Heptagonal Flexagons

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Abstract

Maps and cutouts for a variety of flexagons are presented, emphasizing those which can be cut out, mostly from single sheets of paper. Since TeX may not align front and back images, and in any event if cutting up the booklet is not desired, the .eps files can be printed directly to get sheets suitable for cutting. In the same spirit, only those sheets which are going to be used right away need be printed.

Contents

1 Introduction

Flexagons can become fairly complicated. The ones based on triangles are most conveniently made from long strips of paper; a roll of adding machine or calculator tape is ideal for this purpose given its convenient width. Crooked strips can be gotten by gluing faces together, or just cutting out segments and then joining them together. Leaving one extra triangle in each segment for overlapping and later gluing leads to efficient constructions.

Coloring the triangles is another problem, which can be done with crayons or markers once it is known which colors ought to be used. Aside from copying an already existent design, this is best done by drawing the Tukey triangles and then lettering or numbering the triangles in the strip. That information is sufficient to fold up the strip, since pairs of consecutive numbers are to be hidden by folding them together. Painting can be done before folding by following a color code for the numbers, or after the folding is done, when the faces can be painted wholesale, or even embellished with designs.

Other flexagons, even the ones folded from "straight" strips, require a higher degree of preparation, although it is relatively easy to assemble a collection of primitive components which later can be glued together according to the necessities of the individual flexagon.

2 First Level Heptagonal Flexagon

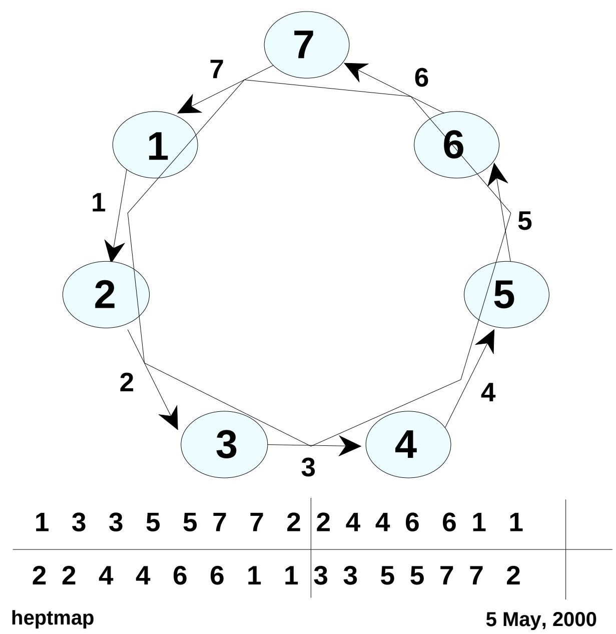

Figure 1: The first level heptagonal flexagon has 7 vertices.

The strip of polygons from which a flexagon is assembled embeds neatly into the plane for triangles through hexagons, although entire polygons will occasionally overlap when the rules of connection demand it. But the first and third of three consecutive heptagons will overlap slightly when successors joined by adjacent edges, which is the rule for normal flexagons.

If a strip is prepared in advance with the intention of folding it later on to get the flexagon, the overlapping heptagons can be trimmed slightly without spoiling the effect. Nevertheless in progressing onwards to octagons, nonagons, and so on, the overlap becomes increasingly severe, and it will probably be a good idea to prepare the polygons separately, or in sparser strips, joining them in later stages of the assembly.

Once folding the strip of polygons begins, the higher order polygons increasingly resemble circles, the hinges between adjacent polygons gradually rotating around the circumference. The result is a four bladed rosette formed from two sectors and four pats, in which the transferrence of subpats from one pat to another procedes with exceptional ease and clarity. However in compensation the flexagon operation of “rotation” requires increasingly more versatile gymnastics. More and more tubulations also become possible as the series progresses.

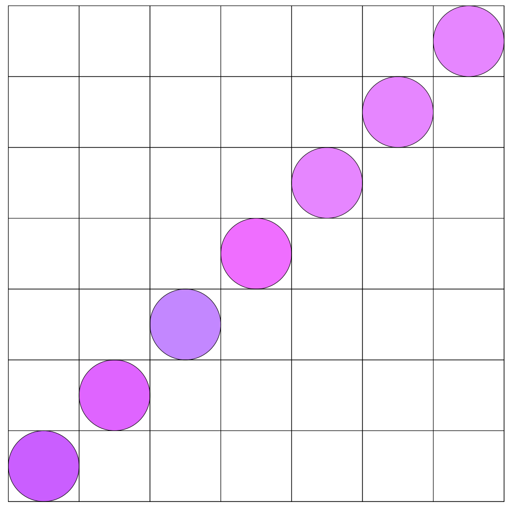

Figure 2: Permutation of the heptagons along the strip for a first level heptagonal flexagon.

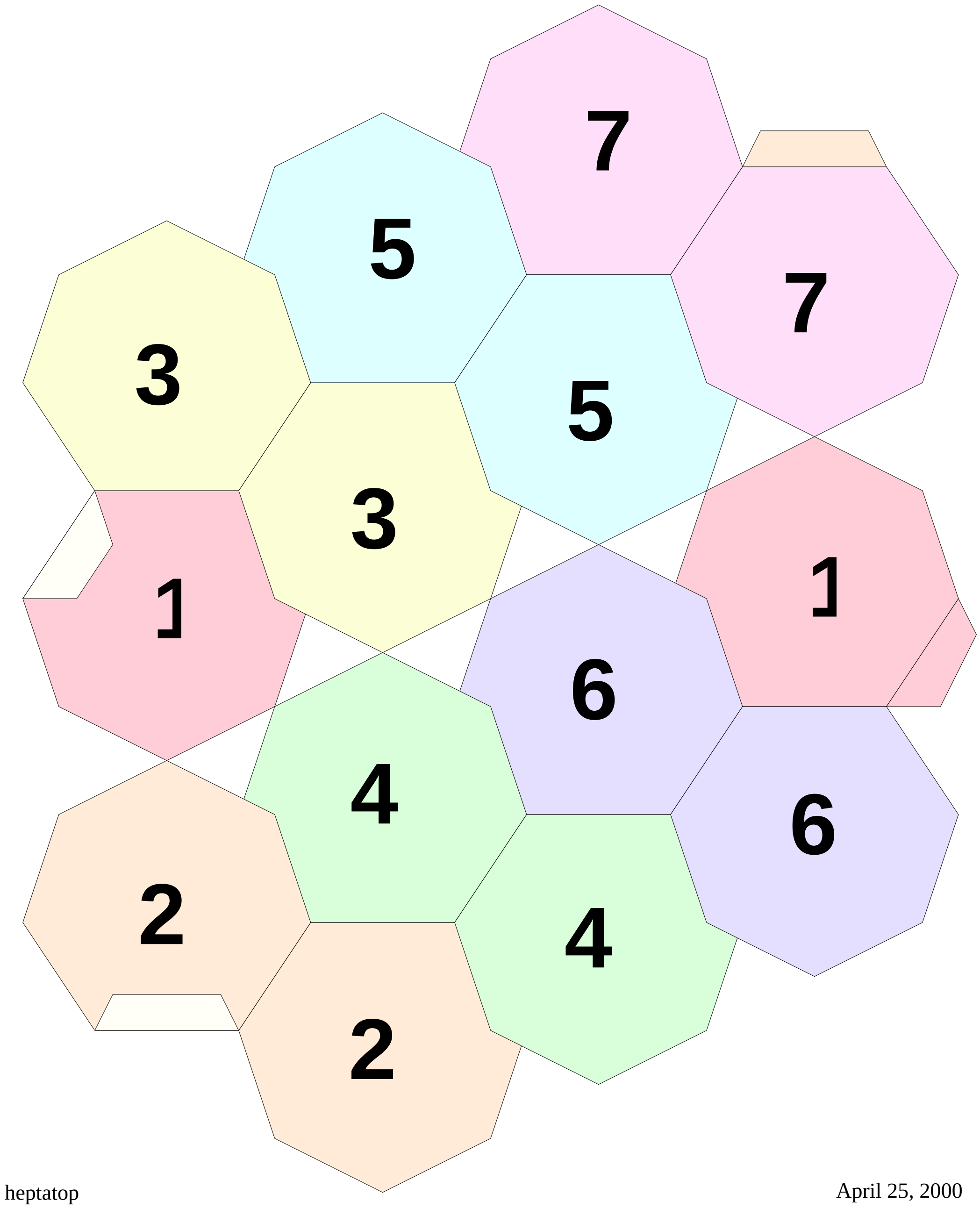

Figure 3: Top side of a heptagonal flexagon consisting of a single cycle.

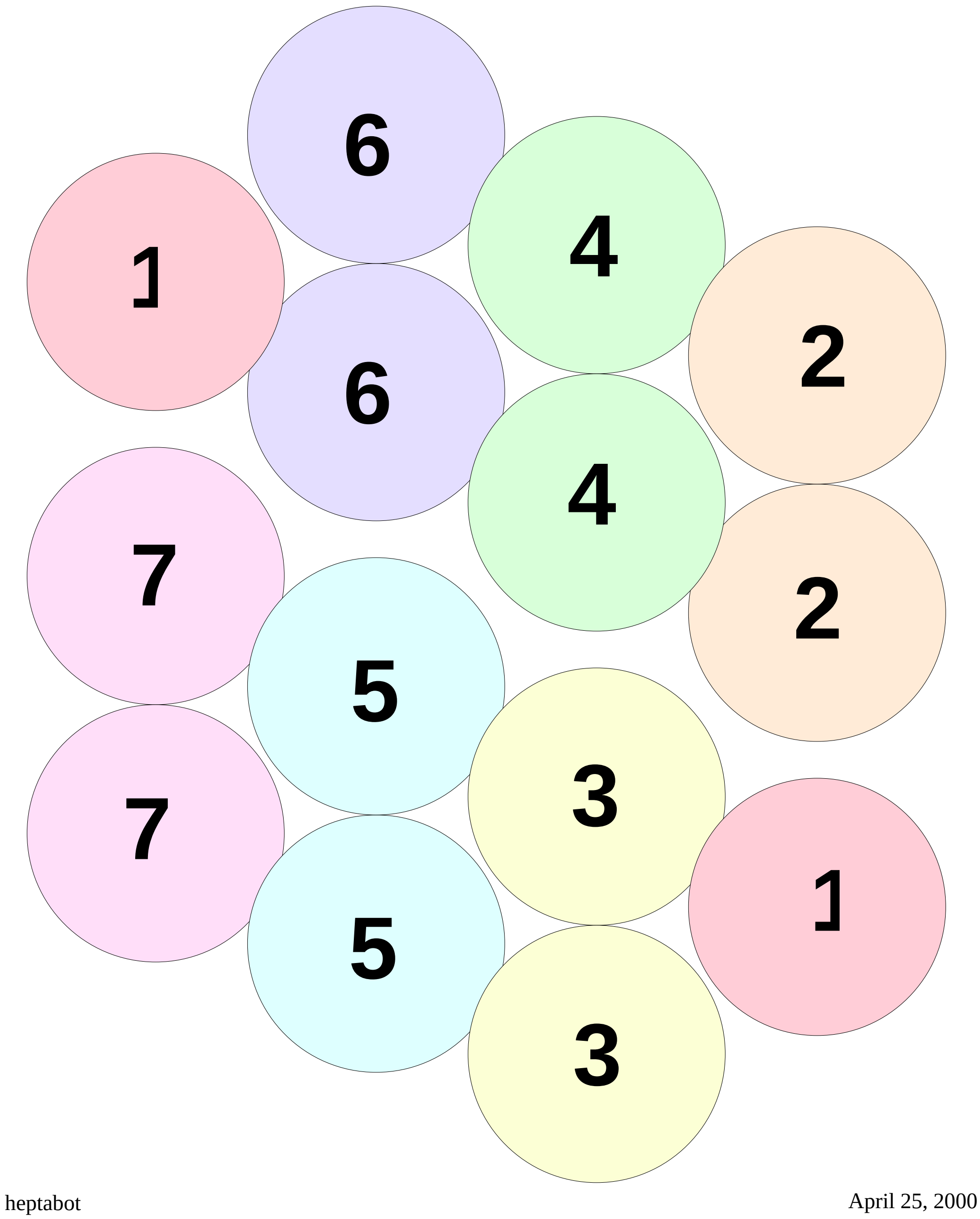

Figure 4: Bottom side of a heptagonal flexagon consisting of a single cycle.

3 Binary Heptagonal Flexagon

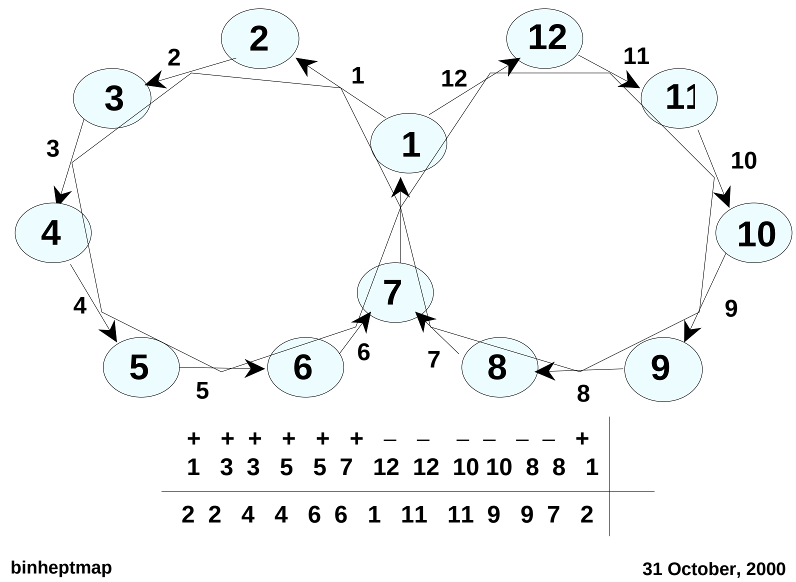

Figure 5: The binary heptagonal flexagon has twelve vertices.

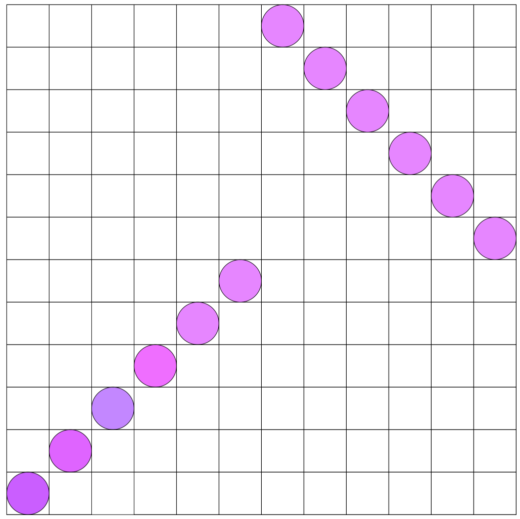

Figure 6: Permutation of the heptagons along the strip for a binary heptagonal flexagon.

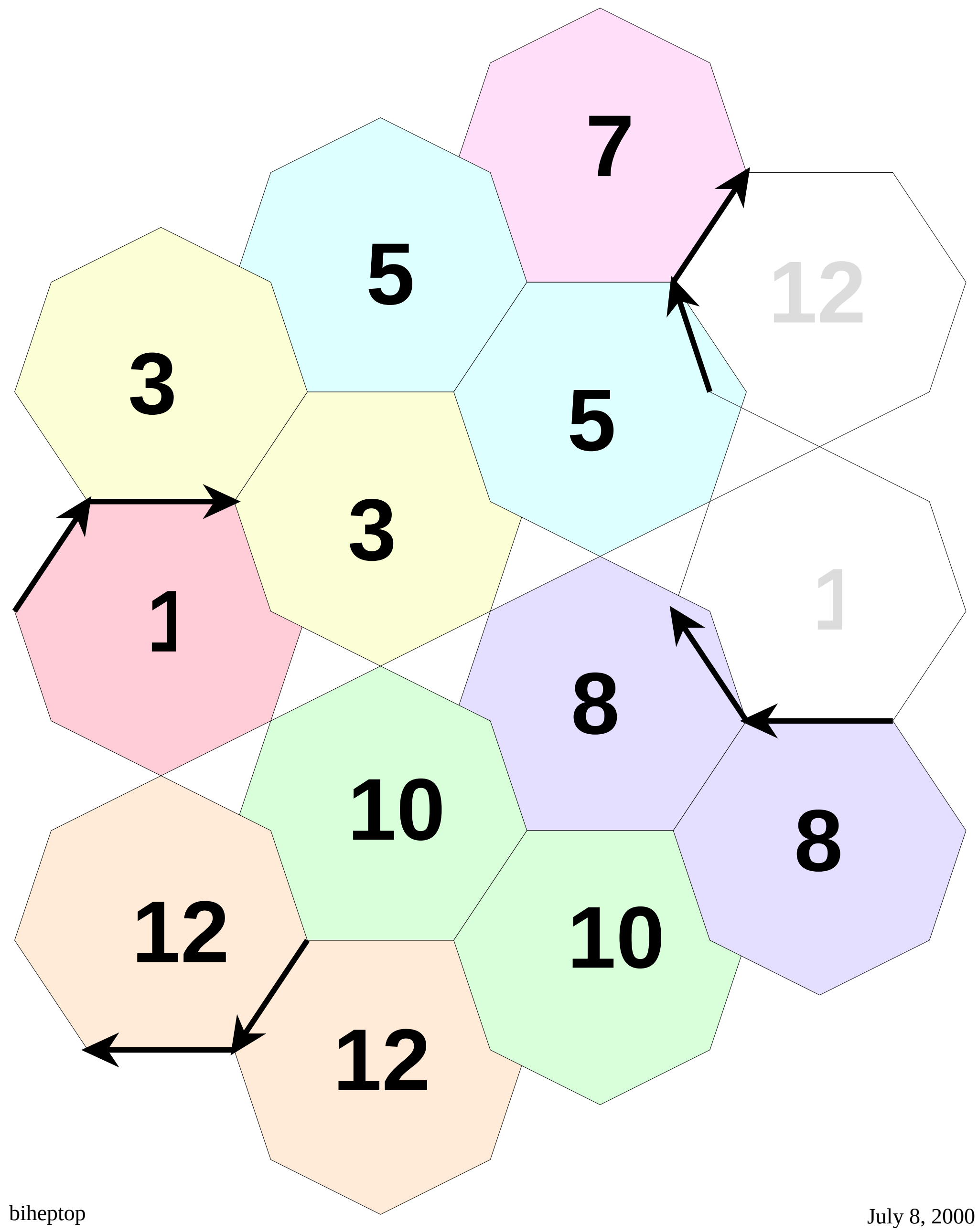

Figure 7: Top side of a binary heptagonal flexagon. First of two sectors.

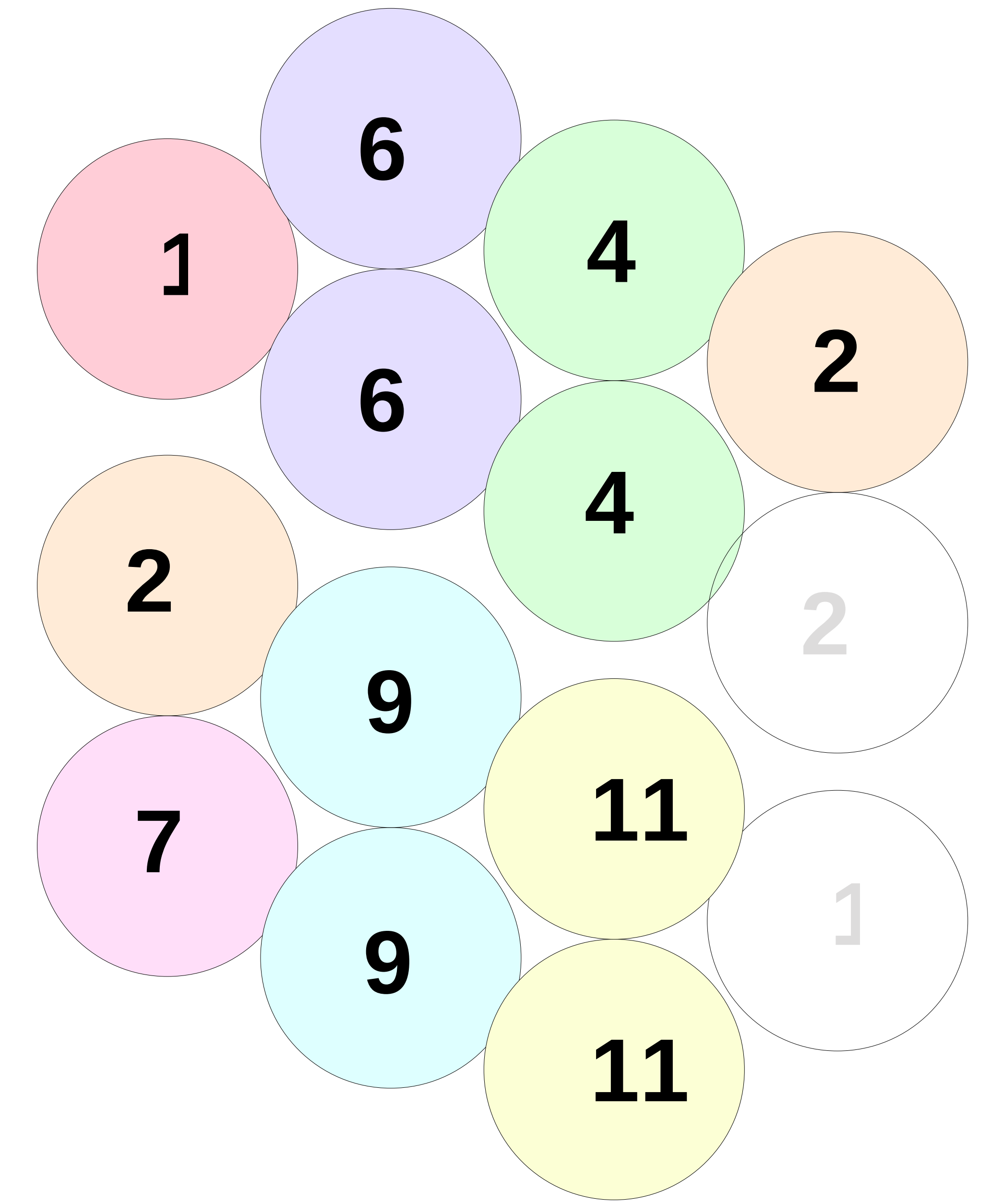

Figure 8: Bottom side of a binary heptagonal flexagon. First of two sectors.

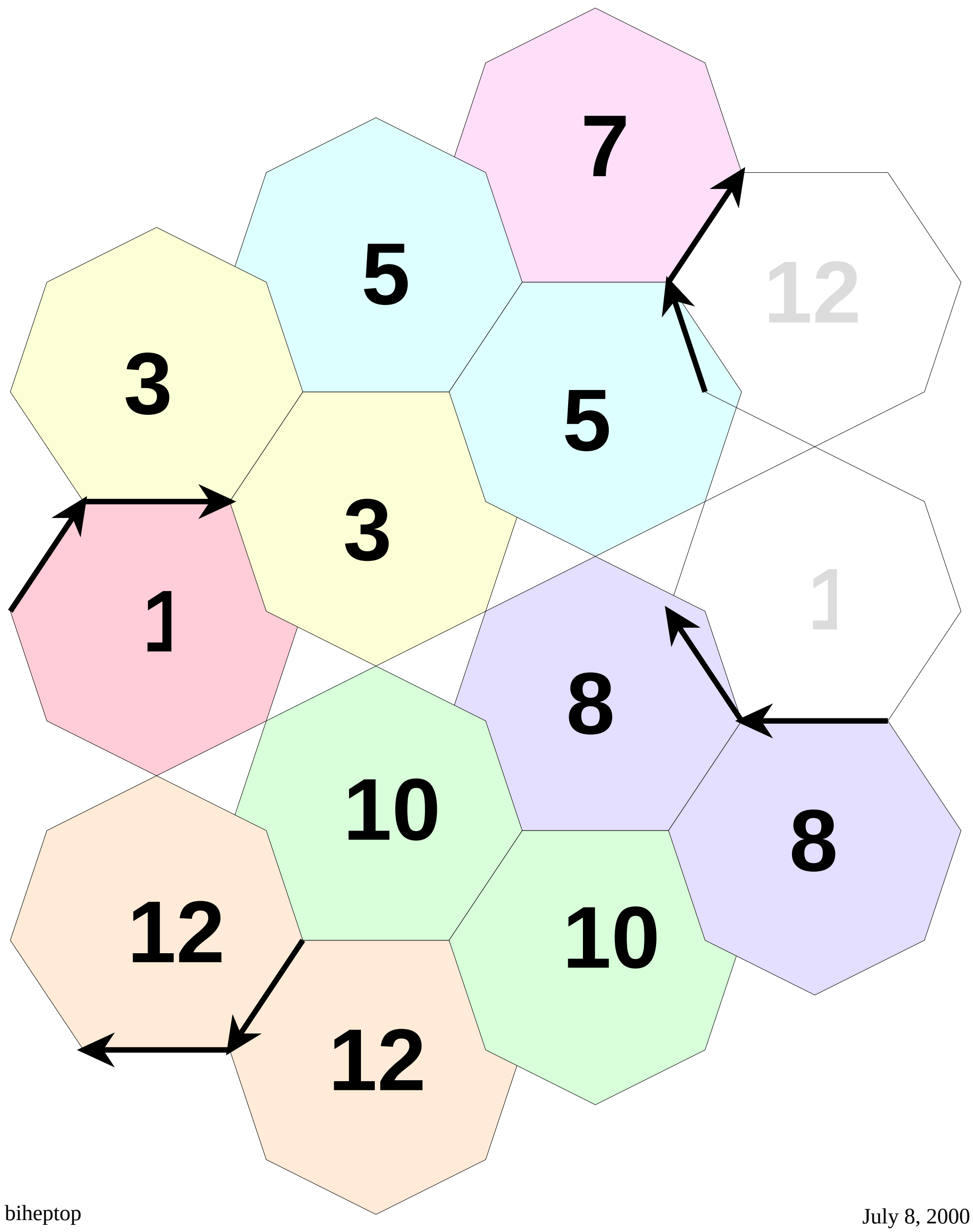

Figure 9: Top side of a binary heptagonal flexagon. Second sector.

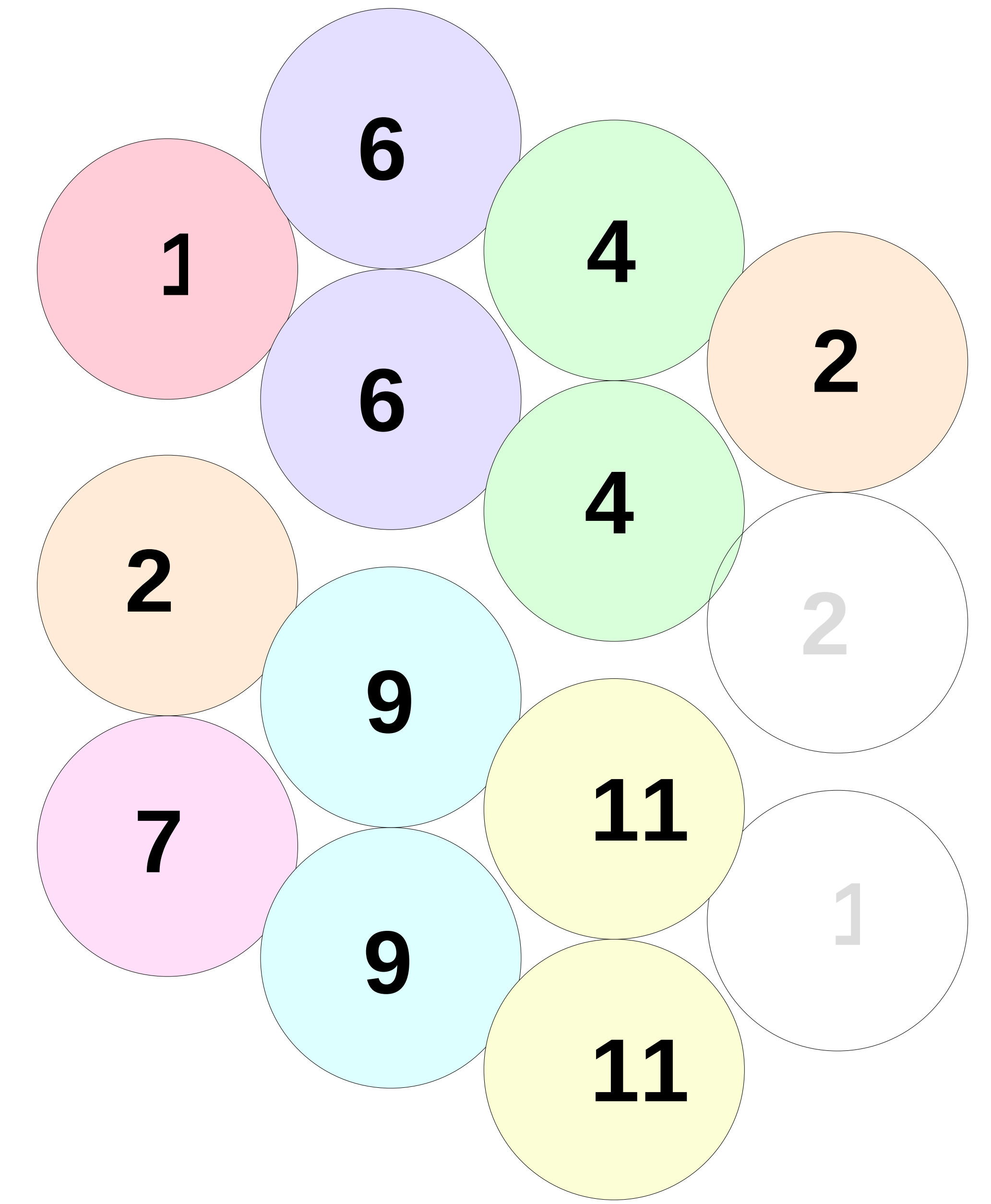

Figure 10: Bottom side of a binary heptagonal flexagon. Second sector.