Fundamentals of Algebra for Computer Science (Home work 1)

September 14, 2006

- Due on Wednesday, September 27, before 4 p.m.
- Late submissions will not be accepted.
- Please give precise arguments for all statements that you write.
- To disprove a fact it is enough to give a counter example.
- Please do not hesitate to contact me if you do not understand the problems.
- Each problem in this homework bear 5 points.

1. Given two sets A and B, their symmetric difference is defined as

$$A \triangle B = (A - B) \cup (B - A)$$

   Prove that $A \triangle B = (A \cup B) - (A \cap B)$

2. For the given set and relations below determine which define equvalence relations.

   (a) S be the set of all people in the world today, $a \sim b$ if a and b have an ancestor in common

   (b) S be the set of all people in the world today, $a \sim b$ if a lives within 100 Km of b.

   (c) S be the set of all straight lines in aplane, $a \sim b$ if a is parallel to b.

   (d) S be the set of all triangles and $a \sim b$ if a and b are similar.

   (e) S be the set of all real numbers, $a \sim b$ if $a = \pm b$.

3. Prove or disprove the following

   (a) If $\sigma \circ \tau$ is onto then both σ and τ are onto.

   (b) If $\sigma \circ \tau$ is one to one then both σ and τ are one to one.

4. If $(m, n) = 1$, given a and b, prove that there exists an x such that $x \equiv a \bmod m$ and $x \equiv b \bmod n$.

5. If $(a, n) = 1$, prove that one can find a $[b] \in \mathbb{Z}_n$ such that $[a][b] = [1]$.

6. Let S^* denote the power set of S. Show that S^* with $\triangle$ as the binary operation forms a group.

7. Let $(G, *)$ be a group such that $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$. Show that G must be abelian.

8. Show that if every element of the group G is its own inverse, then G is abelian.

9. If H and K are subgroups of G, show that $H \cap K$ is also a subgroup of G.

10. Let H and K be subgroups of an abelian group G. Define $HK = \{hk : h \in H, k \in K\}$. Is HK a subgroup of G?