EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION USING A FUZZY-BASED DOMINANCE CONCEPT

Eduardo Fernandez[1], Edy Lopez[1], Sergio Bernal[2], Carlos Coello[3], Jorge Navarro[1]

[1] Autonomous University of Sinaloa, Culiacan, Mexico, eddyf@uas.uasnet.mx; [2] Cornell University, Ithaca, USA; [3] Departamento de Computación, CINVESTAV-IPN, Mexico

Abstract

One aspect that is often disregarded in evolutionary multiobjective research is the fact that the solution of a problem involves not only search but decision making. Most of approaches concentrate on adapting an evolutionary algorithm to generate the Pareto frontier. In this work we present a new idea to incorporate preferences in MOEA. We introduce a binary fuzzy preference relation that expresses the degree of truth of the predicate "***x*** is at least as good as ***y***". On this basis, a strict preference relation with a reasonable high degree of credibility can be established on any population. An alternative ***x*** is not strictly outranked if and only if there does not exist an alternative ***y*** which is strictly preferred to ***x***. It is easy to prove that the best solution is not strictly outranked. We used the Nondominated Sorting Genetic Algorithm II (NSGA-II), but replacing dominance by the above non-outranked concept. So, we search for the no- strictly outranked frontier that is a proper subset of the Pareto frontier. In several instances of a nine-objective knapsack problem our proposal clearly outperforms the standard NSGA-II, achieving non-outranked solutions which are in an obvious privileged zone of the Pareto frontier.

Key Words: multicriteria optimization; evolutionary algorithms; fuzzy preferences.

1. Introduction

In real-world optimization problems the decision-maker (DM) is usually concerned with several criteria which determine the quality of solutions. Often, constraints in mathematical programming problems are not actually mandatory; rather such restrictions are expressing an important desire, a significative DM aspiration level about certain system properties. Therefore, most optimization problems can be represented from a multiple objective perspective.

As a consequence of the conflicting nature of criteria, it is not possible to obtain a common optima, so the ideal solution of a multiobjective problem (MOP) cannot be reached. Hence, to solve a MOP means to find the best compromise solution according to the DM's particular system of preferences (value system). It is easy to prove that the best compromise is a non-dominated solution, a member of the Pareto set. Most operational research methods for MOPs can be classified into the following categories [1]:

1. Techniques which perform a prior articulation of DM's preferences;
2. Interactive methods, which perform a progressive articulation of DM' preferences;
3. Generating techniques, which perform a posteriori articulation of preferences (search before making decisions).

Since the Schaffer's seminal work (cf.[2]), the Multiple Objective Evolutionary Algorithms (MOEAs) have become into a very popular paradigm for solving multiobjective programming problems. MOEAs are very attractive to solve MOPs because those deal simultaneously with a set of possible solutions (the MOEA population) which allows to obtain an approximation of the Pareto frontier in a single algorithm run. Thus, by using MOEAs the DM and/or the decision analyst have not to perform a set of separate single criterion optimization runs as in the case of established operational research methods. Besides, MOEAs are more robust respect to the shape or continuity of the Pareto front, whereas these two issues are a real concern for classical optimization methods (cf.[3]). However, one aspect that is often disregarded in evolutionary multiobjective research is the fact that the solution of a problem involves not only search but decision making. Most of approaches concentrate on adapting an evolutionary algorithm to generate the Pareto frontier. Nevertheless, to find this set does not solve the problem. The DM still has to choose the best compromise solution out of that set. It is not a hard task in problems with 2-3 objectives. But when the number of criteria increases, two important difficulties arise:

a) The algorithm's capacity to converge to this Pareto frontier is degraded;
b) It becomes harder, or even impossible for the DM to establish valid judgments in order to compare options with several conflicting criteria.

Here, we propose a combined approach, with a prior articulation of preferences followed by a generating process of a privileged zone of the Pareto frontier. Using a fuzzy outranking relation, a strict preference relation in the sense of [4] can be established in any population. Our proposal is based on finding a subset of Pareto frontier composed of solutions for which do not exist other solutions preferred to the first ones. This non-outranked concept will be used instead of dominance when the evolutionary search is performed.

2. An Outranking Model of Preferences

Let G be a set of independent criteria and O the objective space. An element $\boldsymbol{x} \in O$ is a vector $(g_1, \dots g_n)$, where g_i is the *i-th* objective value. Let us suppose that for each criterion j there is a relational system of preferences (P_j, I_j) (preference, indifference) which is complete on the domain of *j-th* criterion (G_j). That is, $\forall(g_j(\boldsymbol{x}), g_j(\boldsymbol{y})) \in G_j \times G_j$ one and only one of the following statements is true:

$$\begin{aligned} &- g_j(\boldsymbol{x}) P_j\, g_j(\boldsymbol{y}) \\ &- g_j(\boldsymbol{y}) P_j\, g_j(\boldsymbol{x}) \\ &- g_j(\boldsymbol{x}) I_j\, g_j(\boldsymbol{y}) \end{aligned} \qquad (1)$$

Formulation (1) allows indifference thresholds in order to model some kind of imprecise one-dimensional preferences. It should be noticed that the relational system of preferences given by (1) is more general than usual formulations which consider only true criterion (that is, $g_j(\boldsymbol{x}) \neq g_j(\boldsymbol{y})$ implies non-indifference).

Let us establish the following central premise: For each $(\boldsymbol{x},\boldsymbol{y}) \in O \times O$, the *DM* and the decision analyst (working together) are able to create a fuzzy predicate modeling the degree of truth of the statement "***x*** is at least as good as ***y*** from the DM point of view".
Amongst different ways to create that predicate, we shall describe below an outranking approach based on ELECTRE methods:
A proposition ***xSy*** ("***x*** outranks ***y***") ("***x*** seems at least as good as ***y***") holds if and only if the coalition of criteria in agreement with this proposition is strong enough and there is no important coalition discordant with it (cf.[5]). It can be expressed by the following logical equivalence (cf.[6]):

$$\boldsymbol{x}S\boldsymbol{y} \Leftrightarrow C(\boldsymbol{x},\boldsymbol{y}) \wedge \sim D(\boldsymbol{x},\boldsymbol{y}) \qquad (2)$$

where:
$C(\boldsymbol{x},\boldsymbol{y})$ is the predicate about the strength of concordance coalition;
$D(\boldsymbol{x},\boldsymbol{y})$ is the predicate about the strength of discordance coalition;
$\wedge$ and $\sim$ are logical connectives for conjunction and negation, respectively.
Let $c(\boldsymbol{x},\boldsymbol{y})$ and $d(\boldsymbol{x},\boldsymbol{y})$ denote the degree of truth of the predicates $C(\boldsymbol{x},\boldsymbol{y})$ and $D(\boldsymbol{x},\boldsymbol{y})$. From (2), the degree of truth of ***xSy*** can be calculated as in ELECTRE-III method:

$$\sigma(\boldsymbol{x},\boldsymbol{y}) = c(\boldsymbol{x},\boldsymbol{y}).\, N(d(\boldsymbol{x},\boldsymbol{y})) \qquad (3)$$

where $N(d(\boldsymbol{x},\boldsymbol{y}))$ denotes the degree of truth of the non-discordance predicate .
As in the earlier versions of ELECTRE methods, we shall take

$$c(\boldsymbol{x},\boldsymbol{y}) = \sum_{j \in C_{x,y}} w_j \qquad (4)$$

where $C_{x,y} = \{j \in G$ such that $x_j P_j y_j \vee x_j I_j y_j\}$ and w's denote "weights" ($w_1 + w_2 + \ldots + w_n = 1$).
Let $D_{x,y} = \{j \in G$ such that $y_j P_j x_j\}$ be the discordance coalition with ***xSy.*** The intensity of discordance is measured in comparison with a veto threshold v_j, which is the maximum difference $g_j(\boldsymbol{y}) - g_j(\boldsymbol{x})$ compatible with $\sigma(\boldsymbol{x},\boldsymbol{y}) > 0$. Following Mousseau and Dias ([7]), we shall use here a simplification of the original formulation of the discordance indices in ELECTRE-III method which is given by:

$$Nd(\boldsymbol{x},\boldsymbol{y}) = \min_{j \in D_{x,y}} [1 - d_j(\boldsymbol{x},\boldsymbol{y})] \qquad (5)$$

$$d_j(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} 1 \quad \text{iff } \nabla_j \geq v_j \\ (\nabla_j - u_j)/(v_j - u_j) \quad \text{iff } u_j < \nabla_j < v_j \\ 0 \quad \text{iff } \nabla_j \leq u_j \end{cases} \qquad (6)$$

where $\nabla_j = g_j(\boldsymbol{y}) - g_j(\boldsymbol{x})$ and u_j is a discordance threshold (see Figure 1).

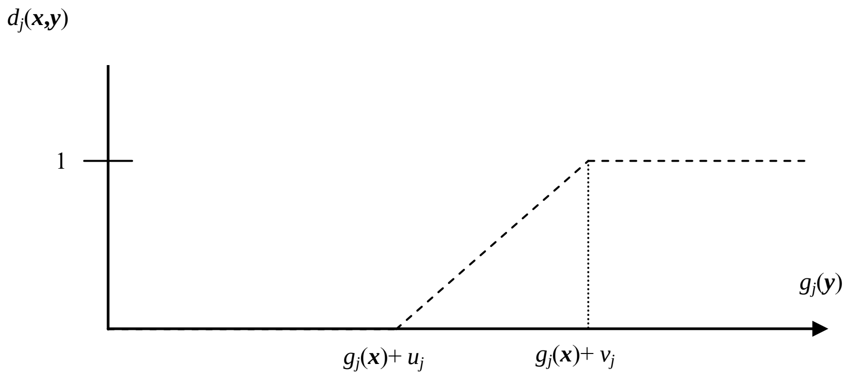

Figure 1 Partial discordance relation $d_j(\boldsymbol{x},\boldsymbol{y})$

The λ-cut $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda$ defines a crisp outranking relation ***xSy***. Credible outranking statements are obtained with λ= 0.75 (strong outranking), even λ==0.67 (weak outranking) [8]. $\sigma(\boldsymbol{x},\boldsymbol{y}) >\approx 0.5$ is identified as a doubtful outranking, and $\sigma(\boldsymbol{x},\boldsymbol{y}) < 0.5$ means a definitive no outranking.
According to Roy (cf.[4])
$\boldsymbol{x}S\boldsymbol{y} \wedge \boldsymbol{y}{\sim}S\boldsymbol{x} \Leftrightarrow \sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < \lambda \Rightarrow$ a presumed preference favoring ***x***.
Following Fernandez et al. ([9]), we assume the existence of a threshold β>0 such that if $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda$ and also $\sigma(\boldsymbol{y},\boldsymbol{x}) \leq (\lambda-\beta)$, then there is an asymmetric preference relation favoring ***x*** that will be denoted by $\boldsymbol{x}P(\lambda,\beta)\boldsymbol{y}$. It can be agreed that for some values of λ and β, the conditions defining P(λ,β) are good arguments for justifying a strict preference relation in the sense proposed by Roy ([4]). β may be a function of λ. In the following we consider that P(λ,β) has been defined on *O*.
Amongst different ways of defining a reasonable strict preference relation we suggest which follows:
$\boldsymbol{x}P(\lambda,\beta)\boldsymbol{y}$ if one of the following propositions is held:
i. ***x*** dominates ***y***
ii. $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq 0.75 \wedge (0.67 \leq \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.75) \wedge (\sigma(\boldsymbol{x},\boldsymbol{y}) - \sigma(\boldsymbol{y},\boldsymbol{x})) \geq \beta$

iii. $\sigma(\boldsymbol{x},\boldsymbol{y})\geq 0.75 \wedge (0.5 \leq \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.67)$
iv. $\sigma(\boldsymbol{x},\boldsymbol{y})\geq 0.75 \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.5$
v. $(0.67 \leq \sigma(\boldsymbol{x},\boldsymbol{y}) < 0.75) \wedge (0.5 \leq \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.67) \wedge (\sigma(\boldsymbol{x},\boldsymbol{y}) - \sigma(\boldsymbol{y},\boldsymbol{x}))\geq\beta$
vi. $(0.67 \leq \sigma(\boldsymbol{x},\boldsymbol{y}) < 0.75) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.5$
vii. $(0.5 < \sigma(\boldsymbol{x},\boldsymbol{y}) < 0.67) \wedge (0.5 \leq \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.67) \wedge (\sigma(\boldsymbol{x},\boldsymbol{y}) - \sigma(\boldsymbol{y},\boldsymbol{x}))> \beta$
viii. $(0.5 < \sigma(\boldsymbol{x},\boldsymbol{y}) < 0.6) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.5 \wedge (\sigma(\boldsymbol{x},\boldsymbol{y}) - \sigma(\boldsymbol{y},\boldsymbol{x}))\geq 2\beta$

Definition 1. $\boldsymbol{x}$ strictly outranks $\boldsymbol{y}$ iff $\boldsymbol{x}P(\lambda,\beta)\boldsymbol{y}$.
Definition 2: Let *A* be a subset of *O*. If there does not exist $\boldsymbol{y}\in A$ such that $\boldsymbol{y}P(\lambda,\beta)\boldsymbol{x}$, we say that $\boldsymbol{x}$ is a non-strictly outranked solution in *A*.
Definition 3: $P(\lambda,\beta)$ is said to be free of inconsistencies iff there is no cycles of that relation in *O*.
Definition 4: $P(\lambda,\beta)$ is said to be minimally free of inconsistencies iff there does exist at least one non-strictly outranked solution in *O*.
Definition 5: For an option $\boldsymbol{x}$, the strictly outranking set is defined as $S_o = \{\boldsymbol{y} \in O$ such that $\boldsymbol{y}P(\lambda,\beta)\boldsymbol{x}\}$.
Definition 6: The weakness of $\boldsymbol{x}$ in a set *A* is $W_e = card\ \{\boldsymbol{y} \in A$ such that $\sigma(\boldsymbol{y},\boldsymbol{x}) > \sigma(\boldsymbol{x},\boldsymbol{y}) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) \geq 0.5\}$.
Definition 7: The strength of $\boldsymbol{x}$ in a set *A* is $W_e = card\ \{\boldsymbol{y} \in A$ such that $\sigma(\boldsymbol{x},\boldsymbol{y}) > \sigma(\boldsymbol{y},\boldsymbol{x}) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) \geq 0.5\}$
Fernandez et al. ([9]) proved that the best alternatives in a set should be found among those in which card(S_o) is minimal. Suppose that $P(\lambda,\beta)$ is minimally free of inconsistencies Hence, the best compromise solution should be a non-strictly outranked solution in *O.* When all solution is strictly outranked by another one, the best compromise should be found among the set with minimum card(S_o).

3. **Adapting NSGA-II to Work with Non-strictly Outranked Classes**

We shall extend the Non-dominated Sorting Genetic Algorithm II (cf.[10]) working with non-strictly outranked individuals instead of non-dominated ones. The "filtering" process is similar, but extracting non-strictly outranked individuals which form classes with the same value of card(S_o). The first front may have card(S_o) $\neq 0$ when $P(\lambda,\beta)$ is fully inconsistent.
Unlike typical MOEAs, we are not interested in obtaining a uniform distribution of solutions representing the Pareto frontier. Therefore, instead of the NSGA-II crowding distance (or other niching operator), we propose to use the above weakness measure. That is, when two individual with equal card(S_o) are compared (in binary tournaments or deciding who will be included into the new generation), the less weaker will be preferred.
This adapted algorithm will be called by Non-Outranked- Sorting Genetic Algorithm (NOSGA), whose pseudocode is presented below:

```
Generate random population (size K)
    Evaluate Objective Values
    Generate fronts of equal values of card(So)
    Assign to these fronts Rank Based on card(So)
    Keep the best front (Rank) in the population memory
    Generate Offspring Population
        Binary Tournament Selection
        Crossover and Mutation
For i = 1 to Number of Generations
    With Parent and Offspring Population
        Generate fronts of equal values of card(So)
        Assign to these fronts Rank Based on card(So)
        Loop (inside) by adding solutions to next generation
        starting from the best front until K individuals found
     Update the population memory
     Select points (elitist) on the better front (with better Rank)
     Form next generation
        Binary Tournament Selection
        Crossover and Mutation
        Increment generation index
End of Loop
```

4. Some Computer Experiments

In order to validate the present proposal we have performed two tests, both nine-objective knapsack problems. The first one is a controlled experiment in which both the true Pareto frontier and the true non-strictly outranked set are known. The second one is a real size problem in which the best sets are unknown.
Let us consider a decision making situation in which the DM is choosing among L different social policies (projects) each with direct social impact. This is measured by using a nine-component vector (N_1, N_2,…. N_9). $N_i = n_{kj}$, the number of people belonging to the k-th social category which receive the j-th level benefit from that policy or project. In those examples k= 1, 2, 3 correspond to (Extreme Poverty, Poverty, Middle Class), and j = 1, 2, 3 to (High Impact, Middle Impact, Low Impact). N_1, N_2, N_3 correspond to extreme poverty people; N_7, N_8, N_9 concern middle class.
Let us denote by N_i^m the value of N_i associated to the m-th project. Let C be a portfolio. The value of N_i for the whole portfolio is $N_i(C) = x_1 N_i^1 + \ldots + x_L N_i^L$ where $x_j = 1$ if the j-th project is supported and $x_j = 0$ otherwise.

We use binary coding; a '1' in the individual j-th allele means that the j-th project belongs to this particular portfolio. Other parameters of the evolutionary search are: crossover probability = 1; mutation probability = 0.02; population size = 100.
Preference model parameters:
Weights: (23, 14, 11, 14, 11, 7, 9, 7, 4); these values express the importance of the differerent objectives.
Indifference thresholds: They are calculated as a measure of the error evaluating each objective.
Veto thresholds: They are settled as 0.5*(Max f_i - Min f_i); operations Max and Min act on a population.

4.1 The Control Test
The information about 20 candidate projects is shown in Table 1. Budget constraints are imposed by area and to the whole portfolio. The different values are given in thousands.

Table 1: Applicant projects

Project	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	Support needed	Area	Region
1	0	0	45	0	15	0	0	18	0	50,000	3	1
2	0	25	0	15	0	0	54	0	0	49,500	1	1
3	0	35	0	0	15	0	0	48	0	49,000	2	1
4	25	0	0	7.5	0	0	0	0	54	48,500	2	1
5	0	25	0	7.5	0	0	0	0	48	48,000	2	2
6	45	0	0	4.5	0	0	0	18	0	47,500	3	2
7	0	0	35	0	4.5	0	0	0	48	47,000	2	2
8	5	0	0	0	4.5	0	54	0	0	46,500	1	2
9	15	0	0	4.5	0	0	12	0	0	46,000	3	1
10	0	0	5	0	13.5	0	36	0	0	45,500	3	2
11	0	0	15	15	0	0	30	0	0	45,000	1	2
12	0	0	35	1.5	0	0	0	36	0	44,500	3	2
13	0	0	15	0	3	0	24	0	0	44,000	3	1
14	40	0	0	0	1.5	0	0	0	24	43,500	3	1
15	0	0	20	0	0	3	0	0	12	43,000	1	2
16	0	40	0	0	15	0	0	42	0	42,500	2	2
17	45	0	0	0	4.5	0	48	0	0	42,000	2	1
18	0	0	30	0	0	4.5	0	0	24	41,500	3	2
19	10	0	0	0	0	3	60	0	0	41,000	2	1
20	0	10	0	15	0	0	30	0	0	40,500	1	2

In this problem the set of feasible portfolios was exhaustively explored. This contains 1,600 non-dominated solutions and only 6 non-strictly outranked ones. These are shown in Table 2.

Table 2: Non strictly outranked portfolios

Project	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9
1	145	110	60	49.5	55.5	3	276	126	24
2	140	110	80	49.5	51	6	222	126	36
3	170	75	60	57	40.5	3	276	78	78
4	140	75	80	61.5	34.5	6	234	78	66
5	165	75	80	57	36	6	222	78	90
6	185	75	15	61.5	25.5	3	288	60	78

A single run of the standard NSGA-II (Population size = 100, mutation probability = 0.02, crossover probability = 1) found 90 non-dominated solutions. All are strictly outranked. Besides, a single run of NOSGA found in the first front the six solutions pointed-out in Table 2. This experiment was replicated several times with similar results.

4.2 An example of real size

Secondly, we solved a portfolio problem with 100 applicant projects characterized by the same nine-objective set as the previous example. In similar way, the feasible region was determined by the total budget and area requirements. The (known) non-outranked front of one random instance of this problem is shown in Table 3. The objective values are given in thousands. Weakness, strength and net flow score were calculated on the final parent-offspring population.

Table 3: Some results

Portfolio	**N_1**	**N_2**	**N_3**	**N_4**	**N_5**	**N_6**	**N_7**	**N_8**	**N_9**	**W**	**S**	**NFS**
1	820	560	725	1005	1080	840	1086	690	576	35	168	18.42
2	820	585	705	1200	930	765	1014	666	804	58	145	22.37
3	820	770	510	1140	1080	660	954	474	828	136	67	-5.38
Ideal	820	890	1000	1260	1380	1215	1248	816	1044			
Nadir	220	135	255	180	345	270	270	174	222			

W.- Weakness ; S.- Strength; NFS.- Net Flow Score

The best solutions seem to be 1 and 2. It is obvious that those solutions are concentrated in a relatively small objective space region. By using the standard NSGA-II an approximation to Pareto front was obtained for the same instance. In fact, the ideal and nadir points in Table 3 were found by NSGA-II. In the following NO_k and ND_k will denote the known non-strictly outranked and non-dominated sets respectively. Let U be $NO_k \cup ND_k$. A comparison between NO_k and ND_k was performed with the following results:

1. Each $\boldsymbol{x} \in NO_k$ is not dominated in U;
2. Each $\boldsymbol{x} \in NO_k$ remains as non-strictly outranked in U;
3. No $\boldsymbol{x} \in ND_k$ is member of NO_k ;
4. No $\boldsymbol{x} \in ND_k$ is non-strictly outranked solution when ND_k is put together the NOSGA final population.
5. After calculating $\sigma(\boldsymbol{x},\boldsymbol{y})$ on U, a ranking of this set considering weakness, strength and net flow was performed. The three solutions belonging to NO_k are the best in U.6. As shown in Table 4, the mean value of weakness, strength, and net flow scores taken on NO_k are clearly better that the respective mean values on ND_k .

From above remarks, it can be concluded that (accepting $\sigma(\boldsymbol{x},\boldsymbol{y})$ as a good model of the outranking statement degree of truth), NO_k is a preference privileged zone in the objective space. Unlike NOSGA, the best front found by NSGA-II (although may be representative of the Pareto frontier) does not contain the best compromise solutions.

Table 4: Mean Values in NO and ND

Set	Weakness	Strength	Net Flow Score
NO	3.88	86.44	58.67
ND	32.63	30.74	-5.28

5. Conclusions

In several instances of different examples our proposal (NOSGA) clearly outperforms the standard NSGA-II, achieving non-outranked solutions which are in an obvious privileged zone of the Pareto frontier. Those solutions are few, concentrated, and satisfactory. A good compromise can be easily detected on the non-outranked front. Additionally, as the overall multiobjective performance is aggregated in $\sigma(\boldsymbol{x},\boldsymbol{y})$, NOSGA shows a weak dependence on the number of objective functions.

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