EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION USING AN OUTRANKING-BASED DOMINANCE GENERALIZATION

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Abstract

One aspect that is often disregarded in the current research on evolutionary multiobjective optimization is the fact that the solution of a multiobjective optimization problem involves not only the search itself, but also a decision making process. Most current approaches concentrate on adapting an evolutionary algorithm to generate the Pareto frontier. In this work, we present a new idea to incorporate preferences into a Multi-Objective Evolutionary Algorithm (MOEA). We introduce a binary fuzzy preference relation that expresses the degree of truth of the predicate "***x*** is at least as good as ***y***". On this basis, a strict preference relation with a reasonably high degree of credibility can be established on any population. An alternative ***x*** is not strictly outranked if and only if there does not exist an alternative ***y*** which is strictly preferred to ***x***. It is easy to prove that the best solution is not strictly outranked. For validating our proposed approach, we used the Nondominated Sorting Genetic Algorithm II (NSGA-II), but replacing Pareto dominance by the above non-outranked concept. So, we search for the non-strictly outranked frontier that is a subset of the Pareto frontier. In several instances of a nine-objective knapsack problem our proposal clearly outperforms the standard NSGA-II, achieving non-outranked solutions which are in an obviously privileged zone of the Pareto frontier.

Key Words: multicriteria optimization; evolutionary algorithms; fuzzy preferences; outranking relations.

1. Introduction

In real-world optimization problems, the decision-maker (*DM*) is usually concerned with several criteria which determine the quality of solutions. Often, constraints in mathematical programming problems are not actually mandatory; instead, such restrictions are expressing an important desire, a significant *DM* aspiration level about certain system properties. Therefore, most optimization problems can be represented from a multiple objective perspective.

As a consequence of the conflicting nature of the criteria, it is not possible to obtain a single optimum, and, consequently, the ideal solution of a multiobjective problem (MOP) cannot be reached. Hence, to solve a MOP means to find the best compromise solution according to the *DM*'s particular system of preferences (value system). It is easy to prove that the best compromise is a non-dominated solution (i.e., a member of the Pareto optimal set). Most operations research methods for MOPs can be classified into the following categories [1]:

1. Techniques which perform an *a priori* articulation of the *DM*'s preferences;
2. Interactive methods, which perform a progressive articulation of the *DM*' preferences;
3. Generating techniques, which perform an *a posteriori* articulation of the *DM*'s preferences (search before making decisions).

Since David Schaffer's seminal work (cf.[2]), Multi-Objective Evolutionary Algorithms (MOEAs) have become a very popular search engine for solving multiobjective programming problems. MOEAs are very attractive to solve MOPs because they deal simultaneously with a set of possible solutions (the MOEA's population) which allows them to obtain an approximation of the Pareto frontier in a single algorithm's run. Thus, by using MOEAs the *DM* and/or the decision analyst does not need to perform a set of separate single-objective optimizations as normally required when using operations research methods. Additionally, MOEAs are more robust regarding the shape or continuity of the Pareto front, whereas these two issues are a real concern for classical optimization methods (cf.[3]). However, one aspect that is often disregarded in the MOEAs literature is the fact that the solution of a problem involves not only the search process, but also (and normally, more important) the decision making process. Most current approaches in the evolutionary multiobjective optimization literature concentrate on adapting an evolutionary algorithm to generate an approximation of the Pareto optimal set. Nevertheless, finding this set does not solve the problem. The *DM* still has to choose the best compromise solution out of that set. This is not a very difficult task when dealing with problems having 2 or 3 objectives. However, as the number of criteria increases, two important difficulties arise:

a) The algorithm's capacity to find this Pareto frontier quickly degrades;
b) It becomes harder, or even impossible for the *DM* to establish valid judgments in order to compare solutions with several conflicting criteria.

Here, we propose a combined approach, with an a priori articulation of preferences followed by a generating process of a specific (i.e., desirable) zone of the Pareto frontier. Using a fuzzy outranking relation, a strict preference relation in the sense of [4] can be established in any population. Our proposal is based on finding a subset of the Pareto frontier composed of solutions for which no other solutions exist which are preferred to the first ones. This non-outranked concept will be used instead of dominance when performing the evolutionary search.

The remainder of this paper is organized as follows. An outranking model of multicriteria preferences is outlined in Section 2, and on this basis the proposed dominance generalization is detailed in Section 3. Our algorithmic proposal is discussed in Section 4 and illustrated by some computer experiments in Section 5. Finally, we draw brief concluding remarks in Section 6.

2. An Outranking Model of Preferences

Let G be the set of objective functions of a multicriteria optimization problem and O its objective space. An element $\boldsymbol{x} \in O$ is a vector $(x_1, \dots x_n)$, where x_i is the *i-th* objective value. Let us suppose that for each criterion j there is a relational system of preferences (P_j, I_j) (preference, indifference) which is complete on the domain of the *j-th* criterion (G_j). That is, $\forall (x_j, y_j) \in G_j \times G_j$ one and only one of the following statements is true:

$$\begin{array}{l} - x_j P_j y_j \\ - y_j P_j x_j \\ - x_j I_j y_j \end{array} \qquad (1)$$

Formulation (1) allows indifference thresholds in order to model some kind of imprecise one-dimensional preferences. It should be noticed that the relational system of preferences given by (1) is more general than the usual formulations which consider only true criteria (that is, $x_j \neq y_j$ implies non-indifference). Without loss of generality, in the following we suppose that $x_j P_j y_j \Rightarrow x_j > y_j$.

Let us establish the following central premise: For each $(\boldsymbol{x},\boldsymbol{y}) \in O \times O$, the *DM* and the decision analyst (working together) are able to create a fuzzy predicate modeling the degree of truth of the statement "$\boldsymbol{x}$ is at least as good as $\boldsymbol{y}$ from the DM's point of view".

Amongst different ways to create that predicate, we shall describe below an outranking approach based on ELECTRE methods:

A proposition $\boldsymbol{xSy}$ ("$\boldsymbol{x}$ outranks $\boldsymbol{y}$") ("$\boldsymbol{x}$ seems at least as good as $\boldsymbol{y}$") holds if and only if the coalition of criteria in agreement with this proposition is strong enough and there is no important coalition discordant with it (cf.[5]). It can be expressed by the following logical equivalence (cf.[6]):

$$\boldsymbol{xSy} \Leftrightarrow C(\boldsymbol{x},\boldsymbol{y}) \wedge \sim D(\boldsymbol{x},\boldsymbol{y}) \qquad (2)$$

where:

$C(\boldsymbol{x},\boldsymbol{y})$ is the predicate about the strength of the concordance coalition;
$D(\boldsymbol{x},\boldsymbol{y})$ is the predicate about the strength of the discordance coalition;
$\wedge$ and ~ are logical connectives for conjunction and negation, respectively.
Let $c(\boldsymbol{x},\boldsymbol{y})$ and $d(\boldsymbol{x},\boldsymbol{y})$ denote the degree of truth of the predicates $C(\boldsymbol{x},\boldsymbol{y})$ and $D(\boldsymbol{x},\boldsymbol{y})$. From (2), the degree of truth of $\boldsymbol{xSy}$ can be calculated as in the ELECTRE-III method:

$$\sigma(\boldsymbol{x},\boldsymbol{y}) = c(\boldsymbol{x},\boldsymbol{y}).\ N(d(\boldsymbol{x},\boldsymbol{y})) \qquad (3)$$

where $N(d(\boldsymbol{x},\boldsymbol{y}))$ denotes the degree of truth of the non-discordance predicate.
As in the earlier versions of the ELECTRE methods, we shall take

$$c(\boldsymbol{x},\boldsymbol{y}) = \sum_{j \in C_{x,y}} w_j \qquad (4)$$

where $C_{x,y} = \{j \in G \text{ such that } x_j P_j y_j \vee x_j I_j y_j\}$; w's denote "weights" ($w_1 + w_2 + \ldots + w_n = 1$) and $\vee$ is the symbol for disjunction.

Let $D_{x,y} = \{j \in G \text{ such that } y_j P_j x_j\}$ be the discordance coalition with $\boldsymbol{xSy}$. The intensity of discordance is measured in comparison with a veto threshold v_j, which is the maximum difference y_j-x_j compatible with $\sigma(\boldsymbol{x},\boldsymbol{y})>0$. Following Mousseau and Dias ([7]), we shall use here a simplification of the original formulation of the discordance indices in the ELECTRE-III method which is given by:

$$N(d(\boldsymbol{x},\boldsymbol{y})) = \min_{j \in D_{x,y}} [1 - d_j(\boldsymbol{x},\boldsymbol{y})] \qquad (5)$$

$$d_j(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} 1 & \text{iff } \nabla_j \geq v_j \\ (\nabla_j - u_j)/(v_j - u_j) & \text{iff } u_j < \nabla_j < v_j \\ 0 & \text{iff } \nabla_j \leq u_j \end{cases} \qquad (6)$$

where $\nabla_j = y_j$-x_j and u_j is a discordance threshold (see Figure 1).

In practical situations the decision-maker supported by a potential decision-analyst should assess the set of model´s parameters which are needed to evaluate σ. This is not an easy task, since decision-makers usually have difficulties in specifying outranking parameters and require an intense support by a decision analyst. To facilitate this process, the pair *DM*-decision analyst can use the preference disaggregation-analysis (*PDA*) paradigm (cf. [8]), which has received an increasing interest from the multicriteria decision support community. *PDA* infers the model's parameters from holistic judgments provided by the *DM*. Those judgments may be obtained from different sources (past decisions, decisions made for a limited set of fictitious objects (actions), or decisions taken for a subset of the objects under consideration for which the *DM* can easily make a judgment [9]). In the framework of outranking methods *PDA* has been recently approached by [10-12].

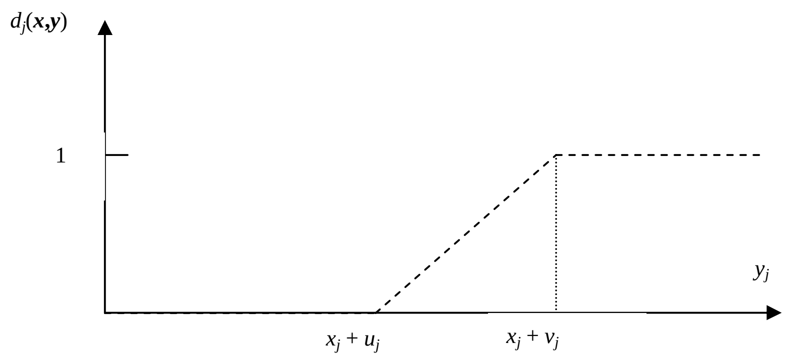

Figure 1 Partial discordance relation $d_j(\boldsymbol{x},\boldsymbol{y})$

3. An Outranking-Based Dominance Generalization

The λ-cut $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda$ defines a crisp outranking relation $\boldsymbol{x}S\boldsymbol{y}$. Credible outranking statements are obtained with λ= 0.75 (strong outranking), and even with λ==0.67 (weak outranking) (cf. [13]). $\sigma(\boldsymbol{x},\boldsymbol{y}) >\approx 0.5$ is identified as a doubtful outranking, and $\sigma(\boldsymbol{x},\boldsymbol{y}) < 0.5$ means a definitive no outranking.
According to Roy (cf.[4]):

$\boldsymbol{x}S\boldsymbol{y} \wedge \sim\boldsymbol{y}S\boldsymbol{x} \Leftrightarrow \sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < \lambda \Rightarrow$ a presumed preference favoring $\boldsymbol{x}$.

Following [14], we assume the existence of a threshold β>0 such that if $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq \lambda$ and also $\sigma(\boldsymbol{y},\boldsymbol{x}) \leq (\sigma(\boldsymbol{x},\boldsymbol{y})-\beta)$, then there is an asymmetric preference relation favoring $\boldsymbol{x}$ what will be denoted by $\boldsymbol{x}\mathrm{P}(\lambda,\beta)\boldsymbol{y}$. It can be agreed that for some values of λ and β, the conditions defining $\mathrm{P}(\lambda,\beta)$ are good arguments for justifying a strict preference relation in the sense proposed by Roy ([4]). β may be a function of $(\sigma(\boldsymbol{x},\boldsymbol{y}), \sigma(\boldsymbol{y},\boldsymbol{x}))$. In the following we consider that $\mathrm{P}(\lambda,\beta)$ has been defined on *O*.
Amongst different ways of defining a reasonable strict preference relation we suggest the following:

$\boldsymbol{x}\mathrm{P}(\lambda,\beta)\boldsymbol{y}$ if one of the following propositions is held:

i. $\boldsymbol{x}$ dominates $\boldsymbol{y}$
ii. $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq 0.67 \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.5$
iii. $\sigma(\boldsymbol{x},\boldsymbol{y}) \geq 0.67 \wedge (0.5 \leq \sigma(\boldsymbol{y},\boldsymbol{x}) < 0.67) \wedge (\sigma(\boldsymbol{x},\boldsymbol{y}) - \sigma(\boldsymbol{y},\boldsymbol{x})) \geq \delta$

δ is a strictly outranking parameter whose value might depend on the number of criteria (cf. ([15]). By consistency of ii and iii, δ should be greater than (0.67 – 0.5) = 0.17.

Definition 1. $\boldsymbol{x}$ strictly outranks $\boldsymbol{y}$ iff $\boldsymbol{x}\mathrm{P}(\lambda,\beta)\boldsymbol{y}$.

Definition 2: Let A be a subset of O. If there does not exist $\boldsymbol{y} \in A$ such that $\boldsymbol{y}P(\lambda,\beta)\boldsymbol{x}$, we say that $\boldsymbol{x}$ is a non-strictly outranked solution in A.

Theorem 1: The set of non-outranked solutions in O is a subset of the Pareto frontier.
Proof:
If the set of non-outranked solutions in O is empty, it is a proper subset of the Pareto frontier. Otherwise, we should prove that

$\boldsymbol{a}$ is a non-strictly outranked solution in $O \Rightarrow \boldsymbol{a}$ is a Pareto solution.

The proof is very simple. Suppose that $\boldsymbol{a}$ is dominated by $\boldsymbol{b} \in O$. By definition of $P(\lambda,\beta)$ we have $\boldsymbol{b}P(\lambda,\beta)\boldsymbol{a}$. Hence, $\boldsymbol{b}$ strictly outranks $\boldsymbol{a}$ in contradiction with the hypothesis.

The reciprocal of Theorem 1 is false. $\boldsymbol{a}$ may be a Pareto solution while being strictly outranked by $\boldsymbol{b}$ simultaneously. It suffices to find $\boldsymbol{b}$ such that $\boldsymbol{b}P(\lambda,\beta)\boldsymbol{a}$ by satisfying ii or iii. In such cases, according to Theorem 1, the set of non-outranked solutions is a proper subset of the Pareto frontier.

Definition 3: $P(\lambda,\beta)$ is said to be free of inconsistencies iff there are no cycles of that relation in O.

Definition 4: $P(\lambda,\beta)$ is said to be minimally free of inconsistencies iff there does exist at least one non-strictly outranked solution in O.

Definition 5: For an element $\boldsymbol{x} \in O$, the strictly outranking set is defined as $S_o = \{\boldsymbol{y} \in O$ such that $\boldsymbol{y}P(\lambda,\beta)\boldsymbol{x}\}$. The cardinal of this set is denoted by $card\,(S_o)$. This is an integer function depending on $\boldsymbol{x}$.

Definition 6: The weakness of $\boldsymbol{x}$ in a set A is $W = card\,\{\boldsymbol{y} \in A$ such that $\sigma(\boldsymbol{y},\boldsymbol{x}) > \sigma(\boldsymbol{x},\boldsymbol{y}) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) \geq 0.5\}$.

Definition 7: The strength of $\boldsymbol{x}$ in a set A is $S_t = card\,\{\boldsymbol{y} \in A$ such that $\sigma(\boldsymbol{x},\boldsymbol{y}) > \sigma(\boldsymbol{y},\boldsymbol{x}) \wedge \sigma(\boldsymbol{y},\boldsymbol{x}) \geq 0.5\}$.

It can be proved that the best alternatives in a set should be found among those in which $card\,(S_o)$ is minimal (cf. [14]). Suppose that $P(\lambda,\beta)$ is minimally free of inconsistencies Hence, the best compromise solution of the multiobjective optimization problem should be a non-strictly outranked solution in O. When every solution is strictly outranked by another one, the best compromise should be found among the set of $\boldsymbol{x}$ with minimum cardinal of S_o.

4. Adapting the NSGA-II to Work with Non-strictly Outranked Classes

We shall extend the Non-dominated Sorting Genetic Algorithm II (cf. [17]) working with non-strictly outranked individuals instead of non-dominated ones. The "filtering" process is similar, but extracting non-strictly outranked individuals which form classes with the same value of card (S_o). The first front may have card$(S_o) \neq 0$ when $P(\lambda,\beta)$ is not minimally free of inconsistencies.

Unlike typical MOEAs, we are not interested in obtaining a uniform distribution of solutions representing the Pareto frontier. Therefore, instead of the NSGA-II's crowding distance (or another density estimator), we propose to use the above weakness measure. That is, when two individual with equal card(S_o) are compared (in binary tournaments or deciding who will be included into the new generation), the least weak will be preferred. This adapted algorithm will be called the Non-Outranked-Sorting Genetic Algorithm (NOSGA), whose pseudocode is shown below:

```
PROCEDURE NOSGA (K, Number_of_Generations)
Initialize Population P
Generate random population with size K
Evaluate objective values
Evaluate σ on P×P
For each x ∈ P, calculate card (So); calculate the weakness of x in P
Generate fronts of equal values of card (So )
Assign to these fronts a rank (level) based on card (So )
Generate Child Population Q with size K
        Perform Binary Tournament Selection
        Perform Recombination and mutation
FOR I = 1 to Number_of_Generations  DO
      Assign P' = P ∪ Q
      Evaluate σ on P'×P'
      FOR each parent and child in P' DO
          Calculate card (So); calculate its weakness in P'
          Assign rank (level) based on card (So )
          Loop (inside) by adding solutions to the
          next generation until K individuals have been found
     End FOR
     Replace P by the K individuals found
     Generate Child Population Q with size K
           Perform Binary Tournament Selection
           Perform Recombination and mutation
End FOR
End PROCEDURE
```

This pseudocode was adapted from the NSGA-II procedures shown in [16, 17]. As in the NSGA-II method, the rank assigned to each individual is the fitness criterion. The main differences are: i) the use of σ in NOSGA; ii) the sorting based on Pareto dominance is replaced by a sorting based on strict outranking; and iii) the use of weakness instead of a density estimator.

NOSGA's selective pressure depends on σ values on the current population. Note that when no veto condition is held, $\sigma(\boldsymbol{x},\boldsymbol{y})$ is determined by the strength of the concordance coalition; its value is obtained from a "weighted-proportion", in which the total amount of criteria is not relevant. Therefore, σ is weakly influenced by the dimension of the objective space, which could be an important advantage in problems with more than a few objective functions. Since in NOSGA the information about objective space is aggregated in the fuzzy outranking relation, such a relative independence should make NOSGA very robust with respect to an increasing number of objective functions.

5. Some Computer Experiments

In order to validate the proposal presented in this paper, we have performed two tests, both using nine-objective knapsack problems. The first one is a controlled experiment in which both the true

Pareto frontier and the true non-strictly outranked set are known. The second one is a more realistic problem in which the best sets are unknown.

Let us consider a decision making situation in which the *DM* is choosing among L different social policies (projects) each with a direct social impact. This is measured by using a nine-component vector (N_1, N_2,…. N_9). $N_i = n_{kj}$, the number of people belonging to the *k*-th social category which receive the *j*-th level benefit from that policy or project. In those examples k= 1, 2, 3 correspond to (Extreme Poverty, Poverty, Middle), and j = 1, 2, 3 to (High Impact, Middle Impact, Low Impact). N_1, N_2, N_3 correspond to extreme poverty people; N_7, N_8, N_9 concern middle class.

Let us denote by N_i^m the value of N_i associated to the *m*-th project. Let C be a portfolio (a subset of the L projects which receives financial support). The value of N_i for the whole portfolio is $N_i(C) = z_1 N_i^1 + \ldots + z_L N_i^L$ where $z_j = 1$ if the *j*-th project is supported and $z_j = 0$, otherwise. The aim of this decision problem is to choose the "best" portfolio satisfying some budget constraints. More formally:

$$\underset{C \in R_F}{\text{Maximize}}\ (N_1(C), N_2(C), \ldots N_9(C)) \qquad (7)$$

where R_F is a feasible region determined by budget constraints.

We use binary encoding; a '1' in the individual *j*-th allele means that the *j*-th project belongs to this particular portfolio. Other parameters of the evolutionary search are: crossover probability = 1; mutation probability = 0.02; population size = 100.

Preference model parameters:

A) The weights; they express the importance of the different objectives. In these experiments, the weights were assessed by a decision-maker following the interpretation of weights as "votes", which is typical of ELECTRE methods (cf. [13]). The assessed values were: (23, 14, 11, 14, 11, 7, 9, 7, 4).

B) Indifference thresholds; usually, those thresholds are used to model some sources of imprecision or uncertainty; here, they were calculated as a measure of the error evaluating each objective.

C) Veto thresholds; they are settled as $0.5*(\text{Max } f_i - \text{Min } f_i)$ as in some applications of ELECTRE methods (cf. [13, 18]); operations Max and Min act on a population.

D) The strict outranking parameter δ was settled as 0.2.

5.1 The Control Test

The information about 20 candidate projects is shown in Table 1. The different values are given in thousands. Budget constraints are imposed by the class of project (educational, health, etc.), geographic region and to the whole portfolio. The total available budget was set as Total_budget= 500 million dollars. The constraints by class and region are given by:

$$\begin{aligned}
&0.3\ \text{Total_budget} \le \text{Budget_Class } 1 \le 0.4\ \text{Total_budget}\\
&0.25\ \text{Total_budget} \le \text{Budget_Class } 2 \le 0.35\ \text{Total_budget}\\
&0.2\ \text{Total_budget} \le \text{Budget_Class } 3 \le 0.3\ \text{Total_budget} \qquad (8)\\
&0.4\ \text{Total_budget} \le \text{Budget_Region } 1 \le 0.6\ \text{Total_budget}
\end{aligned}$$

0.4 Total_budget $\leq$ Budget_Region 2 $\leq$ 0.6 Total_budget

Table 1: Applicant projects

Project	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	Support needed	Class	Region
1	0	0	45	0	15	0	0	18	0	50,000	3	1
2	0	25	0	15	0	0	54	0	0	49,500	1	1
3	0	35	0	0	15	0	0	48	0	49,000	2	1
4	25	0	0	7.5	0	0	0	0	54	48,500	2	1
5	0	25	0	7.5	0	0	0	0	48	48,000	2	2
6	45	0	0	4.5	0	0	0	18	0	47,500	3	2
7	0	0	35	0	4.5	0	0	0	48	47,000	2	2
8	5	0	0	0	4.5	0	54	0	0	46,500	1	2
9	15	0	0	4.5	0	0	12	0	0	46,000	3	1
10	0	0	5	0	13.5	0	36	0	0	45,500	3	2
11	0	0	15	15	0	0	30	0	0	45,000	1	2
12	0	0	35	1.5	0	0	0	36	0	44,500	3	2
13	0	0	15	0	3	0	24	0	0	44,000	3	1
14	40	0	0	0	1.5	0	0	0	24	43,500	3	1
15	0	0	20	0	0	3	0	0	12	43,000	1	2
16	0	40	0	0	15	0	0	42	0	42,500	2	2
17	45	0	0	0	4.5	0	48	0	0	42,000	2	1
18	0	0	30	0	0	4.5	0	0	24	41,500	3	2
19	10	0	0	0	0	3	60	0	0	41,000	2	1
20	0	10	0	15	0	0	30	0	0	40,500	1	2

In this problem the set of feasible portfolios was exhaustively explored by performing an enumerative search. This set contains 1,635 non-dominated solutions and only six non-strictly outranked ones. These are shown in Table 2.

Table 2: Non strictly outranked portfolios

Portfolio	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9
1	145	110	60	49.5	55.5	3	276	126	24
2	140	110	80	49.5	51	6	222	126	36
3	170	75	60	57	40.5	3	276	78	78
4	140	75	80	61.5	34.5	6	234	78	66
5	165	75	80	57	36	6	222	78	90
6	185	75	15	61.5	25.5	3	288	60	78

A single run of the standard NSGA-II (Population size = 100, mutation probability = 0.02, crossover probability = 1) found 93 non-dominated solutions. All are strictly outranked. Additionally, a single run of the NOSGA found in the first front the six solutions is pointed-out in

Table 2. This experiment was replicated in several random instances with similar results, which are pointed-out in Table 3.

Table 3: Results of a control experiment (nine objectives)

Instance	Enumerative Search		NSGA II		NOSGA	
	NO	ND	NO	ND	NO	ND
1	6	1635	3	93	6	6
2	1	2038	1	99	1	6
3	4	1145	0	91	4	4

In Table 3, "NO" and "ND" are associated to non-strictly outranked and non-dominated solutions, respectively. Column NO (ND) below NSGA-II contains the number of individuals which are actually non-strictly outranked (non-dominated) solutions, and which were found in the first rank of such algorithm. Besides, column NO (ND) below NOSGA contains the number of non-strictly outranked (non-dominated) solutions which were found in the first rank of our algorithmic proposal. By comparing the different columns of Table 3, it should be noticed that the NSGA-II approaches the true Pareto front, but fails in finding most of the non-strictly outranked solutions. NOSGA finds the true non-strictly outranked set.

A similar control problem was performed to test the influence of the number of objectives. We used the same information about projects shown in Table 1 but considering only four objective functions (objectives 4, 6, 7, 9 in Problem 7). The criterion weights were updated by using the normalization condition. The budget constraints were imposed as in the above example. Some results are presented in Table 4.

Table 4: Results of a control experiment (four objectives)

Instance	Enumerative Search		NSGA II		NOSGA	
	NO	ND	NO	ND	NO	ND
1	10	276	0	65	7	7
2	3	136	3	97	3	3
3	12	65	7	53	8	8

The NSGA-II shows good results in Instances 2 and 3, but is always outperformed by NOSGA. Comparing Tables 3 and 4, it seems that the NSGA-II results are degraded with nine objectives. Contrarily, NOSGA performs even better in the more complex problem.

5.2 A more realistic example

Secondly, we solved again Problem 7, but now with 100 applicant projects characterized by the same nine-objectives set as in the previous example. In a similar way, the feasible region was determined by the total budget and requirements by class of project and geographic region. The total budget was set as 2.5 billion dollars, and the other constraints were imposed as in (8). The (known) non-outranked front of one random instance of this problem is shown in Table 5. The objective values are given in thousands. Weakness, strength and net flow score were calculated on the final parent-offspring population after 500 generations. Weakness and strength are given by Definitions 6-7. The outranking net flow score was calculated as in [19].

Table 5: Some results in a real size problem

Portfolio	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	**W**	S_t	**NFS**
1	550	950	550	825	1020	660	942	840	564	42	108	16.72
2	555	880	580	975	1035	630	888	798	648	45	105	9.25
3	550	930	550	885	1020	645	936	846	564	45	103	12.14
4	550	1015	490	855	1005	690	882	876	558	59	91	4.85
5	550	935	545	825	975	720	930	858	564	61	89	6.09
6	550	960	530	1080	900	630	888	768	642	65	85	7.42
7	550	1030	490	855	990	690	870	912	558	69	81	-2.29
Ideal	560	1,230	700	1,350	1,410	840	1,008	1,200	834			
Nadir	55	370	80	375	375	120	216	276	162			

W.- Weakness ; S_t.- Strength; NFS.- Net Flow Score

The best solutions seem to be 1, 2 and 3. It is obvious that those solutions are concentrated in a relatively small region of the objective function space. This experiment was replicated in other four random instances, with similar results. Coded in TURBO C++ 3.0, the average run time was 2.5 minutes on a laptop computer with a 1.67 GHz processor, 2 GB RAM and a 120GB hard disk. By using the standard NSGA-II, an approximation to the Pareto front was obtained for the same instances. In fact, the ideal and nadir points in Table 3 were found by the NSGA-II. In the following, NO_k and ND_k will denote the known non-strictly outranked and non-dominated sets, respectively, for the k-th instance. Let U be $NO_k \cup ND_k$. Let NO_U and ND_U be the non-strictly outranked set and the non-dominated set in U, respectively. A comparison between NO_k and ND_k was performed in such five random instances with the results shown in Tables 6, 7 and 8:

Table 6: Mean Values in U

Set	Weakness	Strength	Net Flow Score
NO_1	3.7	71.3	39.4
ND_1	20.9	18.8	-2.8
NO_2	2.0	91.2	55.9
ND_2	34.4	37.3	-2.8
NO_3	3.8	93.6	56.9
ND_3	36.7	36.1	-4.6
NO_4	3.9	86.4	58.7
ND_4	32.6	30.8	-5.3
NO_5	2.0	88	65.7
ND_5	34.0	34.4	-3.3

After calculating $\sigma(\boldsymbol{x},\boldsymbol{y})$ on U, a ranking of this set considering weakness, strength and net flow was performed. In every test instance the solutions belonging to NO_k are the best in U. As shown in Table 6, the mean value of weakness, strength, and net flow scores taken on NO_k are clearly better than the corresponding mean values on ND_k .

Table 7: Robustness of NO_k

Instance	Card(NO_k)	Card(NO_U)	Card($NO_k \cap NO_U$)	Card($NO_k \cap ND_U$)
1	7	7	7	7
2	5	6	5	5
3	8	8	8	8
4	9	9	9	9
5	5	5	5	5

From Table 7, it should be noticed that
1. Each $\boldsymbol{x} \in NO_k$ is not dominated in U;
2. Each $\boldsymbol{x} \in NO_k$ remains as non-strictly outranked in U;
3. Only one non-strictly outranked solution is added by ND_k (in the second instance).

Table 8: Robustness of ND_k

Instance	Card(ND_k)	Card(NO_U)	Card($ND_k \cap NO_U$)	Card($ND_k \cap ND_U$)
1	100	7	0	65
2	100	6	1	89
3	100	8	0	84
4	100	9	0	80
5	100	5	0	82

Additional remarks:

4. In four instances, no $\boldsymbol{x} \in ND_k$ is member of NO_U ; we can conclude that the NSGA-II does not find the non-strictly outranked set. So, it is not possible to guarantee that the best compromise solution is obtained by this algorithm.
5. 11-35% of the solutions belonging to ND_k are actually dominated by some element of NO_k.

From the above remarks, it can be concluded that (accepting $\sigma(\boldsymbol{x},\boldsymbol{y})$ as a good model of the outranking statement degree of truth), NO_k is a preference privileged zone in the objective function space. The best front found by NSGA-II (although may be representative of the Pareto frontier) may not contain the best compromise solutions. In fact, unlike NOSGA, the best front found by NSGA-II is not representative of the non-outranked set.

6. Conclusions

The proposed dominance generalization by using the degree of credibility of an outranking statement helps to find a subset of the Pareto frontier which contains the best compromise solution.

Our proposal (NOSGA) is basically a derivation from the standard NSGA-II in which we replace dominance by its outranking-based generalization. In several instances of different examples, NOSGA clearly outperforms the NSGA-II, achieving non-outranked solutions which are in an obvious privileged zone of the Pareto frontier. Those solutions are few, concentrated, and very satisfactory. A good compromise can be easily detected on the non-outranked front. Additionally, as the overall multiobjective performance is aggregated in $\sigma(\boldsymbol{x},\boldsymbol{y})$, NOSGA seems to be weakly dependent on the number of objective functions. This should be confirmed by more extensive experimentation.

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References

1. Coello Coello, C.A., Lamont, G.B., Van Veldhuizen, D.A. Evolutionary Algorithms for Solving Multi_Objective Problems, Second Edition, Springer, New York, 2007
2. Schaffer, J.D. Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. In Erlbaum, L. (ed.) Genetic Algorithms and their Applications: Proceedings of the First International Conference on Genetic Algorithms, 1985, 93-100.
3. Coello Coello, C. A comprehensive survey of Evolutionary-Based Multiobjective Optimization techniques. Knowledge and Information Systems. An International Journal, 1999 1 (3), 269-308
4. Roy, B. Multicriteria Methodology for Decision Aiding, Kluwer, 1996
5. Roy, B. The Outranking Approach and the Foundations of ELECTRE methods. In Bana e Costa, C.A., (ed.) Reading in Multiple Criteria Decision Aid, 1990, Springer-Verlag, Berlin , 155-183.
6. Perny, P. Multicriteria filtering methods based on concordance and non-discordance principles. Annals of Operations Research, 1998 156 (2), 37-165.
7. Mousseau, V., Dias, L.C. Valued outranking relations in ELECTRE providing manageable disaggregation procedures. European Journal of Operational Research, 2004, 156 (2), 467–482.
8. Jacquet-Lagreze, E., Siskos, Y. Preference disaggregation: Twenty years of MCDA experience. European Journal of Operational Research, 2001 130 (1), 233-245
9. Doumpos, M, Zopounidis, C. Multicriteria decision aid classification methods. Kluwer Academic Publishers, Dordrecht-Boston- London, 2002.
10. Dias, L.C., Mousseau, V. Inferring ELECTRE's veto-related parameters from outranking examples. European Journal of Operational Research, 2006 170 (1), 172–191.
11. Fernandez, E., Navarro, J., Bernal, S. Multicriteria sorting using a valued indifference relation under a preference disaggregation paradigm. European Journal of Operational Research 2008, doi:10.1016/j.ejor.2008.09.020
12. Doumpos, M, Zopounidis, C. An evolutionary approach to construction of outranking models for multicriteria classification: The case of ELECTRE TRI method. European Journal of Operational Research 2008, doi:10.1016/j.ejor.2008.11.035
13. Ostanello, A. Outranking Methods, Proc. of the First Summer School on MCDA, 1983, Sicily, 41-60.
14. Fernandez, E., Cancela, N., Olmedo, R. Deriving a final ranking from fuzzy preferences: an approach compatible with the Principle of Correspondence. Mathematical and Computer Modelling, 2008, 47 (3), 218-234.
15. Fernandez, E., Navarro, J., Duarte, A. Multicriteria sorting using a valued preference closeness relation. European Journal of Operational Research, 2008, 185 (3), 673-686.
16. Deb, K. Multi-Objective Optimization using Evolutionary Algorithms, John Wiley & Sons, Chichester-New York-Weinheim-Brisbane-Singapore-Toronto, 2001.

17. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II, IEEE Transactions on Evolutionary Computation, 6(2), 182-197, April 2002.
18. Opricovic, S., Tzeng, G. Extended VIKOR method in comparison with outranking methods. European Journal of Operational Research 2007, 178 (2), 514-529.
19. Fodor. J., Roubens, M. Fuzzy Preference Modeling and Multicriteria Decision Support, Kluwer, Dordrecht, 1994