# Fundamentals of Algebra for Computer Science (Practice Problems 1)

September 28, 2006

- You need not submit the answers to me.
- These problems would be discussed in class on 2nd October.
- I expect you to try each problem before you come to class on 2nd Oct. (Monday).

1. Prove that the set of $2 \times 2$ non singular matrices forms a group under matrix multiplication. Show that the set of real matrices whose determinant is zero forms a normal subgroup of the above group.

2. Let $G$ be a group and $A$ and $B$ subgroups of $G$. If $x, y \in G$, define $x \sim y$ if $y = axb$. Prove

   (a) The relation so defined is an equivalence relation.

   (b) The equivalence class of $x$ is $AxB = \{axb | a \in A, b \in B\}$

3. $G$ be a group with no non-trivial subgroups, show that $G$ must be finite and of prime order.

4. If $a \in G$, define $N(a) = \{x \in G | xa = ax\}$. Show that $N(a)$ is a subgroup of $G$.

5. Prove that the subgroup of a cyclic group is itself a cyclic group.

6. How many generators do a cyclic group of order $n$ has?

7. If $a \in G$ and $a^m = e$, prove that $o(a)|m$.

8. If $H$ is a subgroup of $G$ such that the product of two right cosets of $H$ in $G$ is again a right coset of $H$ in $G$, prove that $H$ is normal in $G$.

9. If $G$ is a group and $H$ is a subgroup of index 2 in $G$, prove that $H$ is a normal subgroup of $G$.

10. Let $G$ be any group, $g$ be a fixed element in $G$. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that $\phi$ is an isomorphism of $G$ onto $G$.

11. Let $G$ be the group of non-zero complex numbers under multiplication and let $N$ be the set of complex numbers of absolute value 1 (i.e., $a + ib \in N$ if $a^2 + b^2 = 1$). Show that $G|N$ is isomorphic to the group of all positive real numbers under multiplication.