Hybridizing Surrogate Techniques, Rough Sets and Evolutionary Algorithms to Efficiently Solve Multi-Objective Optimization Problems

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ABSTRACT

This paper presents an approach in which a multi-objective evolutionary algorithm (MOEA) is coupled to a surrogate method in order to explore the search space in an efficient manner. A small comparative study among three surrogate methods is conducted: an artificial neural network (ANN), a radial basis function (RBF) and a support vector machine (SVM). The winner in this comparative study was the SVM. However, our results indicated that the spread of solutions achieved by our surrogate-based MOEA was poor. Thus, we decided to introduce a second phase to the algorithm in which it is hybridized with the rough sets in order to improve the spread of solutions and help to reach the true Pareto front. We show that our proposed hybrid approach only requires 2,000 fitness function evaluations in order to solve test problems with up to 30 decision variables.

Introduction.

In recent years, the statement and solution of multi-objective optimization problems has become very common in a wide variety of disciplines in which computational efficiency is a critical issue. However, multi-objective evolutionary algorithms (MOEAs) are normally expensive in the sense that they require a high number of objective function evaluations (e.g., in aeronautical engineering). Through interpolation, extrapolation and/or integration, surrogate models can be used to address complex problems involving experimental design, system analysis and prediction. There exist a variety of techniques for constructing surrogate models [12], models that will probably be built dozens of times. Currently, there exist several evolutionary algorithms that use a meta-model (surrogate) to approximate the real fitness function (see for example [6, 7, 8, 9, 14]).

Our Proposed Approach.

Our proposed approach is divided in two different phases,

each of which consumes a fixed number of fitness function evaluations. In the first phase, our surrogate-based MOEA is applied for 600 fitness function evaluations (this value was empirically derived). In order to determine the most appropriate surrogate method to be adopted, we performed a small comparative study. Although the surrogate-based MOEA can produce reasonably good approximations of the Pareto front, the distribution of solutions is normally poor. This motivated a second phase for our approach, in which rough sets are applied as a local search engine during 1,400 fitness evaluations in order to improve the solutions produced at the previous phase.

Phase 1: Surrogate-based MOEA.

The surrogate model adopted in this work is shown in Figure 1. A multi-objective particle swarm optimizer (MOPSO) is adopted to optimize the approximate model generated by the different surrogates (artificial neural networks (ANNs)[1], radial basis functions (RBFs), and support vector machines (SVMs) [11]). Our MOPSO maintains two populations: the main one (which is used to select the leaders), and a second population that retains some solutions that were dominated by the global nondominated solutions that is needed for the second phase. Both populations use the $pa\epsilon$-dominance grid proposed in [5] to maintain diversity.
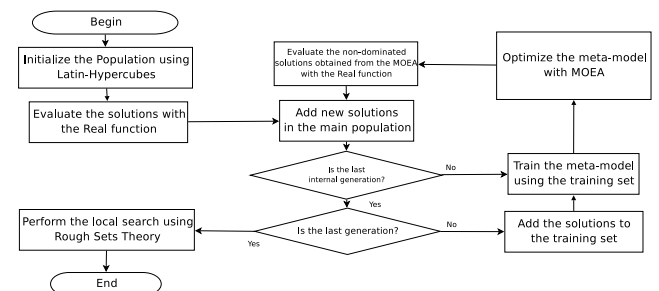

Figure 1: Surrogate model adopted in this paper.

Phase 2:Rough Sets in Multi-Objective Optimization.

Rough Sets theory is a mathematical approach to deal with imperfect knowledge originally proposed by Pawlak [10]. We will try to approximate the Pareto front using a Rough Sets grid. In order to do this, we will use an initial approximation of the Pareto front (provided by the surrogate algorithm) and will implement a grid in order to get more

ZDT1	ANN	RBF	SVM	Mean	ZDT2	ANN	RBF	SVM	Mean	ZDT3	ANN	RBF	SVM	Mean
ANN	–	0.1033	0.9561	0.5297	ANN	–	0.2667	1.0	0.6333	ANN	–	0.1	0.9165	0.5082
RBF	0.8581	–	0.9256	0.8918	RBF	0.75	–	1.0	0.875	RBF	0.8584	–	0.9150	0.8867
SVM	0.0	0.0	–	**0.0**	SVM	0.0	0.0	–	**0.0**	SVM	0.0	0.0	–	**0.0**

ZDT4	ANN	RBF	SVM	Mean	ZDT6	ANN	RBF	SVM	Mean
ANN	–	0.6717	0.83	0.7508	ANN	–	0.4582	0.7864	0.6223
RBF	0.2824	–	0.7774	0.5299	RBF	0.2696	–	0.7280	0.4988
SVM	0.0250	0.1	–	**0.0625**	SVM	0.0	0.0	–	**0.0**

Table 1: Results of Set Coverage (SC) for the ZDT test problems.

Function	Set Coverage				IGD				Spread			
	SVM+RS		NSGA-II		SVM+RS		NSGA-II		SVM+RS		NSGA-II	
	Mean	σ	Mean	σ	Mean	σ	Mean	σ	Mean	σ	Mean	σ
ZDT1	**0.0725**	0.1676	0.8755	0.1785	**0.0061**	0.0046	0.0168	0.0021	0.8308	0.1987	**0.8194**	0.0324
ZDT2	**0.0000**	0.0000	0.9690	0.0505	**0.0026**	0.0027	0.0382	0.0062	**0.5684**	0.2338	0.9711	0.0696
ZDT3	**0.0372**	0.0668	0.6453	0.3343	**0.0145**	0.0071	0.1190	0.0113	**0.8684**	0.0943	0.9475	0.0296
ZDT4	0.7937	0.2512	**0.0167**	0.0500	0.7397	0.1554	**0.1511**	0.0405	**0.9572**	0.0411	1.0795	0.1026
ZDT6	**0.0000**	0.0000	0.9208	0.1886	**0.0135**	0.0025	0.0548	0.0092	1.1000	0.1297	**0.9588**	0.0274

Table 2: Results of SVM+RS and the NSGA-II for the ZDT test problems (2000 evaluations).

information about the front that will let us improve this initial approximation. To create this grid, we use N feasible points divided in two sets: the nondominated points (ES) and the dominated ones (DS). Using these two sets we want to create a grid to describe the set ES in order to intensify the search on it (see [4] for further details).

Analysis of Results.

For our experiments, we adopted the **ZDT** test problems [15]. Three performance measures were adopted in order to allow a quantitative assessment of our results: (1) Inverted Generational Distance (**IGD**) [13]; (2) Two Set Coverage (**SC**) [15]; and (3) Spread (**S**)[2]. For each test problem, 10 independent runs were performed.

In this study, we perform 600 real function evaluations using different surrogate methods: ANNs, RBFs and SVMs. The results reported in Table 1 correspond to SC metric. From the results, we can conclude that the SVM approach is the one that shows the best overall performance in these particular multi-dimensional test functions. So, we decided to choose this approach to hybridize it with the Rough Sets.

We decided to compare our hybrid (using SVM and RS) with an algorithm representative of the state of the art (NSGA-II) [3]. The parameters used by the NSGA-II are the following: crossover rate = 0.9, mutation rate = $1/n$, η_c = 15, η_m = 20, population size = 52 and maximum number of generations = 39. These parameters make the NSGA-II to perform 2,028 fitness function evaluations in total (allowing a fair comparison between both approaches).

The results are reported in Table 2 for the three performance measures (SC, IGD and S) performing the same number of evaluations of both approaches (SVM+RS and NSGA-II). It can be observed that in the ZDT test problems, our approach produced the best results with respect to the SC and IGD in all cases except for ZDT4. The poor performance in ZDT4 might be attributed to the bad scalability presented by both approaches.

Conclusions and Future Work.

We have presented an initial study of surrogate methods to solve bi-objective problems. Three different methods were used in our comparative study: ANNs, RBFs and SVMs, all of them in their regression form to approximate the functions using supervised learning. From this study, we concluded that the SVMs were the most appropriate model for dealing with the kind of problems of our interest, although it is unable to produce a good spread of solutions. Thus, we decided to include a local search procedure based on rough sets theory to intensify the search around the solutions obtained by the SVMs. This hybrid algorithm provides competitive results in most of the test problems adopted.

As part of our future work, we are interested in refining the interaction mechanism between the surrogate method and the MOEA, or use an Ensemble Method to help the surrogate models get a better approximation of the real functions.

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