An Ensemble of S-energy Based Mating Restrictions for Multi-Objective Evolutionary Algorithms

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***Abstract*—Mating restrictions are a mechanism adopted by multi-objective evolutionary algorithms to improve the solution of multi-objective optimization problems (MOPs) by establishing a strategy to mate individuals during the reproduction step of the algorithm. Several mating restrictions have been proposed for MOEAs to solve MOPs having two and three objective functions. However, in the case of many-objective optimization problems (four or more objectives), only a few mating restriction schemes have been proposed so far. The Riesz S-energy is a performance indicator which can be used to evaluate population's diversity, and it is able to provide useful neighborhood information from individuals in MOPs with any number of objectives. This feature has been used in some of our previous work to propose a few different mating restriction schemes based on the s-energy indicator. In this paper, we propose the use of an ensemble of four of these mating restriction mechanisms, which is implemented within the NSGA-III to assess its performance. The ensemble's behavior is guided by two measurements of each mating restriction performance throughout the algorithm's execution. We performed an experimental validation of this ensemble in MOPs with up to seven objective functions, and compared the results obtained using the hypervolume, the s-energy, and the inverted generational distance performance indicators. The results obtained show that the use of our mating restrictions ensemble outperforms the original NSGA-III in most of the test instances adopted.**

***Index Terms*—Multiobjective Optimization, Evolutionary Algorithms, Mating Restrictions.**

I. Introduction

Multi-objective optimization problems (MOPs) can model a variety of real-world problems found in many different fields of knowledge. This has generated a great interest in developing techniques to solve such problems. One of the most commonly used techniques to solve MOPs are the so-called multi-objective evolutionary algorithms (MOEAs), which are population-based heuristics that emulate the biological evolutionary process. MOEAs use the "survival of the fittest" principle found in nature to drive their population from randomly generated individuals to optimal solutions of a problem. Following this analogy, mating restrictions are a bio-inspired mechanism used to bias the way in which individuals mate during the algorithm's reproduction step. They were originally discussed in Goldberg's book on genetic algorithms [1] as a way of avoiding the propagation of individuals with low fitness values (also known as "lethals)". Ever since, different mating restrictions have been proposed to enhance MOEAs. Traditionally, most of these restrictions rely on the individuals similarity (or dissimilarity) to mate them either comparing distances in variable or objective space [2]–[4], or using clustering and neighborhood information [5], [6]. Other metrics have also been used to guide the mating step, such as individuals' survival length [7] or manifold distances [8]. Also, some restrictions have been tailored to work with a specific type of algorithm, such as decomposition-based MOEAs [6], [9], [10].

In this paper, we propose a new mating restriction, which is in fact an ensemble of four previously proposed s-energy based mating restrictions. Our main contribution is this new ensemble as well as the experimental validation of its impact on the solution of MOPs with up to seven objectives.

The remainder of this paper is organized as follows. In Section II some basic concepts are introduced. In Section III we present some previous work concerning mating restrictions and we also define the s-energy based mating restrictions adopted in this work. In Section IV we show, in detail, the way in which our ensemble works. Then, in Section V we present the experimental setup used to validate the performance of the ensemble, as well as the results obtained. Finally, in Section VI we present our conclusions as well as some possible paths for future work.

II. Basic Concepts

Multi-objective optimization problems require the simultaneous optimization of two or more objective functions. Without loss of generality, we will assume only minimization. Formally, a MOP is defined as follows:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

In a MOP, it is normally the case that the objective functions are in conflict, causing that there is no single solution which may simultaneously optimize all objective functions at the same time. Instead, finding a set of solutions that represent the best possible trade-offs among the objective functions is the goal when solving a MOP. Here, we present some useful definitions used to characterize such solutions.

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. Given a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$, it is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. Given a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$, being $\mathcal{F}$ the feasible region, we say $\vec{x}^*$ is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

Hence, the goal of solving a MOP is to obtain the Pareto optimal set ($\mathcal{P}^*$) from the feasible set ($\mathcal{F}$) that satisfy the equality and/or inequality constraints (2) and (3).

III. Previous related work

A. Mating restrictions

A variety of mating restriction mechanisms have been proposed in the literature with the aim of improving the overall performance of MOEAs, either by improving the population's diversity or the convergence speed. However, few of such proposals have been tested with many-objective optimization problems (MaOPs), which are optimization problems having 4 or more objective functions. In this section, we review some mating restriction mechanisms that have been used to solve MaOPs, as well as one mating restriction which considers multiple sources of information, closely related to an ensemble of mating restrictions.

Multi-objective evolutionary algorithms enhancement has been performed in [11], where two decomposition-based MOEAs, namely MOEA/D [12] and EFR [13], were improved using a mating restriction. The resulting algorithms (MOEA/D-DU and EFR-RR) are able to outperform their original versions in most tested problems with up to 13 objective functions. The mechanism used consists in determining neighborhoods for each individual based on their perpendicular distance to weight vectors in objective space. Using this information, the mating restriction allows to balance diversity and convergence by mating individuals within the same neighborhood in MOEA/D or by selecting individuals from the same neighborhood in the ranking performed in EFR.

Another modification to MOEA/D in order to improve the results obtained in many-objective optimization problems is MOEA/D-LWS [14] which implements a localized weighted sum method. This algorithm implements a weighted sum scalarizing function paired with a mating restriction scheme in order to use the function locally (within a hypercone around each weight vector). This modification allows to obtain solutions in non-convex portions of the Pareto fronts, which is the most well-known downside of using a weighted sum. Experimental results comparing against two other MOEA/D variants as well as three other MOEAs show really good results in problems with up to seven objectives.

The Enhanced-Mating-Selection-Many-Objective-NSGA-II (EMS-MO-NSGA-II) [15] enhances MO-NSGA-II mating selection mechanism by utilizing two mating mechanisms. The first one is a reference-point based selection procedure, while the second one is a neighborhood-based selection scheme. These two strategies were experimentally evaluated, both individually and combined, by solving the DTLZ 1-4 test problems [16] with up to 10 objectives. The results obtained showed a significant improvement when using both strategies at the same time.

The many-objective evolutionary algorithm based on directional diversity and favorable convergence (MaOEA-DDFC) [17] uses a scalarizing function to obtain convergence degrees of individuals in the population. Then, using a binary tournament selection scheme, individuals are selected and compared using both Pareto dominance and convergence degrees to create a mating pool in which only the best individuals are chosen. This algorithm was compared with respect to seven other MOEAs, obtaining good results and improving their performance in the majority of the test problems used, which comprise problems with up to ten objectives from the DTLZ and WFG test suites.

The constrained MOEA/D with Directed Mating and Archives of infeasible solutions (CMOEA/D-DMA) [18] relies on useful infeasible solutions which are generated during the search process. Up to eight infeasible solutions per weight vector are stored in an archive, and they are randomly selected to be mated with feasible solutions. This mechanism was coupled to cMOEA/D and used to solve the mCDTLZ problems [19] as well as m objectives k knapsack problems [20] with up to eight objective functions. The results obtained indicate that this algorithm outperforms the original cMOEA/D as well as NSGA-III and TNSDM [21] (Two-stage Nondominated Sorting and Directed Mating) in most of the test instances adopted.

The spectral clustering based multi-source mating selection strategy (SMMS) is designed to detect regularity properties and to balance population diversity and convergence. It was coupled to SMS-EMOA [22] giving rise to the so-called SMMEA [23]. Given an individual $\vec{x}$, this algorithm adopts three different sources for selecting a mate: (1) a sub-population from the same cluster of $\vec{x}$, (2) a sub-population from a cluster adjacent to the cluster containing $\vec{x}$, or (3) the whole population. The selection of one mating source is performed using adaptive probabilities for the first two sources, obtained from each source efficiency. This proposal was compared with respect to six MOEAs in the solution of the MOPF [24], UF [25] and GLT [26] test problems, with two and three objective functions. The results showed that SMMEA had a significantly better performance than the other MOEAs adopted.

B. S-energy based mating restrictions

The mating restrictions that we adopt in this work are based on the Riesz s-energy [27], which has been used in MOEAs as a performance indicator [28]. The s-energy of a population X is obtained using the following expression:

$$E_s(X) = \sum_{i \neq j} \frac{1}{\mid \vec{x}_i - \vec{x}_j \mid^s} \tag{4}$$

where $\mid \cdot \mid$ is the Euclidean distance between two individuals in objective space. A low s-energy value is desirable, since it implies uniformity in the distribution of points. From equation (4), we can define the individual s-energy contribution of a given point $\vec{x}_i$ as follows:

$$C_{E_i} = E_s(X) - E_s(X \setminus \{\vec{x}_i\}). \tag{5}$$

The individual s-energy contribution provides us with neighboring information of each individual in the population. An individual with a low contribution is located in a relatively "non-crowded" region, while an individual with a high contribution has at least one other individual relatively close to it. This information is used by the s-energy based mating restrictions (SMR), proposed in [29]. In this work we adopted four SMRs to create an ensemble of mating restrictions. These restrictions compute the s-energy contribution of each individual in the population and use this information to rank the population according to the following criteria: individuals with a low s-energy contribution are considered the best individuals, conversely, individuals with high contributions are considered the worst ones. Once these contributions are obtained, each mating restriction uses a different strategy to mate parents. The SMRs we used in this ensemble are the following:

- SMR1_SIM: This restriction mates individuals with similar s-energy contribution values. The first pair is formed with the two best individuals from the population. The next pair will select the next two best individuals from the remaining individuals, and so on, until all pairs are formed.
- SMR1_DIS: This restriction pairs individuals with dissimilar s-energy contributions. The first pair selects the best individual and the worst individual. The next pair mates the second best individual with the second worst. This is repeated with the remaining individuals.
- SMR3: This restriction employs a mating pool with size $\sigma_{pool} > 0$, which is a user defined parameter. In this pool are contained the worst individuals from the population which have not been selected yet. SMR3 pairs the best individual from the population with one of the individuals in the mating pool. In order to select one individual from the mating pool, Euclidean distances in objective space are measured and the individual with the smallest distance is selected. In our ensemble we adopted this restriction twice: the first one with $\sigma_{pool} = 5$, and the second one with $\sigma_{pool} = 9$.

We combined these SMRs to create an ensemble of mating restrictions which is described in detail in the following section.

IV. Ensemble of mating restrictions

The ensemble we propose in this work combines the use of four different mating restrictions at each generation. However, the number of pairs selected from each restriction varies according to their individual performance in each problem. In order to determine how many pairs will be selected, we employed two different metrics, which are computed for each mating restriction used at each generation:

- Mating restriction's efficiency: It is the percentage of children generated by each mating restriction mechanism that were selected to survive to the next generation. It is obtained by computing the quotient of the number of selected individuals which are offspring of a certain mating restriction divided by the total number of children generated by this restriction so far.
- Mating restriction's dominance: It is the sum of individuals generated by each mating restriction which dominate either (or both) of their parents. In contrast with efficiency, this value is obtained using only the information from the last few generations, determined by a user-defined parameter $t_r > 0$.

Since one mating restriction scheme may be better than the rest during the first stages of the algorithm, but not in the latter, and one mating restriction may be better in some problems, but not in others, we propose the use of both efficiency and dominance metrics in the following way. We use mating restriction's dominance to obtain a sort of "local" information about the performance of the restrictions since it only considers the last generations. On the other hand, we use mating restriction's efficiency as a "global" information indicator, since it stores how efficient has each restriction been from the beginning of the algorithm. We alternate between these two metrics in the following way.

During the first t_r generations, we use mating restriction's dominance to assign in a directly proportional way the number of pairs to be obtained from each restriction, allowing the ones which have generated better individuals in the last generations to be the ones with more offspring in the next generation. Here, we mention better individuals in the sense of children that dominate their parents. However, every t_r generations the dominance metric will be reset to zero (in order for it to reflect local behavior information), and mating restriction's efficiency will be used to assign the number of pairs to be selected from each restriction instead of dominance. This is to ensure that restrictions which have been proved to be the most useful in the solution of the problem keep being used throughout the execution, even if they may not be the ones with better offspring in a given particular generation. In this work, we propose the use of $t_r = 5$, which proved to be the best value after our experimental validation.

We show the pseudocode of our ensemble in Fig. 1. Since we limit our proposal to an ensemble of mating restrictions which may be applied to different algorithms, we only mention the generic steps of a MOEA, but do not get into the details of them, since they may vary from algorithm to algorithm. Such is the case of the population initialization (line 2), and of the individuals' crossover and mutation (lines 19 and 20) and selection of individuals which will survive to the next generation (line 28).

In lines 3-11 we show the variables initialization, and for the first generation, each mating restriction will be assigned an equal number of pairs (mating restriction size) to be selected with them. Then, during the main loop of the algorithm, the following will occur. In line 13 we will obtain the pairing from each restriction in the ensemble, according to the mating restriction size, previously set. Next, in lines 14-23 we will generate the offspring population using the pairing set from the mating restriction ensemble. In this part, we use a procedure to store the dominance information from each restriction (line 20). In line 24, the final population of this generation is selected (either directly from the offspring population or from a combination of offspring and parents population). Here, we must count how many individuals generated with each mating restriction made it into the final population. Next, we update the efficiency of each mating restriction (lines 25-28). Finally, if t_r generations have already passed, mating restrictions' dominance will be set to zero and mating restrictions' efficiency will be used to assign each restriction size (lines 30-34). If it is not the case, mating restrictions' dominance will be used instead (line 36).

Auxiliary procedures of the algorithm are shown in Figs. 2, 3 and 4. The mating restrictions ensemble selection mechanism is shown in Fig. 2, and it simply consists of obtaining the pairing from each individual mating restriction, and then proceed to select the first pairs from each pairing according to the predefined mating restriction size.

In Fig. 3, we show the update of the dominance metric of each mating restriction. Given a pair of parents and their corresponding children, we count the number of parents for which each child dominates either of its parents, then we update the corresponding mating restriction metric.

Finally, in Fig. 4, we show the procedure used to adjust each mating restriction size according to a given metric (either efficiency or dominance). Given the above-mentioned metric, we compute its total sum and use it to assign a number of pairs in direct proportion of each restriction's contribution to the metric sum.

1: **procedure** MOEA(MOP, t_r)
2: **MOEA_initializePopulation(**$\vec{P_p}$**)**
3: $n_r \leftarrow 4$
4: $t \leftarrow 0$
5: **for** $i \leftarrow 1, n_r$ **do**
6: $mr_{size}[i] \leftarrow \vec{P_p}.size/n_r$
7: $mr_{dom}[i] \leftarrow 0$
8: $mr_{effic}[i] \leftarrow 0$
9: $mr_{offspring}[i] \leftarrow 0$
10: $mr_{selected}[i] \leftarrow 0$
11: **end for**
12: **while** stopping criterion not fulfilled **do**
13: $mr_{pairing} \leftarrow$**MR_select(**$\vec{P_p}, mr_{size}$**)**
14: $\vec{P_o} \leftarrow \emptyset$
15: **for** $i \leftarrow 1, \vec{P}.size$ **do**
16: $p1 \leftarrow \vec{P}[mr_{pairing}[i]]$
17: $p2 \leftarrow \vec{P}[mr_{pairing}[i+1]]$
18: $c1, c2 \leftarrow$ **MOEA_crossover(**$p1, p2$**)**
19: $c1, c2 \leftarrow$ **MOEA_mutate(**$p1, p2$**)**
20: **MR_dominates(**$mr_{dom}, p1, p2, c1, c2$**)**
21: $\vec{P_o} \leftarrow P_o \cup \{c1, c2\}$
22: $i \leftarrow i + 2$
23: **end for**
24: $\vec{P_p}, mr_{selected} \leftarrow$ **MOEA_select(**$\vec{P_p}, \vec{P_o}$**)**
25: **for** $i \leftarrow 1, n_r$ **do**
26: $mr_{offspring}[i] \leftarrow mr_{offspring}[i] + mr_{size}[i]$
27: $mr_{effic}[i] \leftarrow mr_{selected}[i]/mr_{effic}[i]$
28: **end for**
29: **if** $t == t_r$ **then**
30: **MR_adjust(**$\vec{P_p}, mr_{size}, mr_{effic}$**)**
31: **for** $i \leftarrow 1, n_r$ **do**
32: $mr_{dom}[i] \leftarrow 0$
33: **end for**
34: $t \leftarrow 0$
35: **else**
36: **MR_adjust(**$\vec{P_p}, mr_{size}, mr_{dom}$**)**
37: **end if**
38: $t \leftarrow t + 1$
39: **end while**
40: **end procedure**

Fig. 1. Mating restrictions ensemble main algorithm.

The computational cost of our mating restrictions ensemble is directly related to the cost of computing s-energy contributions of each individual in the population, since this is required for each of the four mating restrictions adopted in the ensemble, and it is also the most computational time consuming part of the algorithm. Using a naive approach, the calculation of a single s-energy contribution is $O(n^2)$, being n the size of the population. Then, this process would be repeated for every individual, causing a total computational cost of $O(n^3)$. However, this can be done more efficiently, using the memoization structure proposed in [30], producing a total computational cost of $O(n^2)$ to compute all the individuals' s-energy contributions. Once these contributions are obtained, this information can be used by all four mating restrictions, so it will only be performed once per generation.

```
procedure MR_SELECT(P⃗, mr_size)
    n_r ← 4
    counter ← 0
    mr_pairing ← {0, 0, ..., 0}
    for i ← 1, n_r do
        Pair_i = SMR_i(P⃗)
    end for
    for i ← 1, n_r do
        for j ← 1, mr_size[i] do
            mr_pairing[counter] = Pair_i[j]
            mr_pairing[counter + 1] = Pair_i[j + 1]
            counter ← counter + 2
        end for
    end for
    return mr_pairing
end procedure
```

Fig. 2. Selection of mating restriction according to previously determined values stored in mr_{size}.

```
procedure MR_DOMINATES(mr_dom, p1, p2, c1, c2)
    counter ← 0
    for c_i ∈ {c1, c2} do
        for p_i ∈ {p1, p2} do
            if c_i Pareto dominates p_i then
                counter ← counter + 1
            end if
        end for
    end for
    mr ← mating restriction used to generate p1 and p2
    mr_dom[mr] ← mr_dom[mr] + counter
end procedure
```

Fig. 3. Procedure to increase the dominance counter of a mating restriction from the offspring generated with it.

```
procedure MR_ADJUST(P⃗, mr_size, mr_metric)
    n_r ← 4
    total ← Σ_{i=1}^{n_r} mr_metric[i]
    for i ← 1, n_r do
        mr_size[i] = mr_metric[i]/total * P⃗.size
    end for
end procedure
```

Fig. 4. Procedure to proportionally assign the new size corresponding to each mating restriction in the ensemble according to a given metric.

On the other hand, the mating restriction selection procedure has a computational cost of $O(n)$, while the dominance metric update has a computational cost of $O(nm^2)$, being m the number of objectives. Finally, both efficiency metric update and mating restriction size adjustment procedure have a computational cost of $O(n_r)$, being $n_r = 4$ the number of mating restrictions used in the ensemble. Then, the total cost of our ensemble is $O(n^2 + nm^2)$ per generation, which reduces to $O(n^2)$ for populations with over 100 individuals and problems with up to 10 objectives. In Fig. 5 we show the average execution time required to solve all 16 test problems used with different numbers of objectives. From these values, it can be seen that there is a slight increase due to the mating restriction ensemble used. Nonetheless, such time increase is, in all cases, of less than one second.

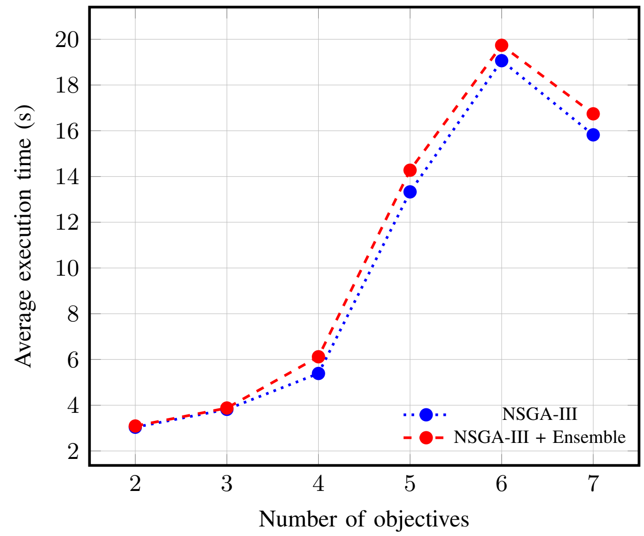

Fig. 5. Average execution time of our proposal compared against the original NSGA-III.

V. EXPERIMENTAL VALIDATION

In order to validate the functionality of our proposed ensemble, we implemented the ensemble in NSGA-III [31], so that we could solve a series of test problems with and without the ensemble to compare the results obtained. We adopted the Deb-Thiele-Laumanns-Ziztler (DTLZ) [16] and the Walking Fish Group (WFG) test suites [32], since they contain a variety of problems with different Pareto front characteristics, such as linear, convex, concave, degenerate or discontinuous Pareto fronts. From these, we used DTLZ1-DTLZ7 and WFG1-WFG9 with 2-7 objectives, to assess how well the ensemble performs in problems with many objectives (more than three). This gives a total of 96 test problems, each of which we solved 30 times using both the original NSGA-III implementation and the one with the ensemble of mating restrictions incorporated within it. We adopted three performance indicators to compare the Pareto approximations obtained in each of these test instances. The indicators used were the hypervolume (HV) [33], the inverted generational distance (IGD) [34], and the s-energy [27]. In the first case, the greater the HV value, the better the approximation, whereas smaller IGD and s-energy values are preferred. The performance indicator values obtained were compared using the Wilcoxon rank-sum test at a confidence interval of 95%.

Our experimental results are shown in Table I. In each indicator pair of columns, the best value is shown in **boldface**, while the cells in gray represent the values that are statistically better according to the Wilcoxon test used.

From Table I, we can observe that the use of our ensemble of mating restrictions produced a better performance in 57 out of the 96 test instances when comparing with respect to the HV, while it only had a worst performance in one test problem (DTLZ3 with 3 objectives). In the remaining 38 problems, results are statistically similar at a confidence interval of 95%.

TABLE I
COMPARISON OF THE AVERAGE HV, IGD AND S-ENERGY VALUES OBTAINED USING THE PROPOSED MATING RESTRICTIONS ENSEMBLE IN NSGA-III. CELLS IN GRAY SHOW THE STATISTICALLY BETTER VALUES, ACCORDING TO THE WILCOXON TEST.

Problem	Number of objectives	Hypervolume		S-energy		IGD	
		NSGA-III	NSGA-III+Ensemble	NSGA-III	NSGA-III+Ensemble	NSGA-III	NSGA-III+Ensemble
DTLZ1	2	5.3357E-01	**5.3368E-01**	1.1956E+05	**1.1723E+05**	1.8466E-03	**1.7915E-03**
DTLZ2		4.1999E-01	**4.2013E-01**	5.3542E+04	**5.3392E+04**	4.0073E-03	**3.9632E-03**
DTLZ3		**4.1811E-01**	4.1800E-01	**5.3496E+04**	5.4529E+04	4.4526E-03	**4.4405E-03**
DTLZ4		3.3742E-01	**3.6844E-01**	**3.9162E+04**	4.4493E+04	2.0079E-01	**1.2698E-01**
DTLZ5		4.1999E-01	**4.2013E-01**	5.3542E+04	**5.3392E+04**	4.0073E-03	**3.9632E-03**
DTLZ6		4.6875E+00	**4.7444E+00**	**5.1342E+04**	7.4879E+04	1.0889E-01	**8.1561E-02**
DTLZ7		7.3494E-01	**7.3499E-01**	**8.6183E+04**	8.8229E+05	5.1686E-03	**5.1674E-03**
WFG1		**6.1667E-01**	5.9821E-01	1.3790E+05	**1.0715E+05**	1.5035E+00	**1.4348E+00**
WFG2		1.3095E+00	**1.3119E+00**	1.3326E+05	**6.6367E+04**	6.5824E-01	**6.5764E-01**
WFG3		3.8506E+00	**3.9094E+00**	2.8483E+04	**2.5233E+04**	6.9185E-02	**3.5092E-02**
WFG4		2.2182E+00	**2.2363E+00**	**2.5083E+04**	2.9992E+04	4.8849E-02	**3.3118E-02**
WFG5		1.9551E+00	**1.9620E+00**	2.3735E+04	**2.1515E+04**	9.2752E-02	**8.4103E-02**
WFG6		**2.6415E+00**	2.6216E+00	**2.3202E+04**	2.4043E+04	**6.3791E-02**	6.7504E-02
WFG7		2.0603E+00	**2.1096E+00**	4.0793E+04	**3.5452E+04**	3.6238E-01	**2.7336E-01**
WFG8		3.2720E+00	**3.3934E+00**	**5.6673E+04**	8.1960E+04	2.1185E-01	**1.7505E-01**
WFG9		**2.2130E+00**	2.2053E+00	**2.3704E+04**	1.9445E+05	**3.9788E-02**	4.0386E-02
DTLZ1	3	**8.7400E-01**	8.7383E-01	**3.6472E+06**	5.0248E+09	**1.8958E-02**	1.9614E-02
DTLZ2		7.4898E-01	**7.4905E-01**	**1.8627E+05**	1.5963E+06	4.9361E-02	**4.9359E-02**
DTLZ3		**7.4327E-01**	7.3947E-01	**1.1968E+06**	1.5548E+07	**4.9667E-02**	5.0218E-02
DTLZ4		6.4110E-01	**6.8052E-01**	5.2214E+11	**2.8705E+11**	2.2900E-01	**1.6367E-01**
DTLZ5		1.3322E-01	**1.3872E-01**	7.4448E+11	**6.9681E+11**	5.9339E-02	**2.3632E-02**
DTLZ6		4.2607E+01	**4.2730E+01**	**1.4683E+11**	5.2693E+11	1.2659E-01	**1.1320E-01**
DTLZ7		1.5000E+00	**1.5079E+00**	**1.7976E+06**	1.9217E+10	**6.8762E-02**	7.0361E-02
WFG1		3.0108E+01	**3.1520E+01**	**1.4561E+06**	1.2302E+07	1.2766E+00	**1.2311E+00**
WFG2		3.9822E+01	**4.0654E+01**	**1.5196E+05**	1.4073E+06	3.3398E-01	**2.9433E-01**
WFG3		2.6393E+01	**2.6680E+01**	**1.9949E+07**	1.1890E+10	1.1101E-01	**1.0206E-01**
WFG4		2.3866E+01	**2.4057E+01**	**5.1545E+03**	5.4314E+03	2.0165E-01	**2.0121E-01**
WFG5		2.2018E+01	**2.2069E+01**	1.1436E+05	**1.3130E+04**	2.1580E-01	**2.1530E-01**
WFG6		2.2226E+01	**2.2362E+01**	1.7143E+04	**1.1931E+04**	2.1430E-01	**2.1276E-01**
WFG7		2.4337E+01	**2.4397E+01**	3.1662E+05	**2.6767E+04**	**2.0034E-01**	2.0050E-01
WFG8		2.4152E+01	**2.4370E+01**	1.1761E+08	**4.1370E+05**	2.6456E-01	**2.5984E-01**
WFG9		**2.3192E+01**	2.2571E+01	1.4636E+04	**8.1013E+03**	**2.2803E-01**	2.3811E-01
DTLZ1	4	**3.1888E+00**	3.1888E+00	2.8918E+09	**4.5400E+06**	**4.0922E-02**	4.1081E-02
DTLZ2		**1.1648E+00**	1.1644E+00	**8.4928E+04**	8.4942E+04	**1.1382E-01**	1.1385E-01
DTLZ3		7.7152E+01	**7.7161E+01**	3.9478E+07	**2.1937E+07**	1.2657E-01	**1.1775E-01**
DTLZ4		1.0208E+00	**1.1058E+00**	2.1598E+11	**6.8190E+10**	2.9346E-01	**1.9139E-01**
DTLZ5		3.8983E+00	**3.9205E+00**	**5.2442E+11**	2.5628E+12	1.1110E-01	**8.8104E-02**
DTLZ6		8.4201E+02	**8.4454E+02**	**1.9505E+11**	4.2948E+11	5.7990E-01	**4.7092E-01**
DTLZ7		2.2224E+00	**2.2505E+00**	**1.7251E+08**	5.6368E+09	2.0624E-01	**2.0128E-01**
WFG1		2.0317E+02	**2.1846E+02**	**1.2278E+09**	2.6709E+09	1.6261E+00	**1.5393E+00**
WFG2		**3.5401E+02**	3.5210E+02	**5.0981E+07**	6.2388E+09	**4.5992E-01**	4.7892E-01
WFG3		3.2675E+02	**3.2734E+02**	**3.8606E+09**	1.1193E+11	**2.9053E-01**	3.0261E-01
WFG4		2.4600E+02	**2.4978E+02**	1.2469E+03	**1.2170E+03**	**5.6932E-01**	5.7066E-01
WFG5		2.3152E+02	**2.3346E+02**	1.4598E+03	**1.4079E+03**	**5.6728E-01**	5.6884E-01
WFG6		2.3278E+02	**2.3480E+02**	1.2446E+03	**1.2099E+03**	5.6924E-01	**5.6907E-01**
WFG7		2.5949E+02	**2.6122E+02**	1.1993E+03	**1.1790E+03**	**5.7119E-01**	5.7204E-01
WFG8		2.8328E+02	**2.8470E+02**	**1.6037E+06**	5.1743E+06	6.2484E-01	**6.2229E-01**
WFG9		**2.3721E+02**	2.3435E+02	1.5574E+03	**1.5563E+03**	5.7438E-01	**5.7430E-01**
DTLZ1	5	**2.9055E-01**	2.9054E-01	1.6853E+11	**6.9391E+10**	**5.0368E-02**	5.0654E-02
DTLZ2		1.3054E+00	**1.3064E+00**	**2.0765E+10**	1.3373E+11	1.5560E-01	**1.5548E-01**
DTLZ3		3.9891E+05	**3.9891E+05**	**1.1463E+11**	1.3830E+11	2.0915E-01	**1.9739E-01**
DTLZ4		1.3021E+00	**1.3021E+00**	**8.0186E+10**	1.3632E+11	1.6338E-01	**1.6333E-01**
DTLZ5		1.2508E+02	**1.2619E+02**	**3.2513E+11**	3.1227E+12	2.3114E-01	**1.9705E-01**
DTLZ6		6.2840E+03	**6.3378E+03**	**2.7431E+10**	1.5967E+11	1.9394E+00	**1.7942E+00**
DTLZ7		3.0123E+00	**3.0622E+00**	**1.8919E+10**	1.0755E+11	2.7940E-01	**2.6670E-01**
WFG1		1.0342E+02	**1.0699E+02**	2.4901E+10	**1.5357E+10**	1.9929E+00	**1.9588E+00**
WFG2		**3.4511E+03**	3.3359E+03	**3.6792E+07**	6.7396E+09	**5.0293E-01**	6.4402E-01
WFG3		**2.5916E+03**	2.5810E+03	**3.5651E+10**	1.6769E+11	**5.9354E-01**	6.2907E-01
WFG4		3.0867E+03	**3.2460E+03**	**1.3402E+03**	5.7594E+05	9.4493E-01	**9.2570E-01**
WFG5		3.0391E+03	**3.1292E+03**	**2.1655E+06**	7.1581E+07	9.5438E-01	**9.3867E-01**
WFG6		2.7510E+03	**2.8592E+03**	**3.3400E+06**	9.9717E+07	9.3486E-01	**9.2226E-01**
WFG7		2.9288E+03	**3.0308E+03**	**1.1611E+06**	2.0022E+10	9.2918E-01	**9.1124E-01**
WFG8		3.2599E+03	**3.3744E+03**	**9.5577E+05**	3.7906E+07	9.9004E-01	**9.6865E-01**
WFG9		2.8869E+03	**2.9086E+03**	8.6278E+04	**3.7789E+03**	**1.0043E+00**	1.0102E+00
DTLZ1	6	6.5860E+00	**6.5860E+00**	2.2901E+05	**2.0608E+05**	6.4333E-02	**6.2869E-02**
DTLZ2		1.7054E+00	**1.7078E+00**	**6.6797E+04**	6.6926E+04	2.1009E-01	**2.0974E-01**
DTLZ3		**1.9893E+06**	1.9893E+06	6.6059E+04	**6.1406E+04**	**2.6801E-01**	3.7702E-01
DTLZ4		1.5375E+00	**1.5453E+00**	**7.7555E+04**	7.8952E+04	2.2195E-01	**2.1492E-01**
DTLZ5		4.1175E+02	**4.1222E+02**	**6.8224E+04**	9.4598E+04	3.5843E-01	**3.3632E-01**
DTLZ6		5.6260E+04	**5.8987E+04**	**1.5554E+04**	1.7993E+05	3.6113E+00	**3.1331E+00**
DTLZ7		3.5956E+00	**3.6867E+00**	**4.1904E+04**	4.3668E+04	4.1830E-01	**3.8385E-01**
WFG1		1.8159E+01	**1.8701E+01**	**8.1161E+04**	1.0444E+05	2.2494E+00	**2.2314E+00**
WFG2		**3.4097E+04**	3.3910E+04	**2.8538E+04**	2.8846E+04	**7.5766E-01**	7.9188E-01
WFG3		2.3652E+04	**2.3992E+04**	**2.3608E+04**	9.0770E+04	**8.5291E-01**	8.7398E-01
WFG4		3.9876E+04	**4.2229E+04**	9.5086E+03	**9.4351E+03**	1.4087E+00	**1.3843E+00**
WFG5		4.0046E+04	**4.1580E+04**	9.5051E+03	**9.4559E+03**	1.4132E+00	**1.3932E+00**
WFG6		4.1404E+04	**4.3121E+04**	9.4639E+03	**9.4293E+03**	1.3901E+00	**1.3730E+00**
WFG7		3.8606E+04	**4.0161E+04**	9.4405E+03	**9.4211E+03**	1.3870E+00	**1.3647E+00**
WFG8		4.2133E+04	**4.3719E+04**	9.6780E+03	**9.6254E+03**	1.4386E+00	**1.4086E+00**
WFG9		4.0286E+04	**4.0794E+04**	1.0119E+04	**1.0014E+04**	1.5206E+00	**1.5023E+00**
DTLZ1	7	**4.0388E+00**	4.0387E+00	**2.8691E+11**	1.8059E+12	**8.8486E-02**	9.3266E-02
DTLZ2		1.7665E+00	**1.7699E+00**	**2.7608E+05**	2.8053E+05	2.8168E-01	**2.8133E-01**
DTLZ3		1.3269E+14	1.3269E+14	**7.1103E+11**	1.8082E+12	**7.6479E-01**	1.0279E+00
DTLZ4		2.5112E+00	**2.5227E+00**	**9.3437E+11**	3.9384E+12	3.1834E-01	**3.0798E-01**
DTLZ5		**7.3010E+02**	7.2946E+02	**1.4659E+12**	3.2859E+12	5.1439E-01	**5.0450E-01**
DTLZ6		3.7784E+05	**3.9944E+05**	**1.4011E+10**	7.1655E+10	3.8287E+00	**3.1421E+00**
DTLZ7		3.7340E+00	**3.8467E+00**	**1.1474E+11**	4.4794E+11	6.0591E-01	**5.9054E-01**
WFG1		3.7775E+00	**4.2824E+00**	**7.5017E+11**	4.0344E+12	2.6143E+00	**2.5898E+00**
WFG2		**4.0164E+05**	3.9833E+05	**2.0529E+11**	2.3815E+11	1.1664E+00	**1.1423E+00**
WFG3		5.5539E+05	**5.6725E+05**	**3.4851E+11**	1.3329E+12	9.0588E-01	**8.3514E-01**
WFG4		5.9929E+05	**6.1767E+05**	**1.3301E+02**	4.5438E+09	**2.0410E+00**	2.0628E+00
WFG5		6.0194E+05	**6.2018E+05**	2.0522E+02	**1.8461E+02**	**2.0456E+00**	2.0481E+00
WFG6		5.4668E+05	**5.6211E+05**	9.7861E+01	**8.4634E+01**	2.0245E+00	**2.0205E+00**
WFG7		5.7509E+05	**5.9665E+05**	9.8455E+01	**8.4844E+01**	2.0327E+00	**2.0233E+00**
WFG8		7.3226E+05	**7.3653E+05**	**4.8377E+09**	2.3037E+11	**2.1442E+00**	2.2305E+00
WFG9		**5.6265E+05**	5.6151E+05	**1.7632E+02**	5.3218E+04	2.1393E+00	**2.1357E+00**

Regarding s-energy values, the ensemble outperformed the original algorithm in 20 test problems, whereas the original algorithm obtained better values in 24 test problems. Finally, the ensemble improved the results obtained in 45 problems when comparing IGD values, while the original algorithm only had better results in 7 test problems.

From these results, we can observe that the use of our ensemble does improve the performance of NSGA-III in more than half of the test problems when comparing HV values. In particular, good results were obtained in problems with 5 and 6 objectives (there were improvements in 68% of the test problems adopted) and in problems with 2 and 3 objectives (improvement of 62%). On the other hand, the worst performing scenarios were the problems with 4 and 7 objectives (43% and 50% of the test problems improved, respectively). Regarding the test problems adopted, the WFG test suite was the one with the largest number of problems improved (around 64%), while the DTLZ test problems had a smaller improvement (52%).

Regarding the IGD values, a similar behavior was obtained, being the problems with 6 and 2 objectives the ones with a larger number of problems improved (68% and 62%, respectively). However, problems with 3 and 4 objectives were the least improved (only 31%).

Concerning the s-energy results, this was the only performance indicator for which the original algorithm obtained a better overall performance than the ensemble. The only exceptions are problems with 2, 4 and 6 objectives, where the ensemble improved more problems than the original algorithm.

VI. Conclusions and Future Work

We proposed here the use of an ensemble of mating restrictions to improve the performance of MOEAs. In particular, we implemented this ensemble in NSGA-III and conducted an experimental validation to explore its effect in solving a series of MOPs with up to seven objective functions. Experimental results in which the HV and the IGD performance indicators were adopted, seem to confirm that the use of our ensemble improves NSGA-III's convergence, hence improving the obtained results.

On the other hand, even though the s-energy is the diversity metric adopted in the mating restriction mechanism, it is not directly minimized throughout the algorithm's execution, resulting in the overall diversity of the population not being particularly improved. Thus, future work may involve refining the ensemble mechanism to improve diversity as well.

Acknowledgment

The first author acknowledges support from CONACyT and CINVESTAV-IPN to pursue graduate studies in Computer Science. The second author gratefully acknowledges support from CONACyT grant no. 2016-01-1920 (*Investigación en Fronteras de la Ciencia 2016*) and from a SEP-Cinvestav grant (application no. 4).

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