on survival length [17] (MRSL) is a self-adaptive mechanism which employs clustering to obtain the structure of the population and then assigns different probabilities to individuals in each cluster based on their corresponding survival length. The underlying idea here is that individuals with a high survival length are high-quality individuals and the surrounding area should be exploited, while individuals with a low survival length are newly generated, and therefore, exploration is needed to assess their quality. Experimental results implementing MRSL in five MOEAs of the state-of-the-art show that its use improves results when solving MOPs having two and three objectives.

The decomposition based multiobjective evolutionary algorithm with self-adaptive mating restriction strategy [18] (MOEA/D-MRS) implements another mating restriction based on survival length. However, this approach is specifically designed for a decomposition-based MOEA. It was compared to other MOEA/D variants, obtaining better results in most of the test problems adopted.

The fuzzy c-means clustering-based mating restriction [19] (FMR) is another scheme in which clustering is used to discover the structure of the population. However, solutions have different degrees of membership to each cluster, resulting in the fact that one solution can belong to more than one cluster. This is used to generate a *mating pool* for each individual, which contains the individuals with which they can mate. FCMMO is an MOEA designed around FMR and it utilizes differential evolution to recombine individuals from a given *mating pool* and the hypervolume-based environmental selection mechanism of the SMS-EMOA [20]. FCMMO obtained good experimental results with respect to five other MOEAs in MOPs with two and three objectives.

The Manifold Learning-Based Mating Restriction Strategy (MRML) is another mechanism which aims to improve the performance of a MOEA by calculating the *manifold distances* between individuals, which considers both objective and decision space distances. MRML employs a niche radius R to obtain the neighborhood of a solution based on the previously calculated *manifold distances*. Once the neighborhood of a solution is obtained, it is paired to the closest solution as its mating companion, and the remaining solutions in the neighborhood are discarded. This causes that some solutions cannot be paired due to missing individuals. However, this is solved by recombining such individuals with mutated versions of themselves. MRML was coupled to three MOEAs and was used to solve MOPs with complicated Pareto sets, having two and three objectives, obtaining good results [21].

All of the proposals mentioned above have been validated adopting test problems with complex Pareto sets or considerably difficult features from the GLT [22], UF [23] and WFG [24] test suites. Concerning the MOEAs adopted to compare results, the most commonly used are well-known MOEAs such as NSGA-II, SMS-EMOA, SPEA/R, SPEA2 and MOEA/D-DE. However, in all cases, three is the maximum number of objectives considered, which may be due to the considerable computational cost involved in the use of clustering techniques or to the additional cost of using hypervolume-based selection in approaches such as FCMMO.

In this paper, we propose new mating restriction schemes which are implemented in NSGA-III to evaluate their impact in solving MOPs with two, three and five objectives.

IV. Proposed Strategies

Riesz s-energy (E_s) was proposed by Hardin and Saff [25], and has been used as a performance indicator to measure the uniformity of the distribution of a set of points [26]. Given a set of m-dimensional points X, its s-energy is defined as follows:

$$E_s(X) = \sum_{i \neq j} \frac{1}{\mid \vec{x}_i - \vec{x}_j \mid^s} \tag{4}$$

where $\mid \cdot \mid$ represents the Euclidean distance and $s > 0$ is a fixed parameter. In this work, we use $s = m - 1$ in all cases. This indicator should be minimized in order to obtain a population with a good diversity. Moreover, the individual contribution (C_{si}) of a given point $\vec{x}_i$ may be computed as:

$$C_{si} = E_s(X) - E_s(X \setminus \{\vec{x}_i\}) \tag{5}$$

where $\vec{x}_i \in X$. Since $E_s(X)$ is to be minimized, a high value of C_{si} means that the individual $\vec{x}_i$ is in a "crowded" region, since at least one other individual in the population is relatively close to it. On the other hand, a low C_{si} value means that the individual $\vec{x}_i$ has a better contribution to the global distribution, since it is in a "non-crowded" region. Then, we can rank the population based on their individuals contribution measured in objective space, being the individual with the highest contribution the worst individual, and the individual with the lowest contribution the best one. Given such a rank, we can establish different mating restriction schemes which may favor individuals with the better contributions. In doing so, we aim to maintain diversity during the evolutionary process. This is the core underlying idea of our proposed s-energy based mating restriction (SMR) schemes, which are described below.

A. SMR1: Similar vs. Dissimilar

The first two schemes we propose are the simplest in terms of considering s-energy contributions. To begin with, at each iteration, we obtain the contribution C_{si} for each individual in the population. Next, we propose to pair individuals according to this contribution. In the first strategy, called SMR1_SIM, we force the best individual to be recombined with the second best individual. Then both individuals are no longer considered in this iteration and the process is repeated until all individuals are paired. This is illustrated in Figure 1. On the other hand, the second strategy SMR1_DIS pairs the best individual with the worst individual. Once again, such individuals are no longer considered and the process is repeated with the second best and second worst individuals, and so on, until all parents are obtained, as depicted in Figure 2.

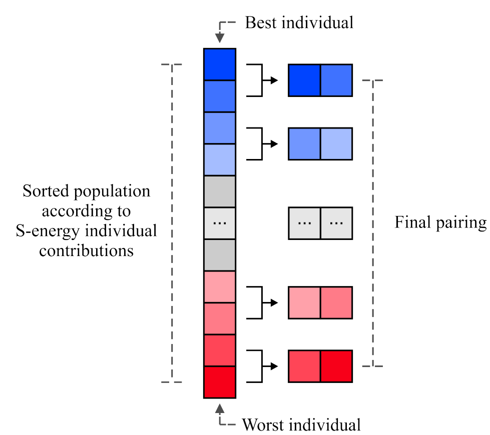

Fig. 1. Example of our proposed SMR1_SIM mechanism.

B. SMR2 and SMR3: Mating Pool and Replacement

The next two mating restriction schemes are variants of SMR1_DIS, in which instead of directly pairing the best individual and the worst individual, a mating pool containing a fixed number of worst solutions is created. The size of the mating pool is set by $\sigma_{pool} > 0$, which is a user-defined parameter. Once the best individual is paired with an individual from the mating pool, they are both no longer considered in the current iteration, and the worst individual gets replaced by the next worst individual available. Next, the second best individual gets paired with an individual from the updated mating pool, and so on until finding all pairs. This mechanism is illustrated in Figure 3.

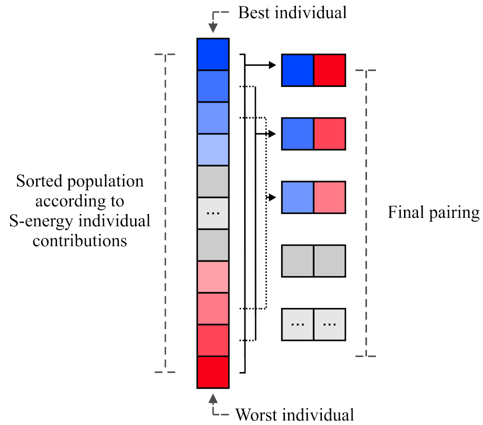

Fig. 2. Example of our proposed SMR1_DIS mechanism.

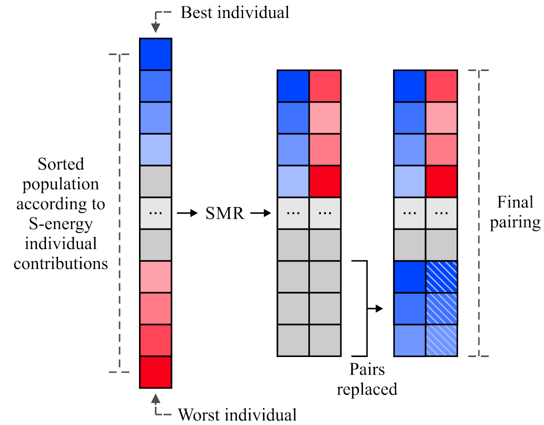

Fig. 4. Example of the SMR2/SMR3 mechanism.

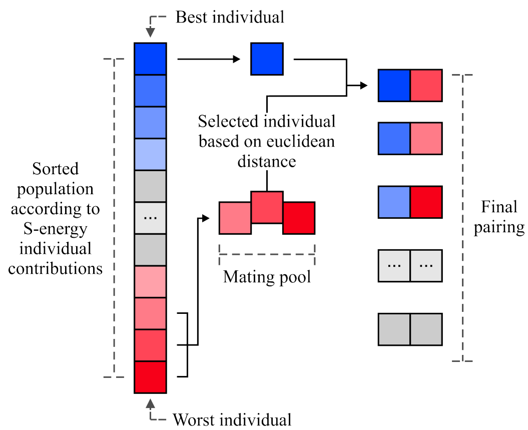

Fig. 3. Example of replacement technique after applying the mating restriction scheme.

We adopted two criteria to select individuals from the mating pool, which yields two other strategies. In both of them the Euclidean distance between the best solution and each of the solutions in the mating pool is obtained in objective space. Then, SMR2 selects the solution with the largest distance, while SMR3 selects the solution with the smallest distance. In both cases the Euclidean distances do not need to be additionally computed, since they were already obtained when computing individual contributions. With both of these strategies we aim to maintain population diversity by generating solutions selecting an individual in a clustered region with an individual in a non-clustered region of the objective space. However, this may not be always the case, since solutions with similar contributions will be paired at some point of the mating restriction strategy. Hence, we propose another feature, which we call replacement. The goal is to replace a percentage of the pairs of parents created with promising new pairs. In order to create such a promising new pair, we select the best individual and we pair it with a newly created individual which is a mutation of itself. Then, we select the second best individual and repeat the process with a mutated version of it, and so on. An example of this mechanism is shown in Figure 4.

With this, we aim to improve the exploitation capability of the algorithm in the vicinity of the individuals with the best contributions. This yields strategies SMR2_R and SMR3_R where replacement of a percentage of the generated pairs is done using the best individuals and mutated versions of themselves. In this work, we used $\frac{1}{6}$ of the population as the percentage to be replaced.

V. Experimental Results

In order to evaluate the impact of our proposed mating restrictions we implemented them in NSGA-III [27] and compared the results obtained results with and without the mating restriction strategy. We adopted the Deb-Thiele-Laumanns-Zitzler (DTLZ) [28] and Walking-Fish-Group (WFG) [24] test suites, since they contain scalable MOPs with solutions that include Pareto sets with different geometrical features.

In order to assess the quality of approximation sets obtained we adopted the hypervolume (HV) performance indicator [29] as well as the inverted generational distance (IGD) [30]. Additionally, we also used s-energy [25] in order to specifically measure the distribution of approximation sets. Even though s-energy is the core of all of our mating restrictions, we only employed it as a quality criterion to rank individuals, and not as a performance indicator which should be improved. Hence, s-energy values are only used for comparison purposes, since its direct minimization is not part of our proposal. In all the tables shown here, the best value for each performance indicator is written in **boldface**, and the cells in which the mating restriction outperformed the original algorithm are shown in gray.

The Hypervolume, IGD and s-energy values obtained when comparing SMR1_SIM and SMR1_DIS in the test problems previously indicated, are presented in Table I. SMR1_SIM outperforms the original NSGA-III in 27 out of 48 test problems when comparing hypervolume values, in 26 problems when using IGD and in 18 test problems when using s-energy. It can be seen that SMR1_SIM shows a clear advantage in the WFG test problems with 2 and 5 objectives. On the other hand, SMR1_DIS only outperforms 19 of the HV values, 22 of the IGD values, and 26 of the s-energy values, being less consistent than SMR_SIM.

TABLE I

COMPARISON OF THE AVERAGE HV, IGD AND S-ENERGY VALUES OBTAINED WHEN USING SMR1_SIM AND SMR1_DIS.

Problem	Number of objectives	Hypervolume			IGD			S-energy		
		NSGA-III	NSGA-III + SMR1_SIM	NSGA-III + SMR1_DIS	NSGA-III	NSGA-III + SMR1_SIM	NSGA-III + SMR1_DIS	NSGA-III	NSGA-III + SMR1_SIM	NSGA-III + SMR1_DIS
DTLZ1		**2.1237E+00**	2.1234E+00	2.1237E+00	**1.7840E-03**	1.8225E-03	1.7841E-03	1.1725E+05	1.1797E+05	**1.1725E+05**
DTLZ2		3.2101E+00	**3.2101E+00**	3.2101E+00	3.9625E-03	**3.9625E-03**	3.9626E-03	5.3391E+04	**5.3391E+04**	5.3392E+04
DTLZ3		3.2098E+00	3.2096E+00	**3.2098E+00**	**3.9867E-03**	4.0062E-03	3.9906E-03	**5.3386E+04**	5.3404E+04	5.3425E+04
DTLZ4	2	**2.8874E+00**	2.8068E+00	2.8874E+00	**2.0079E-01**	2.4999E-01	2.0079E-01	3.9153E+04	**3.5594E+04**	3.9153E+04
DTLZ5		3.2101E+00	**3.2101E+00**	3.2101E+00	3.9625E-03	**3.9625E-03**	3.9626E-03	5.3391E+04	**5.3391E+04**	5.3392E+04
DTLZ6		3.0542E+00	**3.0700E+00**	3.0676E+00	9.2585E-02	**8.4144E-02**	8.5291E-02	5.4260E+04	**4.9077E+04**	4.9814E+04
DTLZ7		**4.4173E+00**	4.4172E+00	4.4173E+00	**5.1851E-03**	5.2685E-03	5.1885E-03	**7.3309E+04**	3.1605E+05	7.2876E+05
DTLZ1		3.3477E+00	**3.3489E+00**	3.3481E+00	2.2906E-02	**1.9120E-02**	2.2479E-02	1.0500E+10	**2.9185E+06**	2.1935E+09
DTLZ2		**7.4184E+00**	7.4184E+00	7.4184E+00	**4.9314E-02**	4.9330E-02	4.9325E-02	**9.1884E+04**	8.3415E+05	2.8241E+05
DTLZ3		7.4116E+00	7.4167E+00	**7.4177E+00**	5.6253E-02	5.0024E-02	**4.9583E-02**	1.6166E+09	**2.8146E+05**	1.2600E+06
DTLZ4	3	**7.0466E+00**	6.7855E+00	7.0335E+00	2.2902E-01	3.0514E-01	**2.0993E-01**	3.0716E+11	2.7960E+11	**9.9681E+10**
DTLZ5		**4.0043E+00**	3.8615E+00	3.9751E+00	6.5059E-02	9.3704E-02	**5.4936E-02**	8.1297E+11	1.8147E+12	**2.4368E+11**
DTLZ6		3.9910E+00	4.0051E+00	**4.0196E+00**	1.0158E-01	9.2757E-02	**8.8402E-02**	**1.1581E+11**	8.2169E+11	1.5469E+11
DTLZ7		1.3202E+01	1.3291E+01	**1.3304E+01**	7.8863E-02	7.0082E-02	**6.8713E-02**	8.0305E+07	2.8752E+09	**3.8662E+06**
DTLZ1		**7.5927E+00**	7.5927E+00	7.5927E+00	**5.0423E-02**	5.2351E-02	5.0776E-02	**2.2272E+10**	2.5017E+11	1.1585E+11
DTLZ2		3.1698E+01	3.1698E+01	**3.1698E+01**	1.5539E-01	1.5540E-01	**1.5537E-01**	**7.7099E+09**	1.7439E+10	8.7488E+09
DTLZ3		3.1694E+01	3.1688E+01	**3.1694E+01**	1.5825E-01	1.6315E-01	**1.5722E-01**	1.1507E+11	**4.4104E+10**	9.6380E+10
DTLZ4	5	3.1679E+01	3.1659E+01	**3.1698E+01**	1.6342E-01	1.7193E-01	**1.5534E-01**	1.4004E+10	2.6691E+11	**1.3712E+10**
DTLZ5		**5.9019E+00**	5.8276E+00	5.8168E+00	2.3194E-01	2.3313E-01	**2.1629E-01**	**2.3861E+11**	2.5894E+12	4.2134E+11
DTLZ6		1.8920E+00	**2.6810E+00**	6.0006E-01	9.3461E-01	**7.7449E-01**	1.2100E+00	4.5468E+11	7.9833E+11	**2.8968E+11**
DTLZ7		7.7036E+01	7.6447E+01	**7.7458E+01**	**2.7009E-01**	2.8704E-01	2.7255E-01	5.3521E+10	1.8419E+11	**1.4913E+10**
WFG1		3.9940E+00	**4.2139E+00**	4.0734E+00	1.4331E+00	**1.3816E+00**	1.4195E+00	**1.2784E+05**	1.5188E+05	1.6825E+05
WFG2		9.2570E+00	**9.3155E+00**	9.2557E+00	6.5750E-01	**6.4416E-01**	6.5751E-01	5.9184E+04	2.1309E+05	**5.8157E+04**
WFG3		1.0437E+01	1.0442E+01	**1.0463E+01**	4.3466E-02	3.9505E-02	**3.9135E-02**	2.9325E+04	**2.6734E+04**	2.6829E+04
WFG4		8.2336E+00	**8.4349E+00**	8.1320E+00	2.8252E-02	**1.7609E-02**	3.5015E-02	**2.2522E+04**	2.3392E+04	2.2684E+04
WFG5	2	7.9344E+00	**7.9741E+00**	7.8600E+00	7.7568E-02	**7.6630E-02**	8.2881E-02	2.2769E+04	2.4107E+04	**2.2023E+04**
WFG6		7.9733E+00	**8.1083E+00**	7.9903E+00	6.8255E-02	**5.9293E-02**	6.6291E-02	2.4668E+04	2.4496E+04	**2.1931E+04**
WFG7		6.6706E+00	**7.1797E+00**	6.6927E+00	2.9508E-01	**1.7524E-01**	2.8986E-01	4.2732E+04	3.9234E+04	**3.6990E+04**
WFG8		7.3725E+00	**7.7891E+00**	7.2931E+00	1.4150E-01	**1.0197E-01**	1.5346E-01	**4.2315E+04**	1.0935E+06	4.6764E+04
WFG9		**8.1696E+00**	8.1685E+00	8.1564E+00	**3.5258E-02**	3.5311E-02	3.6042E-02	2.2445E+04	**2.1248E+04**	2.1764E+04
WFG1		5.4149E+01	**5.5575E+01**	5.5332E+01	1.1700E+00	**1.1456E+00**	1.1514E+00	4.1152E+08	1.4598E+09	**8.0898E+06**
WFG2		**9.6392E+01**	9.5886E+01	9.5399E+01	**2.4560E-01**	2.6005E-01	2.6954E-01	**1.0067E+05**	1.9316E+06	2.3302E+05
WFG3		**2.4037E+01**	2.3975E+01	2.4018E+01	**9.3331E-02**	9.8401E-02	9.3440E-02	**7.1353E+08**	7.4582E+10	9.6926E+09
WFG4		7.6820E+01	**7.6857E+01**	7.6822E+01	2.0001E-01	1.9994E-01	**1.9993E-01**	7.2705E+03	5.8622E+03	**5.1335E+03**
WFG5	3	7.3458E+01	7.3220E+01	**7.3486E+01**	2.1384E-01	2.1422E-01	**2.1292E-01**	5.6060E+03	4.8130E+05	**5.2243E+03**
WFG6		**7.4081E+01**	7.3964E+01	7.4000E+01	**2.0938E-01**	2.1008E-01	2.0995E-01	6.9892E+04	**5.3703E+03**	3.8665E+04
WFG7		**7.7045E+01**	7.7027E+01	7.7035E+01	**2.0004E-01**	2.0012E-01	2.0004E-01	2.8209E+04	**1.0277E+04**	1.0823E+05
WFG8		7.0872E+01	**7.1071E+01**	7.0825E+01	2.5035E-01	**2.4753E-01**	2.5020E-01	**1.2580E+04**	3.1617E+05	5.2681E+04
WFG9		7.0274E+01	6.9398E+01	**7.0321E+01**	**2.2365E-01**	2.3352E-01	2.2455E-01	1.9415E+08	**4.4917E+06**	2.3654E+07
WFG1		4.0787E+03	**4.2274E+03**	4.0783E+03	1.8441E+00	**1.7953E+00**	1.8308E+00	**7.1824E+10**	9.4357E+10	1.6422E+11
WFG2		9.8846E+03	**1.0155E+04**	1.0129E+04	5.6849E-01	**4.5785E-01**	4.6164E-01	**6.8679E+08**	1.9635E+10	8.0177E+09
WFG3		8.0982E+01	**8.5504E+01**	8.0353E+01	5.2549E-01	**4.7552E-01**	5.2642E-01	**6.9978E+10**	6.5755E+11	7.7536E+10
WFG4		8.8925E+03	**8.9640E+03**	8.8805E+03	9.1408E-01	**9.1257E-01**	9.1469E-01	**2.7649E+03**	1.0109E+04	7.4131E+06
WFG5	5	8.6746E+03	**8.6909E+03**	8.6628E+03	9.2626E-01	**9.2389E-01**	9.2761E-01	3.0457E+08	1.1009E+06	**4.7232E+05**
WFG6		8.8660E+03	**8.8872E+03**	8.8594E+03	**9.1416E-01**	9.1438E-01	9.1481E-01	1.4504E+07	**1.0850E+05**	1.2308E+07
WFG7		9.1345E+03	**9.1359E+03**	9.1303E+03	9.0612E-01	**9.0594E-01**	9.0643E-01	**1.1485E+07**	8.1259E+07	4.6796E+07
WFG8		**8.4412E+03**	8.3933E+03	8.4393E+03	9.5154E-01	9.5415E-01	**9.5117E-01**	1.2729E+07	3.2473E+07	**1.1793E+07**
WFG9		**8.0920E+03**	8.0616E+03	8.0749E+03	9.8101E-01	9.8128E-01	**9.8011E-01**	1.5351E+03	1.7351E+05	**1.4390E+03**

In Table II we present the results of comparing SMR2 with and without the replacement feature. SMR2 without replacement produces improvements in 16 ouf of 48 problems regarding the hypervolume, and it produces improvements in 20 problems regarding both IGD and s-energy indicators. SMR2 with replacement improves the results obtained in 21 out of the 48 problems using hypervolume and IGD, while it improves 27 of the results obtained with s-energy. The latter mating restriction obtains particularly good results in the DTLZ problems with 3 objectives as well as in the WFG test problems with 2 objectives. However, both of these fail to obtain good results in problems with 5 objectives, which is an indicator of their poor scaling capability.

The results obtained when comparing SMR3 with and without replacement are shown in Table III. In this case, SMR3 without replacement produces improvements in 26 out of 48 test instances with respect to the hypervolume, and it produces improvements in 22 problems with respect to both IGD and s-energy indicators. This approach was able to produce good results in the WFG test problems with 2 and 5 objectives. In contrast, its version with replacement achieves an improvement in 27 problems with respect to the three indicators used.

In addition to the above results, we decided to evaluate the performance of the two best performing strategies (SMR2_R and SMR3_R) with a different mating pool size ($\sigma_{pool} = 5$). The results obtained with respect to the hypervolume, inverted generational distance and s-energy are shown in Table IV. Concerning SMR2_R, it outperformed the original NSGA-III in 20 out of the 48 test problems with respect to the hypervolume, in 21 problems with respect to the IGD indicator, and in 27 problems with respect to the s-energy. This approach obtained good results in the DTLZ test problems with 3 objectives and in the WFG test problems with 2 objectives. However, it performed poorly in both the DTLZ and the WFG test problems with 5 objectives. On the other hand, SMR3_R improved the results obtained in 29 problems with respect to the hypervolume, in 27 problems with respect to the IGD indicator, and in 21 problems with respect to the s-energy. This approach obtained particularly good results in the DTLZ test problems with 3 objectives, and in the WFG test problems with 2 and 5 objectives.

VI. CONCLUSIONS

In this paper, we have presented three different mating restriction strategies as well as an experimental evaluation of their implementation in the well-known NSGA-III. From the results obtained we can state that the mating restrictions tested in our study do have an impact on the algorithms' final convergence. However, no single mating restriction was able to

TABLE II
COMPARISON OF THE AVERAGE HV, IGD AND S-ENERGY VALUES OBTAINED WHEN USING SMR2 WITH AND WITHOUT REPLACEMENT, WITH $\sigma_{pool} = 3$.

Problem	Number of objectives	Hypervolume			IGD			S-energy		
		NSGA-III	NSGA-III + SMR2	NSGA-III + SMR2_R	NSGA-III	NSGA-III + SMR2	NSGA-III + SMR2_R	NSGA-III	NSGA-III + SMR2	NSGA-III + SMR2_R
DTLZ1		**2.1237E+00**	2.1237E+00	2.1237E+00	**1.7840E-03**	1.7841E-03	1.7841E-03	1.1725E+05	**1.1725E+05**	1.1725E+05
DTLZ2		3.2101E+00	**3.2101E+00**	3.2101E+00	**3.9625E-03**	3.9626E-03	3.9627E-03	5.3391E+04	**5.3391E+04**	5.3392E+04
DTLZ3		3.2098E+00	**3.2098E+00**	3.2097E+00	3.9867E-03	**3.9862E-03**	3.9996E-03	5.3386E+04	**5.3382E+04**	5.3385E+04
DTLZ4	2	2.8874E+00	2.8471E+00	**3.0488E+00**	2.0079E-01	2.2539E-01	**1.0238E-01**	3.9153E+04	**3.7374E+04**	4.6272E+04
DTLZ5		3.2101E+00	**3.2101E+00**	3.2101E+00	**3.9625E-03**	3.9626E-03	3.9627E-03	5.3391E+04	**5.3391E+04**	5.3392E+04
DTLZ6		3.0542E+00	3.0731E+00	**3.0814E+00**	9.2585E-02	8.2071E-02	**7.7864E-02**	5.4260E+04	**4.9079E+04**	4.9343E+04
DTLZ7		**4.4173E+00**	4.4173E+00	4.3635E+00	5.1851E-03	**5.1793E-03**	1.9733E-02	7.3309E+04	7.5555E+04	**6.8418E+04**
DTLZ1		3.3477E+00	3.3489E+00	**3.3491E+00**	2.2906E-02	1.9796E-02	**1.8696E-02**	1.0500E+10	2.5160E+10	**9.4944E+05**
DTLZ2		**7.4184E+00**	7.4183E+00	7.4180E+00	**4.9314E-02**	4.9337E-02	4.9358E-02	**9.1884E+04**	1.9561E+06	2.1869E+05
DTLZ3		7.4116E+00	**7.4179E+00**	7.4178E+00	5.6253E-02	4.9430E-02	**4.9350E-02**	1.6166E+09	9.4484E+04	**7.2603E+04**
DTLZ4	3	7.0466E+00	6.9675E+00	**7.3173E+00**	2.2902E-01	2.4253E-01	**9.8324E-02**	3.0716E+11	4.1412E+11	**2.2249E+11**
DTLZ5		4.0043E+00	3.9703E+00	**4.1862E+00**	6.5059E-02	4.7812E-02	**1.4199E-02**	8.1297E+11	4.0529E+11	**3.4682E+11**
DTLZ6		3.9910E+00	3.9865E+00	**4.0079E+00**	1.0158E-01	1.0413E-01	**9.2818E-02**	**1.1581E+11**	2.7441E+11	1.8917E+11
DTLZ7		1.3202E+01	1.2929E+01	**1.3301E+01**	7.8863E-02	1.0705E-01	**7.0273E-02**	8.0305E+07	7.0115E+06	**1.4823E+06**
DTLZ1		**7.5927E+00**	7.5927E+00	7.5927E+00	**5.0423E-02**	5.0651E-02	5.2614E-02	**2.2272E+10**	6.8670E+10	2.3357E+11
DTLZ2		**3.1698E+01**	3.1697E+01	3.1694E+01	1.5539E-01	**1.5531E-01**	1.5562E-01	7.7099E+09	2.0486E+10	**1.6918E+08**
DTLZ3		3.1694E+01	**3.1694E+01**	3.1691E+01	1.5825E-01	**1.5545E-01**	1.5625E-01	1.1507E+11	**5.0298E+10**	8.9069E+10
DTLZ4	5	3.1679E+01	**3.1698E+01**	3.1698E+01	1.6342E-01	**1.5534E-01**	1.5536E-01	**1.4004E+10**	3.3714E+10	4.1948E+10
DTLZ5		**5.9019E+00**	5.8751E+00	5.6463E+00	2.3194E-01	**2.2842E-01**	2.3629E-01	**2.3861E+11**	5.1000E+11	3.4009E+11
DTLZ6		**1.8920E+00**	2.5272E-01	0.0000E+00	**9.3461E-01**	1.3792E+00	2.4709E+00	4.5468E+11	1.4314E+11	**2.9434E+09**
DTLZ7		7.7036E+01	**7.7616E+01**	6.9739E+01	**2.7009E-01**	2.7200E-01	3.4618E-01	5.3521E+10	5.7123E+10	**3.9399E+10**
WFG1		**3.9940E+00**	3.6979E+00	3.8801E+00	**1.4331E+00**	1.4998E+00	1.4624E+00	1.2784E+05	1.5947E+05	**1.0995E+05**
WFG2		9.2570E+00	9.2427E+00	**9.2880E+00**	6.5750E-01	6.5785E-01	**6.5703E-01**	**5.9184E+04**	7.3249E+04	6.9966E+04
WFG3		1.0437E+01	1.0111E+01	**1.0849E+01**	4.3466E-02	8.0586E-02	**1.2645E-02**	2.9325E+04	3.1228E+04	**2.0534E+04**
WFG4		8.2336E+00	7.8051E+00	**8.2952E+00**	2.8252E-02	6.9665E-02	**2.7971E-02**	**2.2522E+04**	2.4655E+04	2.3629E+04
WFG5	2	**7.9344E+00**	7.6572E+00	7.8556E+00	**7.7568E-02**	1.0236E-01	8.9524E-02	2.2769E+04	3.0152E+04	**2.2608E+04**
WFG6		7.9733E+00	7.9159E+00	**8.3114E+00**	6.8255E-02	6.7294E-02	**3.1370E-02**	2.4668E+04	2.2878E+04	**2.1583E+04**
WFG7		6.6706E+00	6.4966E+00	**7.6059E+00**	2.9508E-01	3.4195E-01	**9.7001E-02**	4.2732E+04	3.9256E+04	**3.2833E+04**
WFG8		7.3725E+00	6.4317E+00	**7.9924E+00**	1.4150E-01	3.3014E-01	**7.3920E-02**	**4.2315E+04**	9.5149E+04	1.1297E+05
WFG9		8.1696E+00	8.1725E+00	**8.1892E+00**	3.5258E-02	3.4784E-02	**3.3108E-02**	2.2445E+04	2.4766E+04	**2.0915E+04**
WFG1		5.4149E+01	5.4087E+01	**5.5611E+01**	1.1700E+00	1.1769E+00	**1.1434E+00**	4.1152E+08	2.6679E+09	**2.4534E+07**
WFG2		9.6392E+01	9.6983E+01	**9.7458E+01**	2.4560E-01	2.3164E-01	**2.2091E-01**	1.0067E+05	**5.0129E+04**	6.4537E+06
WFG3		2.4037E+01	**2.4047E+01**	2.3562E+01	9.3331E-02	**8.7034E-02**	1.2045E-01	**7.1353E+08**	3.7569E+10	4.6382E+10
WFG4		**7.6820E+01**	7.6790E+01	7.6730E+01	2.0001E-01	**1.9990E-01**	2.0005E-01	7.2705E+03	**5.3568E+03**	6.4978E+03
WFG5	3	7.3458E+01	7.3513E+01	**7.3529E+01**	2.1384E-01	**2.1271E-01**	2.1302E-01	**5.6060E+03**	2.8725E+05	1.4801E+05
WFG6		7.4081E+01	7.4086E+01	**7.4172E+01**	2.0938E-01	2.0944E-01	**2.0916E-01**	6.9892E+04	**6.1958E+03**	2.2903E+05
WFG7		**7.7045E+01**	7.7022E+01	7.7005E+01	**2.0004E-01**	2.0008E-01	2.0012E-01	2.8209E+04	2.4284E+04	**8.4907E+03**
WFG8		**7.0872E+01**	7.0336E+01	7.0272E+01	**2.5035E-01**	2.5935E-01	2.6112E-01	**1.2580E+04**	2.5467E+05	2.9949E+08
WFG9		**7.0274E+01**	6.9864E+01	6.9135E+01	**2.2365E-01**	2.2751E-01	2.3545E-01	1.9415E+08	**1.1912E+04**	3.0530E+04
WFG1		**4.0787E+03**	3.9844E+03	4.0727E+03	**1.8441E+00**	1.8623E+00	1.8552E+00	7.1824E+10	**3.9802E+09**	5.8272E+10
WFG2		9.8846E+03	9.9906E+03	**1.0264E+04**	5.6849E-01	5.1831E-01	**4.0606E-01**	6.8679E+08	2.0875E+09	**3.1197E+08**
WFG3		8.0982E+01	**8.2007E+01**	7.5577E+01	5.2549E-01	**4.9372E-01**	5.7865E-01	6.9978E+10	8.1739E+10	**1.1726E+10**
WFG4		**8.8925E+03**	8.7860E+03	8.7304E+03	**9.1408E-01**	9.1886E-01	9.2261E-01	**2.7649E+03**	4.1867E+04	3.5352E+05
WFG5	5	**8.6746E+03**	8.6401E+03	8.6047E+03	9.2626E-01	**9.2503E-01**	9.2816E-01	3.0457E+08	**6.2629E+04**	1.1084E+08
WFG6		**8.8660E+03**	8.8277E+03	8.8222E+03	**9.1416E-01**	9.1648E-01	9.1647E-01	**1.4504E+07**	6.0085E+07	3.3354E+07
WFG7		**9.1345E+03**	9.0887E+03	9.0657E+03	**9.0612E-01**	9.0672E-01	9.0784E-01	**1.1485E+07**	4.2095E+08	9.5148E+07
WFG8		**8.4412E+03**	8.3391E+03	8.3235E+03	**9.5154E-01**	9.5899E-01	9.6068E-01	1.2729E+07	4.2775E+07	**2.9814E+03**
WFG9		**8.0920E+03**	8.0770E+03	8.0551E+03	9.8101E-01	**9.8052E-01**	9.8610E-01	**1.5351E+03**	6.9474E+03	3.1118E+03

improve all the test problem instances adopted. The best overall performing strategy was SMR3_R, which outperformed the original algorithm in 60.4% of the test problems adopted considering the hypervolume indicator.

Regarding similar vs. dissimilar contributions pairing, our experimental results indicated that the combination of individuals with a similar contribution was slightly better than the alternative. Besides, the replacement strategy proved to be very useful in both SMR2 and SMR3, since it not only increased the number of problems improved, but it also produced higher hypervolume values in most of the test instances. This may be a direct consequence of exploiting the area surrounding the individuals with the best s-energy contribution (i.e., the individuals in the least crowded regions). Moreover, the change of mating pool size caused a good performance of SMR3_R in MOPs with 5 objectives. However, it caused a slight worsening in MOPs with 2 objectives. This is more evidence that the mating restriction effect seems to be problem-dependent and scale-dependent.

Because of the above reasons, we consider that the best way of continuing this work is to propose a mating restriction meta-strategy, which is able to employ different strategies depending on information obtained from the particular problem being solved. Alternatively, an adaptive strategy could be used to change the mating pool size depending on the information obtained during the execution of the algorithm.

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TABLE III
COMPARISON OF THE AVERAGE HV, IGD AND S-ENERGY VALUES OBTAINED WHEN USING SMR3 WITH AND WITHOUT REPLACEMENT, WITH $\sigma_{pool} = 3$.

Problem	Number of objectives	Hypervolume			IGD			S-energy		
		NSGA-III	NSGA-III + SMR3	NSGA-III + SMR3_R	NSGA-III	NSGA-III + SMR3	NSGA-III + SMR3_R	NSGA-III	NSGA-III + SMR3	NSGA-III + SMR3_R
DTLZ1		2.1237E+00	2.1237E+00	**2.1237E+00**	**1.7840E-03**	1.7842E-03	1.7842E-03	1.1725E+05	**1.1725E+05**	1.1725E+05
DTLZ2		**3.2101E+00**	3.2101E+00	3.2101E+00	**3.9625E-03**	3.9627E-03	3.9626E-03	5.3391E+04	5.3392E+04	**5.3391E+04**
DTLZ3		**3.2098E+00**	3.2097E+00	3.2096E+00	**3.9867E-03**	3.9960E-03	4.0205E-03	**5.3386E+04**	5.3587E+04	5.3423E+04
DTLZ4	2	2.8874E+00	2.8874E+00	**3.0084E+00**	2.0079E-01	2.0079E-01	**1.2698E-01**	3.9153E+04	**3.9153E+04**	4.4493E+04
DTLZ5		**3.2101E+00**	3.2101E+00	3.2101E+00	**3.9625E-03**	3.9627E-03	3.9626E-03	5.3391E+04	5.3392E+04	**5.3391E+04**
DTLZ6		3.0542E+00	3.0491E+00	**3.0747E+00**	9.2585E-02	9.5855E-02	**8.1569E-02**	5.4260E+04	1.6700E+05	**4.9265E+04**
DTLZ7		**4.4173E+00**	4.4172E+00	4.4173E+00	**5.1851E-03**	5.2280E-03	5.1947E-03	**7.3309E+04**	1.6963E+05	7.4962E+04
DTLZ1		3.3477E+00	3.3491E+00	**3.3491E+00**	2.2906E-02	1.8829E-02	**1.8702E-02**	1.0500E+10	1.1762E+08	**1.6654E+07**
DTLZ2		**7.4184E+00**	7.4184E+00	7.4176E+00	**4.9314E-02**	4.9337E-02	4.9411E-02	**9.1884E+04**	8.6253E+05	1.0465E+05
DTLZ3		7.4116E+00	**7.4180E+00**	7.4179E+00	5.6253E-02	**4.9325E-02**	4.9333E-02	1.6166E+09	**8.6616E+04**	3.2032E+06
DTLZ4	3	7.0466E+00	7.1819E+00	**7.3172E+00**	2.2902E-01	1.6368E-01	**9.8335E-02**	3.0716E+11	9.9760E+10	**5.0508E+10**
DTLZ5		4.0043E+00	3.8906E+00	**4.1683E+00**	6.5059E-02	8.2023E-02	**1.6986E-02**	8.1297E+11	**5.2806E+11**	5.3297E+11
DTLZ6		**3.9910E+00**	3.9880E+00	3.9852E+00	**1.0158E-01**	1.0245E-01	1.0305E-01	**1.1581E+11**	2.9867E+11	1.8305E+11
DTLZ7		1.3202E+01	1.3196E+01	**1.3277E+01**	7.8863E-02	7.9276E-02	**7.1865E-02**	8.0305E+07	1.8347E+07	**1.4284E+07**
DTLZ1		**7.5927E+00**	7.5927E+00	7.5927E+00	**5.0423E-02**	5.1071E-02	5.2628E-02	**2.2272E+10**	1.2492E+11	1.8359E+11
DTLZ2		3.1698E+01	**3.1698E+01**	3.1696E+01	**1.5539E-01**	1.5539E-01	1.5556E-01	7.7099E+09	**2.2677E+09**	9.7765E+09
DTLZ3		3.1694E+01	**3.1695E+01**	3.1693E+01	1.5825E-01	**1.5678E-01**	1.5798E-01	1.1507E+11	**2.8331E+10**	1.1363E+11
DTLZ4	5	3.1679E+01	3.1660E+01	**3.1698E+01**	1.6342E-01	1.7146E-01	**1.5540E-01**	1.4004E+10	1.0865E+11	**7.4339E+09**
DTLZ5		5.9019E+00	5.9270E+00	**5.9463E+00**	2.3194E-01	**2.1696E-01**	2.1813E-01	**2.3861E+11**	2.6928E+11	4.8276E+11
DTLZ6		**1.8920E+00**	1.6630E+00	2.3333E-06	**9.3461E-01**	9.7971E-01	2.0990E+00	4.5468E+11	4.3651E+11	**5.1400E+10**
DTLZ7		7.7036E+01	**7.8003E+01**	7.1112E+01	**2.7009E-01**	2.7161E-01	3.4077E-01	5.3521E+10	9.0555E+10	**3.4682E+10**
WFG1		3.9940E+00	4.4503E+00	**4.5165E+00**	1.4331E+00	1.3207E+00	**1.3035E+00**	1.2784E+05	**8.7193E+04**	1.1013E+05
WFG2		9.2570E+00	9.2665E+00	**9.2876E+00**	6.5750E-01	6.5745E-01	**6.5702E-01**	**5.9184E+04**	6.8224E+04	6.9475E+04
WFG3		1.0437E+01	1.0594E+01	**1.0860E+01**	4.3466E-02	2.7563E-02	**1.2276E-02**	2.9325E+04	2.5815E+04	**2.1805E+04**
WFG4		8.2336E+00	8.2341E+00	**8.3776E+00**	2.8252E-02	2.7391E-02	**2.2568E-02**	2.2522E+04	2.1329E+04	**2.0878E+04**
WFG5	2	**7.9344E+00**	7.8674E+00	7.8262E+00	**7.7568E-02**	8.3617E-02	9.6234E-02	2.2769E+04	2.3775E+04	**2.1972E+04**
WFG6		7.9733E+00	7.9980E+00	**8.2086E+00**	6.8255E-02	6.4347E-02	**4.5157E-02**	2.4668E+04	2.4087E+04	**2.1324E+04**
WFG7		6.6706E+00	6.8319E+00	**7.6937E+00**	2.9508E-01	2.5382E-01	**8.5066E-02**	4.2732E+04	4.8046E+04	**2.7849E+04**
WFG8		7.3725E+00	7.4536E+00	**8.0740E+00**	1.4150E-01	1.2743E-01	**6.7437E-02**	**4.2315E+04**	1.2405E+05	7.1553E+04
WFG9		**8.1696E+00**	8.1420E+00	8.1663E+00	3.5258E-02	3.6425E-02	**3.3411E-02**	2.2445E+04	**2.0999E+04**	2.2023E+04
WFG1		5.4149E+01	5.4513E+01	**5.5103E+01**	1.1700E+00	**1.1577E+00**	1.1593E+00	4.1152E+08	**5.0280E+07**	6.8727E+07
WFG2		9.6392E+01	9.5372E+01	**9.9121E+01**	2.4560E-01	2.6905E-01	**1.8427E-01**	**1.0067E+05**	4.2105E+05	1.7319E+05
WFG3		**2.4037E+01**	2.4025E+01	2.3848E+01	**9.3331E-02**	9.7524E-02	1.0711E-01	7.1353E+08	7.0553E+09	**4.8357E+07**
WFG4		7.6820E+01	**7.6893E+01**	7.6863E+01	2.0001E-01	**1.9998E-01**	2.0010E-01	7.2705E+03	7.5974E+03	**5.1578E+03**
WFG5	3	**7.3458E+01**	7.3416E+01	7.3407E+01	2.1384E-01	**2.1377E-01**	2.1436E-01	**5.6060E+03**	5.6888E+07	6.7134E+03
WFG6		7.4081E+01	**7.4084E+01**	7.4078E+01	**2.0938E-01**	2.0965E-01	2.0966E-01	6.9892E+04	**1.0785E+04**	2.0773E+04
WFG7		7.7045E+01	**7.7047E+01**	7.7034E+01	**2.0004E-01**	2.0014E-01	2.0018E-01	**2.8209E+04**	3.4087E+04	1.8370E+08
WFG8		7.0872E+01	**7.0943E+01**	7.0904E+01	2.5035E-01	**2.4855E-01**	2.4920E-01	**1.2580E+04**	6.7374E+05	4.0960E+04
WFG9		**7.0274E+01**	7.0263E+01	6.8993E+01	**2.2365E-01**	2.2384E-01	2.3688E-01	1.9415E+08	2.4694E+06	**1.9541E+05**
WFG1		4.0787E+03	4.2356E+03	**4.2463E+03**	1.8441E+00	**1.7866E+00**	1.7995E+00	7.1824E+10	**4.7644E+10**	1.0796E+11
WFG2		9.8846E+03	1.0025E+04	**1.0218E+04**	5.6849E-01	5.1666E-01	**4.3216E-01**	**6.8679E+08**	3.3787E+09	5.4226E+09
WFG3		8.0982E+01	8.1019E+01	**8.2890E+01**	5.2549E-01	5.2947E-01	**4.9865E-01**	6.9978E+10	2.7869E+10	**5.6526E+09**
WFG4		8.8925E+03	**8.9669E+03**	8.9299E+03	9.1408E-01	**9.1185E-01**	9.1407E-01	**2.7649E+03**	1.6509E+04	5.3241E+06
WFG5	5	8.6746E+03	**8.6966E+03**	8.6649E+03	**9.2626E-01**	9.2716E-01	9.2983E-01	3.0457E+08	5.2736E+08	**8.4276E+06**
WFG6		8.8660E+03	**8.8836E+03**	8.8627E+03	9.1416E-01	**9.1398E-01**	9.1480E-01	1.4504E+07	**3.1101E+06**	2.8868E+07
WFG7		9.1345E+03	**9.1625E+03**	9.1449E+03	9.0612E-01	**9.0544E-01**	9.0612E-01	1.1485E+07	1.4527E+07	**1.0765E+07**
WFG8		8.4412E+03	**8.4984E+03**	8.4860E+03	9.5154E-01	**9.4711E-01**	9.4829E-01	1.2729E+07	**5.3446E+06**	1.1486E+10
WFG9		**8.0920E+03**	8.0819E+03	8.0532E+03	**9.8101E-01**	9.8359E-01	9.8227E-01	**1.5351E+03**	2.1161E+03	1.7019E+03

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TABLE IV

COMPARISON OF THE AVERAGE HV, IGD AND S-ENERGY VALUES OBTAINED WHEN USING SMR2 AND SMR3 WITH REPLACEMENT WITH $\sigma_{pool} = 5$.

Problem	Number of objectives	Hypervolume			IGD			S-energy		
		NSGA-III	NSGA-III + SMR2_R	NSGA-III + SMR3_R	NSGA-III	NSGA-III + SMR2_R	NSGA-III + SMR3_R	NSGA-III	NSGA-III + SMR2_R	NSGA-III + SMR3_R
DTLZ1		**2.1237E+00**	2.1237E+00	2.1237E+00	**1.7840E-03**	1.7841E-03	1.7843E-03	**1.1725E+05**	1.1725E+05	1.1725E+05
DTLZ2		**3.2101E+00**	3.2101E+00	3.2101E+00	**3.9625E-03**	3.9625E-03	3.9626E-03	5.3391E+04	5.3391E+04	**5.3391E+04**
DTLZ3		**3.2098E+00**	3.2095E+00	3.2096E+00	**3.9867E-03**	4.0309E-03	4.0092E-03	**5.3386E+04**	5.3388E+04	5.3425E+04
DTLZ4	2	2.8874E+00	**3.0891E+00**	2.9681E+00	2.0079E-01	**7.7772E-02**	1.5158E-01	**3.9153E+04**	4.8052E+04	4.2713E+04
DTLZ5		**3.2101E+00**	3.2101E+00	3.2101E+00	**3.9625E-03**	3.9625E-03	3.9626E-03	5.3391E+04	5.3391E+04	**5.3391E+04**
DTLZ6		3.0542E+00	3.0779E+00	**3.0832E+00**	9.2585E-02	7.9947E-02	**7.6878E-02**	5.4260E+04	**4.9193E+04**	4.9453E+04
DTLZ7		**4.4173E+00**	4.2559E+00	4.4172E+00	**5.1851E-03**	4.8976E-02	5.1932E-03	**7.3309E+04**	1.2243E+05	7.9670E+04
DTLZ1		3.3477E+00	3.3491E+00	**3.3491E+00**	2.2906E-02	**1.8692E-02**	1.8701E-02	1.0500E+10	1.0653E+06	**7.1158E+05**
DTLZ2		**7.4184E+00**	7.4180E+00	7.4176E+00	**4.9314E-02**	4.9369E-02	4.9411E-02	**9.1884E+04**	1.7820E+05	5.8420E+05
DTLZ3		7.4116E+00	7.4177E+00	**7.4177E+00**	5.6253E-02	**4.9352E-02**	4.9401E-02	1.6166E+09	**9.9917E+04**	4.0336E+06
DTLZ4	3	7.0466E+00	**7.3174E+00**	7.2495E+00	2.2902E-01	**9.8312E-02**	1.3101E-01	3.0716E+11	**1.0708E+10**	1.5578E+11
DTLZ5		4.0043E+00	4.1776E+00	**4.1783E+00**	6.5059E-02	**1.5283E-02**	1.6123E-02	8.1297E+11	8.5467E+11	**5.4758E+11**
DTLZ6		3.9910E+00	**3.9965E+00**	3.9911E+00	1.0158E-01	**9.8436E-02**	9.9011E-02	**1.1581E+11**	1.8009E+11	2.8578E+11
DTLZ7		1.3202E+01	**1.3303E+01**	1.3258E+01	7.8863E-02	**6.9520E-02**	7.1768E-02	8.0305E+07	**1.8462E+06**	2.6558E+06
DTLZ1		**7.5927E+00**	7.5927E+00	7.5927E+00	**5.0423E-02**	5.0631E-02	5.1905E-02	**2.2272E+10**	5.6993E+10	3.1609E+11
DTLZ2		**3.1698E+01**	3.1694E+01	3.1696E+01	**1.5539E-01**	1.5564E-01	1.5562E-01	7.7099E+09	**1.9388E+08**	3.5682E+10
DTLZ3		3.1694E+01	3.1691E+01	**3.1694E+01**	1.5825E-01	**1.5565E-01**	1.5760E-01	1.1507E+11	**6.5824E+09**	1.5785E+11
DTLZ4	5	3.1679E+01	**3.1698E+01**	3.1679E+01	1.6342E-01	**1.5536E-01**	1.6346E-01	1.4004E+10	**3.4033E+09**	1.0034E+11
DTLZ5		5.9019E+00	5.6280E+00	**6.0829E+00**	2.3194E-01	2.5228E-01	**2.0718E-01**	**2.3861E+11**	4.2277E+11	3.8949E+11
DTLZ6		**1.8920E+00**	0.0000E+00	3.6870E-04	**9.3461E-01**	2.4951E+00	1.9476E+00	4.5468E+11	**1.8908E+10**	3.8261E+10
DTLZ7		**7.7036E+01**	6.9028E+01	7.2048E+01	**2.7009E-01**	3.6074E-01	3.2807E-01	5.3521E+10	**7.1014E+09**	1.5851E+10
WFG1		3.9940E+00	3.8339E+00	**4.6962E+00**	1.4331E+00	1.4698E+00	**1.2558E+00**	1.2784E+05	2.9172E+05	**9.6583E+04**
WFG2		9.2570E+00	9.2874E+00	**9.2883E+00**	6.5750E-01	6.5703E-01	**6.5700E-01**	**5.9184E+04**	1.3126E+05	6.3061E+04
WFG3		1.0437E+01	1.0842E+01	**1.0872E+01**	4.3466E-02	1.2716E-02	**1.1960E-02**	2.9325E+04	2.9934E+04	**2.0208E+04**
WFG4		8.2336E+00	8.2821E+00	**8.3389E+00**	2.8252E-02	3.1142E-02	**2.6048E-02**	2.2522E+04	**2.2348E+04**	2.8639E+04
WFG5	2	**7.9344E+00**	7.8474E+00	7.7655E+00	**7.7568E-02**	9.0569E-02	1.0578E-01	2.2769E+04	2.4668E+04	**2.2315E+04**
WFG6		7.9733E+00	**8.3237E+00**	8.1427E+00	6.8255E-02	**2.9592E-02**	5.5345E-02	2.4668E+04	**2.1575E+04**	2.2302E+04
WFG7		6.6706E+00	7.5509E+00	**7.7180E+00**	2.9508E-01	1.0718E-01	**8.1722E-02**	4.2732E+04	**3.0842E+04**	3.4039E+04
WFG8		7.3725E+00	7.8800E+00	**8.0608E+00**	1.4150E-01	8.5660E-02	**6.9887E-02**	4.2315E+04	4.6592E+04	**3.8096E+04**
WFG9		8.1696E+00	**8.1784E+00**	8.1444E+00	3.5258E-02	**3.3998E-02**	3.5096E-02	**2.2445E+04**	2.4591E+05	2.2531E+04
WFG1		5.4149E+01	**5.4946E+01**	5.4935E+01	1.1700E+00	1.1689E+00	**1.1639E+00**	4.1152E+08	**3.8875E+06**	5.9378E+07
WFG2		9.6392E+01	9.4797E+01	**9.7478E+01**	2.4560E-01	2.8402E-01	**2.2252E-01**	**1.0067E+05**	2.5029E+06	3.1865E+05
WFG3		**2.4037E+01**	2.3528E+01	2.3832E+01	**9.3331E-02**	1.2405E-01	1.1183E-01	7.1353E+08	**1.1163E+08**	2.5357E+09
WFG4		7.6820E+01	7.6714E+01	**7.6866E+01**	2.0001E-01	**1.9992E-01**	2.0019E-01	7.2705E+03	**5.0849E+03**	5.8875E+03
WFG5	3	7.3458E+01	**7.3525E+01**	7.3322E+01	2.1384E-01	**2.1240E-01**	2.1457E-01	**5.6060E+03**	6.6398E+03	3.0764E+04
WFG6		7.4081E+01	7.4043E+01	**7.4101E+01**	**2.0938E-01**	2.1002E-01	2.0965E-01	6.9892E+04	**5.3176E+03**	8.4170E+04
WFG7		**7.7045E+01**	7.6997E+01	7.7031E+01	**2.0004E-01**	2.0005E-01	2.0021E-01	2.8209E+04	**8.5091E+03**	1.7101E+04
WFG8		**7.0872E+01**	7.0276E+01	7.0800E+01	2.5035E-01	2.6113E-01	**2.4855E-01**	**1.2580E+04**	2.0820E+04	6.3507E+09
WFG9		**7.0274E+01**	6.8797E+01	6.9357E+01	**2.2365E-01**	2.3891E-01	2.3173E-01	1.9415E+08	2.5239E+07	**2.7641E+04**
WFG1		4.0787E+03	4.1560E+03	**4.2788E+03**	1.8441E+00	1.8413E+00	**1.8074E+00**	7.1824E+10	**6.2041E+10**	1.0491E+11
WFG2		9.8846E+03	**1.0260E+04**	1.0222E+04	5.6849E-01	**4.0650E-01**	4.3222E-01	6.8679E+08	**3.3836E+05**	2.1037E+10
WFG3		8.0982E+01	7.5389E+01	**8.3216E+01**	5.2549E-01	5.8067E-01	**5.0038E-01**	6.9978E+10	**6.7758E+09**	1.6107E+10
WFG4		8.8925E+03	8.6764E+03	**8.9734E+03**	9.1408E-01	9.2611E-01	**9.1228E-01**	2.7649E+03	**1.6412E+03**	7.5004E+03
WFG5	5	8.6746E+03	8.5996E+03	**8.6801E+03**	**9.2626E-01**	9.2728E-01	9.3455E-01	3.0457E+08	**7.9671E+07**	2.3247E+10
WFG6		8.8660E+03	8.8006E+03	**8.8703E+03**	**9.1416E-01**	9.1731E-01	9.1502E-01	1.4504E+07	**4.4048E+05**	2.4931E+08
WFG7		9.1345E+03	9.0514E+03	**9.1579E+03**	**9.0612E-01**	9.0822E-01	9.0628E-01	**1.1485E+07**	4.6832E+07	1.5277E+09
WFG8		8.4412E+03	8.3023E+03	**8.5127E+03**	9.5154E-01	9.6270E-01	**9.4652E-01**	**1.2729E+07**	5.1636E+07	8.7401E+07
WFG9		**8.0920E+03**	8.0536E+03	8.0699E+03	9.8101E-01	9.8377E-01	**9.8053E-01**	**1.5351E+03**	6.5673E+03	4.2882E+06

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