Design and Analysis of Algorithms 2008
(Home work 2)

September 23, 2008

- Due on Thursday, October 2, before 8 a.m.

- You have a total of 7 late days in the whole term.

- Please give precise arguments for all statements that you write.

- Please do not hesitate to contact me if you do not understand the problems.

- Each problem in this homework bear 10 points.

- Collaboration is encouraged, but you should not copy solutions, but write down your own answers. If copying is detected that may immediately lead to a grade less than 7. (**This would be followed strictly**)

- Credits would be given to partial solutions also.

- The answers should be typed or written clearly and a hard copy is to be submitted.

1. Here is a list of functions in one variable, n.

$$\frac{17n^5}{100}, 8, \frac{n^2}{n+1}, 2^{6\lg n}, n^2 \lg n, n^2 + 27n, 2^n + n$$

Place these functions in a list such that $f(n)$ is any function in the list and $g(n)$ is in right of it then $f(n) = O(g(n))$. You do not need to prove your answer.

2. Prove or disprove the following:

   (a) If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.
   (b) $(n + a)^b = \Theta(n^b)$, for any real constants a and b, where $b > 0$.

3. Solve the following recurrence relations:

   (a) $f(1) = 1$, $f(2) = 1$, $f(n) = f(n-1) + f(n-2)$ for $n > 2$.

(b) $f(1) = f(2) = 1$, and $f(n) = 5f(n-1) - 6f(n-2)$ for $n \geq 3$.

4. Find the number of n digit words generated from the alphabet $\{0, 1, 2, 3\}$ in each of which the number of zeros is even.

5. Give asymptotic tight bounds for $T(n)$ for the following cases. Assume that $T(n)$ is a constant if $n \leq 2$.

   (a) $T(n) = 2T(n/2) + n^3$.
   (b) $T(n) = 7T(n/3) + n^2$.
   (c) $T(n) = 3T(n/2) + n \lg n$.