Design and Analysis of Algorithms 2009
(Home work 5)

November 26, 2009

- Due on December 4, before 10 a.m.
- Late home works will not be accepted.
- Please give precise arguments for all statements that you write.
- Please do not hesitate to contact me if you do not understand the problems.
- Collaboration is discouraged, but not prohibited. It is recommended that you try to solve the problems on your own. You can discuss the questions with your colleagues but you should not copy solutions. Always write down your own answers. If copying is detected that may immediately lead to a grade less than 7. (**This would be followed strictly**)
- Credits would be given to partial solutions also.
- When you write an algorithm, you should briefly discuss the main idea of your algorithm, then write a pseudo code, argue about its correctness and state and prove the running time of your algorithm.
- The answers should be typed or written clearly and a hard copy is to be submitted.

1. [**15 points**] Draw a graph with at least two negative weight edges for which Dijkstra's algorithm produces the wrong answer. Draw another graph with at least one negative weight edge for which Dijkstra's algorithm produces the correct answer. (Try to draw small graphs and also tell the source node so that your solution can be easily verified.)

2. [**15 points**] Show how to find the maximum spanning tree of a given graph, i.e., a spanning tree with the largest total weight.

3. [**15 points**] Describe a linear time algorithm to find shortest paths in directed acyclic graphs.

4. [**15 points**] Recall the algorithm for finding the longest common subsequence studied in class. Show how to reconstruct a LCS from a completed table c and the original sequence $X =< x_1, \ldots, x_m >$ and $Y =< y_1, \ldots, y_n >$ in $O(m + n)$ time without using the table b.

5. [**15 points**] A subsequence is palindromic if it is the same whether read left to right or right to left. For instance, the sequence

$$A, C, G, T, G, T, C, A, A, A, A, T, C, G$$

has many palindromic subsequences, including A, C, G, C, A and A, A, A, A. Devise an algorithm to find the length of the longest common palindromic subsequence of a given sequence. Your algorithm should run in $O(n^2)$ time.