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A TUTORIAL ON MULTI-OBJECTIVE OPTIMIZATION USING METAHEURISTICS

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Abstract. This paper provides an overview of the use of metaheuristics for solving multi-objective optimization problems. The metaheuristics discussed include multi-objective evolutionary algorithms (going from the early approches to the most recent research trends in that area), multi-objective particle swarm optimizers, multi-objective artificial immune systems, multi-objective ant colony systems and multi-objective scatter search. In the final part of the paper, we provide a review of sample applications of multi-objective metaheuristics, and a discussion of some of the topics in which more research is required.

Keywords: multi-objective optimization, metaheuristics.

AMS classification: 90C29, 65K10.

§1. Introduction

A wide variety of problems in engineering and other disciplines have two or more objectives which we wish to minimize simultaneously. Such objectives are normally in conflict with each other (at least partially) and tend to be expressed in different units. These problems are called multi-objective and their solution requires a different notion of optimality that aims for the best possible trade-offs among the objectives (i.e., solutions for which no objective can be improved without worsening another one). For dealing with these problems, it is common to rely on the so-called *Pareto optimality* [92]. This definition gives rise to several compromise solutions, called the *Pareto optimal set*. The objective function values corresponding to the elements of the Pareto optimal set constitute the so-called *Pareto front*.

The algorithms for solving multi-objective optimization problems which are currently available in the mathematical programming literature [87] have a number of limitations, including the facts that some of them have a fairly limited applicability and that others need, in many cases, of problem specific information (e.g., derivatives). Additionally, some of those methods can be easily trapped in local Pareto optimal solutions when dealing with complex search spaces. This has motivated the use of alternative approaches, from which metaheuristics have gained an increasing popularity in the last few years. The main reasons for this popularity are their ease of use and their effectiveness to deal with a wide variety of problems, requiring little or no problem-specific information. Within the many types of metaheuristics currently available, evolutionary algorithms are, with no doubt, the most popular choice [19].[1]

[1] The author maintains the EMOO repository, which currently contains over 4800 bibliographic references on this topic. The EMOO repository is available at: `http://delta.cs.cinvestav.mx/~ccoello/EMOO/`

The remainder of this paper is organized as follows. In Section 2, we provide some basic concepts necessary to understand the rest of the paper. Section 3 is devoted to multi-objective evolutionary algorithms which are, the most popular multi-objective metaheuristic (MOMH) in current use. In Section 4, we talk about four more MOMHs that are relatively popular in the specialized literature. Section 5 summarizes some of the main applications of MOMHs. Two research topics that deserve further exploration are briefly discussed in Section 6. Finally, our conclusions are provided in Section 7.

§2. Basic Concepts

We are interested in solving problems of the type[2]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

Therefore, our aim is to obtain the Pareto optimal set from $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). It is worth indicating, however, that in practice, to obtain all

[2]Without loss of generality, we will assume only minimization problems.

the elements of the Pareto optimal set is normally undesirable and could also be impossible. Thus, our main goal when using a MOMH is to generate a good approximation of the Pareto optimal set (i.e., containing solutions that, when mapped in objective function space, are as close as possible from the true Pareto front of the problem) and having a good distribution (i.e., also in objective function space).

§3. Multi-Objective Evolutionary Algorithms

Since evolutionary algorithms are, by far, the most popular metaheuristic that has been used for solving multi-objective optimization problems, we will devote this entire section to them.

3.1. The Early Days

Evolutionary algorithm (EA) is a generic term used to denote several metaheuristics inspired on the "survival of the fittest" principle from Darwin's evolutionary theory. Their origins can be traced back to the 1960s [68, 100, 49], and have been found to be quite effective in solving a wide variety of complex search, classification and optimization problems [41].

EAs are particularly suitable for solving multi-objective optimization problems because of their capability to operate on a set of solutions (the *population*) at each iteration, which allows them to generate several trade-off solutions in a single run. They have also become popular because of their ease of use and generality (i.e., EAs are less susceptible to the shape and continuity of the Pareto front of a problem than mathematical programming techniques) [19].

It is worth indicating than traditional EAs require some modifications in order to deal with multi-objective optimization problems. The main two are the following:

1. All the nondominated solutions should be considered equally good by the selection mechanism. This means that a different notion of fitness is required for dealing with multi-objective optimization problems. The most popular mechanism to deal with this problem is called Pareto ranking and was introduced by Goldberg [59]. This approach assigns a rank to each solution based on its Pareto dominance, such that nondominated solutions are all sampled at the same rate.

2. EAs tend to converge to a single solution if run long enough, because of stochastic noise [59]. Therefore, a mechanism to maintain diversity is required. This component is known as the *density estimator*. Fitness sharing [60] was the earliest density estimator, but many others have been proposed over time, including clusters [122], entropy [47], adaptive grids [80] and crowding [31], among others.

The first actual implementation of a multi-objective evolutionary algorithm (MOEA) was David Schaffer's *Vector Evaluation Genetic Algorithm* (VEGA), which was introduced in the mid-1980s, mainly aimed for solving problems in machine learning [99].

In the period that goes from the second half of the 1980s to the first half of the 1990s, a few relatively simple and naive MOEAs were introduced. Most of them relied on aggregating functions (mostly linear) [104], lexicographic ordering [51], and target-vector approaches

[113]. Most of these MOEAs did not modify their selection mechanism or any other component, except for the definition of the fitness function. Most of these MOEAs would soon be forgotten.

As indicated before, Pareto ranking was proposed in Goldberg's famous book on genetic algorithms [59]. However, he only provided an informal description of this new selection mechanism but no actual implementation. This gave rise to several MOEAs based on Goldberg's proposal. The three most representative of the early days of MOEAs are:

1. **Nondominated Sorting Genetic Algorithm** (NSGA): This algorithm was proposed by Srinivas and Deb [103] and was the first MOEA to be published in a specialized journal (*Evolutionary Computation*). The NSGA is based on several layers of classification of the individuals as suggested by Goldberg. Before selection takes place, the population is ranked on the basis of nondominance: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. Fitness sharing is used to distribute the population along the Pareto front of the problem.

2. **Niched-Pareto Genetic Algorithm** (NPGA): Proposed by Horn et al. [69]. It uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is the following: Two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. When both competitors are either dominated or nondominated (i.e., there is a tie), the result of the tournament is decided through fitness sharing [60]. In [45], a revised version of this algorithm, called NPGA 2 was proposed. This algorithm relies on a traditional Pareto ranking approach (similar to Fonseca and Fleming's MOGA [50]), but it keeps its tournament selection scheme. Ties are solved through fitness sharing as in its predecessor. However, the niche count of the NPGA 2 is computed using individuals from the next partially filled generation using a technique called "continuously updated fitness sharing" [91].

3. **Multi-Objective Genetic Algorithm** (MOGA): Proposed by Fonseca and Fleming [50]. Here, the rank of a certain individual corresponds to the number of individuals in the current population by which it is dominated. Consider, for example, an individual x_i at generation t, which is dominated by $p_i^{(t)}$ individuals in the current generation. The rank of an individual is given by [50]:

$$\text{rank}(x_i, t) = 1 + p_i^{(t)} \tag{4}$$

All nondominated individuals are assigned rank 1, while dominated ones are penalized according to the population density of the corresponding region of the trade-off surface.

Fitness assignment is performed in the following way [50]:

(a) Sort population according to rank.

(b) Assign fitness to individuals by interpolating from the best to the worst in the way proposed by Goldberg [59], according to some function, usually linear, but not necessarily.

(c) Average the fitnesses of individuals with the same rank, so that all of them are sampled at the same rate. This procedure keeps the global population fitness constant while maintaining appropriate selective pressure, as defined by the function used.

From these 3 algorithms, a few comparative studies undertaken during the mid and late 1990s, indicated that MOGA was the most effective and efficient approach, followed by the NPGA and by the NSGA [22, 108]. It is worth indicating that during the early days (up to the end of the 1990s), most papers would compare MOEAs without using performance measures, but only in a graphical way (plotting the Pareto fronts generated by each MOEA).

3.2. Elitist MOEAs

Towards the end of the 1990s, elitism became a standard mechanism to be provided into any MOEA. The Strengh Pareto Evolutionary Algorithm (SPEA) [118] played a key role in popularizing elitism, since it adopted an external population, and its publication in a specialized journal (the *IEEE Transactions on Evolutionary Computation*), quickly made it a landmark in the field. Consequently, many researchers started to incorporate external populations in their MOEAs, popularizing this mechanism. There are, however, also theoretical reasons for which elitism is a required mechanism in MOEAs (see [97]). Elitism consists of retaining the best solutions found during the search so that they are subject to crossover or mutation. In the context of multi-objective optimization, elitism usually (although not necessarily) refers to the use of an external population (also called secondary population) to retain the nondominated individuals found during the search. External archives can be unbounded but, mainly because of practical reasons, they are normally bounded. Another mechanism that can be used instead of an external archive is the so-called $(\mu + \lambda)$-selection in which parents compete with their children and those which are nondominated (and possibly comply with some additional criterion such as providing a better distribution of solutions) are selected for the following generation. This sort of selection is implicitly elitist, because it will retain the best half of the individuals under consideration.

With the advent of elitist MOEAs, performance measures started to become popular in the specialized literature [27, 119, 109]. It has been found, however, that some of these performance measures are not Pareto-compliant and can provide no reliable assessment [124]. There are also several benchmarks for testing new MOEAs, from which the most popular are: the Zitzler-Deb-Thiele (ZDT) test suite [119], the Deb-Thiele-Laumanns-Zitzler (DTLZ) test suite [32] and the Walkig-Fish-Group (WFG) test suite [70].

The three following approaches are representative of the elitist MOEAs in common use nowadays:

1. **Strength Pareto Evolutionary Algorithm** (SPEA): This algorithm was introduced in [118], and was conceived as a way of integrating different MOEAs. It uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a *strength* value is computed. This strength is similar to the ranking value of MOGA [50], since it is proportional to the number of solutions to which a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. The fitness assignment process of SPEA considers both closeness to the true Pareto front and even distribution of solutions at the same time. Thus, instead of using niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external nondominated set. In fact, since the external nondominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, the authors decided to adopt a technique that prunes the contents of the external nondominated set so that its size remains below a certain threshold (a clustering technique called "average linkage method" [88] was used for that sake). There is a revised version of SPEA, called SPEA2, which has three main differences with respect to its predecessor [121]: (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

2. **Pareto Archived Evolution Strategy** (PAES): This algorithm was introduced in [81]. It consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive (the elitist mechanism) that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. An interesting aspect of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).

3. **Nondominated Sorting Genetic Algorithm II** (NSGA-II): This approach was introduced in [29, 31] as an improved version of the NSGA [103]. In the NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, the NSGA-II uses a crowded-

comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The NSGA-II combines the best parents with the best offspring obtained (i.e., a $(\mu + \lambda)$-selection), instead of using an external archive. Due to its clever mechanisms, the NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good, that it has become very popular in the last few years, becoming a landmark against which other MOEAs have to be compared.

Although many other MOEAs exist (see for example [20, 101, 115]), it is not the intention of this paper to be comprehensive. The interested reader may refer to [19, 28, 105] for more information on this topic.

3.3. Current Trends in MOEAs

During some time, the use of relaxed forms of Pareto dominance became popular as a mechanism to regulate convergence of a MOEA. From these mechanisms, ϵ-dominance was, with no doubt, the most popular [85]. ϵ-dominance allows to control the granularity of the approximation of the Pareto front obtained. As a consequence, it is possible to accelerate convergence using this mechanism (if we are satisfied with a very coarse approximation of the Pareto front). Several MOEAs incorporated ϵ-dominance in their external archives (see for example [33, 65]), and there was even one MOEA fully developed around this concept (see [30]).

However, the main current research trend regarding algorithmic development is to adopt a performance measure in the selection scheme of a MOEA (hypervolume[3] has been the most popular). See for example:

- **Evolution Strategy with Probability Mutation (ESP)**: This approach uses a hypervolume-based, scaling independent, parameterless measure, to truncate overpopulated external archives [71].
- **Indicator-Based Evolutionary Algorithm (IBEA)**: This is a framework that allows any performance indicator to be incorporated into the selection mechanism of a MOEA [120]. Its authors tested it with the hypervolume and with the binary ϵ indicator.
- **S Metric Selection Evolutionary Multiobjective Algorithm (SMS-EMOA)**: This approach is based on the hypervolume performance measure [42, 9].
- **Set Preference Algorithm for Multiobjective optimization (SPAM)**: This can be seen as a generalization of IBEA which allows the use of any sort of set preference relation [123].

The use of hypervolume has some advantages, from which the main one is that it has been proved that the maximization of this performance measure is equivalent to finding the

[3] The hypervolume (also known as the S metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively.

Pareto optimal set [48]. Additionally, empirical studies have shown that (for a certain number of points previously determined) the maximization of the hypervolume does indeed produce subsets of the Pareto front which are well-distributed [82, 42]. Furthermore, hypervolume measures convergence and, to a certain extent, also the spread of solutions along the Pareto front. Finally, it has been shown that hypervolume-selection is less sensitive to scalability in objective function space, which makes it promising to deal with problems having many objectives [75].

Hypervolume has, however, some problems of its own. First, the computation of this performance measure depends on a reference point, which can influence the results in a significant manner. Some people have proposed to use the worst objective function values in the current population, but this requires scaling of the objectives. Its main drawback, however, is that the best algorithms known to compute hypervolume have a polynomial complexity on the number of points used, but such complexity grows exponentially on the number of objectives. This has triggered an important amount of efforts aimed to produce more efficient algorithms to approximate the hypervolume [112, 10, 8, 7, 13].

§4. Other Metaheuristics

Several other metaheuristics have also been used as multi-objective optimizers [19, 23]. Next, we will discuss four of the most popular of them in more detail:

- **Particle Swarm Optimization**: This metaheuristic was inspired on the choreography of a bird flock which aim to find food [78, 79]. The implementation of this algorithm employs a population of particles, whose behavior is affected by either the best local (i.e., within a certain neighborhood) or the best global individual (i.e., with respect to the entire swarm). Particle swarm optimization (PSO) has been successfully used for both continuous nonlinear and discrete binary optimization [43, 44]. An important number of multi-objective versions of PSO currently exist (see for example [21, 94, 95, 46]). However, until relatively recently, most of the research had concentrated on producing new variations of existing algorithms, rather than on studying other (more interesting) topics, such as the role of the main components of a multi-objective particle swarm optimizer. Some recent research in that direction has shown that certain components that had been traditionally disregarded (e.g., the leader selection mechanism and the parameters of the flight formula) play a key role in the performance of a multi-objective particle swarm optimizer [12, 107]. There are also other interesting comparative studies aimed to identify their advantages and limitations [39]. It is expected that more research of this sort will be conducted in the next few years.

- **Artificial Immune Systems**: If considered from a computational point of view, our natural immune system can be considered a distributed intelligent system, which is able to learn and retrieve knowledge previously acquired, in order to solve several (highly complex) recognition and classification tasks [89]. These features make our immune has motivated researchers to develop computational models of our immune system which have been used for a variety of tasks, including classification, pattern recognition, and optimization [26, 89]. Several multi-objective extensions of artificial immune systems have been proposed in the specialized literature (see for example

[17, 53, 14, 52]). Also, several hybrid approaches have been proposed to solve specific tasks (see for example [2], where the authors use a multi-objective immune system hybridized with evolutionary operators and local search, in order to solve a rule extraction problem). More hybrid approaches are still to come, but until now, the high potential of multi-objective artificial immune systems in classification and pattern recognition tasks has not been fully exploited yet [117].

- **Ant Colony Optimization**: This metaheuristic was inspired on the foraging behavior of real ants. It is a distributed, stochastic search procedure based on the indirect communication of a set (called "colony") of artificial ants, which mediate using artificial pheromone trails. These pheromone trails can be seen as distributed information which is used by the ants to construct their solutions to the problem at hand. Such pheromone trails are modified during the algorithm's execution, such that they reflect the search experience acquired by the ants. This metaheuristic is intended for solving difficult (both static and dynamic) combinatorial optimization problems, in which solutions can be generated through the use of a construction procedure [36, 37]. There are several multi-objective extensions of ant colony optimization (ACO) algorithms (see for example [66, 35, 55, 1]), but as multi-objective combinatorial optimization becomes more attractive for researchers [40, 54], it is expected that more multi-objective ant colony optimization approaches (and hybrids of ACO algorithms with other metaheuristics) are proposed within the next few years.

- **Scatter Search**: This approach was originally conceived as an extension of a heuristic called surrogate constraint relaxation, which was designed for solving integer programming problems [56]. Its core idea is to adopt a series of different initializations to generate solutions. A reference set of solutions (the best found so far) is adopted, and then such solutions are "diversified" in order to generate new solutions within the neighborhood of the contents of the reference set. This sort of simple procedure is repeated until no further improvements to the contents of the reference set are detected. In the mid-1990s, some further mechanisms were added to the original scatter search algorithm, which allowed its extension to solve nonlinear, binary and permutation optimization problems [57]. These new applications triggered an important amount of research in the following years [83, 86]. Multi-objective extensions of scatter search are relatively recent, but have been steadily increasing [4, 90]. Scatter search has a lot of potential for hybrid approaches, such as memetic MOEAs [58], since it can act as a powerful local search engine for tasks such as generating missing parts of a Pareto front [98]. Because of its flexibility and ease of use, scatter search is expected to become more commonly adopted in the near future, particularly when designing hybrid MOEAs that rely heavily on good local search engines.

§5. Applications

MOEAs have been applied to a wide variety of domains (see for example [18]). However, and for the purposes of this paper, we can roughly classify the applications of MOEAs into three large groups: engineering, industrial and scientific. Some specific areas within each of these groups are indicated next.

We will start with the engineering applications, which are, by far, the most popular in the literature. This should not be surprising, since engineering disciplines normally have problems with better understood mathematical models, which makes them more suitable for the use of MOEAs. Some sample applications of MOEAs in engineering are the following:

- Electrical engineering [111]
- Hydraulic engineering [5]
- Structural engineering [15]
- Aeronautical engineering [93]
- Robotics [106]
- Control [114]
- Telecommunications [24]
- Civil engineering [38]
- Transport engineering [61]

Now, we will provide some applications of MOEAs in industry:

- Design and manufacture [62]
- Scheduling [64]
- Management [72]

Finally, we have a variety of scientific applications of MOEAs:

- Chemistry [34]
- Physics [96]
- Medicine [125]
- Geography [6]
- Bioinformatics [3]
- Computer science [102]

Although small, this sample should give a good idea of the type of work being done with MOEAs these days. Nevertheless, many other applications exist. The interested reader should refer to the EMOO repository [16] for more information on this topic.

§6. Future Research Paths

In spite of the high volume of research done around multi-objective metaheuristics, several interesting topics remain to be explored in greater depth. Next, we briefly discuss two of them:

1. **Hybridization**: The hybridization of MOMHs with other metaheuristics and with local search mechanisms (either gradient-based or not) aimed to improve their performance is a topic that is currently being explored by many researchers (see for example [63, 67, 76, 84]), because of its high potential. Hybrid MOMHs could be viable alternatives for solving some of the great challenges of today, such as many-objective optimization problems (i.e., problems having 4 or more objective functions) [73, 74]. The use of scalarization methods combined with MOMHs is also another type of interesting hybridization that has a lot of potential for solving highly complex problems (see for example [116]).

2. **Incorporation of user's preferences**: In most real-world applications, users are not interested in the entire Pareto front, but only in a portion of it. Several mechanisms to incorporate user's preferences into a MOMH have been reported in the specialized literature (see for example [25, 77, 110, 11]), but this topic has only been scarcely explored and certainly deserves more attention.

§7. Conclusions

In this paper, we have provided a short (and highly compact) tutorial on the use of metaheuristics for solving multi-objective optimization problems. As such, this tutorial provides a very general overview of the field and is intended to serve as a quick reference for those interested in in this area. The author hopes that, in spite of favoring breadth over depth, this tutorial can be useful for those wishing to do research in multi-objective optimization using metaheuristics, since that has been the purpose of this work.

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