Fundamentals of Algebra for Computer Science
(Home work 4)

November 25, 2006

- Due on December 5, midnight.
- Late submissions will not be accepted.
- Mail your answers to debrup.chakraborty@gmail.com
- Problems 1,2,3 and 4 each bear 5 points and problem 5 bears 10 points.

1. Prove that the center of a group is always a normal subgroup.
2. Prove that any field is an integral domain.
3. Let U and V be ideals in R and let $U+V = \{u+v : u \in U, v \in V\}$. Prove that $U+V$ is also an ideal.
4. If F be a field, prove that its only ideals are (0) and F itself.
5. Let R be a ring with unit element. Using the elements in R we define a new ring $\tilde{R}$ by the following:

$$a \oplus b = a + b + 1$$

$$a \odot b = ab + a + b$$

Where $a, b \in R$ and the addition and multiplication in the right hand side are those of R. Prove that $\tilde{R}$ is a ring under the properties defined. What acts as an unit element in $\tilde{R}$. Also show that R and $\tilde{R}$ are isomorphic.