Design and Analysis of Algorithms 2011
(Home work 6)

November 14, 2011

- Due on November 23, before 10 a.m.
- Late home works will not be accepted.
- Please give precise arguments for all statements that you write.
- Please do not hesitate to contact me if you do not understand the problems.
- Collaboration is discouraged, but not prohibited. It is recommended that you try to solve the problems on your own. You can discuss the questions with your colleagues but you should not copy solutions. Always write down your own answers. If copying is detected that may immediately lead to a grade less than 7. (**This would be followed strictly**)
- Credits would be given to partial solutions also.
- When you write an algorithm, you should briefly discuss the main idea of your algorithm, then write a pseudo code, argue about its correctness and state and prove the running time of your algorithm.
- The answers should be typed or written clearly and a hard copy is to be submitted.

1. [**20 points**] Draw a graph with at least two negative weight edges for which Dijkstra's algorithm produces the wrong answer. Draw another graph with at least one negative weight edge for which Dijkstra's algorithm produces the correct answer. (Try to draw small graphs and also tell the source node so that your solution can be easily verified.)
2. [**25 points**] Describe a linear time algorithm to find shortest paths in directed acyclic graphs. Prove the correctness of the algorithm.
3. [**30 points**] Suppose we have a directed graph $G = (V; E)$ describing a computer network, where vertices correspond to hosts or routers and edges correspond to net-

work links. Also assume that for each network link $(u, v) \in E$, we are given a measure $r(u, v)$ of the reliability of this link: specifically, $r(u, v) =$ the probability that a packet sent across the link (u, v) will not be lost while it is transiting that link. You may assume that these probabilities are independent. So, if we have a path $(v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k)$, the probability that a packet sent along this path makes it from v_0 to v_k successfully is given by $r(v_0, v_1) \times r(v_1, v_2) \times \cdots r(v_{k-1}, v_k)$. Given the graph G, the reliability measure $r(.,.)$, and vertices $s, t \in V$, your job is to find a path from s to t of maximum reliability. Design an efficient algorithm to solve this problem.

4. [**25 points**] Let A be a $m \times n$ matrix and b a m dimensional vector such that $Ax \leq b$ represents a set of difference constraints. Give an efficient algorithm to find $x = (x_1, x_2, \ldots, x_n)$, such that

$$f(x) = \sum_{i=1}^{n} x_i$$

is maximized subject to the constraints $Ax \leq b$ and $x_i < 0$ for all $i = 1, 2, \ldots, n$.

5. [**25 points**] In the class we discussed algorithms to find the minimum spanning trees. Design an algorithm to find the maximum spanning tree, i.e., a spanning tree with maximum weight. Discuss about the correctness and running time of your algorithm.