Use of Genetic Algorithms for the Optimal Design of Reinforced Concrete Beams

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Abstract

In this paper, we extend an optimization model for design of rectangular reinforced concrete beams subject to a specified set of constraints. This extended model is closer to concrete design regulations and practical recipes used by experienced engineers. To solve it, we used an artificial intelligence search technique known as the genetic algorithm (GA), and our results were compared with a mathematical programming technique that deals with the non-linear equations of our model. Some of the issues involved when using GAs such as representation, parameter tuning and genetic operators are also discussed, and we present our own methodology to deal with them. A prototype of a system that follows such methodology is currently being used as a design tool in México.

1. Introduction

The design of a rectangular reinforced concrete beam is normally an interactive process in which the engineer assumes the self-weight of the element beforehand, and chooses a trial section. Then, the moment of resistance of such section is determined, to check its suitability against the given applied bending moment. This process is repeated until a trial section is found suitable. However, some difficulties normally arise when trying to match the moment of resistance of the section with the total applied bending moment due to the self-weight of the beam, which may be quite substantial in many cases. As a consequence, this procedure tends to be slow, and it has a complete lack of economics, since the only concern is to find any section suitable for the given conditions, without even considering the possibility of making it as cheap as possible. In this paper, we'll present a model for optimal design which minimizes the cost of a reinforced concrete beam based not only on the allowable stresses of the element, but also in the costs of concrete, steel and formwork. Our model follows that proposed by Chakrabarty [1] [2], with additional constraints that makes it suitable for practical applications in México. In the next section, we'll introduce some general concepts from reinforced concrete design. Then, our model will be shown and the genetic algorithm approach will be described. Finally, we'll present the results of applying our model to some problems found in the literature.

2. Basic Concepts

According to the strength design method, the nominal moment capacity M_n of a rectangular beam with tension reinforcement only is given by [3]:

$$M_n = bd^2 f_c' w(1 - 0.59w) \tag{1}$$

where b is the width of the beam, d is the distance from the extreme compressive fibre to the centroid of tension reinforcement, f_c' is the compressive strength of concrete, $w = (A_s f_y / bdf_c')$, f_y is the yield strength of reinforcement and A_s is the area of tension reinforcement. There is an infinite amount of solutions to equation (1) that yield the same value of M_n [3]. In the traditional design process, the values of b and/or d are assumed, and the remaining parameters are calculated based on them, iterating until a suitable section is found. An obvious restriction of this approach is that only a few sections can be evaluated in this manner. Since equation (1) does not incorporate any cost parameter, there is no way of achieving a least-cost design. Therefore, we need to include certain cost parameters combined with the design parameters in our optimal design model, so that we can produce least-cost suitable designs.

3. Previous Work

The optimal design of beams was first proposed by Galileo [4], even though his calculations were wrong. Apparently, the doctoral dissertation by E. J. Haug Jr. [5] in 1966 is one of the first modern attempts to use a digital computer as a tool for the optimal design of this structural element. Haug reduced the non-linear optimal design problem to a Lagrange problem in

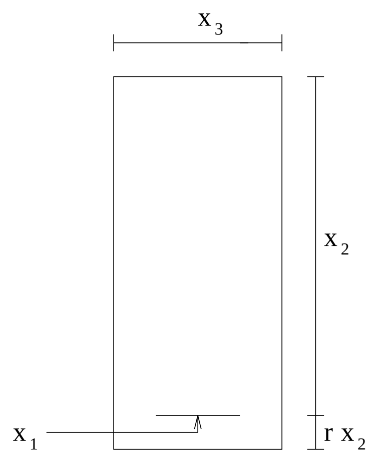

Figure 1: Schematic section of a singly reinforced rectangular beam.

the Calculus of Variations. His model includes restrictions and tries to minimize the weight of the beam in several different situations. Venkayya [6] developed a method based on an energy criteria and a search procedure for design of structures subjected to static loading. He argues that his method can handle very efficiently: (a) designs for multiple load conditions, (b) stress constraints, (c) constraints on displacements, (d) constraints on sizes of the elements. His method has been successfuly applied to the design of trusses, frames and beams. Prakash et al. [7] proposed a model for optimal design of reinforced concrete sections in which the costs of steel, concrete and formwork were included. Chakrabarty's model [1] [2] has some similarities with Prakash's model, but the former is more complete and detailed. That's the main reason why we decided to use Chakrabarty's model as a basis for our implementation, even though we had to slightly modify it in order to produce designs that fall into Mexico's standard regulations for reinforced concrete design, since the original model led in some cases to inconsistent designs.

4. The Optimal Design Model

A schematic section of a rectangular singly reinforced concrete beam is shown in Figure 1 (taken from Chakrabarty [2]). The cost per unit length of the beam will be given by the expression [2] $y(x) = c_1x_1 + c_2x_2x_3 + c_3x_2 + c_4x_3$, where $y(x)$ is the cost per unit length of the beam ($\$/cm$), c_1 is the cost coefficient due to volume of tensile steel reinforcement in the beam ($\$/cm^3$), c_2 is the cost coefficient due to volume of concrete in the beam ($\$/cm^3$), c_3 is the cost coefficient due to formwork along the vertical surfaces of the beam ($\$/cm^2$), c_4 is the cost coefficient due to formwork along the bottom horizontal surface of the beam ($\$/cm^2$), x_1 is the variable giving the area of tensile steel reinforcement as shown in Figure 1 (cm^2), x_2 is the variable giving the depth of the beam as shown in Figure 1 (cm) and x_3 is the variable giving the width of the beam, as shown in Figure 1 (cm). The variables x_1, x_2 and x_3 not only affect the cost of a beam, but will also determine its moment of resistance. Since x_1 may be calculated if we know x_2 and x_3 [3], we'll propose different values for these two variables so that the total cost of the beam is minimum, verifying at the same time that our section has a proper resistant moment. Then, our optimal design model is the following:

minimize: $f(x) = c_1x_1 + c_2x_2x_3 + c_3x_2 + c_4x_3$

subject to:

$$a_1x_1^{-1}x_3x_5 < 1 \quad (equilibrium\ constraint) \tag{2}$$

$$a_2x_4^{-1} + a_3x_2x_3x_4^{-1} < 1$$
$$(bending\ moment\ compatibility\ constraint) \tag{3}$$

$$0.25 \leq x_3/x_2 \leq 0.6$$
$$(width - height\ ratio\ constraint) \tag{4}$$

$$Q(x_2 - a_5x_5)(f_rf_c^{'}x_5x_3 + x_1f_y)a_5/x_4 \geq 1$$
$$(acting\ moment\ constraint) \tag{5}$$

$$a_6/x_3 < 1 \quad (minimum\ width\ constraint) \tag{6}$$

$$x_1, x_2, x_3, x_4, x_5 > 0 \quad (non - negativity\ constraint) \tag{7}$$

Here x_4 is a variable defining the total applied bending moment including the bending moment due to self-weight of the beam; x_5 is a variable defining the depth of the equivalent rectangular stress block. Additionally, $c_1 = w_s \times c_s$ ($\$/cm^3$), where $w_s = 0.00785\ kg/cm^3$ (assumed value) is the unit weight of steel reinforcement, and c_s is the unit cost of steel reinforcement ($\$/kg$). Similarly, $c_2 = (1+r)c_c \times 10^{-6}$ ($\$/cm^3$), where c_c is the unit cost of concrete ($\$/m^3$) and r is the cover ratio; $c_3 = 2(1+r)c_r \times 10^{-4}$ ($\$/cm^2$),where c_r is the unit cost of formwork ($\$/m^2$); $c_4 = c_r \times 10^{-4}$ ($\$/cm^2$). Also, $a_1 = 0.85f_c^{'}/f_y$, where f_y is the yield strength of steel reinforcement (N/cm^2) and $f_c^{'}$ is the compressive strength of concrete (N/cm^2); $a_3 = D(1+r)w_ckL^2$, where $D = 1.4$ (assumed) is the load factor for dead load, $w_c = 0.0228\ N/cm^3$ is the unit weight force of concrete, k is the moment coefficient for the design section ($= 1.8$ for simply supported beam) and L is the span of the beam (cm). Finally, $a_4 = 1/(f_rQf_c^{'})$, where Q is the capacity reduction factor ($= 0.90$ for flexure) and $f_r = 0.85$ (assumed) is the reduction factor of concrete. Also, a_2 is the applied bending moment ($N - cm$), $a_5 = \frac{1}{2}$ (assuming the centroid of compressive force at half the depth of equivalent rectangular stress block), and a_6 is the minimum acceptable width of the beam. To determine the total bending moment (including self-weight of the beam), we use $x_4 = a_2 + a_3 x_2 x_3$. To calculate the area of reinforcement steel, we use $x_1 = \omega x_2 x_3 f_c^{'}/f_y$, where $\omega = \frac{1-\sqrt{1-\frac{4(0.59)x_4}{0.9 x_3 x_2^2 f_c^{'}}}}{1.18}$ This last expression can be derived from equation (1). Finally, the depth of the equivalent stress block is given by $x_5 = x_1/(a_1 x_3)$.

5. Use of Genetic Algorithms

To solve this optimization problem, we used the Simple Genetic Algorithm (SGA) proposed by Goldberg [8]. We won't talk much about the GA itself, since we have done that before [9]. Instead, we'll give some details about the different representation schemes that we used in our experiments. The traditional representation used by the GAs community is the binary scheme. Since the binary alphabet offers the maximum number of schemata per bit of information of any coding [8], its use has been very popular among scientists. However, since the "implicit parallelism" property of GAs does not depend on using bit strings [10] it has been common practice to experiment with larger alphabets. In particular, for optimization problems in which the parameters to be adjusted are continuous, a floating point representation scheme seems a logical choice. One of the advantages of this representation is that it has the property that two points close to each other in the representation space must also be close in the problem space, and vice versa [10]. This is not generally true in the binary approach, where the distance in a representation is normally defined by the number of different bit positions. Such discrepancy may be reduced by using Gray coding representation, which has the property that any two points next to each other in the problem space differ by only one bit. In all our experiments, we used a two-point crossover, and binary tournament selection. The only operator that had to be redefined was *mutation*. Our fitness function was given by the cost function, using a penalty function of the form $fitness = 1/(cost * (v * 500 + 1))$ where v depends on the number of constraints violated, and 500 was a value derived experimentally. Whenever the design doesn't violate any constraint, the fitness function is just the inverse of the cost.

6. Examples

Design a least-cost reinforced concrete rectangular beam simply supported over a span of 10 m supporting a uniform dead load of 15 kN/m and a uniform live load of 20 kN/m. The concrete strength $f_c^{'} = 30\ MPa$ and the steel yield strength $f_y = 300\ MPa$. The unit cost of steel (CS), concrete (CC) and formwork (CFW) are \$ $0.72/kg$, \$ $64.5/m^3$ and \$ $2.155/m^2$,

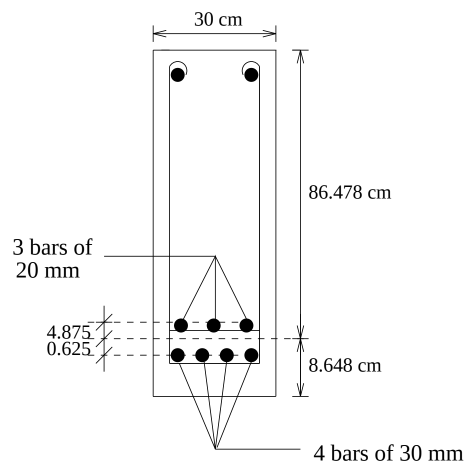

Figure 2: Optimum design of the beam of the first example.

Parameter	Chakrabarty	GA (B)	GA (GC)	GA (FP)
x_1 (cm^2)	37.6926	36.1893	41.5905	37.5205
x_2 (cm)	86.0629	89.5402	78.6177	86.4776
x_3 (cm)	30.0000	30.0162	30.0447	30.0022
$cost$ $(\$/cm)$	0.4435	0.4442	0.4464	0.4436

Table 1: Comparison of the geometric programming approach used by Chakrabarty and the GA using binary (B), Gray coding (GC) and floating point (FP) representation.

respectively. Assume a cover ratio (r) of 0.10, unit weight of concrete of 2323 kg/m^3 and capacity reduction factor as 0.90. The ultimate uniform load is $= 1.4 \times 15 + 1.7 \times 20 = 55\ kN/m$. The ultimate applied bending moment is $= 55 \times 10^2/8 = 687.5\ kN$, $\ m = 687.5 \times 10^5\ N - cm$. Using this information, we can get the values of the cost coefficients and the other model constants [3]: $c_1 = 0.0056520$, $c_2 = 0.00007095$, $c_3 = 0.00047410$, $c_4 = 0.00021550$, $a_1 = 0.08500$, $a_2 = 68'750,000$, $a_3 = 438'233,950$, $a_4 = 0.00043573$, $a_5 = 0.50$, $a_6 = 30.00$.

Our results and their comparison with the method used by Chakrabarty [1] (geometric programming) are shown in Table 1. As we can see, the floating point representation produced the best results and Gray coding the worst. Our final design for this problem is shown in Figure 2, and has a total height of 95.125, which is about 1% more than Chakrabarty's design. This slight difference is due to the fact that Chakrabarty's model considers the area of reinforcement steel as a variable, even when this is a parameter that depends on the beam section, and can't take any arbitrary value.

Method	x_1 (cm^2)	x_2 (cm)	x_3 (cm)	cost ($/cm)
Constants	b=40 cm	$CS = 0.36$	$CC = 64.5$	$CFW = 2.155$
Chakrabarty	57.0072	59.8678	40.000	0.3680
GA (FP)	50.2583	66.7029	40.0033	0.3716
Constants	b=40 cm	$CS = 0.72$	$CC = 129.0$	$CFW = 2.155$
Chakrabarty	55.4240	61.2698	40.000	0.6987
GA (FP)	49.9278	67.0981	40.0050	0.7035
Constants	b=40 cm	$CS = 0.72$	$CC = 64.5$	$CFW = 1.0775$
Chakrabarty	42.3510	78.3650	40.000	0.4847
GA (FP)	42.5568	78.0625	40.0001	0.4848

Table 2: Comparison of the geometric programming approach used by Chakrabarty and the GA using floating point representation.

On the other hand, our costs of steel, concrete and formwork represent the 47.80%, 41.50% and 10.70% of the total cost, which corresponds almost exactly to the costs obtained by Chakrabarty. Floating point representation was used in all the further experiments, since it provided the best results overall. Notice that our model has more constraints than Chakrabarty's model, in order to make it more realistic. For example, we require the relation x_3/x_2 to be between 0.25 and 0.60 in order to have a "reasonable" amount of reinforcement steel in our designs, so that we can guarantee a good adherence between steel and concrete, and we can provide a good control of the beam's deflection. Since Chakrabarty doesn't impose this constraint in his model, some of his results violate it. The results of experimenting with different values for the costs of reinforcement steel, concrete and formwork are shown in Table 2. The discrepancies between our results and those produced by Chakrabarty's method will indicate some violation of the constraints imposed by our model.

Future Work and Conclusions

Even when we already have some concrete results in this research, a lot of work remains to be done. For example, we are currently exploring techniques for adjusting the parameters of the GA, such as fuzzy logic. Also, we are interested on doing a theoretical analysis of the search space of this optimization problem, so that we can devise some strategies to solve it more efficiently. Nevertheless, our current results are very promising, and the system has called the attention of more than one engineer both in the academia and the building industry in México. We have been working in the use of GAs for structural optimization problems during the last two years, and so far, we have implemented systems to generate optimal designs of beams, columns and plane and space trusses. However, our final goal is to develop a complete structural optimization system that uses GAs, and that probably incorporates also the traditional mathematical programming techniques available, together with some other powerful heuristics such as tabu search and simulated annealing. Such a system intends to be a very powerful tool for computer aided structural design that will allow to reduce costs without sacrificing safety.

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