Discrete Mathematics 2015
(Home work 2)

September 25, 2015

- Due on Wednesday, October 7, before 10 a.m.(You can submit in the class)
- Late home works will not be accepted.
- Please give precise arguments for all statements that you write.
- Please do not hesitate to contact me if you do not understand the problems.
- Collaboration is discouraged, but not prohibited. It is recommended that you try to solve the problems on your own. You can discuss the questions with your colleagues but you should not copy solutions. Always write down your own answers. If copying is detected that may immediately lead to a grade less than 7. (**This would be followed strictly**)
- Credits would be given to partial solutions also.
- The answers should be typed or written clearly and a hard copy is to be submitted.

1. [**5 points**] Prove that for any sets A, B, C, D if $(A \times B) \cap (C \times D) = \emptyset$, then either $A \cap C = \emptyset$ or $B \cap D = \emptyset$.
2. [**3 × 5 = 15 points**] Let R and S be arbitrary equivalence relations on a set X. Decide which of the following relations are necessarily equivalence relations (if yes, prove; if not, give a counterexample).
   (a) $R \cap S$
   (b) $R \cup S$
   (c) $R \setminus S$
3. [**10 points**] How many (binary) relations are possible on the two element set $\{0, 1\}$. Out of these how many are reflexive, symmetric, antisymmetric and transitive.

4. [**10 points**] We define composition of relations as we defined composition of functions in class, i.e., if R be a relation between the sets A and B and S a relation between B and C(we will also write this as $R : A \to B$ and $S : B \to C$, as we did for functions), we define $S \circ R$ as follows:

   For $a \in A$ and $c \in C$, $(a, c) \in S \circ R$ (or analogously $a(S \circ R)b$) if there exists a $b \in B$, such that $(a, b) \in R$ and $(b, c) \in R$.

   We can represent a relation S between two finite sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$ by a $n \times m$ matrix M_S, where $M_S(i, j) = 1$ if $(a_i, b_j) \in S$ and $M_S(i, j) = 0$ otherwise.

   A Boolean matrix is a matrix whose elements are from the set $\{0, 1\}$. If M_1, M_2 are two Boolean matrices of size $m \times n$ and $n \times p$ respectively, by $M_2 \otimes M_2$ we mean the Boolean product of M_1 and M_2. To compute $M_1 \otimes M_2$, we multiply M_1 and M_2 by the rules of ordinary matrix multiplication, with the difference that instead of addition we use the Boolean OR operation and instead of multiplication we use the Boolean AND operation.

   Let M_R and M_S be the matrix representation of two relations R and S. Prove that the matrix representation of $S \circ R$ is $M_R \otimes M_S$.

5. [**2 × 5 = 10 points**] Let $\sigma : S \to T$ and $\tau : T \to W$ be functions. Prove or disprove the following

   (a) If $\tau \circ \sigma$ is onto then both σ and τ are onto.

   (b) If $\tau \circ \sigma$ is one to one then both σ and τ are one to one.