

## Problem sheet

These problems are for your practice. You need not submit them.

- 16 people 4 each from Scotland, England, Ireland and Wales are present in a meeting. A committee of 6 people are to be selected among these 16 people. How many ways the selection can be made such that
  - Each country must be represented
  - No country can have more than 2 representatives.
- Three integers are selected from the integers  $1, 2, \dots, 1000$ . In how many ways they can be selected so that their sum is divisible by 3.
- Prove the following identities:
  - $\binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k}$
  - $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n \binom{n}{n} = 0$ .
  - $\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots + (n+1)\binom{n}{n} = 2^{n-1}(2+n)$ .
- Solve the following recurrence relations:
  - $a_0 = 1, a_1 = 2$ , and  $a_n = 4a_{n-2}$ , for  $n \geq 2$
  - $a_0 = 0, a_1 = a_2 = 1$ ,  $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ , for  $n \geq 3$
  - $F_0 = 2$ , and  $F_{n+1} = 4F_n + 5^n$ , for  $n > 0$ .
- There are  $n$  married couples attending a dance. How many ways are there to form  $n$  pairs for dancing if no wife should dance with her husband.
- How many permutations of the numbers  $1, 2, 3, \dots, 10$  exist that map no even number to itself.
- Find the number of integers between 1 and 1000 which are divisible by none of 3, 7 and 11.
- Among all  $n$  digit integers, how many of them contain the digits 0 and 1 but not the digits 8 and 9.