Fuzzy Rule-based Design of Evolutionary Algorithm for Optimization (Supplementary Materials)

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I. MODE

A. operators.

(1) DE_1: current-to-$p$best/bin with archive:

$$u_{z,j} = \begin{cases} x_{z,j} + F_z \left(x_{p,j} - x_{z,j} + x_{r_1,j} - \widetilde{x}_{r_2,j} \right) \\ \qquad\qquad if \left(rand \leq cr_z \text{ or } j = j_{rand} \right) \\ x_{z,j} \qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{1}$$

(2) DE_2: current-to-$p$best/bin without archive:

$$u_{z,j} = \begin{cases} x_{z,j} + F_z \left(x_{p,j} - x_{z,j} + x_{r_1,j} - x_{r_3,j} \right) \\ \qquad\qquad if \left(rand \leq cr_z \text{ or } j = j_{rand} \right) \\ x_{z,j} \qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{2}$$

(3) DE_3: wighted-rand-to-ϕbest/bin:

$$u_{z,j} = \begin{cases} F_z x_{r_1,j} + F_z \left(x_{\phi,j} - x_{r_3,j} \right), \\ if \left(rand \leq cr_z \text{ or } j = j_{rand} \right) \\ \\ x_{z,j} \qquad\qquad\qquad \text{otherwise} \end{cases} \tag{3}$$

where $r_1 \neq r_2 \neq r_3 \neq z$ are random integer numbers, with $\overrightarrow{x}_{r_1}$ and $\overrightarrow{x}_{r_2}$ randomly selected from X_1, $\overrightarrow{x}_p$ is selected from the best 10% individuals in X_1 [1], while $\widetilde{x}_{r_2,j}$ is chosen from the union of the entire X_1 and archive AR. Similar to JADE, initially, the archive was empty, then parent vectors which failed in the selection process were added to it and, once its size exceeded a threshold, $2.6PS$ [2], randomly selected elements were deleted to make space for the newly inserted ones [1], $\overrightarrow{x}_\phi$ is selected from the best 50% individuals in X_1 [3].

B. Adaptation of F and Cr

- A historical memory with H entries for both parameters (M_{Cr}, M_F) is initialized, where all values are initially set to a value of 0.5.
- Each individual $\overrightarrow{x}_z$ is associated with its own Cr_z and F_z, such that

$$Cr_z = \text{randni}(M_{Cr,r_z}, 0.1) \tag{4}$$

$$F_z = \text{randci}(M_{F,r_z}, 0.1) \tag{5}$$

  where r_z is randomly selected from $[1, H]$, randni and randci are values randomly selected from normal and Cauchy distributions with mean M_{Cr,r_z} and M_{F,r_z}, respectively, and variance 0.1.
- At the end of each generation, Cr_z and F_z used by the successful individuals are recorded in S_{Cr} and S_F, and then the contents of memory are updated as follows

$$M_{Cr,d} = mean_{WA} \left(S_{Cr} \right) \; if \; S_{Cr} \neq \text{null} \tag{6}$$

$$M_{F,d} = mean_{WL} \left(S_F \right) \; if \; S_F \neq \text{null} \tag{7}$$

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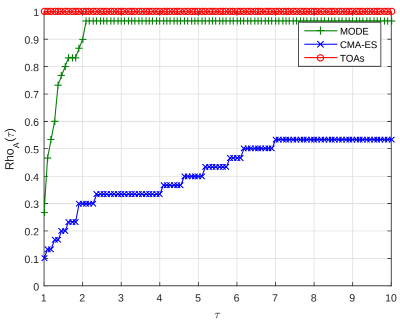
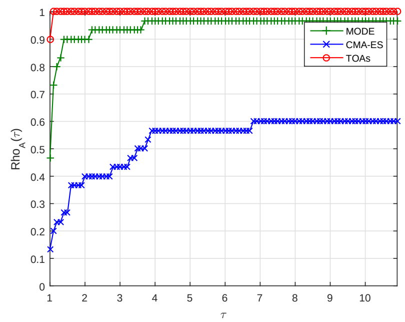
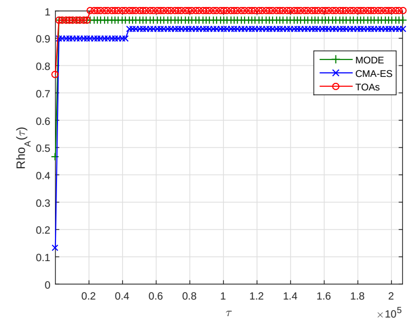


(a) 10D



(b) 30D



(c) 50D

Figure 1. Performance profiles of TOAs, MODE and CMA-ES ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, with Rho_A cumulative distribution function for ℓ_A)

where $1 \leq d \leq H$ is the the position in the memory to be updated. It is initialized to 1, and then incremented whenever a new element is inserted into the history, and if it is greater than H, it is set to 1. $\text{mean}_{WA}(S_{Cr})$ and $\text{mean}_{WL}(S_F)$ are computed as follows

$$\text{mean}_{WA}(S_{Cr}) = \sum_{\gamma=1}^{|S_{Cr}|} w_\gamma S_{cr,\gamma} \tag{8}$$

$$\text{mean}_{WL}(S_F) = \frac{\sum_{\gamma=1}^{|S_F|} w_\gamma S_{F,\gamma}^2}{\sum_{\gamma=1}^{|S_F|} w_\gamma S_{F,\gamma}} \tag{9}$$

where $|S_{Cr}|$ is the number of successful Cr recorded in S_{Cr}, with $|S_{Cr}|$=$|S_F|$ and

$$w_\gamma = \frac{\Delta f_\gamma}{\sum_{\gamma=1}^{|S_{cr}|} \Delta f_\gamma} \tag{10}$$

$\Delta f_\gamma = |f_{\gamma,old} - f_{\gamma,new}|$.

II. CMAES STEPS

The main steps in the CMA-ES used in this paper are described in the supplementary materials.

1) Generate initial solutions ($X_2 = \{\overrightarrow{x}_{2,1}, \overrightarrow{x}_{2,2}, ..., \overrightarrow{x}_{2,z}, ..., \overrightarrow{x}_{2,PS_2}\}$), then calculate the mean vector $\left(\overrightarrow{x}_m = \sum_{z=1}^{PS_2} w_z \overrightarrow{x}_{2,z},\right)$, where $\sum_{z=1}^{PS_2} w_z = 1$ and $w_1 = w_2 = ... = w_{PS_2} = \frac{1}{PS_2}$.
2) Sample new individuals, such that

$$\overrightarrow{x}_{z,t+1} = N\left(\overrightarrow{x}_{m,t}, \sigma_t^2 C_t\right) = \overrightarrow{x}_m + \sigma_t N\left(0, C_t\right), \quad \forall z = 1 : PS_2 \tag{11}$$

3) Evaluate the new offspring and sort them based on the fitness values.
4) The best μ individuals are selected as a parental vector, and their center is calculated according to

$$\overrightarrow{x}_{m,t+1} = \sum_{k=1}^{\mu} w_k \overrightarrow{x}_{k,t} \tag{12}$$

where $\sum_{k=1}^{\mu} w_k = 1$ and $w_1 \geq w_2 \geq ... \geq w_\mu$.
5) Update the evolution paths p_{t+1}^s and p_{t+1}^σ .
6) Adapt the covariance matrix (C_{t+1}).
7) Update global step size (σ_{t+1}).
8) Repeat steps 2 to 7 until a stopping criterion is met.

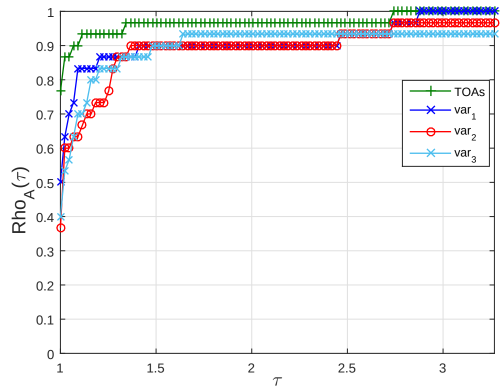


Figure 2. Performance profiles of TOAs, var_1, var_2 and var_3 ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, with Rho_A cumulative distribution function for ℓ_A)

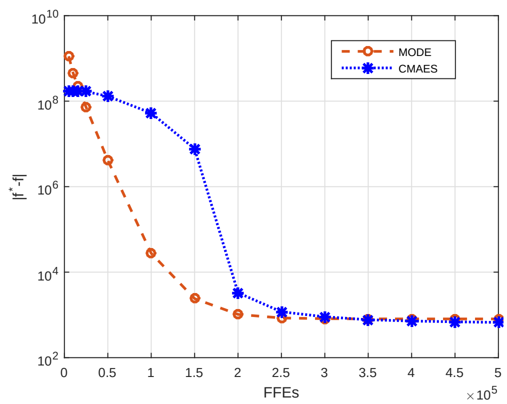


Figure 3. Average fitness errors, of 51 runs, of MODE and CMA-ES for F_{29} with 50D

Table I. FITNESS ERRORS $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^{*}}\right)\right|\right)$ OBTAINED BY MODE, CMA-ES AND TOAS FOR 10D TEST PROBLEMS (BETTER BEST AND MEAN FITNESS ERRORS SHOWN IN BOLDFACE)

Prob.	Best fitness error			Mean (std.)			Stat. test (p,Dec.)	
	MODE	CMA-ES	TOAs	MODE	CMA-ES	TOAs	MODE	CMA-ES
F_1	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000 (≈)	1.0000(≈)
F_2	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_3	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_4	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	2.8643E+01 (1.339E+01)	2.7534E+01 (1.399E+01)	**2.2675E+01** (1.657E+01)	0.0000(+)	0.0003(+)
F_5	2.7064E-01	1.9998E+01	**1.2972E-05**	1.4709E+01 (8.310E+00)	2.0000E+01 (3.794E-04)	**1.3781E+01** (8.914E+00)	0.0144(+)	0.0000(+)
F_6	**0.0000E+00**	1.0929E+00	**0.0000E+00**	**0.0000E+00** (0.000E+00)	9.2810E+00 (3.398E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0000(+)
F_7	**0.0000E+00**	0.0000E+00	**0.0000E+00**	3.9673E-03 (8.124E-03)	1.2071E-02 (1.048E-02)	**1.9806E-03** (4.596E-03)	0.1983(≈)	0.0000(+)
F_8	**0.0000E+00**	2.7859E+01	**0.0000E+00**	**0.0000E+00** (0.000E+00)	6.3989E+01 (1.386E+01)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0000(+)
F_9	9.9817E-01	2.7859E+01	**0.0000E+00**	2.4830E+00 (1.000E+00)	5.8780E+01 (1.533E+01)	**1.9509E-01** (6.299E-01)	0.0000(+)	0.0000(+)
F_{10}	**0.0000E+00**	4.8058E+02	**0.0000E+00**	**3.6738E-03** (1.484E-02)	1.4417E+03 (3.873E+02)	**3.6738E-03** (1.484E-02)	0.3438(≈)	0.0000(+)
F_{11}	6.4027E-01	7.0611E+02	**3.1227E-01**	3.5707E+01 (3.770E+01)	1.4937E+03 (3.732E+02)	**2.8616E+01** (4.027E+01)	0.1029(≈)	0.0000(+)
F_{12}	4.1796E-02	4.8175E-03	**0.0000E+00**	8.1927E-02 (1.759E-02)	8.4101E-01 (1.115E+00)	**4.0102E-02** (3.200E-02)	0.0000(+)	0.0000(+)
F_{13}	3.5007E-02	1.3831E-02	**7.5786E-03**	5.5321E-02 (1.564E-02)	1.6328E-01 (6.512E-02)	**3.5294E-02** (1.567E-02)	0.0000(+)	0.0000(+)
F_{14}	3.0691E-02	2.0464E-01	**9.1761E-03**	8.4115E-02 (2.582E-02)	4.2834E-01 (8.516E-02)	**7.4381E-02** (2.711E-02)	0.0719(≈)	0.0000(+)
F_{15}	1.8711E-01	3.3372E-01	**1.5440E-01**	3.7787E-01 (6.951E-02)	1.1406E+00 (4.482E-01)	**2.8330E-01** (8.335E-02)	0.0000(+)	0.0000(+)
F_{16}	3.5194E-01	3.6066E+00	**1.5218E-01**	1.2514E+00 (3.431E-01)	4.2145E+00 (2.408E-01)	**8.2050E-01** (3.394E-01)	0.0000(+)	0.0000(+)
F_{17}	**0.0000E+00**	1.4601E+02	**0.0000E+00**	1.1255E+00 (1.161E+00)	6.1030E+02 (2.383E+02)	**9.7349E-01** (9.550E-01)	0.3833(≈)	0.0000(+)
F_{18}	8.0636E-03	6.0008E+00	**1.3351E-04**	2.0703E-01 (1.960E-01)	3.6959E+01 (2.512E+01)	**1.5507E-01** (2.205E-01)	0.1049(≈)	0.0000(+)
F_{19}	2.9148E-02	1.5096E+00	**2.1274E-03**	9.9348E-02 (1.124E-01)	3.5568E+00 (1.285E+00)	**5.1849E-02** (5.679E-02)	0.0001(+)	0.0000(+)
F_{20}	3.3216E-02	4.8622E+00	**4.2546E-03**	1.7856E-01 (1.394E-01)	5.1140E+01 (4.715E+01)	**1.3701E-01** (1.135E-01)	0.1111(≈)	0.0000(+)
F_{21}	**2.1356E-06**	1.7688E+01	2.5858E-05	3.6829E-01 (3.015E-01)	3.4837E+02 (2.075E+02)	**2.5423E-01** (2.471E-01)	0.0512(≈)	0.0000(+)
F_{22}	6.0563E-02	2.0658E+01	**1.8704E-02**	1.1427E-01 (3.258E-02)	2.1006E+02 (1.099E+02)	**6.0563E-02** (3.049E-02)	0.0000(+)	0.0000(+)
F_{23}	**2.0000E+02**	**2.0000E+02**	**2.0000E+02**	**2.0000E+02** (0.000E+00)	3.2438E+02 (2.538E+01)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{24}	1.0006E+02	1.1496E+02	**1.0000E+02**	1.0743E+02 (2.500E+00)	1.9189E+02 (2.939E+01)	**1.0464E+02** (3.158E+00)	0.0000(+)	0.0000(+)
F_{25}	1.0001E+02	1.8832E+02	**1.0000E+02**	1.1084E+02 (8.728E+00)	1.9903E+02 (2.180E+00)	**1.0842E+02** (1.030E+01)	0.0412(+)	0.0000(+)
F_{26}	**1.0001E+02**	1.0006E+02	**1.0001E+02**	1.0006E+02 (1.522E-02)	1.0350E+02 (1.762E+00)	**1.0004E+02** (1.959E-02)	0.0005(+)	0.0000(+)
F_{27}	1.1760E+00	3.0491E+02	**7.3930E-01**	3.2697E+01 (7.288E+01)	5.4072E+02 (1.275E+02)	**2.4672E+01** (6.466E+01)	0.0001(+)	0.0000(+)
F_{28}	**2.0000E+02**	5.2245E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	1.3825E+03 (3.052E+02)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{29}	2.2154E+02	**1.4921E+02**	2.0000E+02	2.2200E+02 (4.896E-01)	6.8912E+05 (1.042E+06)	**2.1899E+02** (7.623E+00)	0.0648(≈)	0.0000(+)
F_{30}	4.6231E+02	6.1872E+02	**2.0000E+02**	4.6288E+02 (3.889E+00)	1.0554E+03 (2.675E+02)	**4.6054E+02** (3.984E+01)	0.7287(≈)	0.0000(+)

Table II. FITNESS ERRORS $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^{*}}\right)\right|\right)$ OBTAINED BY MODE, CMA-ES AND TOAS FOR 30D (BETTER BEST AND MEAN FITNESS ERRORS SHOWN IN BOLDFACE)

Prob.	Best			mean (std.)			Stat. Test (p,Dec.)	
	MODE	CMA-ES	TOAs	MODE	CMA-ES	TOAs	MODE	CMA-ES
F_1	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_2	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_3	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_4	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	2.5170E+00 (1.258E+01)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.5000(≈)
F_5	2.0000E+01	1.9998E+01	**1.9996E+01**	2.0143E+01(4.738E-02)	1.99994E+01 (4.044E-04)	**1.99988E+01** (7.612E-04)	0.0000(+)	0.0002(+)
F_6	**0.0000E+00**	5.8760E+00	**0.0000E+00**	2.8429E-04 (2.025E-03)	2.3286E+01 (9.688E+00)	**1.0975E-06** (6.422E-06)	0.7344(≈)	0.0000(+)
F_7	**0.0000E+00**	0.0000E+00	**0.0000E+00**	**0.0000E+00** (0.000E+00)	1.0631E-03 (3.203E-03)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0313(+)
F_8	**0.0000E+00**	1.4029E+02	**0.0000E+00**	**0.0000E+00** (0.000E+00)	1.8276E+02 (2.101E+01)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0000(+)
F_9	4.1423E+00	4.1788E+01	**0.0000E+00**	7.7047E+00 (1.766E+00)	2.1975E+02 (3.541E+01)	**2.0935E+00** (1.468E+00)	0.0000(+)	0.0000(+)
F_{10}	**0.0000E+00**	2.7481E+03	**0.0000E+00**	**0.0000E+00** (0.000E+00)	4.2107E+03 (6.699E+02)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{11}	8.3758E+02	2.8929E+03	**4.9584E+02**	1.2662E+03 (1.806E+02)	4.5663E+03 (8.661E+02)	**1.2317E+03** (2.614E+02)	0.5802(≈)	0.0000(+)
F_{12}	1.2742E-01	1.6508E-02	**1.2133E-02**	1.8692E-01 (2.853E-02)	2.9684E-01 (3.152E-01)	**8.5045E-02** (5.704E-02)	0.0000(+)	0.0000(+)
F_{13}	8.8498E-02	1.3030E-01	**3.0967E-02**	1.2778E-01 (1.796E-02)	2.6356E-01 (7.336E-02)	**9.6216E-02** (3.660E-02)	0.0000(+)	0.0000(+)
F_{14}	1.5644E-01	1.8148E-01	**1.6515E-01**	2.3397E-01 (2.529E-02)	3.5879E-01 (6.341E-02)	**2.3147E-01** (3.276E-02)	0.6528(≈)	0.0000(+)
F_{15}	1.7105E+00	2.1425E+00	**1.0527E+00**	2.2126E+00 (2.375E-01)	3.6764E+00 (9.865E-01)	**1.8911E+00** (2.908E-01)	0.0000(+)	0.0000(+)
F_{16}	**6.8232E+00**	1.2294E+01	7.6222E+00	**8.6792E+00** (4.490E-01)	1.3541E+01 (3.712E-01)	8.7338E+00 (4.435E-01)	0.5738(≈)	0.0000(+)
F_{17}	5.1059E+01	6.3563E+02	**4.4657E+01**	1.6389E+02 (7.830E+01)	1.5746E+03 (4.683E+02)	**1.4176E+02** (1.091E+02)	0.1220(≈)	0.0000(+)
F_{18}	**1.2063E+00**	2.3239E+01	2.1841E+00	**4.5678E+00** (2.007E+00)	1.0875E+02 (4.552E+01)	4.7361E+00 (1.833E+00)	0.9179(≈)	0.0000(+)
F_{19}	2.3767E+00	6.1327E+00	**1.5929E+00**	3.4718E+00 (5.308E-01)	1.2903E+01 (1.213E+01)	**3.3211E+00** (7.760E-01)	0.4938(≈)	0.0000(+)
F_{20}	**5.5291E-01**	5.7388E+01	9.6560E-01	2.9524E+00 (1.443E+00)	1.6900E+02 (7.064E+01)	**2.8593E+00** (1.169E+00)	0.8075(≈)	0.0000(+)
F_{21}	1.3867E+00	3.7831E+02	**1.3209E+00**	7.4364E+01 (6.901E+01)	1.0266E+03 (3.491E+02)	**5.5289E+01** (6.598E+01)	0.2266(≈)	0.0000(+)
F_{22}	2.2477E+01	1.5274E+02	**1.2704E+00**	3.0816E+01 (9.859E+00)	5.2232E+02 (2.402E+02)	**2.5394E+01** (7.237E+00)	0.0001(+)	0.0000(+)
F_{23}	**2.0000E+02**	3.1524E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	3.1524E+02 (4.019E-13)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{24}	**2.0000E+02**	**2.0000E+02**	**2.0000E+02**	**2.0000E+02** (0.000E+00)	2.2267E+02 (8.032E+00)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{25}	**2.0000E+02**	**2.0000E+02**	**2.0000E+02**	**2.0000E+02** (0.000E+00)	2.0797E+02 (4.238E+00)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{26}	1.0008E+02	1.0061E+02	**1.0003E+02**	1.0012E+02 (1.598E-02)	1.3252E+02 (3.532E+01)	**1.0009E+02** (2.967E-02)	0.0000(+)	0.0000(+)
F_{27}	**2.0000E+02**	4.0059E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	1.3543E+03 (4.763E+02)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{28}	**2.0000E+02**	2.7417E+03	**2.0000E+02**	**2.0000E+02** (0.000E+00)	5.1140E+03 (1.126E+03)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{29}	7.1382E+02	**3.2381E+02**	7.1363E+02	7.1697E+02 (3.707E+00)	**6.5633E+02** (1.627E+02)	7.1607E+02 (1.908E+00)	0.3682(≈)	0.1682(≈)
F_{30}	**4.1361E+02**	1.4221E+03	4.1786E+02	9.0701E+02 (3.737E+02)	2.9185E+03 (1.009E+03)	**8.9290E+02** (3.574E+02)	0.6800(≈)	0.0000(+)

Table III. Fitness errors $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^*}\right)\right|\right)$ obtained by MODE, CMA-ES and TOAs for 50D (better best and mean fitness errors shown in boldface)

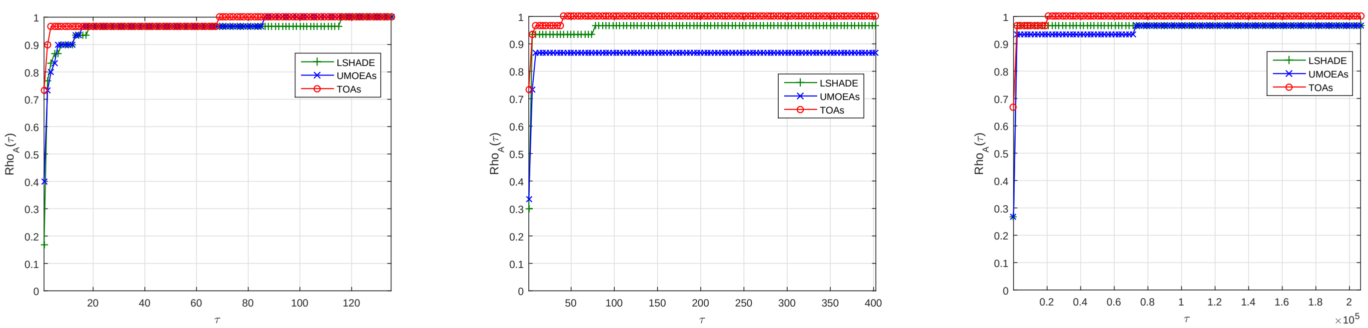
Prob.	Best fitness errors			mean (std.) fitness errors			Stat. Test (p,Dec.)	
	MODE	CMA-ES	TOAs	MODE	CMA-ES	TOAs	MODE	CMA-ES
F_1	3.3205E+01	**0.0000E+00**	5.1221E-04	1.2472E+03 (1.345E+03)	**0.0000E+00** (0.000E+00)	4.0049E-03 (1.844E-03)	0.0000(+)	0.0000(−)
F_2	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_3	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	1.0000(≈)	1.0000(≈)
F_4	7.0316E-01	**0.0000E+00**	**0.0000E+00**	3.9908E+01 (4.539E+01)	2.2824E+01 (4.126E+01)	**7.7803E+00** (2.662E+01)	0.0000(+)	0.0928(≈)
F_5	2.0000E+01	1.99993E+01	**1.99988E+01**	2.0283E+01(5.529E-02)	2.0000E+01 (1.409E-04)	**2.0000E+01** (2.206E-04)	0.0000(+)	0.0512(≈)
F_6	**0.0000E+00**	1.6134E+01	**7.8746E-06**	3.0782E-01(5.243E-01)	5.0596E+01 (1.582E+01)	**2.7796E-01** (5.455E-01)	0.8808(≈)	0.0000(+)
F_7	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00** (0.000E+00)	4.3477E-04 (2.187E-03)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.5000(≈)
F_8	**0.0000E+00**	2.6864E+02	**0.0000E+00**	**0.0000E+00** (0.000E+00)	3.3606E+02 (3.121E+01)	**0.0000E+00** (0.000E+00)	1.0000(≈)	0.0000(+)
F_9	6.7492E+00	3.8007E+02	**2.9849E+00**	1.3768E+01(2.113E+00)	4.6241E+02 (4.660E+01)	**7.0641E+00** (2.228E+00)	0.0000(+)	0.0000(+)
F_{10}	**1.1204E-05**	6.0102E+03	2.4992E-02	**6.0755E-03** (7.297E-03)	7.7879E+03 (7.233E+02)	1.2809E-01 (4.602E-02)	0.0000(−)	0.0000(+)
F_{11}	2.5460E+03	6.1268E+03	**2.3631E+03**	3.3554E+03 (3.129E+02)	8.2890E+03 (8.949E+02)	**3.2110E+03** (4.634E+02)	0.0349(+)	0.0000(+)
F_{12}	1.7650E-01	2.4502E-02	**2.2802E-02**	2.4585E-01 (2.998E-02)	2.2591E-01 (2.438E-01)	**1.0218E-01** (7.514E-02)	0.0000(+)	0.0008(+)
F_{13}	1.2106E-01	2.3830E-01	**4.7506E-02**	1.7016E-01 (2.077E-02)	3.7422E-01 (8.578E-02)	**1.4538E-01** (3.775E-02)	0.0006(+)	0.0000(+)
F_{14}	2.6031E-01	2.6115E-01	**2.1066E-01**	3.0606E-01 (2.559E-02)	3.9604E-01 (1.568E-01)	**2.9633E-01** (3.150E-02)	0.1832(≈)	0.0000(+)
F_{15}	4.3286E+00	**2.7889E+00**	2.8622E+00	5.3701E+00 (4.699E-01)	6.5155E+00 (1.696E+00)	**4.7024E+00** (8.799E-01)	0.0000(+)	0.0000(+)
F_{16}	1.5648E+01	2.1740E+01	**1.5309E+01**	1.6895E+01 (4.595E-01)	2.2693E+01 (5.590E-01)	1.7364E+01 (5.717E-01)	0.00097(−)	0.0000(+)
F_{17}	3.9375E+02	1.8399E+03	**4.7186E+02**	1.1240E+03 (3.415E+02)	2.5545E+03 (4.675E+02)	**1.1052E+03** (3.469E+02)	0.8002(≈)	0.0000(+)
F_{18}	3.5918E+01	1.0574E+02	**3.0184E+01**	8.2736E+01 (1.617E+01)	2.1747E+02 (6.443E+01)	**7.5725E+01** (1.710E+01)	0.0318(+)	0.0000(+)
F_{19}	5.9661E+00	1.3829E+01	**6.1175E+00**	**8.6468E+00** (1.730E+00)	2.0419E+01 (7.001E+00)	8.9844E+00 (1.623E+00)	0.1894(≈)	0.0000(+)
F_{20}	**5.5841E+00**	1.8771E+02	7.1767E+00	**1.3051E+01** (3.543E+00)	3.1421E+02 (8.280E+01)	1.3235E+01 (3.649E+00)	0.5931(≈)	0.0000(+)
F_{21}	2.4828E+02	7.3555E+02	**2.4659E+02**	**4.6234E+02** (1.135E+02)	1.6521E+03 (4.134E+02)	4.7211E+02 (1.157E+02)	0.6460(≈)	0.0000(+)
F_{22}	4.1851E+01	1.6784E+02	**2.4381E+01**	1.4957E+02 (8.469E+01)	7.2859E+02 (2.910E+02)	**1.4231E+02** (9.209E+01)	0.8075(≈)	0.0000(+)
F_{23}	**2.0000E+02**	3.4400E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	3.4400E+02 (2.805E-04)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{24}	**2.0000E+02**	2.5449E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	2.7036E+02 (9.341E+00)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{25}	**2.0000E+02**	2.0000E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	2.1083E+02 (1.102E+01)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{26}	1.0013E+02	1.0042E+02	**1.0007E+02**	1.3149E+02 (4.678E+01)	1.2592E+02 (4.377E+01)	**1.1195E+02** (3.247E+01)	0.0092(+)	0.0000(+)
F_{27}	**2.0000E+02**	7.5031E+02	**2.0000E+02**	**2.0000E+02** (0.000E+00)	2.7833E+03 (3.286E+02)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{28}	**2.0000E+02**	6.7422E+03	**2.0000E+02**	**2.0000E+02** (0.000E+00)	1.0157E+04 (1.828E+03)	**2.0000E+02** (0.000E+00)	1.0000(≈)	0.0000(+)
F_{29}	**7.6303E+02**	4.5224E+02	7.2343E+02	8.0544E+02 (5.130E+01)	**6.6699E+02** (1.189E+02)	7.9795E+02 (3.872E+01)	0.1313(≈)	0.0000(−)
F_{30}	7.9696E+03	8.6443E+03	**2.0000E+02**	8.6958E+03 (3.954E+02)	9.5305E+03 (5.331E+02)	**8.2148E+03** (1.677E+03)	0.0734(≈)	0.0000(+)

Table IV. Fitness errors $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^*}\right)\right|\right)$ obtained by LSHADE, UMOEAs and TOAs for 10D and 30D problems (better mean fitness errors are shown in boldface)

Prob.	mean (std.)					
	10D			30D		
	LSHADE	UMOEAs	TOAs	LSHADE	UMOEAs	TOAs
F_1	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_2	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_3	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_4	2.9410E+01 (1.259E+01)	**1.0825E+01** (1.568E+01)	2.2675E+01 (1.657E+01)	**0.0000E+00** (0.000E+00)	3.7295E+00 (1.507E+01)	**0.0000E+00** (0.000E+00)
F_5	1.4146E+01 (8.761E+00)	1.6190E+01 (7.688E+00)	**1.3781E+01** (8.914E+00)	2.0115E+01 (3.681E-02)	2.0171E+01 (1.684E-01)	**1.99988E+01** (7.612E-04)
F_6	1.7540E-02 (1.253E-01)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**1.3843E-07** (9.886E-07)	6.7207E-01 (8.805E-01)	1.0975E-06 (6.422E-06)
F_7	3.0429E-03 (6.508E-03)	**0.0000E+00** (0.000E+00)	1.9806E-03 (4.596E-03)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_8	**0.0000E+00 (0.000E+00)**	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	2.4800E+00 (1.638E+00)	**0.0000E+00** (0.000E+00)
F_9	2.3446E+00 (8.402E-01)	2.4191E+00 (1.620E+00)	**1.9509E-01** (6.299E-01)	6.7849E+00 (1.483E+00)	1.0184E+01 (6.921E+00)	**2.0935E+00** (1.468E+00)
F_{10}	8.5722E-03 (2.171E-02)	3.1455E-01 (1.062E+00)	**3.6738E-03** (1.484E-02)	1.6329E-02 (1.575E-02)	4.4893E+00 (2.967E+00)	**0.0000E+00** (0.000E+00)
F_{11}	3.2056E+01 (3.831E+01)	1.5840E+02 (1.224E+02)	**2.8616E+01** (4.027E+01)	**1.2295E+03** (1.825E+02)	1.4717E+03 (7.622E+02)	1.2317E+03 (2.614E+02)
F_{12}	6.8167E-02 (1.922E-02)	**5.9289E-04** (2.964E-03)	4.0102E-02 (3.200E-02)	1.6058E-01 (2.295E-02)	**2.1121E-03** (2.398E-03)	8.5045E-02 (5.704E-02)
F_{13}	5.1562E-02 (1.509E-02)	**1.3604E-02** (9.966E-03)	3.5294E-02 (1.567E-02)	1.2412E-01 (1.748E-02)	**7.0444E-02** (2.831E-02)	9.6216E-02 (3.660E-02)
F_{14}	8.1362E-02 (2.554E-02)	1.1435E-01 (4.026E-02)	**7.4381E-02** (2.711E-02)	2.4170E-01 (2.984E-02)	**1.9518E-01** (3.473E-02)	2.3147E-01 (3.276E-02)
F_{15}	3.6610E-01 (6.920E-02)	6.4726E-01 (2.051E-01)	**2.8330E-01** (8.335E-02)	2.1464E+00 (2.513E-01)	3.1897E+00 (5.676E-01)	**1.8911E+00** (2.908E-01)
F_{16}	1.2408E+00 (3.033E-01)	1.6821E+00 (5.144E-01)	**8.2050E-01** (3.394E-01)	**8.4990E+00** (4.581E-01)	1.0654E+01 (5.938E-01)	8.7338E+00 (4.435E-01)
F_{17}	9.7666E-01 (1.076E+00)	1.4210E+01 (2.135E+01)	**9.7349E-01** (9.550E-01)	1.8751E+02 (7.496E+01)	8.6304E+02 (4.225E+02)	**1.4176E+02** (1.091E+02)
F_{18}	2.4409E-01 (3.140E-01)	9.4175E-01 (8.591E-01)	**1.5507E-01** (2.205E-01)	5.9101E+00 (2.890E+00)	2.7505E+01 (1.736E+01)	**4.7361E+00** (1.833E+00)
F_{19}	7.7300E-02 (6.405E-02)	1.0249E-01 (1.917E-01)	**5.1849E-02** (5.679E-02)	3.6818E+00 (6.804E-01)	3.6593E+00 (1.273E+00)	**3.3211E+00** (7.760E-01)
F_{20}	1.8488E-01 (1.802E-01)	4.4108E-01 (4.840E-01)	**1.3701E-01** (1.135E-01)	3.0819E+00 (1.468E+00)	1.6137E+01 (5.696E+00)	**2.8593E+00** (1.169E+00)
F_{21}	4.0807E-01 (3.094E-01)	7.1703E-01 (2.349E+00)	**2.5423E-01** (2.471E-01)	8.6833E+01 (8.989E+01)	4.2187E+02 (2.157E+02)	**5.5289E+01** (6.598E+01)
F_{22}	**4.4103E-02** (2.816E-02)	2.1726E-01 (2.199E-01)	6.0563E-02 (3.049E-02)	2.7601E+01 (1.787E+01)	9.7648E+01 (7.609E+01)	**2.5394E+01** (7.237E+00)
F_{23}	3.2946E+02 (2.870E-13)	3.2184E+02 (3.076E+01)	**2.0000E+02** (0.000E+00)	3.1524E+02 (4.019E-13)	3.1524E+02 (5.741E-14)	**2.0000E+02** (0.000E+00)
F_{24}	1.0749E+02 (2.283E+00)	1.0860E+02 (2.918E+00)	**1.0464E+02** (3.158E+00)	2.2405E+02 (1.057E+00)	2.1252E+02 (1.150E+01)	**2.0000E+02** (0.000E+00)
F_{25}	1.3274E+02 (4.036E+01)	1.2550E+02 (1.159E+01)	**1.0842E+02** (1.030E+01)	2.0261E+02 (4.957E-02)	**2.0000E+02** (0.000E+00)	**2.0000E+02** (0.000E+00)
F_{26}	1.0005E+02 (1.629E-02)	**1.0002**E+02 (1.210E-02)	1.0004**E+02** (1.959E-02)	1.0011E+02 (1.552E-02)	**1.00089E+02** (4.324E-02)	1.00094E+02 (2.967E-02)
F_{27}	5.8064E+01 (1.341E+02)	**7.7098E+00** (4.175E+01)	2.4672E+01 (6.466E+01)	3.0000E+02 (2.400E-13)	3.1491E+02 (2.983E+01)	**2.0000E+02** (0.000E+00)
F_{28}	3.8081E+02 (3.167E+01)	2.9760E+02 (1.335E+02)	**2.0000E+02** (0.000E+00)	8.4030E+02 (1.402E+01)	8.2709E+02 (8.511E+01)	**2.0000E+02** (0.000E+00)
F_{29}	2.2199E+02 (4.628E-01)	**1.8346E+02** (4.424E+01)	2.1899E+02 (7.623E+00)	7.1688E+02 (5.132E+00)	**7.1588E+02** (2.513E+02)	7.1607E+02 (1.908E+00)
F_{30}	4.6488E+02 (1.332E+01)	4.6365E+02 (4.914E+01)	**4.6054E+02** (3.984E+01)	1.2464E+03 (6.199E+02)	9.1421E+02 (4.379E+02)	**8.9290E+02** (3.574E+02)

Table V. FITNESS ERRORS $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^*}\right)\right|\right)$ OBTAINED BY LSHADE, UMOEAS AND TOAS FOR 50D (BEST MEAN FITNESS ERRORS ARE SHOWN IN BOLDFACE)

Prob.	mean (std.)		
	LSHADE	UMOEAs	TOAs
F_1	1.2411E+03 (1.515E+03)	**0.0000E+00** (0.000E+00)	2.0663E-04 (1.476E-03)
F_2	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_3	**0.0000E+00** (0.000E+00)	0.0000E+00 (0.000E+00)	**0.0000E+00** (0.000E+00)
F_4	5.8866E+01 (4.561E+01)	8.0871E+01 (3.781E+01)	**7.7803E+00** (2.662E+01)
F_5	2.0249E+01 (4.592E-02)	2.0155E+01 (2.058E-01)	**2.0000E+01** (2.206E-04)
F_6	**2.6408E-01** (5.228E-01)	1.4615E+00 (1.520E+00)	2.7796E-01 (5.455E-01)
F_7	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)	**0.0000E+00** (0.000E+00)
F_8	2.5817E-09 (7.484E-09)	7.6281E+00 (5.207E+00)	**0.0000E+00** (0.000E+00)
F_9	1.1447E+01 (2.134E+00)	2.0965E+01 (5.063E+00)	**7.0641E+00** (2.228E+00)
F_{10}	1.2188E-01 (4.129E-02)	8.7297E+01 (1.234E+02)	**1.2247E-03** (3.752E-03)
F_{11}	3.2219E+03 (3.298E+02)	3.4497E+03 (1.537E+03)	**3.2110E+03** (4.634E+02)
F_{12}	2.1891E-01 (2.818E-02)	**1.3021E-03** (8.754E-04)	1.0218E-01 (7.514E-02)
F_{13}	1.6043E-01 (1.832E-02)	**1.1041E-01** (3.277E-02)	1.4538E-01 (3.775E-02)
F_{14}	2.9724E-01 (2.469E-02)	**2.7956E-01** (3.289E-02)	2.9633E-01 (3.150E-02)
F_{15}	5.1500E+00 (5.077E-01)	5.5483E+00 (9.456E-01)	**4.7024E+00** (8.799E-01)
F_{16}	1.6854E+01 (4.812E-01)	1.9333E+01 (7.955E-01)	**1.6615E+01** (4.650E-01)
F_{17}	1.4026E+03 (5.130E+02)	2.5196E+03 (5.453E+02)	**1.1052E+03** (3.469E+02)
F_{18}	9.7265E+01 (1.384E+01)	1.1394E+02 (6.610E+01)	**7.5725E+01** (1.710E+01)
F_{19}	**8.2960E+00** (1.814E+00)	1.0376E+01 (1.535E+00)	8.9844E+00 (1.623E+00)
F_{20}	1.3914E+01 (4.564E+00)	8.1586E+01 (3.267E+01)	**1.3235E+01** (3.649E+00)
F_{21}	5.1533E+02 (1.491E+02)	1.5664E+03 (3.613E+02)	**4.7211E+02** (1.157E+02)
F_{22}	**1.1447E+02** (7.505E+01)	2.9400E+02 (1.607E+02)	1.4231E+02 (9.209E+01)
F_{23}	3.4400E+02 (4.442E-13)	3.4400E+02 (4.593E-13)	**2.0000E+02** (0.000E+00)
F_{24}	2.7522E+02 (6.617E-01)	2.7158E+02 (4.499E+00)	**2.0000E+02** (0.000E+00)
F_{25}	2.0529E+02 (3.649E-01)	2.0011E+02 (7.851E-01)	**2.0000E+02** (0.000E+00)
F_{26}	1.0212E+02 (1.398E+01)	**1.0210E+02** (1.398E+01)	1.1195E+02 (3.247E+01)
F_{27}	3.3275E+02 (3.028E+01)	3.9515E+02 (5.103E+01)	**2.0000E+02 (0.000E+00)**
F_{28}	1.1114E+03 (2.910E+01)	1.2523E+03 (7.463E+01)	**2.0000E+02 (0.000E+00)**
F_{29}	**7.9463E+02** (2.401E+01)	1.0795E+03 (2.760E+02)	7.9795E+02 (3.872E+01)
F_{30}	8.6599E+03 (4.131E+02)	8.8415E+03 (9.781E+02)	**8.2148E+03** (1.677E+03)



(a) 10D (b) 30D (c) 50D

Figure 4. Performance profiles of TOAs, LSHADE and UMOEAs ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, where Rho_A cumulative distribution function for ℓ_A)

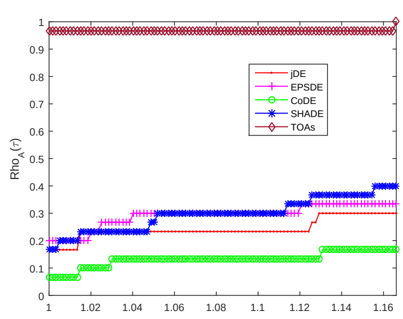


Figure 5. Performance profiles of TOAs and other state-of-the-art algorithm based on CEC2014 problems ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, where Rho_A cumulative distribution function for ℓ_A)

Table VI. FITNESS ERRORS $\left(\left|f\left(\overrightarrow{x}_{best}\right)-f\left(\overrightarrow{x^{*}}\right)\right|\right)$ OBTAINED BY JDE, EPSDE, CODE, SHADE AND TOAS FOR 30D PROBLEMS (BEST MEAN FITNESS ERRORS ARE SHOWN IN BOLDFACE)

Prob	jDE	EPSDE	CoDE	SHADE	TOAs
F_1	6.7084E+04	6.3464E+05	2.5918E+04	4.1611E+02	**0.0000E+00**
F_2	**0.0000E+00**	**0.0000E+00**	5.8925E+00	**0.0000E+00**	**0.0000E+00**
F_3	**0.0000E+00**	**0.0000E+00**	1.4523E-04	**0.0000E+00**	**0.0000E+00**
F_4	5.9627E+00	**0.0000E+00**	1.8442E+01	7.8169E-02	**0.0000E+00**
F_5	2.0342E+01	2.0394E+01	2.0602E+01	2.0103E+01	**1.9999E+01**
F_6	2.1312E+00	1.4043E+01	2.1568E+01	3.1190E-01	**1.0975E-06**
F_7	**0.0000E+00**	**0.0000E+00**	7.0317E-05	**0.0000E+00**	**0.0000E+00**
F_8	**0.0000E+00**	**0.0000E+00**	1.8380E+01	**0.0000E+00**	**0.0000E+00**
F_9	4.4038E+01	5.9240E+01	1.4064E+02	1.5385E+01	**2.0935E+00**
F_{10}	8.1644E-04	2.2751E-02	7.8600E+02	3.6740E-03	**0.0000E+00**
F_{11}	2.4873E+03	3.1293E+03	4.8239E+03	1.3709E+03	**1.2317E+03**
F_{12}	4.6583E-01	5.5979E-01	1.0051E+00	1.6407E-01	**8.5045E-02**
F_{13}	3.0295E-01	2.6050E-01	4.7126E-01	2.2049E-01	**9.6216E-02**
F_{14}	2.8543E-01	2.3989E-01	2.8360E-01	2.4228E-01	**2.3147E-01**
F_{15}	5.8219E+00	6.7872E+00	1.3687E+01	2.5948E+00	**1.8911E+00**
F_{16}	9.8511E+00	1.0343E+01	1.1619E+01	9.1787E+00	**8.7338E+00**
F_{17}	1.2286E+03	3.2522E+03	1.4411E+03	1.0509E+03	**1.4176E+02**
F_{18}	2.1862E+01	3.2907E+01	4.9418E+01	5.8931E+01	**4.7361E+00**
F_{19}	4.6651E+00	5.0989E+00	6.9954E+00	4.3396E+00	**3.3211E+00**
F_{20}	1.1460E+01	2.1118E+01	3.0450E+01	1.4134E+01	**2.8593E+00**
F_{21}	2.9357E+02	7.3785E+02	7.1930E+02	2.9156E+02	**5.5289E+01**
F_{22}	1.0320E+02	1.6613E+02	1.3024E+02	9.7177E+01	**2.5394E+01**
F_{23}	3.1524E+02	3.1524E+02	3.1524E+02	3.1524E+02	**2.0000E+02**
F_{24}	2.2475E+02	2.2373E+02	2.2579E+02	2.2531E+02	**2.0000E+02**
F_{25}	2.0339E+02	2.0509E+02	2.0291E+02	2.0318E+02	**2.0000E+02**
F_{26}	1.0029E+02	1.0027E+02	1.0045E+02	1.0020E+02	**1.0009E+02**
F_{27}	3.4892E+02	4.2230E+02	4.0054E+02	3.1748E+02	**2.0000E+02**
F_{28}	7.8817E+02	8.8393E+02	9.3314E+02	8.3275E+02	**2.0000E+02**
F_{29}	8.1303E+02	1.1435E+03	**6.1364E+02**	7.0945E+02	7.1607E+02
F_{30}	1.4600E+03	1.8667E+03	1.1461E+03	1.7713E+03	**8.9290E+02**

Table VII. BASIC CHARACTERISTICS OF 13 PROBLEMS

Problems	Name	$f(x)$	Initial range	f_{min}
F_1	Sphere model	$\sum_{j=1}^{D} x_j^2$	$[-100,100]^D$	0
F_2	Schwefel 2.22	$\sum_{j=1}^{D} \|x_j\| + \prod_{j=1}^{D} x_j^2$	$[-10,10]^D$	0
F_3	Schwefel 1.2	$\sum_{j=1}^{D} \left(\sum_{o=1}^{j} x_o\right)^2$	$[-100,100]^D$	0
F_4	Schwefel 2.21	$max_j \left\{\|x_j\|\right\}$	$[-100,100]^D$	0
F_5	Rosenbrock's valley	$\sum_{j=1}^{D-1} \left(100\left(x_j^2-x_{j+1}\right)+(x_j-1)^2\right)$	$[-30,30]^D$	0
F_6	Step function	$\sum_{j=1}^{D} \lfloor x_j+0.5\rfloor^2$	$[-100,100]^D$	0
F_7	Noisy quartic	$\sum_{j=1}^{D} jx_j^4 + \text{rand}(0,1)$	$[-1.28,1.28]^D$	0
F_8	Schwefwl 2.26	$\sum_{j=1}^{D} -x_j \sin\left(\|x_j\|\right)$	$[-500,500]^D$	$-418.98288728D$
F_9	Rastring	$\sum_{j=1}^{D} \left(x_j^2 - 10\cos\left(2\pi x_j\right)+10\right)$	$[-5.12,5.12]^D$	0
F_{10}	Ackley's path	$-20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{j=1}^{D} x_j^2}\right) - \exp\left(\frac{1}{D}\sum_{j=1}^{D} cos\left(2\pi x_j\right)\right)+20+e$	$[-32,32]^D$	0
F_{11}	Griewank	$\frac{1}{4000}\sum_{j=1}^{D} x_j^2 - \prod_{j=1}^{D} \cos\left(\frac{x_j}{\sqrt{j}}\right)+1$	$[-600,600]^D$	0
F_{12}	Generalized penalized	$\frac{\pi}{D}\Big\{10\sin^2\left(\pi y_1\right)+\sum_{j=1}^{D-1}\left(y_j-1\right)^2\left[1+10\sin^2\left(\pi y_{j+1}\right)\right] + \left(y_D-1\right)^2\Big\} + \sum_{j=1}^{D} u\left(x_j,10,1000,4\right)$, where $y_j = 1+\frac{(x_j+1)}{4}$ and $u\left(x_j,a,b,c\right)=\begin{cases} b\left(x_j-a\right)^c, & x_j>a \\ 0, & -a\le x_j\le a \\ b\left(-x_j-a\right)^c, & x_j<-a\end{cases}$	$[-50,50]^D$	0

Table VIII. AVERAGE FITNESS ERRORS OBTAINED BY IDE, JADE/EIG, JADEEP,CODE, EPSDE, DYNNP_JDE, SHADE AND TOAS FOR CEC2013 TEST PROBLEMS WITH $D = 30$ (BEST MEAN FITNESS ERRORS SHOWN IN BOLDFACE)

Prob.	IDE	JADE/eig	JADEEP	CoDE	EPSDE	dynNP_jDE	SHADE	CIPDE	TOAs
F_1	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00** (0.00E+00)	**0.00E+00**(0.00E+00)
F_2	2.42E+05(1.40E+05)	2.18E-05(3.81E-05)	5.43E+03(4.22E+03)	9.78E+04(4.81E+04)	1.37E+06(5.23E+06)	9.52E+04(4.09E+04)	9.00E+03(7.47E+03)	1.00E+04 (7.54E+03)	**8.41E-10**(4.57E-09)
F_3	2.08E+04(1.46E+05)	8.67E+01(2.70E+02)	6.19E+06(7.51E+06)	1.08E+06(3.03E+06)	1.75E+08(5.39E+08)	1.71E+06(2.54E+06)	4.02E+01(2.13E+02)	6.76E+05 (1.85E+06)	**9.07E-02**(3.87E-01)
F_4	9.75E+00(3.26E+02)	**0.00E+00(0.00E+00)**	5.00E+03(1.03E+04)	8.18E-02(1.09E-01)	8.08E+03(2.56E+04)	4.76E+01(4.75E+01)	1.92E-04(3.01E-04)	5.13E+03 (9.06E+03)	**0.00E+00**(0.00E+00)
F_5	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00** (0.00E+00)	**0.00E+00**(0.00E+00)
F_6	5.00E+00(2.82E+00)	5.28E+00(1.11E+01)	9.09E-01(3.65E+00)	4.16E+00(9.00E+00)	9.27E+00(1.33E+00)	1.19E+01(1.66E+00)	5.96E-01(3.73E+00)	**0.00E+00** (0.00E+00)	**0.00E+00**(0.00E+00)
F_7	5.55E-01(5.07E-01)	1.03E+00(7.59E-01)	4.67E+00(3.72E+00)	9.32E+00(6.34E+00)	5.88E+01(4.29E+01)	2.62E+00(1.59E+00)	4.60E+00(5.39E+00)	2.80E+00 (2.37E+00)	**5.45E-01**(3.95E-01)
F_8	2.09E+01(4.78E-02)	2.09E+01(3.64E-02)	2.09E+01(4.90E-02)	2.08E+01(1.18E-01)	2.09E+01(5.32E-02)	2.10E+01(3.98E-02)	**2.07E+01**(1.76E-01)	2.09E+01 (4.05E-02)	2.08E+01(1.41E-01)
F_9	1.76E+01(3.39E+00)	2.67E+01(1.41E+00)	2.69E+01(1.63E+00)	**1.45E+01**(2.90E+00)	3.50E+01(4.21E+00)	2.20E+01(5.12E+00)	2.75E+01(1.77E+00)	1.93E+01 (3.18E+00)	2.44E+01(3.04E+00)
F_{10}	3.42E-02(1.47E-02)	1.55E-02(8.21E-03)	3.75E-02(1.82E-02)	2.71E-02(1.50E-02)	1.02E-01(5.65E-02)	3.63E-02(2.34E-02)	7.69E-02(3.58E-02)	6.10E-02 (3.51E-02)	**1.98E-03**(4.52E-03)
F_{11}	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	1.95E-02(1.39E-01)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00** (0.00E+00)	**0.00E+00**(0.00E+00)
F_{12}	2.73E+01(4.59E+00)	3.40E+01(6.17E+00)	2.06E+01(3.95E+00)	3.98E+01(1.21E+01)	4.94E+01(9.28E+00)	4.07E+01(8.81E+00)	2.30E+01(3.73E+00)	1.57E+01 (4.50E+00)	**1.91E+00(1.19E+00)**
F_{13}	5.13E+01(1.14E+01)	4.74E+01(1.24E+01)	4.16E+01(1.36E+01)	8.04E+01(2.74E+01)	7.68E+01(1.72E+01)	7.10E+01(1.72E+01)	5.03E+01(1.34E+01)	1.93E+01 (9.05E+00)	**1.93E+00(1.31E+00)**
F_{14}	2.34E+01(3.18E+01)	4.92E+01(6.55E+00)	4.39E-02(3.18E-02)	3.60E+00(4.09E+00)	3.99E-01(6.00E-01)	**9.39E-03(1.40E-02)**	3.18E-02(2.33E-02)	5.82E-01 (4.96E-01)	2.28E-02(2.13E-02)
F_{15}	2.93E+03(3.88E+02)	4.04E+03(3.52E+02)	3.20E+03(3.31E+02)	3.36E+03(5.31E+02)	6.75E+03(7.60E+02)	4.39E+03(4.72E+02)	3.22E+03(2.64E+02)	2.75E+03 (6.87E+02)	**2.64E+03(3.95E+02)**
F_{16}	1.12E+00(1.67E-01)	1.85E+00(1.10E+00)	1.75E+00(6.59E-01)	3.38E-01(2.03E-01)	2.48E+00(2.88E-01)	2.32E+00(2.83E-01)	9.13E-01(1.85E-01)	2.10E+00 (7.74E-01)	**1.20E-01(6.78E-02)**
F_{17}	3.13E+01(3.80E-01)	3.13E+01(1.18E-01)	**3.04E+01**(1.54E-11)	**3.04E+01**(1.17E-02)	**3.04E+01**(2.51E-02)	**3.04E+01**(1.78E-03)	**3.04E+01**(3.83E-14)	3.05E+01 (3.05E-02)	**3.04E+01**(6.49E-05)
F_{18}	6.48E+01(9.65E+00)	9.42E+01(8.80E+00)	7.31E+01(6.00E+00)	6.69E+01(9.23E+00)	1.37E+02(1.12E+01)	1.35E+02(1.24E+01)	7.25E+01(5.58E+00)	3.96E+01 (3.34E+00)	**3.40E+01(1.33E+00)**
F_{19}	1.14E+00(1.63E-01)	2.29E+00(7.98E-02)	1.43E+00(1.19E-01)	1.61E+00(3.58E-01)	1.84E+00(2.00E-01)	1.27E+00(1.09E-01)	1.36E+00(1.20E-01)	**1.02E+00** (1.31E-01)	1.09E+00(1.51E-01)
F_{20}	9.94E+00(4.97E-01)	1.46E+01(1.31E+00)	1.01E+01(6.04E-01)	1.06E+01(6.69E-01)	1.30E+01(6.33E-01)	1.13E+01(4.14E-01)	1.05E+01(6.04E-01)	**9.74E+00** (7.23E-01)	1.08E+01(1.86E+00)
F_{21}	3.17E+02(6.01E+01)	3.09E+02(8.20E+01)	2.98E+02(5.52E+01)	3.02E+02(9.02E+01)	3.05E+02(8.06E+01)	2.94E+02(8.29E+01)	3.09E+02(5.65E+01)	2.90E+02 (4.82E+01)	**2.82E+02**(5.18E+01)
F_{22}	1.21E+02(4.39E+00)	2.95E+02(8.37E+01)	1.03E+02(2.37E+01)	1.17E+02(9.96E+00)	3.09E+02(1.12E+02)	1.03E+02(2.57E+01)	**9.81E+01**(2.52E+01)	1.11E+02 (1.42E+01)	1.07E+02(1.67E+00)
F_{23}	3.28E+03(3.80E+02)	4.13E+03(3.99E+02)	3.25E+03(4.11E+02)	3.56E+03(6.12E+02)	6.74E+03(8.20E+02)	4.36E+03(4.61E+02)	3.51E+03(4.11E+02)	2.54E+03 (5.59E+02)	**2.53E+03**(3.26E+02)
F_{24}	2.00E+02(3.60E-01)	2.17E+02(1.64E+01)	2.10E+02(6.66E+00)	2.21E+02(9.28E+00)	2.91E+02(7.08E+00)	2.04E+02(4.22E+00)	2.05E+02(5.29E+00)	2.07E+02 (4.16E+00)	**1.997E+02**(3.28E+00)
F_{25}	**2.14E+02**(2.09E+01)	2.84E+02(5.18E+00)	2.63E+02(1.57E+01)	2.57E+02(6.55E+00)	2.99E+02(3.29E+00)	2.55E+02(7.91E+00)	2.59E+02(1.96E+01)	2.63E+02 (1.02E+01)	2.40E+02(3.89E+00)
F_{26}	2.00E+02(6.39E-03)	2.16E+02(5.20E+01)	2.09E+02(2.96E+01)	2.18E+02(4.48E+01)	3.56E+02(6.49E+01)	2.00E+02(3.06E-03)	2.02E+02(1.48E+01)	2.00E+02 (8.07E-03)	**1.93E+02**(1.90E+01)
F_{27}	3.06E+02(5.37E+00)	8.26E+02(2.16E+02)	5.34E+02(2.24E+02)	6.20E+02(1.01E+02)	1.21E+03(7.42E+01)	3.90E+02(9.12E+01)	3.88E+02(1.09E+02)	4.16E+02 (8.36E+01)	**3.05E+02**(8.00E+00)
F_{28}	3.00E+02(0.00E+00)	3.00E+02(3.28E-12)	3.00E+02(2.67E-13)	3.00E+02(0.00E+00)	3.00E+02(0.00E+00)	3.00E+02(0.00E+00)	3.00E+02(0.00E+00)	3.00E+02 (4.55E-14)	**2.65E+02**(7.70E+01)

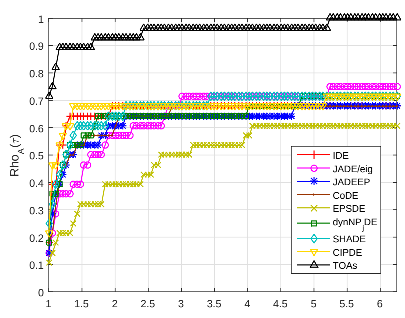


Figure 6. Performance profiles of TOAs and other state-of-the-art algorithm based on CEC2013 problems ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, where Rho_A cumulative distribution function for ℓ_A)

Table IX. AVERAGE FITNESS ERRORS OBTAINED BY DMPSADE, AEPD-JADE, HSDE, EPS-DPSO, TCCS-AC, DE-DPS, CODE AND TOAS FOR CEC2005 TEST PROBLEMS WITH $D = 30$ (BEST MEAN FITNESS ERRORS SHOWN IN BOLDFACE AND ANY FITNESS ERROR LESS THAN 1E-08 CONSIDERED ZERO)

Prob.	DMPSADE	AEPD-JADE	HSDE	EPS-dPSO	TCCS-AC	DE-DPS	CoDE	TOAs
F_1	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)
F_2	**0.00E+00**(0.00E+00)	**0.00E+00**(1.22E-08)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)
F_3	3.50E+05(1.34E+05)	**0.00E+00**(0.00E+00)	5.37E+04(2.33E+04)	1.31E+04(0.00E+00)	1.58E+06(9.51E+05)	5.02E+04(2.24E+04)	1.05E+05(6.25E+04)	**0.00E+00**(0.00E+00)
F_4	2.87E+02 (1.80E+02)	3.59E-02(4.51E-02)	1.50E-04(7.11E-04)	2.17E+01(5.50E−01)	1.33E+04(4.64E+03)	0.00E+00(0.00E+00)	5.81E-03(1.38E-02)	**0.00E+00**(0.00E+00)
F_5	2.19E+03(3.52E+02)	2.45E+01(2.90E+01)	5.33E+02(3.90E+02)	5.42E+03(1.89E+02)	4.92E+03(9.38E+02)	**0.00E+00**(2.05E-13)	3.31E+02(3.44E+02)	1.43E-08(3.29E-08)
F_6	6.23E-01(1.60E+00)	3.27E+00(5.55E+00)	1.59E-01(7.97E-01)	2.19E+01(1.06E+00)	3.55E+01(5.01E+01)	**0.00E+00**(0.00E+00)	1.60E-01(7.85E-01)	**0.00E+00**(0.00E+00)
F_7	4.61E-04(1.73E-03)	2.1E-02(1.97E-02)	1.00E-02(1.13E-02)	1.17E−02(1.05E−02)	1.20E-07(3.32E-07)	3.75E-03(4.39E-03)	7.46E-03(8.55E-03)	**0.00E+00**(0.00E+00)
F_8	2.11E+01(4.29E-02)	2.09E+1(8.80E-02)	2.09E+01(3.37E-02)	2.01E+01(1.30E−01)	2.02E+01(4.93E-02)	2.09E+01(4.08E-02)	2.01E+01(1.41E-01)	**2.00E+01**(5.08E-08)
F_9	**0.00E+00**(0.00E+00)	4.91E+00(4.57E+00)	**0.00E+00**(0.00E+00)	3.23E+01(1.56E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)	**0.00E+00**(0.00E+00)
F_{10}	5.92E+01(1.47E+01)	1.34E+02(3.63E+001)	4.70E+01(1.16E+01)	2.90E+02(1.16E+01)	1.09E+02(4.20E+01)	2.29E+01(6.14E+00)	4.15E+01(1.16E+01)	**3.43E+00**(1.35E+00)
F_{11}	3.99E+01(7.50E+00)	2.15E+01(3.74E+00)	1.97E+01(5.22E+00)	2.87E+01(5.12E−01)	1.72E+01(2.26E+00)	**9.27E+00**(3.24E+00)	1.18E+01(3.40E+00)	1.23E+01(2.99E+00)
F_{12}	1.21E+04(9.38E+03)	4.23E+03(4.78E+03)	**8.24E+02**(1.37E+03)	2.37E+05(4.51E+04)	3.85E+03(3.38E+03)	1.01E+03(1.40E+03)	3.05E+03(3.80E+03)	1.24E+03(1.62E+03)
F_{13}	3.82E+00(3.69E-01)	3.00E+03(1.36E+00)	3.38E+00(2.27E-01)	2.26E+00(7.33E−01)	1.34E+00(1.90E-01)	1.99E+00(4.13E-01)	1.57E+00(3.27E-01)	**9.46E-01**(2.09E-01)
F_{14}	2.19E+01(4.21E-01)	1.26E+01(4.46E-01	1.29E+01(3.51E-01)	1.26E+01(3.61E−01)	1.36E+01(3.01E-01)	1.23E+01(5.05E-01)	1.23E+01(4.81E-01)	**1.19E+01**(3.26E-01)
F_{15}	2.33E+02(8.90E+01)	3.40E+02(2.99E+02)	3.76E+02(1.09E+02)	3.70E+02(3.35E+01)	2.48E+02(1.96E+02)	3.08E+02(9.092E+01)	3.88E+02(6.85E+01)	3.77E+02(3.72E+01)
F_{16}	5.12E+01(1.14E+01)	7.92E+02(3.87E+01)	1.28E+02(1.09E+02)	2.34E+02(3.84E+01)	2.37E+02(1.37E+02)	5.20E+01(8.80E+00)	7.37E+01(5.13E+01)	**1.87E+01**(2.77E+00)
F_{17}	5.76E+01(1.20E+01)	7.91E+02(3.64E+02)	1.67E+02(1.50E+02)	3.33E+02(2.55E+01)	2.58E+02(1.66E+02)	5.35E+01(2.40E+01)	6.67E+01(2.12E+01)	**2.50E+01**(3.41E+00)
F_{18}	**9.00E+02**(0.00E+00)	1.20E+03(3.52E+02)	8.96E+02(3.62E+01)	9.04E+02(1.69E+01)	9.07E+02(8.08E+00)	9.04E+02(2.85E-01)	9.04E+02(1.04E+00)	**9.00E+02**(0.00E+00)
F_{19}	**9.00E+02**(0.00E+00)	1.16E+03(5.92E+02)	9.01E+02(3.05E+01)	9.16E+02(5.72E+00)	9.08E+02(8.36E+00)	9.04E+02(3.64E-01)	9.04E+02(9.42E-01)	**9.00E+02**(0.00E+00)
F_{20}	**9.00E+02**(0.00E+00)	1.12E+03(3.20E+01)	9.05E+02(2.20E+01)	9.18E+02(2.47E+01)	9.11E+02(7.29E+00)	9.04E+02(3.25E-01)	9.04E+02(9.01E-01)	**9.00E+02**(0.00E+00)
F_{21}	5.00E+02(5.21E-13)	1.26E+03(3.10E+01)	5.00E+02(1.16E-13)	5.00E+02(0.00E+00)	5.73E+02(1.95E+02)	5.00E+02(0.00E+00)	5.00E+02(4.88E-13)	**4.96E+02**(1.81E+01)
F_{22}	9.15E+02(9.06E+00)	1.17E+03(4.48E+01)	9.10E+02(9.60E+00)	9.36E+02(2.23E+01)	9.12E+02(3.38E+01)	8.60E+02(3.61E+01)	8.63E+02(2.43E+01)	**8.40E+02**(2.27E+01)
F_{23}	5.39E+02(4.17E-05)	1.29E+03(2.19E+01)	5.34E+02(4.28E-04)	5.34E+02(8.75E+01)	6.41E+02(1.98E+02)	5.34E+02(3.86E-04)	5.34E+02(4.12E-04)	**5.34E+02**(6.51E-13)
F_{24}	**2.00E+02**(7.09E-13)	1.09E+03(1.41E+01)	**2.00E+02**(1.23E-12)	**2.00E+02**(5.60E−03)	2.12E+02(6.00E+01)	**2.00E+02**(0.00E+00)	**2.00E+02**(2.85E-14)	**2.00E+02**(2.90E-14)
F_{25}	2.20E+02(1.96E+00)	1.18E0+3(2.39E+01)	2.11E+02(8.55E-01)	**2.00E+02**(1.45E−03)	2.04E+02(5.77E+00)	2.10E+02(4.85E-01)	2.11E+02(9.02E-01)	2.09E+02(2.50E-01)

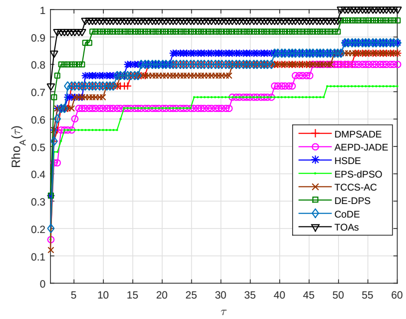


Figure 7. Performance profiles of TOAs and other state-of-the-art algorithm based on CEC2005 unconstrained problems ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, where Rho_A cumulative distribution function for ℓ_A)

Table X. AVERAGE FITNESS VALUES OF TOAS, JDE, NSDE, ODE, DEGL/SAW AND MGBDE ON 12 CLASSICAL UNCONSTRAINED PROBLEMS (BEST FITNESS VALUES SHOWN IN BOLDFACE)

Prob.	D	FFE_{max}	jDE	NSDE	ODE	DEGL/SAW	MGBDE	TOAs
F_1	25	500,000	4.05E-35	9.55E-35	1.64E-38	8.78E-37	6.21E-128	**0.00E+00**
F_2	25	500,000	8.34E-26	8.94E-30	3.51E-11	4.95E-36	5.86E-72	**0.00E+00**
F_3	25	500,000	4.28E-14	3.06E-09	7.18E-04	1.21E-26	9.50E-24	**0.00E+00**
F_4	25	500,000	3.02E-14	2.09E-11	1.28E-27	4.99E-15	2.67E-24	**1.18E-280**
F_5	25	500,000	5.65E-26	2.65E-25	1.09E-04	6.89E-25	2.83E-11	**0.00E+00**
F_6	25	500,000	1.67E-36	4.04E-28	**0.00E+00**	9.56E-48	**0.00E+00**	**0.00E+00**
F_7	25	500,000	3.76E-02	4.35E-03	1.31E-03	**1.05E-07**	4.56E-04	3.84E-05
F_8	25	500,000	**-10475**	**-10475**	-5982.1	**-10475**	**-10475**	**-10475**
F_9	25	500,000	6.73E-24	4.84E-21	6.53E+01	5.85E-25	4.97E-17	**0.00E+00**
F_{10}	25	500,000	7.83E-15	5.97E-10	4.14E-15	**5.98E-23**	4.14E-15	8.88E-16
F_{11}	25	500,000	1.83E-28	7.93E-26	**0.00E+00**	2.99E-36	**0.00E+00**	**0.00E+00**
F_{12}	25	500,000	9.37E-24	5.85E-21	**1.88E-32**	7.21E-27	**1.88E-32**	**1.88E-32**

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[2] R. Tanabe and A. Fukunaga, "Improving the search performance of shade using linear population size reduction," in *IEEE Congress on Evolutionary Computation*, July 2014, pp. 1658–1665.

[3] R. Sarker, S. Elsayed, and T. Ray, "Differential evolution with dynamic parameters selection for optimization problems," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 5, pp. 689–707, Oct 2014.

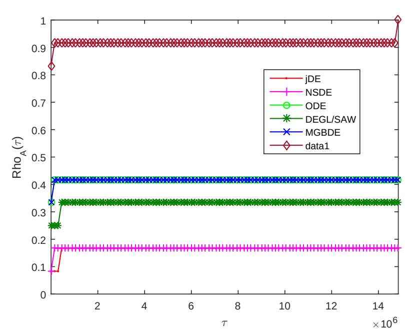

Figure 8. Performance profiles of TOAs and other state-of-the-art algorithm based on 12 unconstrained problems ($Rho_A(\tau) = \frac{1}{\text{number of problems}}$ probability of algorithm A's performance ratio $\left(\ell_A = \frac{f_A}{f_{best}}\right)$ less than $\tau \in \mathbb{R}$ of best possible ratio, where Rho_A cumulative distribution function for ℓ_A)

Table XI. COMPUTATIONAL RESULTS OF TOAS, μ-JADE AND GADE ON 12 CLASSICAL UNCONSTRAINED PROBLEMS WITH $D = 30$ VARIABLES (BEST FITNESS VALUES SHOWN IN BOLDFACE)

Prob.	μ-JADE[4]		GaDE[5]		TOAs	
	FFE_{max}	average fitness value (standard deviation)	FFE_{max}	average fitness value (standard deviation)	FFE_{max}	average fitness value (standard deviation)
F_1	3,000,000	9.00E-13(2.50E-12)	150,000	**0.00E+00**(0.00E+00)	150,000	8.88E-145(5.37E-144)
F_2	3,000,000	1.30E-06(1.60E-06)	200,000	**0.00E+00**(0.00E+00)	200,000	3.34E-115(1.83E-114)
F_3	3,000,000	9.70E+02(4.20E+02)	500,000	2.42E-01(1.95E-01)	500,000	**0.00E+00(0.00E+00)**
F_4	3,000,000	4.90E+00(2.50E+00)	500,000	3.89E-02(1.18E-01)	500,000	**7.55E-237(0.00E+00)**
F_5	3,000,000	4.30E+01(2.90E+01)	2,000,000	1.39E+01(4.11E+00)	2,000,000	**0.00E+00(0.00E+00)**
F_6	3,000,000	2.00E-02(1.40E-01)	150,000	**0.00E+00**(0.00E+00)	150,000	**0.00E+00(0.00E+00)**
F_7	3,000,000	4.60E-02(1.40E-02)	300,000	4.86E-03(1.22E-03)	300,000	**5.14E-05(3.35E-05)**
F_8	3,000,000	4.00E+03(4.50E+02)	900,000	1.34E-02(5.26E-18)	900,000	**-1.26E+04(3.78E-12)**
F_9	3,000,000	1.20E+02(1.40E+01)	500,000	**0.00E+00**(0.00E+00)	500,000	**0.00E+00(0.00E+00)**
F_{10}	3,000,000	2.30E-02(1.60E-01)	200,000	1.07E-15(1.64E-15)	200,000	**8.88E-16(0.00E+00)**
F_{11}	3,000,000	2.10E-03(4.80E-03)	150,000	**0.00E+00**(0.00E+00)	150,000	**0.00E+00(0.00E+00)**
F_{12}	3,000,000	2.60E-03(1.40E-02)	150,000	**1.57E-32**(2.76E-48)	150,000	**1.38E-32(5.73E-34)**

Table XII. MEAN RANK OF μ-JADE,GADE AND TOAS BASED ON FRIEDMAN TEST FOR THE 12 CLASSICAL PROBLEMS WITH D=30

μ-JADE	GaDE	TOAs
3.00	1.67	**1.33**

[4] C. Brown, Y. Jin, M. Leach, and M. Hodgson, "\mu jade: adaptive differential evolution with a small population," *Soft computing*, vol. 20, no. 10, pp. 4111–4120, 2016.

[5] Y. Fan, Q. Liang, C. Liu, and X. Yan, "Self–adapting control parameters with multi–parent crossover in differential evolution algorithm," *International Journal of Computing Science and Mathematics*, vol. 6, no. 1, pp. 40–48, 2015.