Complexity Theory 2010
(Home work 2)

July 5, 2012

- Due on Monday, July 24, before 10 a.m.
- Late home works will not be accepted.
- Please give precise arguments for all statements that you write.
- Please do not hesitate to contact me if you do not understand the problems.
- Collaboration is discouraged, but not prohibited. It is recommended that you try to solve the problems on your own. You can discuss the questions with your colleagues but you should not copy solutions. Always write down your own answers. If copying is detected that may immediately lead to a grade less than 7. (**This would be followed strictly**)
- Credits would be given to partial solutions also.
- The answers should be typed or written clearly and a hard copy is to be submitted.

1. Describe the error in the following proof for $\mathbf{P} \neq \mathbf{NP}$.

   We assume $\mathbf{P} = \mathbf{NP}$ and obtain a contradiction. If $\mathbf{P} = \mathbf{NP}$, then $\mathsf{SAT} \in \mathbf{P}$ and so for some k, $\mathsf{SAT} \in \mathbf{DTIME}(n^k)$. As every language in $\mathbf{NP}$ is polynomial time reducible to SAT, we have $\mathbf{NP} \subseteq \mathbf{DTIME}(n^k)$. Therefore $\mathbf{P} \subseteq \mathbf{DTIME}(n^k)$. But, by the time hierarchy theorem $\mathbf{DTIME}(n^{k+1})$ contains a language that is not in $\mathbf{DTIME}(n^k)$ which contradicts $\mathbf{P} \subseteq \mathbf{DTIME}(n^k)$. Hence $\mathbf{P} \neq \mathbf{NP}$.

2. Define the unique satisfiability problem as follows:

   $$\mathsf{USAT} = \{\phi : \phi \text{ is a CNF formula that has a single satisfying assignment}\}.$$

   Show that $\mathsf{USAT} \in \mathbf{P}^{\mathsf{SAT}}$.

3. Suppose A and B be two oracles, one of them is an oracle for TQBF but you don't know which. Give an algorithm that has access to both A and B and that is guaranteed to solve TQBF in polynomial time.

4. A complexity class $\mathbf{X}$ is called *polynomial time downward Karp reducible* if for all languages L_1, L_2, if $L_1 \leq_p L_2$ and $L_2 \in \mathbf{X}$ then $L_1 \in \mathbf{X}$. Let $\mathbf{E} = \mathbf{DTIME}(2^{O(n)})$. Show that $\mathbf{E}$ is not polynomial time downward Karp reducible.
(Hint:Recall, for any language L we defined a language L_{pad} by padding appropriate amount of ones to the end of every string $x \in L$. Such a construction may help.)

5. Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$

6. A language L is called *polynomial time downward self reducible* if there exist a polynomial time oracle TM M such that M^L decides L, but a restriction on M is that if M is running on input x then it can only query its oracle with strings whose lengths are less than $|x|$.

   (a) Show that SAT and TQBF are polynomial time downward self reducible.

   (b) If L is polynomial time downward self reducible then $L \in \mathbf{PSPACE}$.