Decomposition-Based Multiobjective Optimization with Bicriteria Assisted Adaptive Operator Selection

Wu Lin[1], Qiuzhen Lin[1], Junkai Ji[1], Zexuan Zhu[1], Carlos A. Coello Coello[2], Ka-Chun Wong[3]

[1] *College of Computer Science and Software Engineering, Shenzhen University, Shenzhen, P.R. China*

[2] *CINVESTAV-IPN, Department of Computer Science, Mexico, D.F., 07360, Mexico*

[3] *Department of Computer Science, City University of Hong Kong, Hong Kong, PR China*

Abstract

This paper proposes a novel bicriteria assisted adaptive operator selection (B-AOS) strategy for decomposition-based multiobjective evolutionary algorithms (MOEA/Ds). In this approach, two operator pools are employed to focus on exploitation and exploration, each of which includes two DE operators with distinct search patterns. Then, two criteria, one (called the Pareto criterion) emphasizing convergence and the other (called the crowding criterion) focusing on diversity, are collaboratively used to assist the selection of a suitable DE operator for the current solution, which aims to obtain a good balance between exploitation and exploration during the evolutionary search of each solution. Specifically, the Pareto criterion is used to decide whether exploration or exploitation is preferred for the current solution, which will help to select an operator pool. After that, from the selected operator pool, the crowding criterion is used to further assist the selection of the DE operator based on a binary tournament strategy. The experimental results show that our proposed B-AOS performs better than other existing state-of-the-art adaptive operator selection methods, and several MOEA/Ds embedded with B-AOS can significantly improve their performance on most of the benchmark problems adopted.

Keywords: Multiobjective optimization; Recombination operator; Adaptive operator selection.

1. Introduction

Many real-world engineering applications need to find the optimality of several (often conflicting) objectives simultaneously [1-4], which are often termed multiobjective optimization problems (MOPs). In general, an MOP is defined mathematically as follows:

$$\min_{x \in \Omega} F(x) = (f_1(x), ..., f_m(x))^T, \tag{1}$$

where $\Omega \in \mathbb{R}^n$ is the *n*-dimensional decision (variable) space (*n* is the number of decision variables) and $x = (x_1, ..., x_n)^T$ is a decision vector in Ω. An objective vector $F(x)$ defines *m* objective functions $f_1(x), ..., f_m(x)$, which are used to obtain *m* objective values after inputting a decision vector *x*. Due to the conflict among the objectives, there exists a set of equally optimal solutions termed the Pareto-optimal set (PS), and the mapping of PS in the objective space is termed the Pareto-optimal front (PF). The PF can help decision makers perform the final selection based on their preference for the objectives [5-6]. During recent decades, multiobjective evolutionary algorithms (MOEAs) have been regarded as a popular and effective approach to solve MOPs, as their population-based nature can obtain a set of solutions in a single run [7-10]. Most MOEAs can be classified into three main types according to their environmental selection mechanisms, i.e., Pareto-based MOEAs [11-14], indicator-based MOEAs [15-18] and decomposition-based MOEAs (MOEA/Ds) [19-22]. In particular, MOEA/Ds have become very popular in recent years due to their promising performance in various complicated MOPs. In MOEA/Ds, MOPs are decomposed into multiple single-objective optimization subproblems by a set of uniformly distributed weight vectors. Then, all the subproblems are simultaneously optimized in a collaborative manner [19].

Generally, MOEA/Ds [21-22] contain several components in their algorithmic design, including the decomposition approaches (defined by the used aggregation function and weight vectors), the selection of evolved subproblems, the recombination operators, and the solution update methods for each subproblem. Recent studies of MOEA/Ds have been conducted to improve the above components to further enhance the optimization performance, which are briefly introduced below.

Regarding decomposition approaches, two relevant schemes are those adopted in MOEA/D-UDM [23] and UMOEA/D [24] to generate uniform weight vectors for subproblems. In MOEA/D-ACD [25], some constraints are embedded into the decomposition functions, which reduces the improvement regions for subproblems. In MOEA/D-M2M [26], the decomposition approach is modified by transforming an MOP into a set of simple MOPs, which simplifies the solution of the original MOP. Moreover, a hierarchical decomposition approach is presented in MOEA/HD [27] to address ill-defined MOPs with pointed, long-tailed, disconnected, or degenerate PFs, and an adversarial decomposition method is proposed in MOEA/AD [28] to exploit the complementary characteristics of different subproblems.

On the selection of evolved subproblems, a dynamic resource allocation strategy is introduced in MOEA/D-DRA [29], which selects the subproblems to be evolved based on the improvements for these subproblems. Its more generalized version is presented in MOEA/D-GRA [30], which uses a probability vector of improvements to assign computational resources. In MOEA/D-IRA [31], the diversity status of subproblems is further considered in order to attain a more reasonable allocation. Moreover, the computational resources to each subspace are allocated by measuring the subspace contributions to the population convergence in OPE-MOEA [32].Regarding the recombination operators, many studies have been conducted in MOEA/Ds [33-39], which exploit the genetic information of parent solutions to produce a better offspring solution. In the original MOEA/D [19], simulated binary crossover (SBX) [33] was first studied for a continuous search space. In [40], another commonly used crossover operator, i.e., differential evolution [34] (DE), is used to address some complicated MOPs, showing a strong global search ability. In AHX [39], a hybrid crossover operator was designed to combine the advantages of the local search ability in SBX and the strong global search capability in DE.

Finally, for the subproblem solution update methods, the largest number of subproblems that can be replaced is constrained as *nc* in MOEA/D-DE [40] (*nc* is a preset integer value). Then, a stable matching model is designed in MOEA/D-STM [41] to associate solutions for subproblems, aiming to balance the convergence (measured by the aggregated function values of solutions for subproblems) and the diversity (reflected by the distances between solutions and the subproblem direction vectors). Furthermore, an interrelationship-based selection mechanism is proposed in MOEA/D-IR [42] based on the mutual preference of solutions and subproblems, which follows the principle of diversity first and convergence second. In ENS-MOEA/D [43], different neighborhood settings are adaptively selected for solution selection and update. In MOEA/D-AGR [38], an adaptive global replacement scheme studies the effect of replacement neighborhoods, which concludes that the sigmoid function-based adaptive scheme is the best choice to control replacement neighborhoods. More recently, the concept of incomplete preference lists was introduced in [44], which can be used in the stable matching model [41] to remedy its loss of population diversity.

This paper mainly focuses on the recombination operators of MOEA/Ds. As reflected by the studies reported in [36, 45, 46], the use of a single DE operator may show some shortcomings in solving MOPs with certain complicated features; thus, multiple DE operators are suggested for MOEA/Ds. There are few research studies that exploit the convergence and diversity status of each solution in recombination operators of MOEA/Ds, which may be a promising path to combine their advantages. Thus, this paper designs a novel bicriteria assisted adaptive operator selection (B-AOS) strategy for MOEA/Ds, aiming to further improve their search capability. Two operator pools are used in the B-AOS to focus on exploitation and exploration. Then, in each operator pool, two DE operators with distinct search patterns are included to compete for computational resources. Two criteria, i.e., the Pareto criterion and crowding criterion, are employed to adaptively select DE operators. The Pareto dominance status of the selected solutions (called the Pareto criterion) is exploited to assist the selection of an operator pool in B-AOS, which is used to reflect whether exploration or exploitation is preferred for the solution at the current evolutionary stage. After that, a crowding-based binary tournament competition strategy (called the crowding criterion) is used to further assist the DE operator selection from the above operator pool, which aims to strike a good balance between exploration and exploitation. To conclude, our proposed B-AOS consists of the following contributions:

1) Based on the distinct search characteristics of four common DE operators, two operator pools are adopted to focus on exploitation and exploration, each of which includes two DE operators with distinct search patterns.

2) Different from the existing AOS methods, a new bicriteria method is proposed. One (Pareto-based criterion) emphasizes convergence, and the other (called crowding criterion) focuses on diversity. Both criteria are collaboratively used to assist AOS in selecting a DE operator for the current solution, with the aim of obtaining a good balance between exploitation and exploration during the evolutionary search of each solution.

The rest of this paper is organized as follows. Section 2 introduces the related background, including different types of DE operators, related works about AOS, and an analysis of their search characteristics. The details of B-AOS are described in Section 3, including a Pareto-assisted operator pool selection and a crowding-assisted operator selection used in each pool. In Section 4, the experimental settings are provided, including the used parameter settings and performance indicators. All the experimental results are summarized and discussed in Section 5. Finally, this study is concluded, and some paths for future research are provided in Section 6.

2. Preliminaries

2.1 Different Types of DE Operators

Recombination is an important operator in MOEAs, as it allows inheritance of the genetic information of the parents to their offspring with the aim of improving their quality. In general, SBX [33] and DE [34] are two popular recombination operators used in MOEAs. Specifically, the DE operator has a strong global search ability, which is commonly used in MOEA/Ds [40]. Several DE operators have been proposed, and each of them is advantageous for solving certain types of MOPs [36, 45].

As suggested in recent studies of AOS in MOEA/D-FRRMAB [36] and MOEA/D-CDE [45], four popular DE mutation strategies with distinct search characteristics (DE/rand/1, DE/current-to-rand/1, DE/rand/2 and DE/current-to-rand/2) are used in this paper and are defined in Eqs. (2)-(5), as follows:

$$v^i = x^i + F \times (x^{r_1} - x^{r_2}) \tag{2}$$

$$v^i = x^i + K \times (x^i - x^{r_1}) + F \times (x^{r_2} - x^{r_3}), \tag{3}$$

$$v^i = x^i + F \times (x^{r_1} - x^{r_2}) + F \times (x^{r_3} - x^{r_4}), \tag{4}$$

$$v^i = x^i + K \times (x^i - x^{r_1}) + F \times (x^{r_2} - x^{r_3}) + F \times (x^{r_4} - x^{r_5}), \tag{5}$$

where x^i is called the target vector, usually set as each individual in the current population, and v^i denotes the mutant vector. x^{r_1}, x^{r_2}, x^{r_3}, x^{r_4}, x^{r_5} are five distinct solutions randomly selected from the population, which are different from x^i. The scaling factors $F \in [0,1]$ and $K \in [0,1]$ are positive real values to control the weighting of difference vectors.

After producing the above mutant vectors in DE, a crossover strategy is run in DE to generate each dimension of offspring u^i from the current individual x^i or the mutant vector v^i. Here, the binomial crossover commonly used in DE is given by

$$u_j^i = \begin{cases} v_j^i, & \text{if } rand \le CR \text{ or } j = j_{rand} \\ x_j^i, & \text{otherwise} \end{cases} \tag{6}$$

where $i \in \{1,...,N\}$ is the index of an individual (*N* is the population size), $j \in \{1,...,n\}$ is the dimension index of solutions (*n* is the dimension of the decision variable space), *rand* is a uniformly distributed random number in [0, 1], the crossover rate *CR* is a user-defined control parameter in [0, 1], and j_{rand} is a random index selected from $\{1,...,n\}$ to ensure that at least one variable is inherited from v^i. When u_j^i is beyond its allowable bounds, it will be randomly initialized within the feasible range. Thus, the generated offspring can simultaneously inherit parts of the information from the original and mutant vectors.

Based on the above DE mutation strategies in Eqs. (2)-(5), it is easy to find that the difference vectors are composed of two random individuals, which provide a wide range of exploration. With the increase in difference vectors, the exploration range increases. As the DE/rand/2 and DE/current-to-rand/2 strategies contain two difference vectors with four random individuals, they provide more extensive exploration capabilities than the strategies of DE/rand/1 and DE/current-to-rand/1. However, the DE/rand/1 and DE/current-to-rand/1 strategies show stronger exploitation abilities. To demonstrate their different exploitation abilities, assume that the target vector x^i is a constant, and the values of *K* and *F* are set the same in Eqs. (2)-(5), while other vectors (x^{r_1}, x^{r_2}, x^{r_3}, x^{r_4}, x^{r_5}) are uniformly and randomly selected from [0, 1]. Then, according to the property of uniform distribution, it is easily concluded that the expectations of x^{r_1}, x^{r_2}, x^{r_3}, x^{r_4}, x^{r_5} will be 0.5 with variances of 1/12. Thus, it is easy to obtain that the expectations of v^i are equal to the expectations of their corresponding target vector x^i for Eqs. (2)-(5), while the variances of v^i are given below:

$$D(v^i)=F^2(D(x^{r_1})+D(x^{r_2})), \tag{7}$$

$$D(v^i)=F^2(D(x^{r_1})+D(x^{r_2})+D(x^{r_3})), \tag{8}$$

$$D(v^i)=F^2(D(x^{r_1})+D(x^{r_2})+D(x^{r_3})+D(x^{r_4})), \tag{9}$$

$$D(v^i)=F^2(D(x^{r_1})+D(x^{r_2})+D(x^{r_3})+D(x^{r_4})+D(x^{r_5})), \tag{10}$$

where $D(x^{r_1})=D(x^{r_2})=D(x^{r_3})=D(x^{r_4})=D(x^{r_5})=1/12$ is the variance in random difference vectors. For clarity, the statistical experiments are performed 200,000 times for four DE mutation strategies, which visually show their exploration capabilities during the evolutionary search. Assuming that the target vector x^i is set to 0.5, the values of F and K are all set to 0.5, and other vectors ($x^{r_1}, x^{r_2}, x^{r_3}, x^{r_4}, x^{r_5}$) are uniformly and randomly selected from [0, 1], the probability distributions of v^i using different DE mutation strategies are plotted in Fig. (1). Thus, with the increase in their variances, the exploration capabilities will become stronger from DE/rand/1, DE/current-to-rand/1, DE/rand/2, to DE/current-to-rand/2, which can be used in different evolutionary stages.

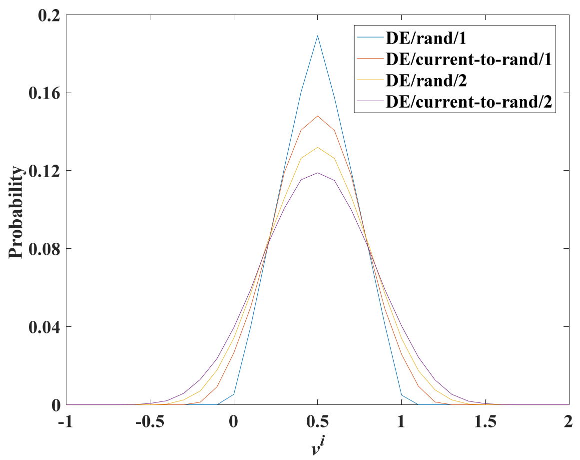

Fig. 1: The exploration capabilities of the four different DE mutation strategies

2.2 Related Works of Adaptive Operator Selection

In recent studies of MOEA/Ds [36, 45], some AOS strategies have been employed to adaptively select recombination operators for generating new solutions during the evolutionary search, which can significantly improve their performance when compared with a single recombination operator. In general, AOS involves two main components: credit assignment and operator selection. The first defines how to reward an operator based on its recent performance in the search process, while the second uses these rewards to decide which operator should be selected.

Some studies [46-49] have been performed to design credit assignment schemes. For example, in [46], the improvement of the average fitness was regarded as the reward amount. However, in [47], each recombination operator was rewarded based on its maximal fitness improvement recently achieved. Moreover, both diversity and fitness improvements were considered in [48] to reward operators, and two rank-based credit assignment schemes were designed in [49] to better estimate the performance of operators during the evolutionary search.

Regarding operator selection, some related studies can be found in [36, 45, 50-52]. For example, the probability matching method [50] was designed to give high probabilities for selecting the operators with average performance, while the adaptive pursuit method [51] adopts a winner-take-all strategy to increase the probability for selecting the best operator. In ADEMO/D [52], both probability matching and adaptive pursuit methods are combined with four credit assignment schemes based on the improvements of the relative fitness. Moreover, four different DE mutation strategies with a sliding window are employed in MOEA/D-FRRMAB [36] to select operators based on their enhancements on subproblems. Similarly, four DE operator pools are organized in MOEA/D-CDE [45], which includes two DE mutation strategies with complementary search patterns in each pool to provide an enhanced search capability. In addition, a hybrid control strategy of parameters and mutation operators was reported in AMODE-RAVM [53], which combined a reference axis vicinity mechanism to speed up convergence by using both randomness and guided information derived from the evolutionary search.

2.3 Motivations

As introduced in Section 2.2, most of the existing AOS methods [36, 45, 50-52] adaptively select recombination operators based on the current performance of the operators during the evolutionary search process. In fact, due to the uncertainty and randomness during the evolutionary search, it is not always effective to predict which operator is more suitable for recombination. In this paper, the characteristics of four commonly used DE mutation strategies are first specifically analyzed and redefined in a statistical sense, as introduced in Section 2.1. Considering the advantages of DE mutation strategies in exploitation and exploration, the convergence and diversity status of each solution during the current evolutionary stage is used to assist the selection of an appropriate operator for their recombination. In general, when the status of solutions is far from convergence, MOEAs often need to enhance their exploration capabilities, attempting to expand their diversity. Otherwise, MOEAs should emphasize their exploitation abilities to speed up convergence. Therefore, the Pareto dominance status and the crowding status are used as two different criteria in this paper to assist the selection of recombination operators in MOEA/Ds; it is helpful to exert their advantages, especially when exploration or exploitation is preferred during the current evolutionary process of each solution.

Inspired by the above analyses and discussions, two operator pools are used in the proposed B-AOS, which focus on exploitation and exploration according to the Pareto criterion. In each of the two pools, two DE mutation strategies with distinct search patterns are used to allow the computational resources to compete according to the crowding criterion. It can be easily found that the obvious distinction between the proposed B-AOS and the existing AOS methods is that the status of each solution during the current evolutionary stage is employed to select suitable operators in B-AOS, rather than considering the recent operators' performance.

3. Bicriteria Assisted Adaptive Operator Selection

In this section, the details of the proposed B-AOS are introduced, which helps to select a suitable DE strategy for each solution during the evolutionary search. First, a Pareto-assisted operator pool selection in B-AOS is presented, which estimates the Pareto dominance status of the current solution to decide which operator pool is better for recombination at the current evolutionary stage. Then, a crowding-assisted operator selection is introduced to further select a DE strategy for the current solution, which evaluates the crowding status of solutions in the neighborhood to assist the selection of a DE strategy from each operator pool. Finally, the proposed B-AOS is embedded into a simple MOEA/D variant (i.e., MOEA/D-DE [40]), which is used as an example to illustrate how to embed B-AOS into other MOEA/Ds, and the corresponding pseudocode is provided to facilitate the implementation of MOEA/D-BAOS.

3.1 Pareto-Assisted Operator Pool Selection

Based on the characteristics of the four different DE mutation strategies described in Section 2.1, they are classified into two different operator pools, which focus on exploitation and exploration as follows:

$$Pool\text{-I} = \{\text{DE/rand/1 } (\mathrm{DE}_1^{\mathrm{I}}), \text{ DE/current-to-rand/1 } (\mathrm{DE}_2^{\mathrm{I}})\} \tag{11}$$

$$Pool\text{-II} = \{\text{DE/rand/2 } (\mathrm{DE}_1^{\mathrm{II}}), \text{ DE/current-to-rand/2 } (\mathrm{DE}_2^{\mathrm{II}})\} \tag{12}$$

Each operator pool includes two different DE mutation strategies with distinct search patterns. The DE mutation strategies of *Pool*-I emphasize exploitation, while the DE mutation strategies of *Pool*-II focus on exploration. To be more detailed, a parameter *type* is used to indicate which operator pool will be used as a candidate pool, which depends on the status of the selected solution. If exploitation is needed for the selected solution in the evolutionary stage, *Pool*-I should be used, which helps to inherit the advantages of parents due to the relatively smaller disturbance. In B-AOS, the Pareto-assisted operator pool selection is first used to determine the setting of the *type* based on the Pareto dominance status of the currently selected solution x, as follows:

$$type = \begin{cases} \mathrm{I}, & \text{if } x \text{ is non-dominated solution} \\ \mathrm{II}, & \text{otherwise} \end{cases} \tag{13}$$

The settings of *type* = I and *type* = II indicate the use of *Pool*-I and *Pool*-II, respectively. In (13), the Pareto dominance status of x is used to indicate whether exploitation or exploration is preferred for recombination at the current evolutionary stage of x. If x is a nondominated solution, exploitation should be needed to find more superior solutions near x. Otherwise, the status of x is far from convergence, and thus, more exploration should be required to search for more nondominated solutions in uncovered areas. In this way, this strategy can select DE strategies to adaptively focus on exploitation and exploration based on the Pareto dominance status of different solutions.

For clarity, the pseudocode of B-AOS is presented in **Algorithm 1** with the inputs x (the selected solution), P (the population), B (the index sets of the neighbors of all solutions) and an *operator pool* (four distinct DE mutation strategies in Eqs. (2)-(5)). In line 1, the solution x is checked to determine whether it is nondominated in P. Then, as shown in line 2, if x is a nondominated solution, *type* = I in line 3 indicates that *Pool*-I = $\{\mathrm{DE}_1^{\mathrm{I}}, \mathrm{DE}_2^{\mathrm{I}}\}$ will be used as the candidate pool. Otherwise, *type* = II in line 6 indicates that *Pool*-II = $\{\mathrm{DE}_1^{\mathrm{II}}, \mathrm{DE}_2^{\mathrm{II}}\}$ will be used as the candidate pool. After that, crowding-assisted operator selection (**Algorithm 2**) will be further used to adaptively select one DE mutation strategy from the candidate pool, i.e., *Pool*-I = $\{\mathrm{DE}_1^{\mathrm{I}}, \mathrm{DE}_2^{\mathrm{I}}\}$ as shown in line 4 or *Pool*-II = $\{\mathrm{DE}_1^{\mathrm{II}}, \mathrm{DE}_2^{\mathrm{II}}\}$ as shown in line 7. Finally, in line 9, the *operator* selected by B-AOS is returned as the recombination operator for the solution x.

Algorithm 1: Bi-criteria Assisted Adaptive Operator Selection (x, P, B, *Operator Pool*)

Input:

x: the current selected solution

P: the population

B: the index sets of the neighbors of all solutions

Operator Pool=$\{\mathrm{DE}_1^{\mathrm{I}}, \mathrm{DE}_2^{\mathrm{I}}, \mathrm{DE}_1^{\mathrm{II}}, \mathrm{DE}_2^{\mathrm{II}}\}$

Output: *operator*

1. calculate the Pareto dominance status of x in P;
2. **if** x is a non-dominated solution
   // exploitation abilities are needed for recombination
3. *type* = I //*Pool*-I = $\{\mathrm{DE}_1^{\mathrm{I}}, \mathrm{DE}_2^{\mathrm{I}}\}$ will be used as candidate pool ;
4. *operator* = Crowding-Assisted Operator Selection (x, P, B, *Pool*-I);
5. **else**
   // exploration abilities are needed for recombination
6. *type* = II //*Pool*-II = $\{\mathrm{DE}_1^{\mathrm{II}}, \mathrm{DE}_2^{\mathrm{II}}\}$ will be used as candidate pool;
7. *operator* = Crowding-Assisted Operator Selection (x, P, B, *Pool*-II);
8. **End**
9. **return** *operator*

3.2 Crowding-Assisted Operator Selection

In this crowding-assisted operator selection, a binary tournament competition strategy is used to select one DE strategy from each candidate pool according to the crowding status of two selected solutions. Here, the *HAD* method [54] is used to reflect the crowding status of solutions in their neighborhoods. Given the neighbor of one solution x^i, i.e., $B(x^i)$, the HAD value of x^i can be computed as follows:

$$HAD(x^i) = \frac{|B(x^i)| - 1}{\sum_{x^j \in B(x^i), x^j \neq x^i} 1/d(x^i, x^j)}, \tag{14}$$

where $d(x^i, x^j)$ denotes the Euclidean distance between solutions x^i and x^j in objective space. Note that $B(x^i)$ is formed by these solutions $x^{i_1}, ..., x^{i_T}$ associated with their corresponding weight vector $w^{i_1}, ..., w^{i_T}$ (where $w^{i_1}, ..., w^{i_T}$ are the T closest weight vectors to w^i) in MOEA/Ds, which is introduced in Section 3.3. This *HAD* approach considers the Euclidean distances to other solutions' neighbors, which is more accurate in reflecting the crowding status of each solution in the local search space. To visually show the calculation of the *HAD* value, one simple example is provided in Fig. 2, where a solution x^1 is assumed to have three neighboring solutions (i.e., x^2, x^3 and x^4). Then, $HAD(x^1)$ is computed as follows:

$$HAD(x^1) = \frac{3}{1/d^1 + 1/d^2 + 1/d^3}, \tag{15}$$

where d^1, d^2 and d^3 represent the Euclidean distance between x^1 and its neighboring solutions (x^2, x^3 and x^4) in objective space.

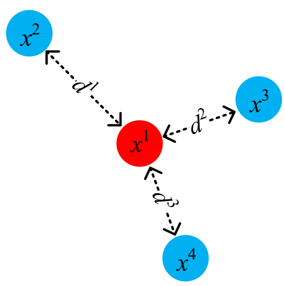

Fig. 2: An example of *HAD* approach

The pseudocode of crowding-assisted operator selection is introduced in **Algorithm 2** with the inputs x (the currently selected solution), P (the population), B (the sets of the neighbors of all solutions) and a *candidate pool* (such as *Pool*-I or *Pool*-II, each of which contains two distinct DE mutation strategies). In line 1, a solution y is randomly selected from P, which is used to compare with the selected solution x. After that, the crowding status of the two solutions is estimated by Eq. (14) in line 2. As shown in lines 3-13, the binary tournament competition strategy is used to decide whether to adopt DE_1^{type} or DE_2^{type} as the recombination operator for x. As shown in line 3, if the crowding distance of x is larger than that of y, one DE mutation strategy (i.e., DE_1^{type}) with more emphasis on exploitation is selected as the recombination operator for x in line 4, which helps to exploit the local area. As shown in line 5, if the crowding distance of x is smaller than that of y, due to the similarity among these neighborhood solutions of x, then one DE mutation strategy (i.e., DE_2^{type}) is selected as the recombination operator for x in line 6, which could extend the exploration on this crowded area. Otherwise, as shown in lines 8-12, if the crowding distances of x and y are equal, one DE mutation strategy will be randomly selected from the candidate pool. Finally, the *operator* chosen from the *candidate pool* is returned in line 14.

Algorithm 2: Crowding-Assisted Operator Selection (x, P, B, *Candidate Pool*)

Input:
  x: the current selected solution
  P: the population
  B: the sets of the neighbors of all solutions
  Candidate Pool= $\{\mathrm{DE}_1^{type}, \mathrm{DE}_2^{type}\}$
Output: *operator*

```
1.  Randomly select one solution y from P;
2.  Estimate the crowding status of x and y using their respective neighbors by (14);
3.  if HAD(x) > HAD(y)
4.      operator = DE_1^type;
5.  else if HAD(x) < HAD(y)
6.      operator = DE_2^type;
7.  Else
8.      if random < 0.5
9.          operator = DE_1^type;
10.     Else
11.         operator = DE_2^type;
12.     End
13. End
14. return operator
```

3.3 Incorporation with MOEA/D

In this subsection, the proposed B-AOS is embedded into MOEA/D as shown by the pseudocode in **Algorithm 3** with the inputs *MOP* (the target MOP), N (the population size), T (the neighborhood size), δ (the probability of local mating) and *operator pool* (including four DE mutation strategies). First, as shown in line 1, the population P is initialized by randomly sampling N solutions from the decision space of the *MOP*. In line 2, a set of weight vectors, i.e., W, is generated, which consists of N uniformly distributed weight vectors in objective space. Then, in lines 3-6, for the weight vector w^i ($i \in \{1,...,N\}$), the indexes of its T closest weight vectors are saved into the set $B(i)=\{i_1,i_2,...,i_T\}$, and then the solutions $x^{i_1},...,x^{i_T}$ associated with the weight vectors $w^{i_1},...,w^{i_T}$ are also saved into the neighbor of x^i, i.e., $B(x^i)$. In line 7, the ideal point z^{ide} is initialized by finding the current best value of each objective. After that, as shown in lines 9-14, for each solution $x \in P$, its neighbor $B(i)$ is selected as candidate set E with a high probability δ in line 11. Otherwise, the whole population P is selected as candidate set E with a lower probability 1-δ in line 13. Then, as shown in line 15, the bicriteria assisted adaptive operator selection (**Algorithm 1**) is used to select one DE mutation strategy from the *operator pool*, which is used for recombination of x. Next, two indexes are randomly selected from E to form parent solutions in line 16, and then a new offspring y is generated by using the selected DE mutation strategy (one of Eqs. (2)-(5)) and the binomial crossover (Eq. (6)) in line 17. Then, the polynomial mutation operator is run in line 18. Then, y is used to update the ideal point z^{ide} and replace its neighboring solutions in lines 19 and 20. As shown in line 8, while the stopping criterion is not satisfied, the above steps in lines 9-21 are run iteratively. Finally, the final population P is returned in line 23.

Algorithm 3: The framework of MOEA/D with B-AOS method

Input:
    MOP: the multiobjective optimization problem
    N: the population size
    T: the neighborhood size
    δ: the probability of local mating
    Operator Pool=$\{\mathrm{DE}_1^{\mathrm{I}}, \mathrm{DE}_2^{\mathrm{I}}, \mathrm{DE}_1^{\mathrm{II}}, \mathrm{DE}_2^{\mathrm{II}}\}$
Output: a set of solutions
1. Initialize a set of solutions $P=\{x^1,...,x^N\}$;
2. Generate a set of uniformly distributed weight vectors $W=\{w^1,...,w^N\}$;
3. **for** $i \leftarrow 1:N$ **do**
4. $B(i) \leftarrow \{i_1,...,i_T\}$, where $w^{i_1},...,w^{i_T}$ are T closest weight vectors to w^i
5. $B(x^i) \leftarrow \{x^{i_1},...,x^{i_T}\}$, where $x^{i_1},...,x^{i_T}$ are solutions associated with $w^{i_1},...,w^{i_T}$
6. **End**
7. Initialize the ideal point z^{ide};
8. **while** stopping criterion is not satisfied **do**
9.   **for** $i \leftarrow 1:N$ **do**
10.   **if** $rand < \delta$ **then**
11. $E \leftarrow B(i)$;
12.   **Else**
13. $E \leftarrow \{1,...,N\}$;
14.   **End**
15.   *op*=Bi-criteria Assisted Adaptive Operator Selection (x^i, *P*, *B*, *Operator Pool*); //**Algorithm 1**
16.   Randomly select two index r_1 and r_2 from *E*;
17.   Generate candidate $\bar{y}$ by DE mutation strategy *op* and the binomial crossover over x^i, x^{r1}, x^{r2};
18.   Apply polynomial mutation operator on $\bar{y}$ with probability p_m to produce the offspring y;
19.   Update the ideal reference point z^{ide};
20.   Update the neighboring solutions;
21.   **end;**
22. **End**
23. **return** *P*

From the above pseudocode in **Algorithm 3**, it can be found that the difference between MOEA/D embedded with B-AOS (termed MOEA/D-BAOS) and the original MOEA/D [40] is that four distinct DE mutation strategies can be adaptively selected for each solution by B-AOS (i.e., **Algorithm 1**). Thus, it is very easy to incorporate the proposed B-AOS into various MOEA/Ds.

4. Experimental Settings

4.1 Test Problems and Parameter Settings

In this study, 19 unconstrained test MOPs (ten UF instances (UF1-UF10) from the CEC2009 MOEA competition [55] and nine F instances (F1-F9) [40]) are used to evaluate the performance of our proposed B-AOS. These test problems have been widely adopted to validate the performance of many MOEA/D variants, such as MOEA/D-DE [40], MOEA/D-DRA [29], MOEA/D-IR [42], and MOEA/D-IRA [31]. It is worth noting that UF1-UF7, F1-F5, and F7-F9 are biobjective problems, while UF8-UF10 and F6 are three-objective problems. The number of decision variables is set to 30 for F1-F5 and F9 and all UF test problems. The number of decision variables is set to 10 for F6-F8.

Table 1

Parameters settings of all the compared algorithms

Algorithms	Parameters settings
FRRMAB	T=20, δ=0.9, nr=2,C=5, W=0.5*N, D=1.0, CR=1.0, F=0.5 and K=0.5
ACOS	T=20, δ=0.9, nr=2, C=5, W=0.5*N
Uniform	T=20, δ=0.9, nr=2, p=0.25, CR=1.0, F=0.5 and K=0.5
BAOS	T=20, δ=0.9, nr=2
MOEA/D-DE	T=20, δ=0.9, nr=2
MOEA/D-IR	T=20, δ=0.9, K_d =2, ϑ =8
MOEA/D-IRA	T=20, δ=0.8,□t =20, β =8, $pn_{\min}$ =0.05

As suggested in [36, 45], the parameters settings in the compared AOS methods (FRRMAB and ACOS) are listed in Table 1. Because MOEA/D-DRA is used as the baseline algorithm in FRRMAB [36] and ACOS [45], uniform and B-AOS are also embedded into MOEAD/DRA to have a fair comparison. Thus, their common parameters are set as follows: T (the neighborhood size) is set to 20, δ (the probability of selecting the neighbors as the candidate set for evolution) is set to 0.9 and nr (the maximum number of solutions replaced by each new solution) is set to 2. As suggested in [36], the control parameters for FRRMAB are set as follows: C (scaling factor) is set to 5.0, W (size of sliding window) is set to 0.5*N (N is the population size), and D (decaying factor) is set to 1.0. As suggested in [45], C (scaling factor) is set to 5.0 and W (size of sliding window) is set to 0.5*N in ACOS. In uniform, each DE operator is randomly selected from the operator pool with the same probability (p=0.25). For the crossover operator, as suggested in [36], the control parameters CR, F and K are set to 1.0, 0.5 and 0.5, respectively, for all DE operators in FRRMAB, uniform and B-AOS. However, in ACOS, the settings of the control parameters CR, F and K in the four operator pools adopted were suggested in [48] as follows. In the operator pool op_1, F uses an adaptive parameter control strategy, and CR is set to 1 for DE/rand/1, while the parameters F and CR are set to 0.2 and 0.8, respectively, and K is a uniformly distributed random value generated in [0, 1] for DE/rand/2. In operator pool op_2, F uses an adaptive parameter control strategy, and CR is set to 1 for DE/rand/2, while the parameters F and CR are all set to 1 for DE/current-to-rand/1. In operator pool op_3, F uses an adaptive parameter control strategy, CR is set to 1 and K is a uniformly distributed random value in [0, 1] for DE/current-to-rand/2. In operator pool op_4, F uses an adaptive parameter control strategy, and CR is set to 1 for DE/current-to-rand/1. In addition, for the polynomial-based mutation operator, as suggested in [36], the distribution index η is set to 20 and the mutation probability p_m is set to $1/n$ (n is the number of decision variables). In addition, for MOEA/D-DE [40], MOEA/D-IR [42], MOEA/D-IRA [31], and their corresponding improved versions (called MOEA/D-DE-BAOS, MOEA/D-IR-BAOS and MOEA/D-IRA-BAOS), all parameters settings are kept the same as those in their original papers. In MOEA/D-DE, T is set to 20, δ is set to 0.9 and nr is set to 2. In MOEA/D-IR, T is set to 20, δ is set to 0.9, K_d (the number of subproblems selected for a solution) is set to 2, and ϑ (the number of subproblems selected for a subproblem) is set to 8. In MOEA/D-IRA, T is set to 20, δ is set to 0.8, □t (the updating period) is set to 20, β (the weight parameter) is set to 0.98, and $pn_{\min}$ (the minimal selection probability) is set to 0.05.

The population size N is set to 600 for all the biobjective UF and F test problems and to 1,000 for all of the three-objective problems. The maximum number of function evaluations is set to 150,000 for F1-F5 and F7-F9 and to 300,000 for F6 and UF1-UF10. All the compared algorithms performed 51 independent runs on each benchmark problem.

4.2 Performance Measures

In this paper, to assess the performance of B-AOS, the following two widely used performance indicators are adopted:

1) Inverted generational distance (IGD) indicator [56]: Let us assume that a subset of the true PF is represented by P and the final solution set obtained by an MOEA is A. The IGD indicator is computed as follows:

$$IGD(P, A) = \frac{\sum_{i=1}^{|P|} d(P_i, A)}{|P|}, \tag{15}$$

where $|P|$ represents the size of P and $d(P_i, A)$ denotes the minimum Euclidean distance between P_i and the solutions of A in objective space. When calculating this indicator, 1,000 and 10,000 samples are uniformly sampled from the true PFs for the biobjective and three-objective UF instances, respectively, while 5,000 samples are used for all the F instances. A lower value of IGD suggests that the obtained set is closer to the true PF and more uniformly distributed along the true PF.

2) Hypervolume (HV) indicator [57]: Let $z^r = (z_1^r, ..., z_m^r,)^T$ be a set of reference points in objective space that is dominated by all the Pareto-optimal solutions. The HV indicator calculates the objective space size, which is dominated by the solutions in S and bounded by z^r, as follows:

$$HV(S, z^r) = VOL\left(\bigcup_{x \in S} \left[f_1(x), z_1^r\right] \times ... \times \left[f_m(x), z_m^r\right]\right), \tag{16}$$

where $VOL(\cdot)$ represents the Lebesgue measure. When computing the HV indicator, it is suggested to set a reference point slightly larger than the worst value of each objective on the true PF, which can obtain a better balance for convergence and diversity of the approximation set. Thus, the reference point is set to $(1.1, ..., 1.1)^T$ for all the test problems. A larger value of HV is preferred, which indicates a better quality of S to approximate all of the true PF.

5. Experimental Studies

In this section, some experimental results are provided to validate the superiority of B-AOS. First, the comparison results of B-AOS with other state-of-the-art AOS methods are given, i.e., FRRMAB [36], ACOS [45] and uniform (a uniformly random AOS method). Then, the proposed B-AOS is embedded in MOEA/D-DE [40], MOEA/D-IR [42] and MOEA/D-IRA [31], which is used to further confirm the effectiveness of B-AOS in other MOEA/D variants. Moreover, B-AOS is further compared with each single operator based on the baseline algorithm (MOEA/D-DRA [29]), which aims to show the rationality and effectiveness of the operator selection mechanism in B-AOS.

5.1. Comparing B-AOS with Other AOS Methods

In our experiments, FRRMAB, ACOS and uniform are compared with our proposed B-AOS. For a fair comparison, as suggested in [36], all AOS methods are embedded into the same baseline algorithm, i.e., MOEA/D-DRA. The parameters of the AOS methods are set the same as those in their original papers [36, 45], as listed in Table 1. The experimental results of the four different AOS methods in terms of IGD and HV are listed in Tables 2-3. With respect to IGD, as summarized in the second to last row of Table 2, B-AOS obtained the best results in 8 out of 19 cases, while FRRMAB, ACOS, and uniform performed best in 4, 5, and 2 out of 19 cases, respectively. With respect to HV, as shown in the second to last row of Table 3, B-AOS obtained the best results in 6 out of 19 cases, while FRRMAB, ACOS, and uniform performed best in 4, 6, and 3 out of 19 cases, respectively.

Table 2
Mean and standard deviation of IGD values obtained by MOEA/D-DRA with different AOS

Problem	FRRMAB	ACOS	Uniform	B-AOS
UF1	9.763e-04(8.21e-05)-	1.036e-03(7.32e-05)-	9.674e-04(7.25e-05)~	**9.423e-04(9.23e-05)**
UF2	**2.030e-03(7.51e-04)~**	2.033e-03(9.57e-04)~	2.161e-03(8.26e-04)~	2.034e-03(7.81e-04)
UF3	4.641e-03(5.54e-03)~	5.661e-03(3.52e-03)-	**3.447e-03(3.00e-03)~**	3.537e-03(2.55e-03)
UF4	5.435e-02(4.14e-03)~	**3.532e-02(4.94e-04)+**	5.317e-02(3.74e-03)~	5.388e-02(3.02e-03)
UF5	3.176e-01(8.00e-02)-	**1.920e-01(3.44e-02)+**	3.038e-01(4.84e-02)~	2.888e-01(4.41e-02)
UF6	8.399e-02(5.31e-02)~	1.152e-01(1.70e-01)-	9.440e-02(1.01e-01)~	**6.710e-02(2.67e-02)**
UF7	1.132e-03(1.23e-04)-	1.169e-03(1.36e-04)-	1.133e-03(2.02e-04)-	**1.005e-03(1.05e-04)**
UF8	3.013e-02(6.26e-03)+	7.859e-02(1.75e-02)-	**2.861e-02(5.99e-03)+**	3.414e-02(7.13e-03)
UF9	**4.766e-02(4.58e-02)~**	5.701e-02(4.74e-02)-	4.869e-02(4.63e-02)~	5.681e-02(5.14e-02)
UF10	4.980e-01(6.65e-02)~	**3.823e-01(5.51e-02)+**	4.826e-01(6.43e-02)~	4.933e-01(5.92e-02)
F1	7.318e-04(1.24e-05)-	7.178e-04(7.25e-06)~	7.350e-04(1.79e-05)-	**7.177e-04(1.36e-05)**
F2	3.621e-02(2.49e-02)-	3.707e-02(2.22e-02)-	3.830e-02(2.72e-02)-	**2.490e-02(1.19e-02)**
F3	7.659e-03(3.02e-03)~	**6.455e-03(2.93e-03)+**	7.444e-03(2.45e-03)~	7.466e-03(2.31e-03)
F4	8.123e-03(4.50e-03)-	1.210e-02(5.51e-03)-	5.554e-03(2.44e-03)-	**3.898e-03(1.79e-03)**
F5	1.108e-02(3.47e-03)~	**9.486e-03(3.71e-03)+**	1.165e-02(3.09e-03)~	1.239e-02(4.19e-03)
F6	2.583e-02(1.43e-03)-	3.746e-02(2.25e-03)-	2.558e-02(1.18e-03)-	**2.482e-02(1.97e-03)**
F7	**2.859e-01(4.47e-02)+**	3.077e-01(5.70e-02)~	3.164e-01(4.10e-02)~	3.150e-01(5.36e-02)
F8	**4.875e-02(1.53e-02)~**	6.396e-02(1.22e-02)-	5.538e-02(1.97e-02)~	5.297e-02(1.83e-02)
F9	3.554e-02(1.01e-02)-	2.514e-02(1.77e-02)-	3.406e-02(1.00e-02)-	**1.761e-02(1.33e-02)**
Best/All	4/19	5/19	2/19	8/19
+/–/~	2/8/9	5/11//3	1/6/12	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that the results of the compared AOS methods are significantly better than, worse than or similar to that of B-AOS, respectively. The best results are highlighted in **boldface**.

Table 3
Mean and standard deviation of HV values obtained by MOEA/D-DRA with different AOS

Problem	FRRMAB	ACOS	Uniform	B-AOS
UF1	8.746e-01(2.47e-04)-	8.745e-01(1.86e-04)-	8.746e-01(2.03e-04)~	**8.747e-01(2.29e-04)**
UF2	8.729e-01(1.48e-03)~	**8.730e-01(1.57e-03)~**	8.727e-01(1.68e-03)~	8.727e-01(1.79e-03)
UF3	8.688e-01(8.55e-03)~	8.666e-01(6.53e-03)-	**8.706e-01(4.50e-03)~**	8.703e-01(4.30e-03)
UF4	4.499e-01(6.47e-03)~	4.837e-01(1.10e-03)+	**4.520e-01(6.43e-03)~**	4.509e-01(5.04e-03)
UF5	1.474e-01(8.28e-02)~	**3.135e-01(8.15e-02)+**	1.576e-01(6.96e-02)~	1.503e-01(7.21e-02)
UF6	**4.371e-01(5.83e-02)~**	4.097e-01(8.34e-02)-	4.268e-01(6.73e-02)~	4.425e-01(3.50e-02)
UF7	7.074e-01(4.82e-04)-	7.075e-01(3.64e-04)-	7.074e-01(7.95e-04)-	**7.078e-01(3.26e-04)**
UF8	7.516e-01(1.30e-02)+	6.758e-01(1.91e-02)-	**7.535e-01(1.33e-02)+**	7.449e-01(1.64e-02)
UF9	**1.047e+00(7.25e-02)~**	1.009e+00(7.05e-02)-	1.045e+00(7.66e-02)~	1.031e+00(8.20e-02)
UF10	6.992e-02(3.50e-02)~	**1.748e-01(2.74e-02)+**	7.957e-02(3.87e-02)+	6.295e-02(3.79e-02)
F1	8.753e-01(6.28e-05)~	**8.754e-01(2.52e-05)+**	8.753e-01(6.36e-05)~	8.753e-01(6.78e-05)
F2	7.921e-01(3.79e-02)-	7.953e-01(4.04e-02)-	7.873e-01(4.77e-02)-	**8.182e-01(3.01e-02)**
F3	8.639e-01(5.39e-03)~	**8.661e-01(4.63e-03)+**	8.644e-01(4.11e-03)~	8.628e-01(4.67e-03)
F4	8.611e-01(7.53e-03)-	8.550e-01(8.96e-03)-	8.654e-01(4.57e-03)-	**8.684e-01(3.61e-03)**
F5	8.587e-01(5.09e-03)~	**8.616e-01(5.62e-03)+**	8.578e-01(5.52e-03)~	8.558e-01(7.23e-03)
F6	7.533e-01(3.58e-03)-	7.254e-01(4.14e-03)-	7.535e-01(3.08e-03)-	**7.558e-01(3.76e-03)**
F7	**4.569e-01(4.48e-02)+**	4.052e-01(6.75e-02)+	4.352e-01(4.37e-02)+	3.367e-01(9.14e-02)
F8	**7.824e-01(2.55e-02)~**	7.595e-01(2.01e-02)-	7.718e-01(3.34e-02)~	7.732e-01(3.16e-02)
F9	4.674e-01(2.21e-02)-	4.924e-01(3.88e-02)-	4.708e-01(2.25e-02)-	**5.068e-01(2.96e-02)**
Best/All	4/19	6/19	3/19	6/19
+/–/~	2/6/11	7/11/1	3/5/11	\

According to Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that the results of the compared AOS methods are significantly better than, worse than or similar to that of B-AOS, respectively. The best results are highlighted in **boldface**.

Moreover, in the last row of Table 2 and Table 3, the one-by-one comparisons of B-AOS with the other three competitors are summarized. "+/–/~" indicate the numbers of problems in which the competitors performed better than, worse than and similarly to B-AOS, respectively. In terms of IGD, B-AOS performed significantly better than FRRMAB, ACOS, and uniform in 8, 11, and 6 out of 19 cases, while it performed significantly worse than FRRMAB, ACOS, and uniform in 2, 5, and 1 out of 19 cases. Compared with FRRMAB, B-AOS obtained significantly better or similar results on most problems except for UF8 and F7. Compared with ACOS, B-AOS obtained significantly better or similar results on most problems except for UF4, UF5, UF10, F3 and F5. Compared with uniform, B-AOS only obtained a significantly worse result on UF8. Similarly, in terms of HV, as shown in Table 3, B-AOS achieved significantly better or similar results than FRRMAB, ACOS, and uniform in 17 (6 better and 11 similar), 12 (11 better and 1 similar), and 16 (5 better and 11 similar) out of 19 cases, respectively, while it was worse than FRRMAB, ACOS, and uniform in 2, 7, and 3 out of 19 cases. Based on the above analysis and discussions, it is reasonable to conclude that B-AOS showed superior performance over its three competitors when dealing with most of these used test problems.

Moreover, to quantify the overall performance of each algorithm, Friedman's test from the software tool KEEL [51] was used to show the average performance ranks of other AOS methods and B-AOS for solving all the test problems, which are plotted in Fig. 3. Obviously, the average performance ranks (2.0526 on IGD and 2.3947 on HV) of B-AOS are smaller than those of other competitors, which confirms our advantages when considering all the test problems.

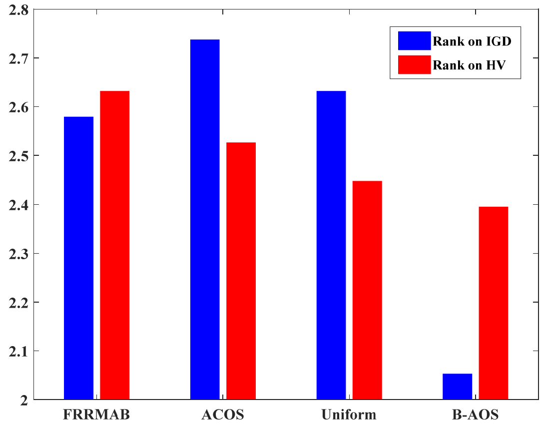

Fig. 3: Average ranking of Friedman's test for other AOS methods and B-AOS

Furthermore, to illustrate the performance of B-AOS, the final solution sets with the median IGD values from 51 independent runs were plotted. Due to page limitations, only F4 and F9 are used to show the superiority of B-AOS with respect to other AOS methods. As shown in Figs. 4 (a)-(d), B-AOS was able to obtain a better approximation of the true PF on F4, while FRRMAB, ACOS, and uniform could not find the lower boundary part of the true PF. Similarly, as shown in Figs. 5 (a)-(d), it is obvious that the approximation solutions obtained by FRRMAB, ACOS, and uniform failed to cover the whole true PF on F9, while the final solutions obtained by B-AOS could better converge to the true PF. Therefore, it can be concluded that B-AOS showed superiority over other AOS approaches on the test problems used.

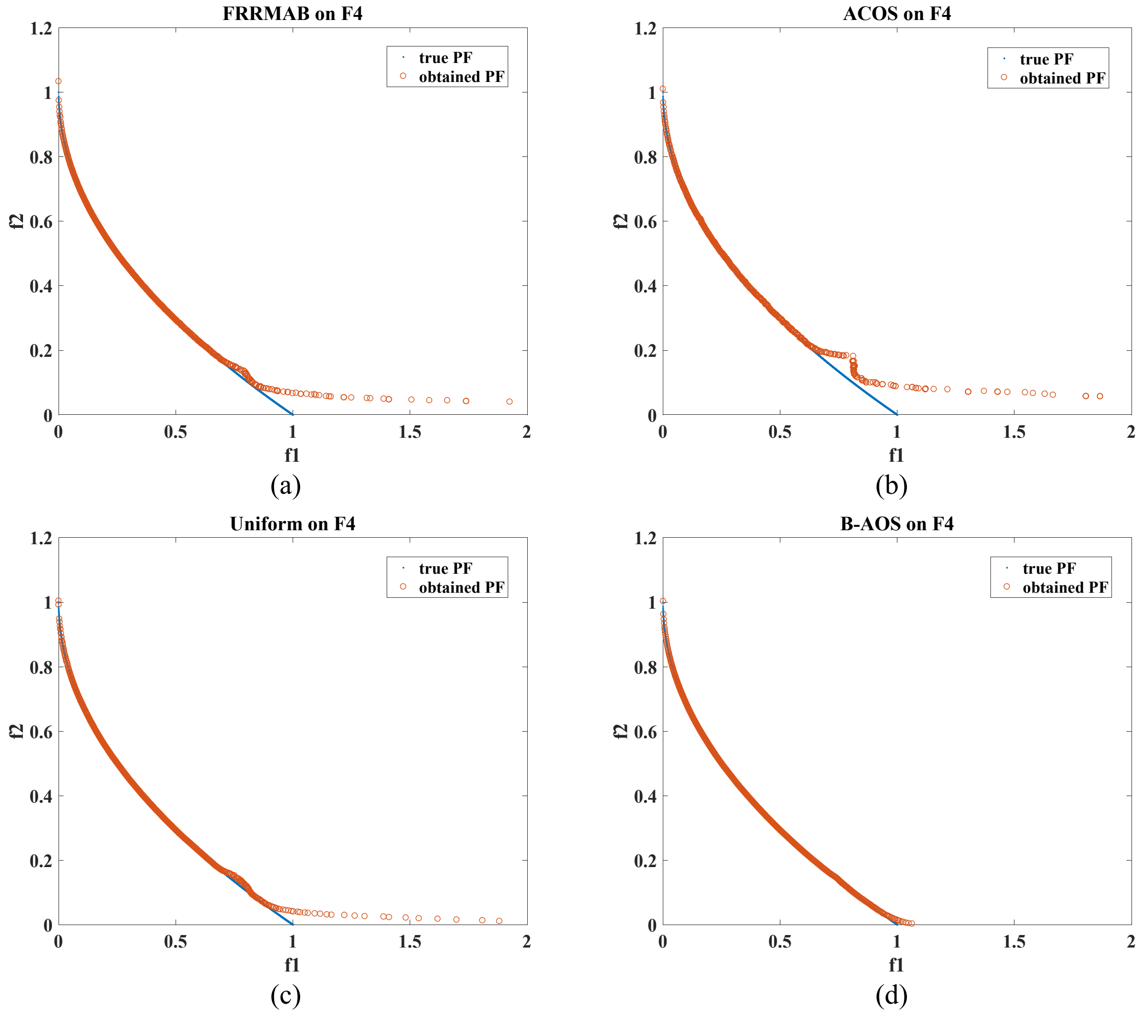

Fig. 4: Final solutions obtained by different AOS methods on F4

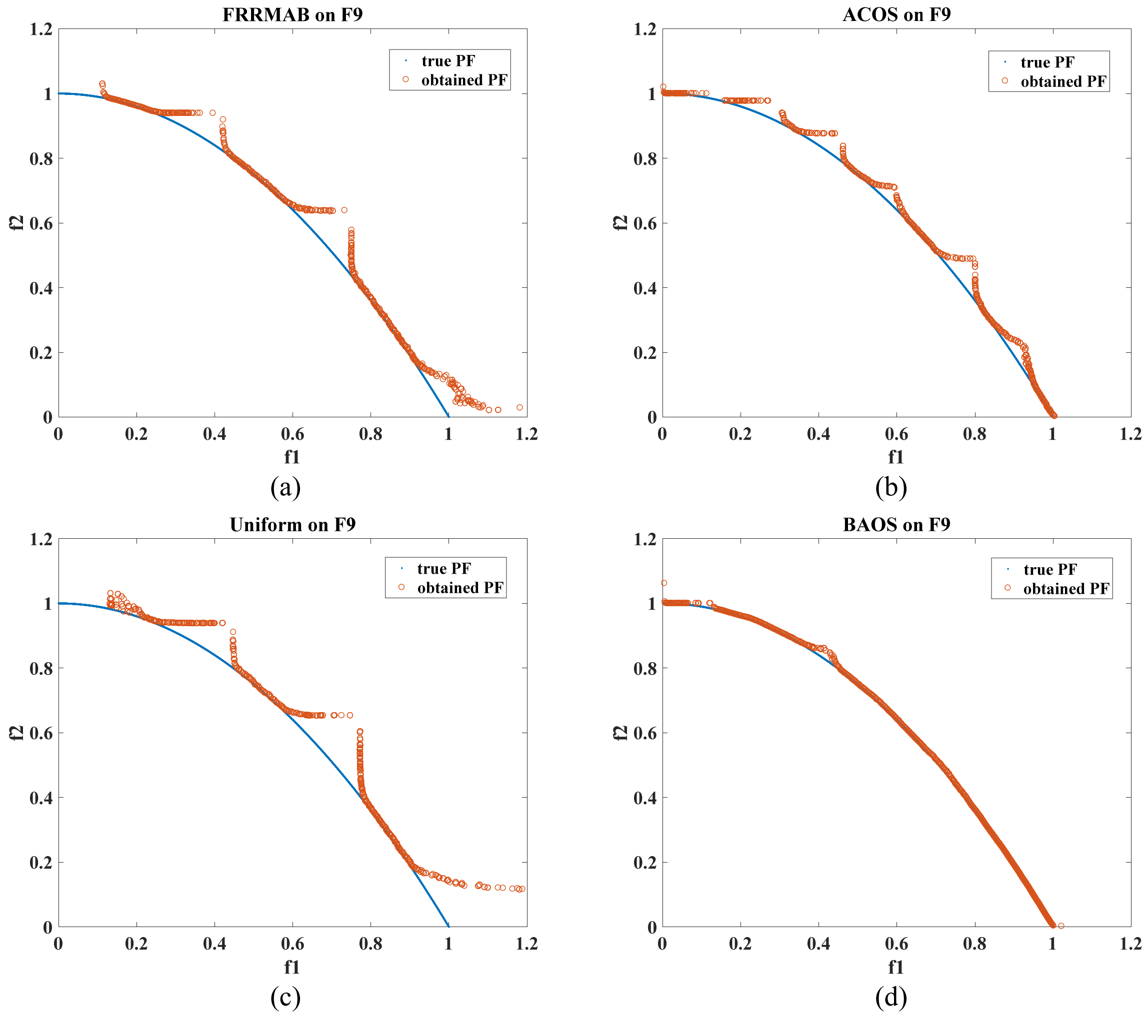

Fig. 5: Final solutions obtained by different AOS methods on F9

5.2. Performance of B-AOS on Existing MOEA/Ds

To further verify the effectiveness of the proposed B-AOS in other MOEA/Ds, it was embedded into MOEA/D-DE [40], MOEA/D-IR [42] and MOEA/D-IRA [31] and then compared with the original versions of three MOEA/Ds. Table 4 and Table 5 give the comparison results between the original algorithms and their variants with B-AOS in terms of IGD and HV, respectively. As shown in the second to last row of Table 4 and Table 5, the best results obtained by the original MOEA/Ds and their variants with B-AOS are summarized. With respect to IGD, as shown in the second to last row of Table 4, the improvement of B-AOS based on MOEA/D-DE, MOEA/D-IR and MOEA/D-IRA can be found on 15, 15 and 16 out of 19 test instances, respectively. With respect to HV, as shown in the second to last row of Table 5, the improvement of B-AOS based on MOEA/D-DE, MOEA/D-IR and MOEA/D-IRA can be found on 14, 14 and 16 out of 19 test instances, respectively. Moreover, in the last row of Table 4 and Table 5, the one-by-one comparisons of the original MOEA/Ds and their corresponding variants with B-AOS are summarized. "+/–/~" indicate the numbers of problems in which the competitors performed better than, worse than and similar to MOEA/Ds with B-AOS, respectively. As shown in Table 4 and Table 5, MOEA/D-DE-AOS obtained significantly better or similar results than MOEA/D-DE on 17 (11 better and 6 similar results) out of 19 test instances in terms of IGD and 16 (11 better and 5 similar results) out of 19 test instances in terms of HV, respectively. MOEA/D-IR-AOS obtained significantly better or similar results than MOEA/D-IR on 15 (11 better and 4 similar results) out of 19 test instances in terms of IGD and 16 (12 better and 4 similar results) out of 19 test instances in terms of HV. MOEA/D-IRA-AOS obtained significantly better or similar results than MOEA/D-IRA on 16 (13 better and 3 similar results) out of 19 test instances in terms of IGD and 16 (14 better and 2 similar results) out of 19 test instances in terms of HV, respectively. However, for some three-objective problems, as more nondominated solutions exist during the evolutionary process, B-AOS failed to show its superiority. As shown in Table 4 and Table 5, in terms of IGD and HV, MOEA/D-DE-AOS was significantly worse than MOEA/D-DE on UF8 and UF10. For UF8-UF10, MOEA/D-IR achieved better results than MOEA/D-IR-AOS in IGD and HV. For UF8, UF10 and F6, MOEA/D-IRA achieved better results than MOEA/D-IRA-AOS in IGD and HV. In these cases, most individuals were nondominated solutions during the evolutionary process, which caused little usage of Pool-II. Thus, B-AOS only focuses on exploitation in some cases. From the above analysis and discussions, it is evident that B-AOS is a very promising AOS, which helps to obtain a good balance of exploitation and exploration for recombination when solving most of the test problems, even though it failed to show its superiority on some three-objective problems.

Table 4
Mean and standard deviation of IGD values obtained by the three original MOEA/Ds and their variants with B-AOS

Problem	MOEA/D-DE	MOEA/D-DE-BAOS	MOEA/D-IR	MOEA/D-IR-BAOS	MOEA/D-IRA	MOEA/D-IRA-BAOS
UF1	9.634e-04(9.90e-05)-	**9.055e-04(7.28e-05)**	1.048e-03(1.05e-04)-	**9.506e-04(7.34e-05)**	8.335e-04(3.78e-05)-	**7.915e-04(3.85e-05)**
UF2	5.617e-03(1.72e-03)~	**5.340e-03(1.88e-03)**	2.928e-03(1.53e-03)~	**2.657e-03(1.50e-03)**	1.250e-03(2.07e-04)-	**1.160e-03(9.77e-05)**
UF3	1.654e-02(1.62e-02)-	**5.115e-03(6.19e-03)**	1.593e-02(1.67e-02)-	**5.590e-03(9.01e-03)**	4.398e-03(3.56e-03)-	**2.478e-03(3.17e-03)**
UF4	**5.544e-02(3.16e-03)~**	5.672e-02(3.92e-03)	5.208e-02(2.95e-03)~	**5.100e-02(3.19e-03)**	4.891e-02(3.25e-03)~	**4.792e-02(2.46e-03)**
UF5	3.112e-01(6.76e-02)~	**3.070e-01(5.16e-02)**	2.691e-01(6.97e-02)~	**2.617e-01(3.65e-02)**	2.442e-01(1.72e-02)~	**2.361e-01(3.20e-02)**
UF6	9.493e-02(8.36e-02)~	**7.506e-02(3.50e-02)**	6.863e-02(2.82e-02)-	**6.086e-02(2.38e-02)**	7.039e-02(2.75e-02)-	**6.686e-02(3.62e-02)**
UF7	1.553e-03(4.88e-04)~	**1.533e-03(2.55e-04)**	1.099e-03(9.96e-05)~	**1.074e-03(8.67e-05)**	8.788e-04(3.40e-05)~	**8.669e-04(4.50e-05)**
UF8	**5.408e-02(8.74e-03)+**	6.596e-02(1.08e-02)	**2.368e-02(2.39e-03)+**	2.953e-02(6.96e-03)	**2.967e-02(5.77e-03)+**	5.839e-02(2.27e-02)
UF9	5.302e-02(3.89e-02)-	**4.702e-02(3.74e-02)**	**3.257e-02(3.54e-02)+**	4.692e-02(5.46e-02)	2.806e-02(2.90e-02)-	**2.057e-02(1.93e-02)**
UF10	**5.246e-01(6.44e-02)+**	5.613e-01(7.83e-02)	**7.508e-01(2.23e-01)+**	1.233e+00(6.90e-01)	**4.832e-01(8.45e-02)+**	2.133e+00(1.58e-01)
F1	7.804e-04(2.09e-05)-	**7.174e-04(1.23e-05)**	8.698e-04(4.05e-05)-	**7.743e-04(2.35e-05)**	7.282e-04(1.29e-05)-	**6.920e-04(6.12e-06)**
F2	6.155e-02(4.48e-02)-	**2.358e-02(1.05e-02)**	5.866e-02(4.54e-02)-	**1.128e-02(9.55e-03)**	5.706e-03(4.79e-03)-	**2.456e-03(5.18e-04)**
F3	2.773e-02(2.67e-02)-	**9.774e-03(4.09e-03)**	1.759e-02(2.79e-02)-	**3.431e-03(2.07e-03)**	3.582e-03(1.65e-03)-	**2.154e-03(1.05e-03)**
F4	2.592e-02(6.88e-03)-	**1.964e-02(1.01e-02)**	2.218e-02(7.06e-03)-	**1.676e-02(5.94e-03)**	2.064e-03(1.57e-04)-	**1.606e-03(9.78e-05)**
F5	1.907e-02(1.27e-02)-	**1.233e-02(3.88e-03)**	1.557e-02(1.92e-02)-	**1.000e-02(4.76e-03)**	9.936e-03(2.63e-03)-	**7.026e-03(1.96e-03)**
F6	3.991e-02(4.21e-03)-	**3.677e-02(2.38e-03)**	2.560e-02(1.51e-03)-	**2.383e-02(2.20e-03)**	**2.868e-02(2.04e-03)+**	3.623e-02(9.43e-03)
F7	**2.911e-01(5.08e-02)~**	3.099e-01(6.09e-02)	**4.239e-01(7.57e-02)+**	5.723e-01(2.01e-01)	2.717e-01(6.20e-02)-	**2.220e-01(6.48e-02)**
F8	8.055e-02(1.89e-02)-	**5.741e-02(1.52e-02)**	1.043e-01(5.29e-02)-	**9.811e-02(6.79e-02)**	6.746e-02(1.95e-02)-	**5.515e-02(2.04e-02)**
F9	4.366e-02(2.76e-02)-	**1.766e-02(1.30e-02)**	3.074e-02(1.70e-02)-	**1.265e-02(8.71e-03)**	3.384e-03(8.93e-04)-	**2.780e-03(6.73e-04)**
Best/All	4/19	15/19	4/19	15/19	3/19	16/19
+/–/~	2/11/6	\	4/11/4	\	3/13/3	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that original MOEA/Ds are significantly better than, worse than or similar to their respective MOEA/Ds with B-AOS. The best results are highlighted in **boldface**.

Table 5
Mean and standard deviation of HV values obtained by three original MOEA/Ds and their variants with B-AOS

Problem	MOEA/D-DE	MOEA/D-DE-BAOS	MOEA/D-IR	MOEA/D-IR-BAOS	MOEA/D-IRA	MOEA/D-IRA-BAOS
UF1	8.746e-01(2.62e-04)-	**8.747e-01(1.82e-04)**	8.744e-01(3.40e-04)-	**8.746e-01(2.42e-04)**	8.750e-01(1.25e-04)-	**8.751e-01(1.12e-04)**
UF2	8.673e-01(2.60e-03)~	**8.680e-01(2.93e-03)**	8.713e-01(2.34e-03)-	**8.718e-01(2.34e-03)**	8.741e-01(6.46e-04)~	**8.744e-01(2.30e-04)**
UF3	8.498e-01(2.36e-02)-	**8.679e-01(9.59e-03)**	8.493e-01(2.40e-02)-	**8.670e-01(1.34e-02)**	8.682e-01(6.46e-03)-	**8.717e-01(6.56e-03)**
UF4	**4.478e-01(5.22e-03)~**	4.459e-01(6.24e-03)	4.549e-01(5.06e-03)~	**4.566e-01(5.54e-03)**	4.594e-01(5.14e-03)-	**4.617e-01(4.09e-03)**
UF5	**1.632e-01(6.12e-02)~**	1.459e-01(7.51e-02)	**1.865e-01(5.11e-02)~**	1.825e-01(5.97e-02)	2.015e-01(3.58e-02)~	**2.185e-01(6.94e-02)**
UF6	4.178e-01(8.18e-02)~	**4.261e-01(5.83e-02)**	4.383e-01(5.16e-02)-	**4.595e-01(3.80e-02)**	4.320e-01(5.75e-02)-	**4.453e-01(5.47e-02)**
UF7	7.066e-01(1.09e-03)~	**7.067e-01(7.00e-04)**	7.075e-01(2.95e-04)-	**7.076e-01(2.21e-04)**	7.082e-01(1.36e-04)-	**7.082e-01(1.23e-04)**
UF8	**6.882e-01(1.90e-02)+**	6.720e-01(1.64e-02)	**7.633e-01(3.90e-03)~**	7.607e-01(8.65e-03)	**7.511e-01(1.12e-02)+**	7.136e-01(3.37e-02)
UF9	1.012e+00(6.04e-02)-	**1.023e+00(5.81e-02)**	**1.077e+00(5.98e-02)+**	1.057e+00(8.36e-02)	1.086e+00(4.80e-02)-	**1.098e+00(2.84e-02)**
UF10	**7.349e-02(3.19e-02)+**	4.150e-02(2.72e-02)	**3.598e-02(3.46e-02)+**	1.282e-02(2.50e-02)	**2.820e-02(3.42e-02)+**	0.000e+00(0.00e+00)
F1	8.751e-01(8.39e-05)-	**8.753e-01(7.40e-05)**	8.751e-01(7.11e-05)-	**8.753e-01(5.50e-05)**	8.753e-01(6.78e-05)-	**8.754e-01(6.50e-05)**
F2	7.636e-01(6.07e-02)-	**8.224e-01(2.83e-02)**	7.906e-01(4.26e-02)-	**8.542e-01(2.12e-02)**	8.664e-01(8.69e-03)-	**8.719e-01(7.93e-04)**
F3	8.381e-01(2.84e-02)-	**8.599e-01(6.97e-03)**	8.543e-01(2.34e-02)-	**8.707e-01(3.00e-03)**	8.703e-01(2.89e-03)-	**8.725e-01(2.18e-03)**
F4	8.345e-01(9.89e-03)-	**8.439e-01(1.53e-02)**	8.399e-01(1.03e-02)-	**8.475e-01(8.96e-03)**	8.728e-01(2.80e-04)-	**8.736e-01(1.81e-04)**
F5	8.473e-01(1.57e-02)-	**8.564e-01(6.24e-03)**	8.547e-01(1.43e-02)-	**8.613e-01(5.25e-03)**	8.607e-01(4.02e-03)-	**8.650e-01(3.02e-03)**
F6	7.113e-01(9.25e-03)-	**7.202e-01(5.95e-03)**	7.543e-01(2.91e-03)-	**7.597e-01(4.78e-03)**	**7.467e-01(4.12e-03)+**	7.274e-01(2.38e-02)
F7	**4.363e-01(4.50e-02)+**	3.254e-01(9.84e-02)	**2.581e-01(8.84e-02)+**	1.303e-01(1.27e-01)	3.956e-01(7.09e-02)-	**4.523e-01(9.21e-02)**
F8	7.312e-01(2.85e-02)-	**7.656e-01(2.51e-02)**	7.142e-01(5.09e-02)~	**7.212e-01(6.99e-02)**	7.507e-01(3.36e-02)-	**7.731e-01(3.36e-02)**
F9	4.609e-01(3.13e-02)-	**5.062e-01(2.98e-02)**	4.788e-01(2.68e-02)-	**5.172e-01(2.05e-02)**	5.373e-01(1.47e-03)-	**5.383e-01(1.03e-03)**
Best/All	5/19	14/19	5/19	14/19	3/19	16/19
+/–/~	3/11/5	\	3/12/4	\	3/14/2	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that original MOEA/Ds are significantly better than, worse than or similar to their respective MOEA/Ds with B-AOS. The best results are highlighted in **boldface**.

In addition, to show the performance improvement of B-AOS in MOEA/Ds during the evolutionary process more clearly, the curves of the mean IGD values from 20 runs versus the number of function evaluations on the benchmark problems are plotted in Fig. 6 and Fig. 7, where the blue curves represent the original MOEA/Ds while the red curves represent their variants with B-AOS. Note that curves with different shapes indicate different MOEA/D variants, i.e.,MOEA/D-DE, MOEA/D-IR and MOEA/D-IRA. Due to page limitations, some of the test instances, i.e., UF1-UF3, UF6, F2-F3, F5, and F9, are selected to show the significant improvement of MOEA/Ds by using B-AOS to replace the single recombination operator. As shown in Figs. 6 (a)-(d), for the UF test problems, i.e., UF1, UF2, UF3 and UF6, replacing the original recombination operator with B-AOS can improve the performance of MOEA/Ds during the evolutionary process. Similarly, when dealing with the F test instances, i.e., F2, F3, F5, and F9, as plotted in Figs. 7 (a)-(d), MOEA/Ds embedded with B-AOS also achieve better results during the evolutionary process when compared to their original MOEA/Ds. To summarize, B-AOS can improve the performance of these original MOEA/Ds on most of the test problems used because the proposed B-AOS can adaptively select DE mutation strategies to balance exploitation and exploration during the evolutionary search.

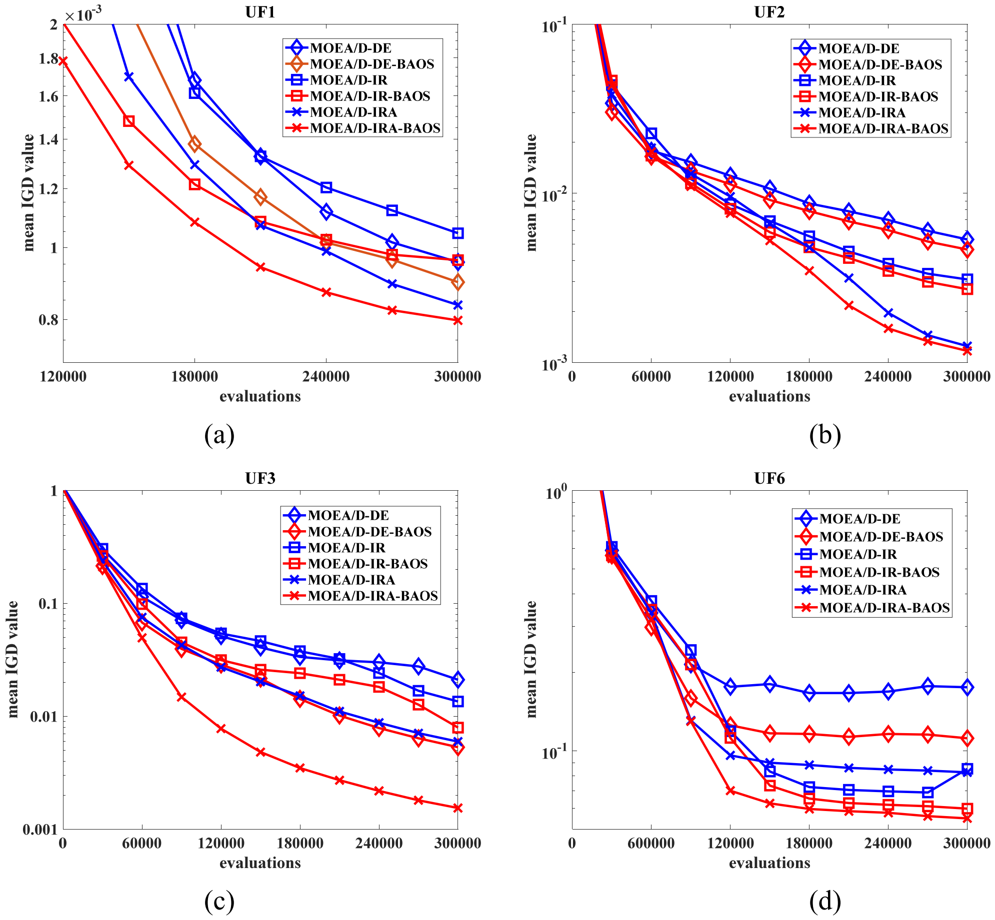

Fig. 6: Convergence curves of three original MOEA/Ds and their variants with B-AOS on UF test instances

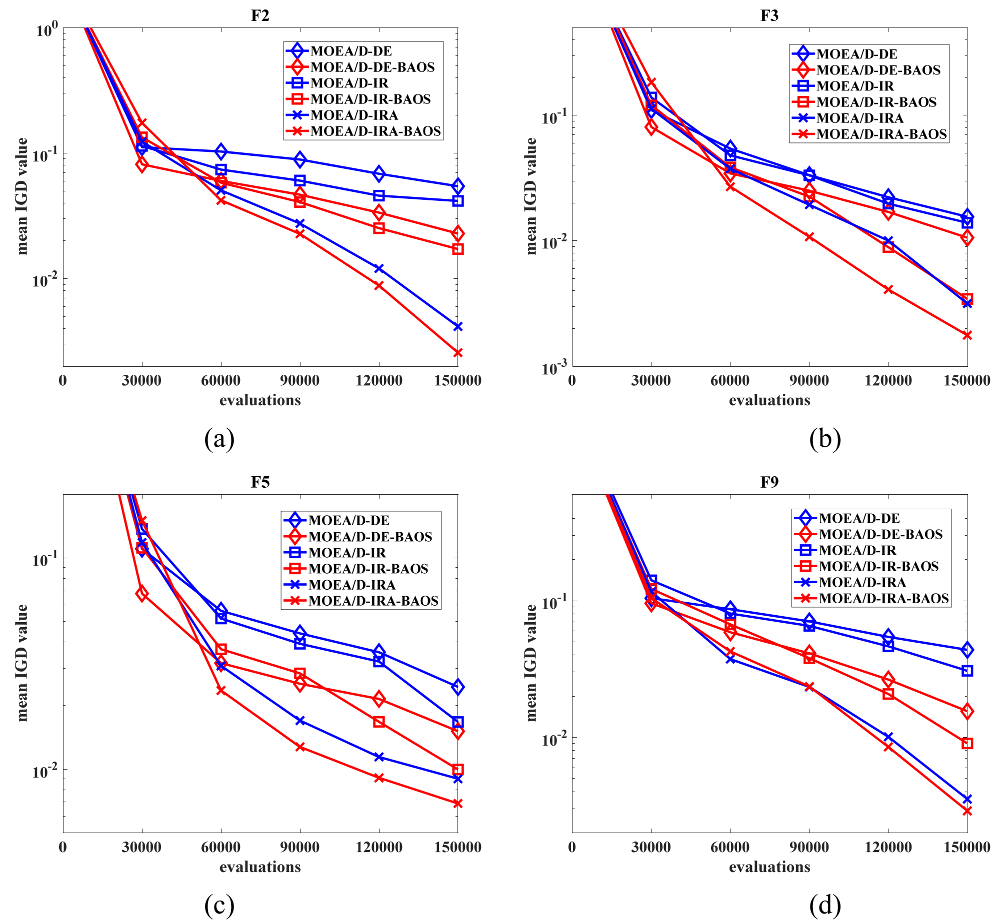

Fig. 7: Convergence curves of three original MOEA/Ds and their variants with B-AOS on F test instances

5.3. Operator Pool versus a Single Operator

In this subsection, the effectiveness of using a pool of operators in MOEA/Ds is studied and analyzed. In experiments, the classical MOEA/D-DRA is used as the baseline algorithm for the operator pool or a single operator. In this case, MOEA/D-DRA-BAOS is compared against four variants (i.e., Variants I-IV), each of which adopts a single DE operator from the operator pool, i.e., DE/rand/1, DE/current-to-rand/1, DE/rand/2, and DE/current-to-rand/2. To allow a fair comparison, all the parameter settings in Variants I-IV are kept the same as those in MOEAD-DRA-BAOS, except that MOEAD-DRA-BAOS uses an operator pool while each of Variants I-IV only employs a single DE operator.

Table 6

Mean and standard deviation of IGD values obtained by four single DE variants and B-AOS under MOEA/D-DRA

Problem	Variant I	Variant II	Variant III	Variant IV	B-AOS
UF1	9.853e-04(7.69e-05)-	1.172e-03(4.59e-04)-	1.250e-03(8.29e-05)-	1.067e-03(8.12e-05)-	**9.423e-04(9.23e-05)**
UF2	2.390e-03(1.04e-03)-	1.105e-02(1.59e-02)-	2.426e-03(3.01e-04)-	2.404e-03(9.93e-04)-	**2.034e-03(7.81e-04)**
UF3	8.325e-03(1.01e-02)-	4.998e-02(2.72e-02)-	5.440e-03(4.65e-03)-	**2.042e-03(1.83e-03)+**	3.537e-03(2.55e-03)
UF4	5.569e-02(3.75e-03)-	6.038e-02(4.41e-03)-	**4.895e-02(2.31e-03)+**	5.489e-02(3.92e-03)~	5.388e-02(3.02e-03)
UF5	2.981e-01(4.94e-02)~	4.676e-01(8.97e-02)-	3.044e-01(4.81e-02)~	3.039e-01(5.59e-02)~	**2.888e-01(4.41e-02)**
UF6	9.138e-02(7.09e-02)-	1.725e-01(8.96e-02)-	8.267e-02(8.76e-02)~	9.057e-02(1.35e-01)-	**6.710e-02(2.67e-02)**
UF7	1.099e-03(2.24e-04)-	2.644e-03(2.17e-03)-	1.398e-03(7.75e-05)-	1.292e-03(1.39e-04)-	**1.005e-03(1.05e-04)**
UF8	2.917e-02(4.61e-03)+	**2.865e-02(4.38e-03)+**	4.018e-02(9.38e-03)-	2.989e-02(4.56e-03)+	3.414e-02(7.13e-03)
UF9	5.384e-02(4.65e-02)~	6.994e-02(4.66e-02)-	3.513e-02(2.66e-02)~	**3.411e-02(3.09e-02)+**	5.681e-02(5.14e-02)
UF10	4.949e-01(6.64e-02)~	**4.329e-01(5.11e-02)+**	1.005e+00(1.73e-01)-	7.589e-01(1.45e-01)-	4.933e-01(5.92e-02)
F1	7.695e-04(1.95e-05)-	**7.028e-04(1.16e-05)+**	1.018e-03(3.77e-05)-	7.784e-04(2.21e-05)-	7.177e-04(1.36e-05)
F2	5.152e-02(2.56e-02)-	1.029e-01(5.82e-02)-	**1.994e-02(1.05e-02)+**	3.274e-02(6.62e-03)-	2.490e-02(1.19e-02)
F3	2.232e-02(2.00e-02)-	3.028e-02(2.52e-02)-	6.301e-03(2.73e-03)+	**5.573e-03(2.59e-03)+**	7.466e-03(2.31e-03)
F4	4.444e-03(2.15e-03)~	2.001e-02(6.30e-03)-	4.987e-03(2.37e-03)-	6.553e-03(3.88e-03)-	**3.898e-03(1.79e-03)**
F5	1.691e-02(8.77e-03)-	3.657e-02(2.23e-02)-	**1.044e-02(2.96e-03)+**	1.136e-02(3.17e-03)~	1.239e-02(4.19e-03)
F6	2.707e-02(1.66e-03)-	2.657e-02(1.48e-03)-	3.141e-02(1.99e-03)-	2.616e-02(1.27e-03)-	**2.482e-02(1.97e-03)**
F7	3.063e-01(4.10e-02)~	**3.041e-01(2.61e-02)~**	3.112e-01(4.11e-02)~	3.057e-01(6.22e-02)~	3.150e-01(5.36e-02)
F8	7.111e-02(1.56e-02)-	9.540e-02(5.82e-02)-	7.733e-02(1.02e-02)-	**4.835e-02(1.56e-02)~**	5.297e-02(1.83e-02)
F9	5.560e-02(3.86e-02)-	8.772e-02(5.81e-02)-	**1.647e-02(1.17e-02)~**	2.906e-02(1.05e-02)-	1.761e-02(1.33e-02)
Best/Second/All	0/6/19	4/0/19	4/3/19	4/3/19	7/7/19
+/–/~	1/13/5	3/15/1	4/10/5	4/10/5	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that the performances of Variants I-IV are significantly better than, worse than or similar to those of B-AOS, respectively. The best and second-best results are highlighted in **boldface** and gray background, respectively.

The experimental results obtained by Variants I-IV and B-AOS under MOEA/D-DRA are given in Table 6 and Table 7, respectively, showing the mean IGD and HV values. With respect to IGD, as summarized in the second to last row of Table 6, B-AOS obtained the best or second-best results in 14 (7 best and 7 second-best) out of 19 cases, while Variants I-IV performed best or second-best in 6 (0 best and 6 second-best), 4 (4 best and 0 second-best), 7 (4 best and 3 second-best), and 7 (4 best and 3 second-best) out of 19 cases. With respect to HV, as shown in the second to last row of Table 3, B-AOS obtained the best or second-best results in 13 (6 best and 7 second-best) out of 19 cases, while Variants I-IV performed best or second-best in 7 (1 best and 6 second-best), 5 (4 best and 1 second-best), 7 (4 best and 3 second-best), and 6 (4 best and 2 second-best) out of 19 cases, respectively. From the above-summarized comparison results, it is obvious that B-AOS achieves superior performance on most of the test problems adopted. Moreover, the one-by-one comparisons of B-AOS and Variants I-IV under MOEA/D-DRA are summarized in the last rows of Table 6 and Table 7, where "+/–/~" indicate the numbers of problems that the competitors are significantly better than, worse than and similar to B-AOS, respectively. In terms of IGD, B-AOS was better than Variants I-IV in 13, 15, 10 and 10 out of 19 cases, respectively, while it was worse than Variants I-IV in 1, 3, 4 and 4 out of 19 cases, respectively. In addition, when compared with Variants I-IV, B-AOS obtained better HV results on 11, 15, 9 and 9 out of 19 cases, while it only showed poor performance on 3, 4, 4 and 4 out of 19 cases.

Table 7
Mean and standard deviation of HV values obtained by four single DE variants and B-AOS under MOEA/D-DRA

Problem	Variant I	Variant II	Variant III	Variant IV	B-AOS
UF1	8.746e-01(2.22e-04)-	8.738e-01(1.21e-03)-	8.741e-01(2.14e-04)-	8.745e-01(1.82e-04)-	**8.747e-01(2.29e-04)**
UF2	8.723e-01(2.04e-03)~	8.620e-01(1.47e-02)-	8.724e-01(8.75e-04)-	8.724e-01(1.82e-03)~	**8.727e-01(1.79e-03)**
UF3	8.620e-01(1.56e-02)-	7.674e-01(5.78e-02)-	8.674e-01(6.94e-03)-	**8.729e-01(2.64e-03)+**	8.703e-01(4.30e-03)
UF4	4.476e-01(6.01e-03)-	4.393e-01(6.27e-03)-	**4.592e-01(4.23e-03)+**	4.488e-01(6.57e-03)-	4.509e-01(5.04e-03)
UF5	**1.629e-01(6.64e-02)~**	8.617e-02(5.61e-02)-	1.414e-01(7.82e-02)~	1.447e-01(8.43e-02)~	1.503e-01(7.21e-02)
UF6	4.192e-01(8.27e-02)~	4.305e-01(4.98e-02)-	4.297e-01(7.40e-02)~	4.357e-01(7.04e-02)~	**4.425e-01(3.50e-02)**
UF7	7.076e-01(5.77e-04)-	7.032e-01(4.83e-03)-	7.071e-01(2.99e-04)-	7.070e-01(5.91e-04)-	**7.078e-01(3.26e-04)**
UF8	7.521e-01(9.89e-03)+	**7.571e-01(9.66e-03)+**	7.211e-01(2.40e-02)-	7.482e-01(1.11e-02)~	7.449e-01(1.64e-02)
UF9	1.035e+00(7.59e-02)~	9.984e-01(6.63e-02)-	1.069e+00(4.62e-02)~	**1.072e+00(5.34e-02)+**	1.031e+00(8.20e-02)
UF10	9.411e-02(3.35e-02)+	**1.401e-01(2.79e-02)+**	4.126e-04(2.33e-03)-	6.850e-03(1.82e-02)-	6.295e-02(3.79e-02)
F1	8.752e-01(8.37e-05)-	**8.754e-01(8.01e-05)+**	8.747e-01(8.83e-05)-	8.752e-01(6.10e-05)-	8.753e-01(6.78e-05)
F2	7.676e-01(4.29e-02)-	7.138e-01(7.37e-02)-	**8.358e-01(2.70e-02)+**	7.983e-01(1.46e-02)-	8.182e-01(3.01e-02)
F3	8.417e-01(2.70e-02)-	8.381e-01(2.68e-02)-	8.657e-01(4.91e-03)+	**8.670e-01(4.42e-03)+**	8.628e-01(4.67e-03)
F4	8.676e-01(3.94e-03)~	8.422e-01(1.02e-02)-	8.673e-01(4.20e-03)~	8.635e-01(6.60e-03)-	**8.684e-01(3.61e-03)**
F5	8.489e-01(1.45e-02)-	8.311e-01(2.41e-02)-	**8.593e-01(5.31e-03)+**	8.581e-01(4.95e-03)~	8.558e-01(7.23e-03)
F6	7.488e-01(4.02e-03)-	7.533e-01(3.28e-03)-	7.350e-01(4.90e-03)-	7.505e-01(3.03e-03)-	**7.558e-01(3.76e-03)**
F7	4.312e-01(3.81e-02)+	**4.605e-01(2.75e-02)+**	3.251e-01(5.64e-02)~	3.950e-01(8.23e-02)+	3.367e-01(9.14e-02)
F8	7.441e-01(2.67e-02)-	7.283e-01(6.12e-02)-	7.306e-01(1.87e-02)-	**7.827e-01(2.54e-02)~**	7.732e-01(3.16e-02)
F9	4.478e-01(3.94e-02)-	4.116e-01(5.27e-02)-	**5.113e-01(2.48e-02)~**	4.810e-01(2.36e-02)-	5.068e-01(2.96e-02)
Best/Second/All	1/6/19	4/1/19	4/3/19	4/2/19	6/7/19
+/–/~	3/11/5	4/15/0	4/9/6	4/9/6	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that the performances of Variants I-IV are significantly better than, worse than or similar to those of B-AOS, respectively. The best and second-best results are highlighted in **boldface** and gray background, respectively.

Moreover, to quantify the overall performance of each algorithm, Friedman's test from the software tool KEEL [51] was used to show the average performance ranks of Variants I-IV and B-AOS for solving all the test problems, which are plotted in Fig. 8. The average performance ranks (2.1053 on IGD and 2.1579 on HV) of B-AOS are obviously much smaller than those of Variants I-IV, which confirms our advantages when considering all the test problems.

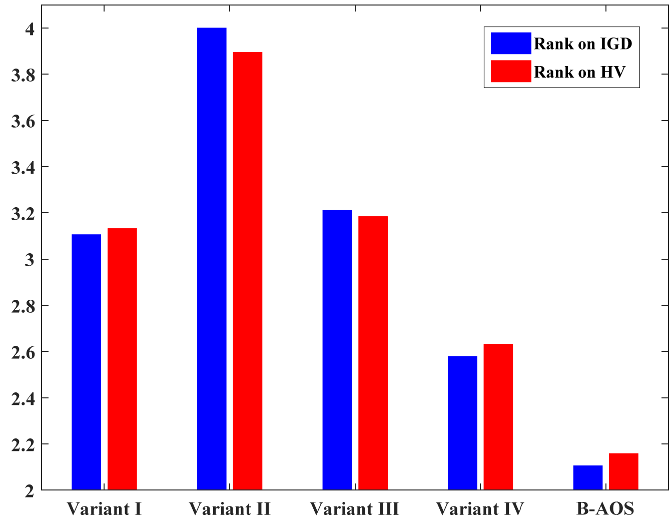

Fig. 8: Average ranking of Friedman's test for the compared variants and B-AOS

According to the above-summarized comparison results, we can conclude that B-AOS was able to obtain better results on most of the test problems adopted, which suggests that adopting B-AOS to manage an operator pool is better than only employing a single operator in balancing exploitation and exploration during the evolutionary search.

5.4. Further Analysis of Crowding-Assisted Operator Selection

In this subsection, we further analyze and discuss crowding-assisted operator selection in B-AOS, aiming to demonstrate its effective mechanism.

Table 8
Mean and standard deviation of IGD values obtained by MOEA/D-DRA with B-AOS and their variants

Problem	B-AOS_S-I	B-AOS_S-II	B-AOS_S-III	B-AOS_neighbor	B-AOS_average	B-AOS
UF1	9.688e-04(6.82e-05)-	9.445e-04(1.92e-04)~	9.674e-04(7.25e-05)~	9.599e-04(7.21e-05)~	9.116e-04(6.42e-05)~	9.423e-04(9.23e-05)
UF2	2.158e-03(6.58e-04)~	2.966e-03(2.05e-03)~	2.161e-03(8.26e-04)~	2.123e-03(6.73e-04)~	2.280e-03(1.29e-03)~	2.034e-03(7.81e-04)
UF3	9.418e-03(9.72e-03)-	6.363e-03(9.64e-03)~	3.447e-03(3.00e-03)~	3.039e-03(2.22e-03)~	4.010e-03(8.11e-03)-	3.537e-03(2.55e-03)
UF4	5.278e-02(3.00e-03)~	5.873e-02(3.89e-03)-	5.317e-02(3.74e-03)~	5.415e-02(3.82e-03)~	5.665e-02(4.47e-03)-	5.388e-02(3.02e-03)
UF5	2.920e-01(4.66e-02)~	3.132e-01(4.55e-02)-	3.038e-01(4.84e-02)~	3.178e-01(5.05e-02)-	3.095e-01(5.12e-02)-	2.888e-01(4.41e-02)
UF6	8.276e-02(8.46e-02)~	7.186e-02(4.75e-02)~	9.440e-02(1.01e-01)~	8.055e-02(8.64e-02)~	6.374e-02(1.72e-02)~	6.710e-02(2.67e-02)
UF7	1.009e-03(5.59e-05)~	1.246e-03(3.77e-04)-	1.133e-03(2.02e-04)-	1.014e-03(6.78e-05)~	1.014e-03(9.38e-05)~	1.005e-03(1.05e-04)
UF8	3.619e-02(1.02e-02)~	3.865e-02(1.11e-02)-	2.861e-02(5.99e-03)+	3.711e-02(1.00e-02)~	3.536e-02(9.65e-03)~	3.414e-02(7.13e-03)
UF9	4.417e-02(4.25e-02)~	5.933e-02(4.76e-02)-	4.869e-02(4.63e-02)~	4.529e-02(4.15e-02)~	4.874e-02(4.23e-02)~	5.681e-02(5.14e-02)
UF10	5.261e-01(6.73e-02)-	4.474e-01(5.83e-02)+	4.826e-01(6.43e-02)~	5.061e-01(7.13e-02)~	4.849e-01(6.46e-02)~	4.933e-01(5.92e-02)
F1	7.672e-04(2.12e-05)-	7.032e-04(1.37e-05)+	7.350e-04(1.79e-05)-	7.582e-04(2.20e-05)-	7.135e-04(1.55e-05)~	7.177e-04(1.36e-05)
F2	1.687e-02(1.12e-02)+	3.306e-02(2.37e-02)-	3.830e-02(2.72e-02)-	2.497e-02(1.13e-02)~	2.809e-02(9.06e-03)~	2.490e-02(1.19e-02)
F3	6.914e-03(2.75e-03)~	1.291e-02(1.11e-02)-	7.444e-03(2.45e-03)~	7.752e-03(2.70e-03)~	8.397e-03(2.88e-03)~	7.466e-03(2.31e-03)
F4	3.724e-03(1.60e-03)~	1.696e-02(5.91e-03)-	5.554e-03(2.44e-03)-	5.512e-03(3.24e-03)-	7.576e-03(3.81e-03)-	3.898e-03(1.79e-03)
F5	1.162e-02(2.91e-03)~	1.505e-02(5.65e-03)-	1.165e-02(3.09e-03)~	1.163e-02(3.07e-03)~	1.267e-02(3.52e-03)~	1.239e-02(4.19e-03)
F6	2.653e-02(1.59e-03)-	2.487e-02(1.69e-03)~	2.558e-02(1.18e-03)-	2.532e-02(1.42e-03)-	2.475e-02(1.19e-03)~	2.482e-02(1.97e-03)
F7	3.535e-01(6.47e-02)-	2.971e-01(4.99e-02)~	3.164e-01(4.10e-02)~	3.230e-01(5.26e-02)~	3.174e-01(5.69e-02)~	3.150e-01(5.36e-02)
F8	6.698e-02(1.20e-02)-	4.714e-02(1.91e-02)~	5.538e-02(1.97e-02)~	5.456e-02(1.59e-02)~	4.929e-02(1.68e-02)~	5.297e-02(1.83e-02)
F9	1.434e-02(1.33e-02)~	3.127e-02(1.02e-02)-	3.406e-02(1.00e-02)-	2.033e-02(1.33e-02)~	2.677e-02(1.42e-02)-	1.761e-02(1.33e-02)
+/–/~	1/7/11	2/10/7	1/6/12	0/4/15	0/5/14	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that other variants are significantly better than, worse than or similar to MOEA/D-DRA with B-AOS.

Table 9
Mean and standard deviation of HV values obtained by MOEA/D-DRA with B-AOS and their variants

Problem	B-AOS_S-I	B-AOS_S-II	B-AOS_S-III	B-AOS_neighbor	B-AOS_average	B-AOS
UF1	8.746e-01(2.00e-04)~	8.745e-01(4.04e-04)-	8.746e-01(2.03e-04)~	8.746e-01(2.29e-04)~	8.747e-01(2.42e-04)~	8.747e-01(2.29e-04)
UF2	8.725e-01(1.69e-03)~	8.712e-01(3.73e-03)~	8.727e-01(1.68e-03)~	8.728e-01(1.29e-03)~	8.723e-01(2.54e-03)~	8.727e-01(1.79e-03)
UF3	8.599e-01(1.63e-02)-	8.654e-01(1.69e-02)~	8.706e-01(4.50e-03)~	8.712e-01(4.04e-03)~	8.701e-01(1.15e-02)-	8.703e-01(4.30e-03)
UF4	4.529e-01(4.70e-03)~	4.421e-01(6.44e-03)-	4.520e-01(6.43e-03)~	4.503e-01(6.59e-03)~	4.458e-01(7.07e-03)-	4.509e-01(5.04e-03)
UF5	1.420e-01(7.21e-02)~	1.339e-01(6.79e-02)~	1.576e-01(6.96e-02)~	1.223e-01(6.36e-02)-	1.398e-01(7.63e-02)~	1.503e-01(7.21e-02)
UF6	4.184e-01(7.46e-02)-	4.443e-01(4.52e-02)~	4.268e-01(6.73e-02)~	4.367e-01(5.51e-02)~	4.467e-01(3.24e-02)~	4.425e-01(3.50e-02)
UF7	7.078e-01(2.07e-04)~	7.070e-01(1.03e-03)-	7.074e-01(7.95e-04)-	7.078e-01(2.91e-04)~	7.077e-01(3.57e-04)~	7.078e-01(3.26e-04)
UF8	7.383e-01(2.20e-02)-	7.391e-01(2.33e-02)~	7.535e-01(1.33e-02)+	7.365e-01(2.26e-02)-	7.434e-01(1.95e-02)~	7.449e-01(1.64e-02)
UF9	1.053e+00(7.02e-02)~	1.020e+00(6.93e-02)-	1.045e+00(7.66e-02)~	1.048e+00(6.86e-02)~	1.040e+00(6.61e-02)~	1.031e+00(8.20e-02)
UF10	5.202e-02(3.35e-02)~	7.567e-02(4.78e-02)~	7.957e-02(3.87e-02)+	4.898e-02(3.52e-02)~	4.841e-02(3.64e-02)-	6.295e-02(3.79e-02)
F1	8.752e-01(9.33e-05)-	8.754e-01(5.92e-05)+	8.753e-01(6.36e-05)~	8.752e-01(7.74e-05)-	8.754e-01(6.66e-05)+	8.753e-01(6.78e-05)
F2	8.398e-01(2.92e-02)+	8.024e-01(4.11e-02)~	7.873e-01(4.77e-02)-	8.186e-01(2.94e-02)~	8.093e-01(2.36e-02)~	8.182e-01(3.01e-02)
F3	8.645e-01(5.22e-03)+	8.569e-01(1.07e-02)-	8.644e-01(4.11e-03)~	8.630e-01(4.80e-03)~	8.621e-01(4.96e-03)~	8.628e-01(4.67e-03)
F4	8.692e-01(3.17e-03)~	8.471e-01(8.58e-03)-	8.654e-01(4.57e-03)-	8.656e-01(5.83e-03)-	8.616e-01(6.37e-03)-	8.684e-01(3.61e-03)
F5	8.575e-01(5.44e-03)~	8.536e-01(6.22e-03)~	8.578e-01(5.52e-03)~	8.575e-01(5.09e-03)~	8.561e-01(6.39e-03)~	8.558e-01(7.23e-03)
F6	7.496e-01(3.43e-03)-	7.570e-01(3.59e-03)~	7.535e-01(3.08e-03)-	7.532e-01(3.32e-03)-	7.566e-01(2.47e-03)~	7.558e-01(3.76e-03)
F7	2.601e-01(8.75e-02)-	4.003e-01(8.32e-02)+	4.352e-01(4.37e-02)+	3.247e-01(7.83e-02)~	3.439e-01(9.94e-02)~	3.367e-01(9.14e-02)
F8	7.481e-01(2.19e-02)-	7.861e-01(3.14e-02)~	7.718e-01(3.34e-02)~	7.710e-01(2.79e-02)~	7.815e-01(2.84e-02)~	7.732e-01(3.16e-02)
F9	5.140e-01(2.92e-02)~	4.757e-01(2.29e-02)-	4.708e-01(2.25e-02)-	4.998e-01(3.05e-02)~	4.861e-01(3.18e-02)-	5.068e-01(2.96e-02)
+/–/~	2/7/10	2/7/10	3/5/11	0/5/14	1/5/13	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that other variants are significantly better than, worse than or similar to MOEA/D-DRA with B-AOS.

First, to validate its effectiveness in B-AOS, some experiments are conducted. For a fair comparison, the classical MOEA/D-DRA is used as the baseline algorithm. One variant (i.e., B-AOS_V-I) always selects DE_1^{type} from the *candidate pool* in the crowding-assisted operator selection. However, another variant (i.e., B-AOS_V-II) always selects DE_2^{type} from the *candidate pool* in the crowding-assisted operator selection. In addition, the variant (i.e., B-AOS_V-III) randomly selects one DE mutation strategy from the operator pool. The experimental results obtained by B-AOS_V-I, B-AOS_V-II, B-AOS_V-III and B-AOS under MOEA/D-DRA are given in Table 8 and Table 9, respectively, showing the mean IGD and HV values. With respect to IGD, as summarized in the last row of Table 8, B-AOS obtained better results than B-AOS_V-I, B-AOS_V-II, B-AOS_V-III in 7, 10, and 6 cases out of 19 cases and achieved similar results with these variants in 11, 7, and 12 cases out of 19 cases. B-AOS was only worse than these variants in 1, 2, and 1 case(s) out of 19 cases. From the above-summarized results, it is obvious that crowding-assisted operator selection contributes to improving the performance of B-AOS on these test problems.

In addition, we also use different crowding-assisted operator selections for comparison. For example, one solution is selected from the neighborhood of the current solution to compute the crowding status in **Algorithm 4**, and the average crowding status of all solutions in the population is computed in **Algorithm 5**. Note that **Algorithms 4-5** can be used as the crowding-assisted operator selection. The compared experiments are run by using the classical MOEA/D-DRA as the baseline algorithm. Tables 8-9 compare the results of the original B-AOS in **Algorithm 2**, B-AOS_neighbor in **Algorithm 4**, and B-AOS_average in **Algorithm 5**. With respect to IGD, as shown in the second to last row of Table 8, B-AOS performed better than B-AOS_neighbor and B-AOS_average on 4 and 5 cases, and similarly to B-AOS_neighbor and B-AOS_average on 15 and 14 cases. B-AOS_neighbor and B-AOS_average could not perform better than B-AOS in any case. With respect to HV, as shown in the last row of Table 9, B-AOS was better than, worse than, and similar to B-AOS_neighbor on 5, 14 and 0 case(s). B-AOS was better than, worse than, and similar to B-AOS_neighbor in 5, 13 and 1 case(s). The above-summarized results validate the effectiveness of the crowding-assisted operator selection (**Algorithm 2**) in B-AOS.

Algorithm 4: B-AOS_neighbor (*x*, *P*, *B*, *Candidate Pool*)

Input: *x* (the current selected solution)
P (the population)
B (the index sets of the neighbors of all solutions)
Candidate Pool=$\{\mathrm{DE}_1^{type}, \mathrm{DE}_2^{type}\}$
Output: *operator*
randomly select one solution *y* from *B*(*x*);
estimate the crowding status of *x* and *y* in their respective neighbors using (14);
if $HAD(x) > HAD(y)$
    operator = DE_1^{type};
else if $HAD(x) < HAD(y)$
    operator = DE_2^{type};
Else
    if random < 0.5
        operator = DE_1^{type};
    Else
        operator = DE_2^{type};
    End
End
return *operator*

Algorithm 5: B-AOS_average (*x*, *P*, *B*, *Candidate Pool*)

Input: *x* (the current selected solution)
P (the population)
B (the index sets of the neighbors of all solutions)
Candidate Pool=$\{\mathrm{DE}_1^{type}, \mathrm{DE}_2^{type}\}$
Output: *operator*
for each solution *x* in *P*
    estimate the crowding status of *x* in their respective neighbors using (14);
End
compute the average crowding distance (*HAD_mean*) of all solutions in population;
if $HAD(x) > HAD\_mean$
    operator = DE_1^{type};
else if $HAD(x) < HAD\_mean$
    operator = DE_2^{type};
Else
    if random < 0.5
        operator = DE_1^{type};
    Else
        operator = DE_2^{type};
    End
End
return *operator*

5.5. Competition with Some Other Recent MOEAs

In this subsection, four popular or recently proposed MOEAs (i.e., I_{SDE}^{+} [16], MOEA/AD [28], MOEA/D-MUP [58] and MOEA/D-DDS [59]) are used for performance comparison in our experimental studies. Please note that the settings of the four compared MOEAs are suggested in their references and B-AOS is embedded into MOEA/D-IRA for comparison. IGD and HV are used as the performance indicators forevaluating the performance of the compared algorithms. Table 10 and Table 11 give the experimental results of MOEA/D-IRA-BAOS with the other four MOEAs regarding IGD and HV, respectively. "+/–/~" indicate that the competitors performed better than, worse than and similar to MOEA/D-IRA-BAOS, respectively. MOEA/D-IRA-BAOS obtained the best results on 14 cases on both IGD and HV. Considering IGD, I_{SDE}^{+}, MOEA/AD and MOEA/D-DDS only achieved the best results in 1, 1 and 3 cases, respectively. MOEA/D-MUP failed to obtain the best result. For HV, MOEA/AD, MOEA/D-MUP and MOEA/D-DDS only achieved the best results in 2, 1 and 2 cases, respectively. I_{SDE}^{+} failed to obtain the best result. Using one-by-one comparisons on IGD, MOEA/D-IRA-BAOS achieved significantly better or similar results than I_{SDE}^{+}, MOEA/AD, MOEA/D-MUP and MOEA/D-DDS in 16 (16 better and 0 similar), 17 (17 better and 0 similar), 18 (17 better and 1 similar), and 15 (15 better and 0 similar) out of 19 cases, respectively, while it was worse than these compared MOEAs in 3, 2, 1 and 4 out of 19 cases. Considering HV, MOEA/D-IRA-BAOS was better than or similar to I_{SDE}^{+}, MOEA/AD, MOEA/D-MUP and MOEA/D-DDS in 17 (17 better and 0 similar), 17 (16 better and 1 similar), 16 (16 better and 0 similar), and 17 (15 better and 2 similar) out of 19 cases. I_{SDE}^{+}, MOEA/AD, MOEA/D-MUP and MOEA/D-DDS were better than MOEA/D-IRA-BAOS in 2, 2, 3 and 2 out of 19 cases, respectively. From the above comparison results, the advantages of B-AOS embedded into MOEA/D-IRA are confirmed when solving the adopted test problems.

Table 10

Mean and standard deviation of IGD values obtained by the recent MOEAs and MOEA/D-IRA with B-AOS

Problem	I_{SDE}^{+}	MOEA/AD	MOEA/D-MUP	MOEA/D-DDS	MOEA/D-IRA-BAOS
UF1	8.468e-02(1.38e-02)-	1.296e-01(6.60e-02)-	1.816e-03(1.51e-04)-	9.836e-04(9.81e-05)-	**7.915e-04(3.85e-05)**
UF2	3.233e-02(6.92e-03)-	6.157e-02(4.46e-02)-	6.368e-03(1.50e-03)-	5.395e-03(2.36e-03)-	**1.160e-03(9.77e-05)**
UF3	1.785e-01(2.14e-02)-	3.116e-01(2.49e-02)-	8.671e-03(9.86e-03)-	1.713e-02(1.09e-02)-	**2.478e-03(3.17e-03)**
UF4	4.017e-02(7.41e-04)+	**3.679e-02(1.19e-03)+**	6.892e-02(5.74e-03)-	5.388e-02(3.26e-03)-	4.792e-02(2.46e-03)
UF5	3.772e-01(1.06e-01)-	4.656e-01(1.17e-01)-	3.339e-01(1.32e-01)-	2.667e-01(2.05e-02)-	**2.361e-01(3.20e-02)**
UF6	2.312e-01(8.70e-02)-	4.505e-01(1.27e-01)-	3.509e-01(2.08e-01)-	7.860e-02(2.82e-02)-	**6.686e-02(3.62e-02)**
UF7	4.615e-02(9.48e-03)-	3.660e-01(1.72e-01)-	3.529e-03(1.93e-03)-	3.070e-03(1.05e-03)-	**8.669e-04(4.50e-05)**
UF8	3.431e-01(9.04e-02)-	1.409e-01(4.23e-02)-	8.251e-02(1.77e-02)-	**4.542e-02(7.54e-03)+**	5.839e-02(2.27e-02)
UF9	1.833e-01(6.89e-02)-	9.283e-02(5.20e-02)-	1.209e-01(4.00e-02)-	3.429e-02(6.12e-03)-	**2.057e-02(1.93e-02)**
UF10	**3.307e-01(6.60e-02)+**	3.339e-01(1.50e-01)+	4.648e-01(6.07e-02)+	1.793e+00(2.65e-01)+	2.133e+00(1.58e-01)
F1	1.434e-02(2.67e-03)-	8.753e-04(3.12e-04)-	9.258e-04(3.42e-05)-	8.759e-04(2.50e-05)-	**6.920e-04(6.12e-06)**
F2	7.678e-02(1.29e-02)-	1.617e-01(7.88e-02)-	1.577e-02(2.14e-02)-	5.255e-03(4.01e-03)-	**2.456e-03(5.18e-04)**
F3	5.583e-02(6.24e-03)-	1.049e-01(5.50e-02)-	5.031e-03(1.61e-03)-	4.700e-03(2.00e-03)-	**2.154e-03(1.05e-03)**
F4	6.157e-02(8.91e-03)-	7.523e-02(8.30e-03)-	3.051e-03(3.07e-04)-	5.792e-03(2.30e-03)-	**1.606e-03(9.78e-05)**
F5	3.809e-02(3.88e-03)-	8.683e-02(5.18e-02)-	1.321e-02(4.13e-03)-	9.297e-03(3.16e-03)-	**7.026e-03(1.96e-03)**
F6	9.084e-02(1.60e-02)-	5.303e-02(7.99e-03)-	5.064e-02(8.47e-03)-	**1.765e-02(7.93e-05)+**	3.623e-02(9.43e-03)
F7	1.174e-01(2.39e-02)+	3.780e-01(1.19e-01)-	2.369e-01(6.57e-02)~	**9.429e-02(4.68e-02)+**	2.220e-01(6.48e-02)
F8	1.848e-01(2.09e-02)-	3.139e-01(1.04e-01)-	1.510e-01(1.02e-01)-	2.511e-01(3.64e-02)-	**5.515e-02(2.04e-02)**
F9	8.164e-02(1.97e-02)-	1.092e-01(6.45e-02)-	1.303e-02(2.87e-02)-	2.152e-02(6.76e-03)-	**2.780e-03(6.73e-04)**
Best/All	1/19	1/19	0/18	3/19	14/19
+/–/~	3/16/0	2/17/0	1/17/1	4/15/0	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that MOEAs are better than, worse than or similar to their respective MOEA/D-IRA-BAOS. The best results are highlighted in **boldface**.

Table 11
Mean and standard deviation of HV values obtained by the recent MOEAs and MOEA/D-IRA with B-AOS

Problem	I_{SDE}^+	MOEA/AD	MOEA/D-MUP	MOEA/D-DDS	MOEA/D-IRA-BAOS
UF1	6.150e-01(1.55e-02)-	6.039e-01(1.18e-01)-	8.737e-01(2.31e-04)-	8.745e-01(2.32e-04)-	**8.751e-01(1.12e-04)**
UF2	6.853e-01(5.66e-03)-	8.203e-01(2.98e-02)-	8.671e-01(2.08e-03)-	8.677e-01(3.48e-03)-	**8.744e-01(2.30e-04)**
UF3	4.597e-01(2.64e-02)-	4.241e-01(7.09e-02)-	8.619e-01(1.81e-02)-	8.447e-01(2.02e-02)-	**8.717e-01(6.56e-03)**
UF4	3.952e-01(8.67e-04)-	**4.689e-01(8.30e-03)+**	4.260e-01(8.54e-03)-	4.517e-01(5.38e-03)-	4.617e-01(4.09e-03)
UF5	1.235e-01(8.20e-02)-	1.839e-01(8.85e-02)~	**2.971e-01(8.03e-02)+**	1.794e-01(3.34e-02)-	2.185e-01(6.94e-02)
UF6	2.321e-01(7.82e-02)-	2.614e-01(8.94e-02)-	3.401e-01(1.19e-01)-	4.184e-01(5.72e-02)-	**4.453e-01(5.47e-02)**
UF7	5.228e-01(1.11e-02)-	3.064e-01(1.72e-01)-	7.032e-01(3.96e-03)-	7.039e-01(2.06e-03)-	**7.082e-01(1.23e-04)**
UF8	3.767e-01(2.88e-02)-	5.118e-01(8.07e-02)-	6.697e-01(2.09e-02)-	7.055e-01(1.87e-02)~	**7.136e-01(3.37e-02)**
UF9	6.169e-01(5.10e-02)-	9.423e-01(8.37e-02)-	9.246e-01(6.99e-02)-	1.046e+00(1.45e-02)-	**1.098e+00(2.84e-02)**
UF10	2.061e-01(8.75e-02)+	**2.456e-01(1.26e-01)+**	1.370e-01(4.00e-02)+	0.000e+00(0.00e+00)~	0.000e+00(0.00e+00)
F1	7.017e-01(3.26e-03)-	8.749e-01(1.47e-03)-	8.751e-01(8.14e-05)-	8.747e-01(1.06e-04)-	**8.754e-01(6.50e-05)**
F2	6.061e-01(1.65e-02)-	5.404e-01(1.39e-01)-	8.531e-01(2.62e-02)-	8.672e-01(6.77e-03)-	**8.719e-01(7.93e-04)**
F3	6.551e-01(5.79e-03)-	7.903e-01(3.27e-02)-	8.690e-01(2.42e-03)-	8.686e-01(2.90e-03)-	**8.725e-01(2.18e-03)**
F4	6.541e-01(5.71e-03)-	8.135e-01(5.92e-03)-	8.719e-01(4.33e-04)-	8.674e-01(3.28e-03)-	**8.736e-01(1.81e-04)**
F5	6.767e-01(4.14e-03)-	8.017e-01(3.08e-02)-	8.561e-01(8.17e-03)-	8.618e-01(4.73e-03)-	**8.650e-01(3.02e-03)**
F6	5.239e-01(1.44e-02)-	6.828e-01(2.42e-02)-	7.126e-01(1.94e-02)-	**7.830e-01(7.39e-04)+**	7.274e-01(2.38e-02)
F7	5.338e-01(3.33e-02)+	3.777e-01(1.11e-01)-	5.446e-01(7.55e-02)+	**6.966e-01(8.40e-02)+**	4.523e-01(9.21e-02)
F8	4.479e-01(2.72e-02)-	3.790e-01(1.11e-01)-	6.647e-01(8.99e-02)-	4.386e-01(5.57e-02)-	**7.731e-01(3.36e-02)**
F9	3.329e-01(1.88e-02)-	3.178e-01(5.68e-02)-	5.243e-01(3.44e-02)-	5.055e-01(1.27e-02)-	**5.383e-01(1.03e-03)**
Best/All	0/19	2/19	1/19	2/19	14/19
+/–/~	2/17/0	2/16/1	3/16/0	2/15/2	\

According to the Wilcoxon's rank sum test with a significant level $\alpha = 0.05$, +, − and ~ indicate that MOEAs are better than, worse than or similar to their respective MOEA/D-IRA-BAOS. The best results are highlighted in **boldface**.

6. Conclusions

In this paper, a novel bicriteria assisted adaptive operator selection (B-AOS) strategy is proposed, in which one criterion (i.e., the Pareto criterion) reflecting convergence and another criterion (i.e., the crowding criterion) reflecting diversity are used to adaptively select DE mutation strategies. First, two operator pools are designed in our algorithm, which focus on exploitation and exploration. Each operator pool consists of two complementary DE mutation strategies. In addition, the selection of two operator pools is adaptively adjusted for each solution during different evolutionary search stages according to the convergence status of solutions, in which the Pareto dominance relationship is used to judge which pool is preferred for the current solution. After that, the crowding criterion is further used to assist the selection of DE mutation strategies from the pool. In the proposed B-AOS strategy, the Pareto criterion shows its effectiveness in two cases: 1) the solutions that are far away from the convergence status can help to promote the exploration capabilities of the population during the evolutionary search process, which aims to find more uncovered areas; 2) the solutions having better convergence aim to emphasize their exploitation abilities to speed up convergence. Another crowding criterion is used based on the binary tournament selection strategy to provide a competitive mechanism for the operators within the same pool, which aims to exploit their relative advantages in exploration and exploitation. In our experimental studies, 19 complicated test MOPs are used for performance comparison, and the proposed B-AOS shows superior performance over other existing AOS methods, i.e., FRRMAB ACOS and uniform. Moreover, our experimental results also validated that B-AOS can further improve the performance of MOEA/Ds with a single DE operator (MOEA/D-DE, MOEA/D-DRA, MOEA/D-IR, and MOEA/D-IRA) and that B-AOS embedded into MOEA/D-IR performs better than four recently proposed MOEAs (I_{SDE}^{+}, MOEA/AD, MOEA/D-MUP and MOEA/D-DDS) when dealing with most of the benchmark problems adopted. The advantages of B-AOS are also confirmed when compared to the static method with only one single DE mutation strategy.

In our future work, other evolutionary operators, such as SBX, other DE mutation strategies (e.g., DE/current-to-best), or some other recombination operators used in particle swarm optimization and ant colony optimization, will be further studied in B-AOS. How to effectively utilize and combine these recombination operators will also be an interesting research direction, especially for tackling various novel many and multi-objective benchmark problems with challenging difficulties [60].

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant 61876110, 61836005, and 61672358, the Joint Funds of the National Natural Science Foundation of China under Key Program Grant U1713212, and the Shenzhen Technology Plan under Grant JCYJ20190808164211203. The last author acknowledges support from CONACyT Grant no. 2016-01-1920 (Investigación en Fronteras de la Ciencia) and from an SEP-Cinvestav Grant (application no. 4).

References:

[1] X. Sun, L. Zhao, P. Zhang, L. Bao, and Y. Chen, Enhanced NSGA-II with evolving directions prediction for interval multi-objective optimization, Swarm and Evolutionary Computation, 49 (2019) 124-133.

[2] X. Tian, and J. Li, A novel improved fruit fly optimization algorithm for aerodynamic shape design optimization, Knowledge-Based Systems, 179(1) (2019) 77-91.

[3] Q. Fan, Y, Jin, W. Wang, and X. Yan, A performance-driven multi-algorithm selection strategy for consumption optimization of sea-rail intermodal transportation, Swarm and Evolutionary Computation, 44 (2019) 1-17.

[4] J. Wang, G. Liang and J. Zhang, Cooperative differential evolution framework for constrained multiobjective optimization, IEEE Trans. Cybern., 49(6) (2019) 2060-2072.

[5] C. A. C. Coello, G. B. Lamont, and D. A. V. Veldhuizen, Evolutionary Algorithms for Solving Multi-Objective Problems, second edition, Springer, US, 2007.

[6] E. Zitzler, K. Deb, and L. Thiele, Comparison of multiobjective evolutionary algorithms: empirical results, Evolutionary Computation, 8(2) (2000) 173-195.

[7] A. Trivedi, D. Srinivasan, K. Sanyal, and A. Ghosh, A survey of multiobjective evolutionary algorithms based on decomposition, IEEE Trans. Evol. Comput., 21(3) (2017) 440-462.

[8] X. Cai, M. Hu, D. Gong, Y. Guo, Y. Zhang, Z. Fan, and Y. Huang. A decomposition-based coevolutionary multiobjective local search for combinatorial multiobjective optimization, Swarm and Evolutionary Computation, 49 (2019) 178-193.

[9] H. Zhang, J. Sun, T. Liu, K. Zhang, and Q. Zhang, Balancing exploration and exploitation in multiobjective evolutionary optimization, Information Sciences, 497 (2019) 129-148.

[10] F. Wang, Y. Li, H. Zhang, T. Hu, and X. Shen, An adaptive weight vector guided evolutionary algorithm for preference-based multiobjective optimization, Swarm and Evolutionary Computation, 49 (2019) 220-233.

[11] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput., 6(2) (2002) 182-197.

[12] K. Deb, and H. Jain, An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, part I: solving problems with box constraints, IEEE Trans. Evol. Comput., 18(4) (2014) 577-601.

[13] S. Jiang, and S. Yang, A strength Pareto evolutionary algorithm based on reference direction for multiobjective and many-objective optimization, IEEE Trans. Evol. Comput., 21(3) (2017) 329-346.

[14] Y. Song, X. Ma, X. Li, L. Xing, and P. Wang, Learning-guided nondominated sorting genetic algorithm II for multi-objective satellite range scheduling problem, Swarm and Evolutionary Computation, 49(2019) 194-205.

[15] W. Hong, K. Tang, A. Zhou, H. Ishibuchi, X. Yao, A scalable indicator-based evolutionary algorithm for large-scale multi-objective optimization, IEEE Trans. Evol. Comput., 23(3) (2019) 525-537.

[16] T. Pamulapati, R. Mallipeddi, and P. N. Suganthan, I_{SDE}^{+} - An indicator for multi and many-objective optimization, IEEE Trans. Evol. Comput., 23(2) (2019) 346-352.

[17] Y. Tian, R. Cheng, X. Zhang, F. Cheng, and Y. Jin, An indicator-based multiobjective evolutionary algorithm with reference point adaptation for better versatility, IEEE Trans. Evol. Comput., 22(4) (2018) 609-622.

[18] H. Chen, Y. Tian, W. Pedrycz, G. Wu, R. Wang and L. Wang, Hyperplane assisted evolutionary algorithm for many-objective optimization problems, IEEE Trans. Cybern., 50(7) (2020) 3367-3380.

[19] Q. Zhang, and H. Li, MOEA/D: A multiobjective evolutionary algorithm based on decomposition, IEEE Trans. Evol. Comput., 11(6) (2007) 712-731.

[20] X. Ma, Q. Zhang, G. Tian, J. Yang, and Z. Zhu, On Tchebycheff decomposition approaches for multiobjective evolutionary optimization, IEEE Trans. Evol. Comput., 22(2) (2018) 226-244.

[21] L. Chen, H.-L. Liu, K. C. Tan, Y.-M. Cheung, Y. Wang, Evolutionary many-objective algorithm using decomposition-based dominance relationship, IEEE Trans. Cybern., 49(12) (2019) 4129-4139.

[22] D. Han, W. Du, W. Du, Y. Chu, and C. Wu, An adaptive decomposition-based evolutionary algorithm for many-objective optimization, Information Sciences, 491 (2019) 204-222.

[23] X. Ma, Y. Qi, L. Li, F. Liu, L. Jiao, and J. Wu, MOEA/D with uniform decomposition measurement for many-objective problems, Soft Comput., 18(12) (2014) 2541-2564.

[24] Y. Tan, Y. Jiao, H. Li, and X. Wang, MOEA/D + uniform design: A new version of MOEA/D for optimization problems with many objectives, Comput. Oper. Res., 40(6) (2013) 1648-1660.

[25] L. Wang, Q. Zhang, A. Zhou, M. Gong, and L. Jiao, Constrained subproblems in a decomposition-based multiobjective evolutionary algorithm, IEEE Trans. Evol. Comput., 20(3) (2016) 475-480.

[26] H. Liu, F. Gu, and Q. Zhang, Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems, IEEE Trans. Evol. Comput., 18(3) (2014) 450–455.

[27] H. Xu, W. Zeng, D. Zhang, and X. Zeng, MOEA/HD: A multiobjective evolutionary algorithm based on hierarchical decomposition, IEEE Trans. Cybern., 49(2) (2019) 517-526.

[28] M. Wu, K. Li, S. Kwong, and Q. Zhang, Evolutionary many-objective optimization based on adversarial decomposition," IEEE Trans. Cybern., 50(2) (2020) 753-764.

[29] Q. Zhang, W. Liu, and H. Li, The performance of a new version of MOEA/D on CEC09 unconstrained MOP test instances, in Proc. IEEE Congr. Evol. Comput., (2009) 203-208.

[30] A. Zhou, and Q. Zhang, Are all the subproblems equally important? Resource allocation in decomposition-based multiobjective evolutionary algorithms, IEEE Trans. Evol. Comput., 20(1) (2016) 52-64.

[31] Q. Lin, G. Jin, Y. Ma, K. C. W, C. A. C. Coello, J. Li, J. Chen and J. Zhang, A diversity-enhanced resource allocation strategy for decomposition-based multiobjective evolutionary algorithm, IEEE Trans. Cybern., 48(8) (2018) 2388-2401.

[32] H. Chen, G. Wu, W. Pedrycz, P. N. Suganthan, L. Xing and X. Zhu, An adaptive resource allocation strategy for objective space partition-based multiobjective optimization, IEEE Transactions on Systems, Man, and Cybernetics: Systems, doi: 10.1109/TSMC.2019.2898456.

[33] K. Deb, and R. B. Agrawal, Simulated binary crossover for continuous search space, Complex System, 9 (1995) 115-148.

[34] S. Das, and P. N. Suganthan, Differential Evolution: A survey of the state-of-the-art, 15(1) (2011) 4-31.

[35] K. Deb, A. Anand, and D. Joshi, A computationally efficient evolutionary algorithm for real-parameter optimization, Evolut. Comput. 10 (2002) 371-395.

[36] K. Li, A. Fialho, S. Kwong, and Q. Zhang, Adaptive operator selection with bandits for a multiobjective evolutionary algorithm based on decomposition, IEEE Trans. Evol. Comput., 18(1) (2014) 114-130.

[37] C. Qian, Y. Yu, and Z. Zhou, An analysis on recombination in multi-objective evolutionary optimization, Artificial Intelligence, 204 (2013) 99-119.

[38] Z. Wang, Q. Zhang, A. Zhou, M. Gong, and L. Jiao, Adaptive replacement strategies for MOEA/D, IEEE Trans. Cybern., 46(2) (2016) 474-486.

[39] Q. Zhu, Q. Lin, Z. Du, Z. Liang, W. Wang, Z. Zhu, J. Chen, P. Huang, and Z. Ming, A novel adaptive hybrid crossover operator for multiobjective evolutionary algorithm, Information Science, 345 (2016) 177-198.

[40] H. Li and Q. Zhang, Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II, IEEE Trans. Evol. Comput., 13(2) (2009) 284-302.

[41] K. Li, Q. Zhang, S. Kwong, M. Li, and R. Wang, Stable matching based selection in evolutionary multiobjective optimization, IEEE Trans. Evol. Comput., 18(6) (2014) 909-923.

[42] K. Li, S. Kwong, Q. Zhang, and K. Deb, Interrelationship-based selection for decomposition multiobjective optimization, IEEE Trans.Cybern., 45(10) (2015) 2076-2088.

[43] S. Zhao, P. N. Suganthan, and Q. Zhang, Decomposition-based multiobjective evolutionary algorithm with an ensemble of neighborhood sizes, IEEE Trans. Evol. Comput., 16(3) (2012) 442-446.

[44] M. Wu, K. Li, S. Kwong, Y. Zhou, and Q. Zhang, Matching-based selection with incomplete lists for decomposition multiobjective optimization, IEEE Trans. Evol. Comput., 21(4) (2017) 554-568.

[45] Q. Lin, Z. Liu and Q. Yan, Adaptive composite operator selection and parameter control for multiobjective evolutionary algorithm, Information Sciences, 339 (2015) 332-352.

[46] J. Whitacre, T. Pham, and R. Sarker, Use of statistical outlier detection method in adaptive evolutionary algorithms, in Proc. Annu. Conf. Genet. Evol. Comput., (2006) 1345-1352.

[47] A. Fialho, R. Ros, M. Schoenauer, and M. Sebag, Comparison-based adaptive strategy selection in differential evolution, in Proc. 11th Int. Conf. Parallel Problem Solving From Nature, (2010) 194-203.

[48] J. Maturana, F. Lardeux, and F. Saubion, Autonomous operator management for evolutionary algorithms, J. Heuristics, 16(6) (2010) 881-909.

[49] A. Fialho, M. Schoenauer, and M. Sebag, Toward comparison-based adaptive operator selection, in Proc. Annu. Conf. Genet. Evol. Comput., (2010) 767-774.

[50] D. E. Goldberg, Probability matching, the magnitude of reinforcement, and classifier system bidding, Mach. Learning, 5(4) (1990) 407-425.

[51] D. Thierens, An adaptive pursuit strategy for allocating operator probabilities, in Proc. Annu. Conf. Genet. Evol. Comput., (2005) 1539-1546.

[52] S. M. Venske, R. A. Gonçalves, and M. R. Delgado, ADEMO/D: Multiobjective optimization by an adaptive differential evolution algorithm, Neurocomputing, 127 (2014) 65-77.

[53] L. Tang, X. Wang, and Z. Dong, Adaptive multiobjective differential evolution with reference axis vicinity mechanism, IEEE Trans. Cybern., 49(9) (2019) 3571-3585.

[54] V. L. Huang, P. N. Suganthan, K. Qin, and S. Baskar, Differential evolution with external archive and harmonic distance-based diversity measure, (2008) https://www.researchgate.net/publication/228967624.

[55] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari, Multiobjective optimization test instances for the CEC 2009 special session and competition, Univ. Essex and Nanyang Technological University, Essex, U.K./Singapore, Tech. Rep. CES-487, (2008).

[56] P. Bosman, and D. Thierens, The balance between proximity and diversity in multiobjective evolutionary algorithms, IEEE Trans. Evol. Comput., 7 (2) (2003) 174-188.

[57] E. Zitzler and L. Thiele, Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach, IEEE Trans. Evol. Comput., 3(4) (1999) 257-271.

[58] Wu Lin, Qiuzhen Lin, Zexuan Zhu, Jianqiang Li, Jianyong Chen, and Zhong Ming, Evolutionary search with multiple utopian reference points in decomposition-based multiobjective optimization", Complexity, (2019) 1-22.

[59] S. Liu, Q. Lin, K. Wong, L. Ma, C. A. C. Coello, and D. Gong, A novel multi-objective evolutionary algorithm with dynamic decomposition strategy, Swarm and Evolutionary Computation, 48 (2019) 182-200.

[60] H. Li, K. Deb, Q. Zhang, P. Suganthan, and L. Chen, Comparison between MOEA/D and NSGA-III on a set of novel many and multi-objective benchmark problems with challenging difficulties, Swarm and Evolutionary Computation, 46 (2019) 104-117.