Hexagon Flexagons

Harold V. McIntosh
Departamento de Aplicación de Microcomputadoras,
Instituto de Ciencias, Universidad Autónoma de Puebla,
Apartado postal 461, 72000 Puebla, Puebla, México.

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Abstract

Maps and cutouts for a variety of flexagons are presented, emphasizing those which can be cut out, mostly from single sheets of paper. Since TeX may not align front and back images, and in any event if cutting up the booklet is not desired, the .eps files can be printed directly to get sheets suitable for cutting. In the same spirit, only those sheets which are going to be used right away need be printed.

Contents

1 Introduction

Flexagons can become fairly complicated. The ones based on triangles are most conveniently made from long strips of paper; a roll of adding machine or calculator tape is ideal for this purpose given its convenient width. Crooked strips can be gotten by gluing faces together, or just cutting out segments and then joining them together. Leaving one extra triangle in each segment for overlapping and later gluing leads to efficient constructions.

Coloring the triangles is another problem, which can be done with crayons or markers once it is known which colors ought to be used. Aside from copying an already existent design, this is best done by drawing the Tukey triangles and then lettering or numbering the triangles in the strip. That information is sufficient to fold up the strip, since pairs of consecutive numbers are to be hidden by folding them together. Painting can be done before folding by following a color code for the numbers, or after the folding is done, when the faces can be painted wholesale, or even embellished with designs.

Other flexagons, even the ones folded from "straight" strips, require a higher degree of preparation, although it is relatively easy to assemble a collection of primitive components which later can be glued together according to the necessities of the individual flexagon.

2 First Level Hexagonal Flexagon

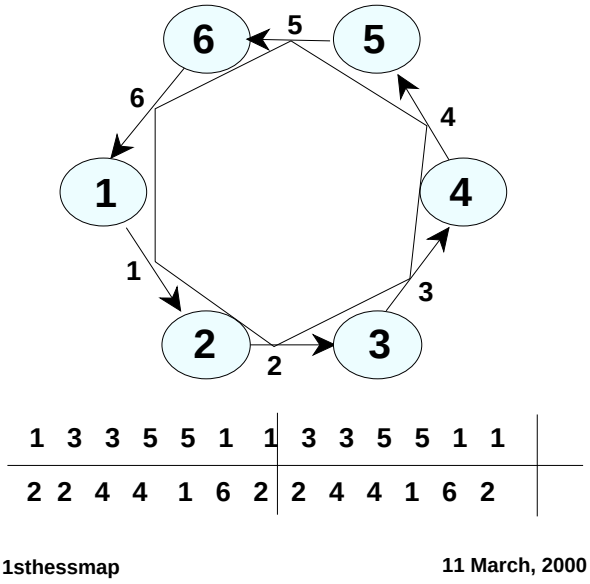

Figure 1: The first level hexagonal flexagon has 6 vertices.

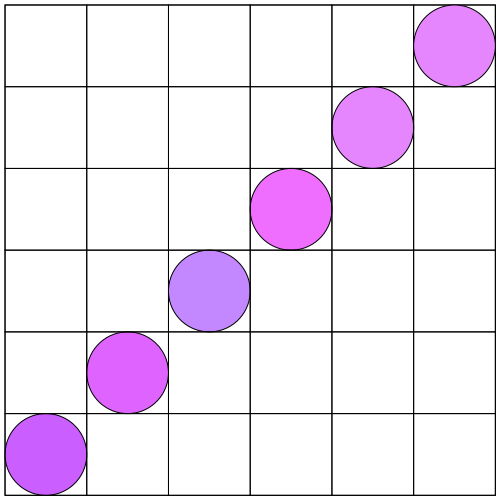

Figure 2: Permutation of the hexagons along the strip for a first level hexagonal flexagon.

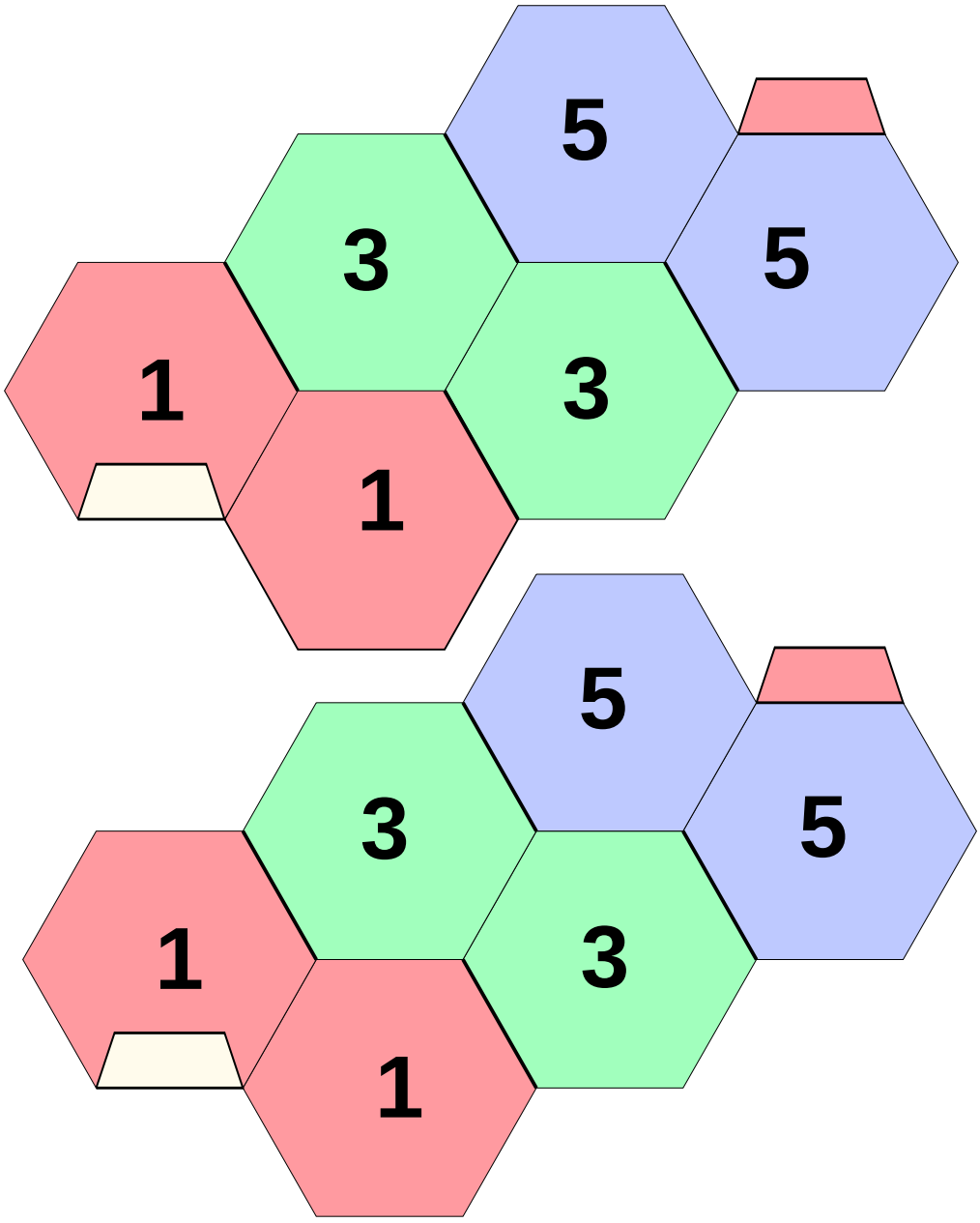

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Figure 3: Top side of a hexagonal flexagon consisting of a single cycle.

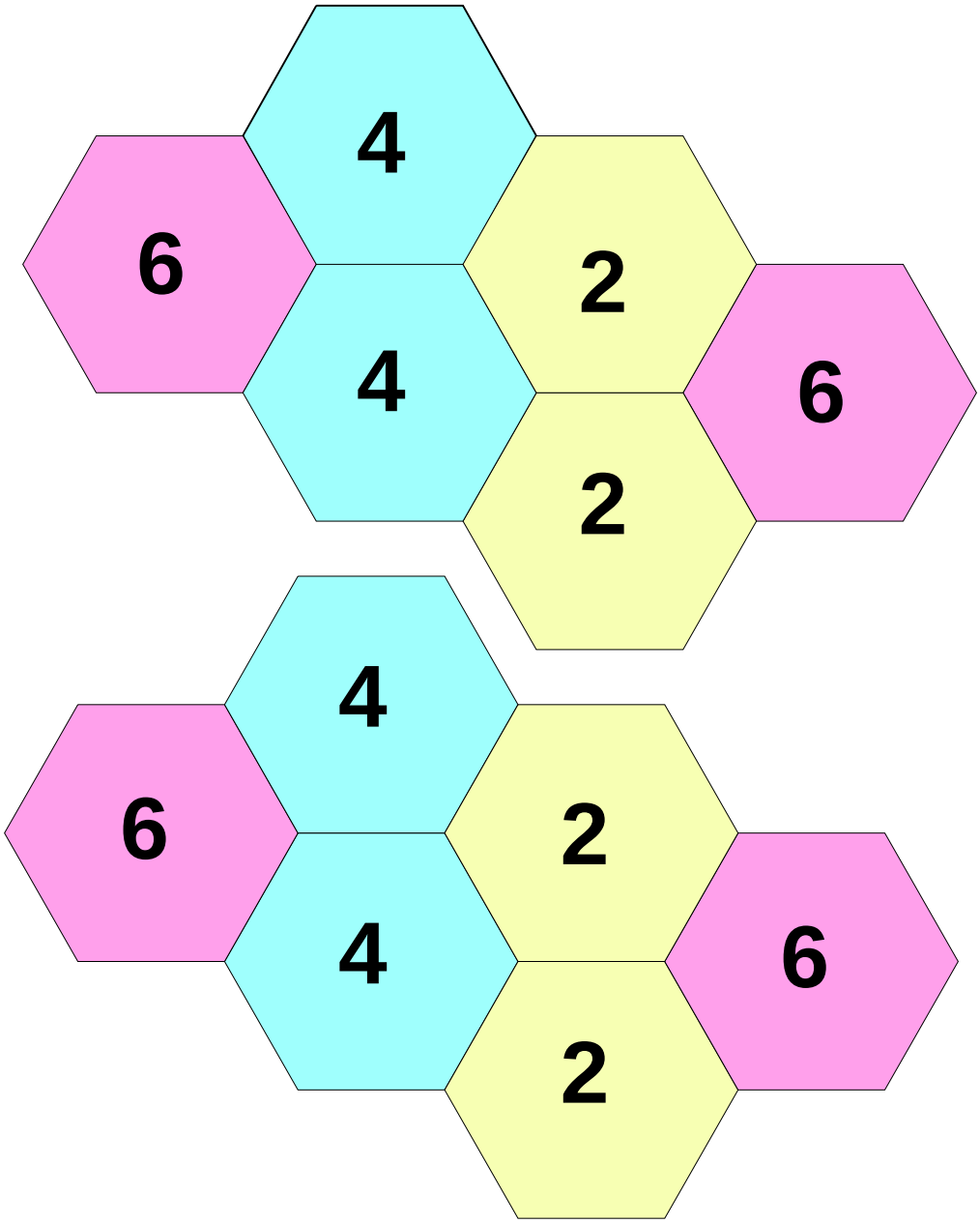

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Figure 4: Bottom side of a hexagonal flexagon consisting of a single cycle.

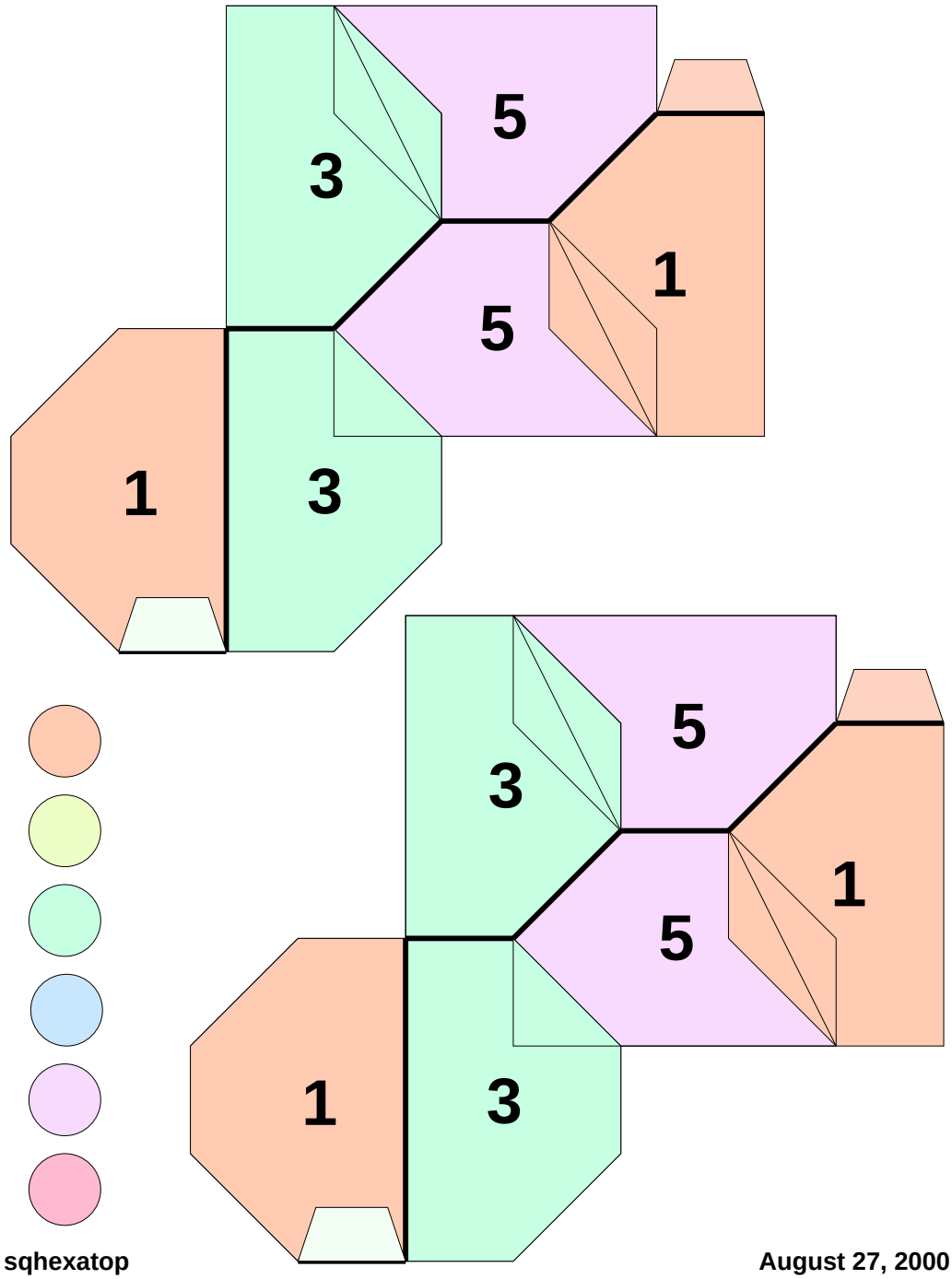


Figure 5: Top side of a hexagonal flexagon whose hexagon has been squared up.

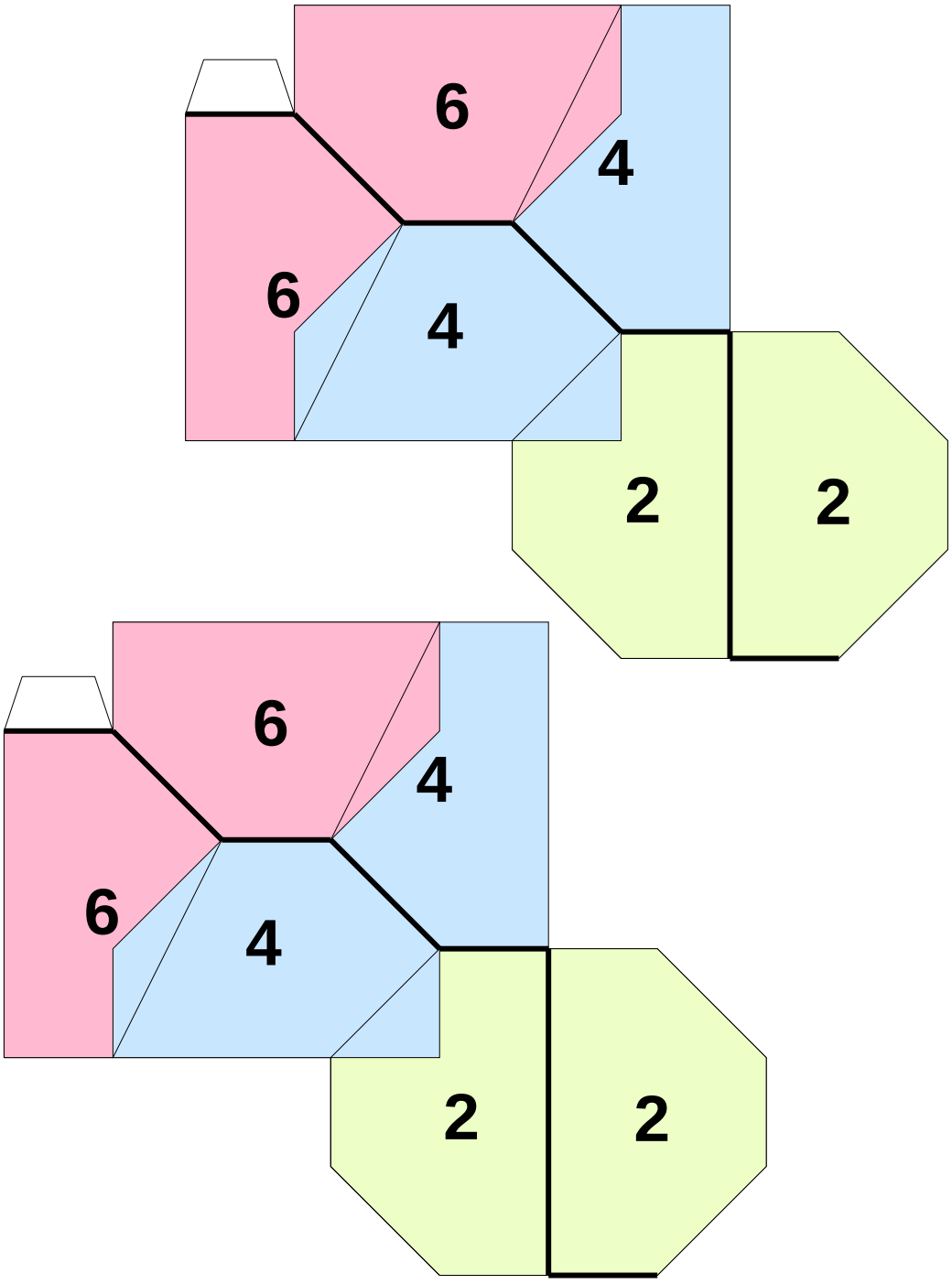


Figure 6: Bottom side of a hexagonal flexagon whose gexagon has been squared up.

3 Binary Hexagonal Flexagon

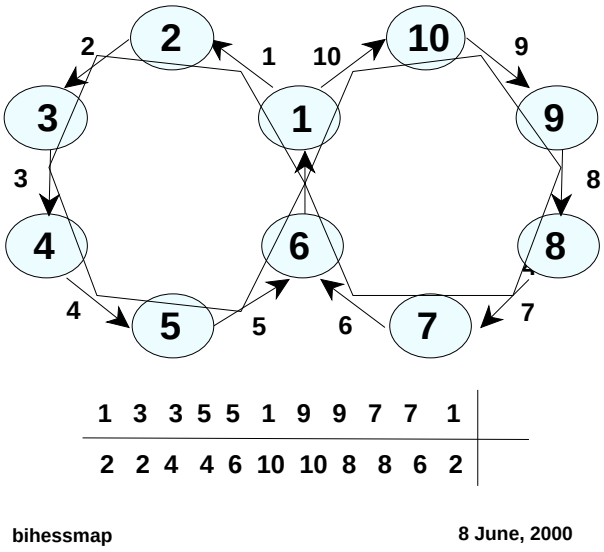


Figure 7: The binary hexagonal flexagon has two cycles, each of which has two vertices in common with the other one, for a total of ten.

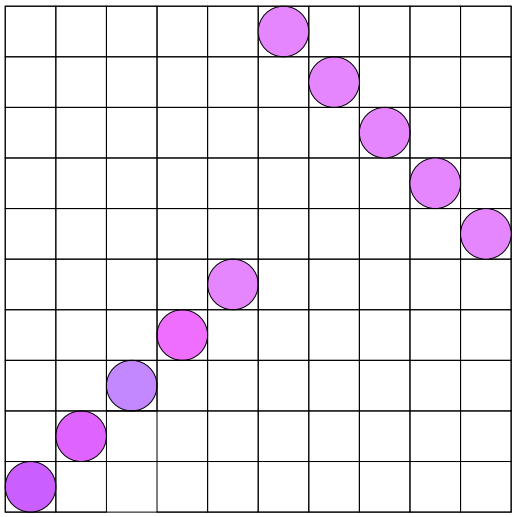


Figure 8: Permutation of the hexagons along the strip for a binary hexagonal flexagon.

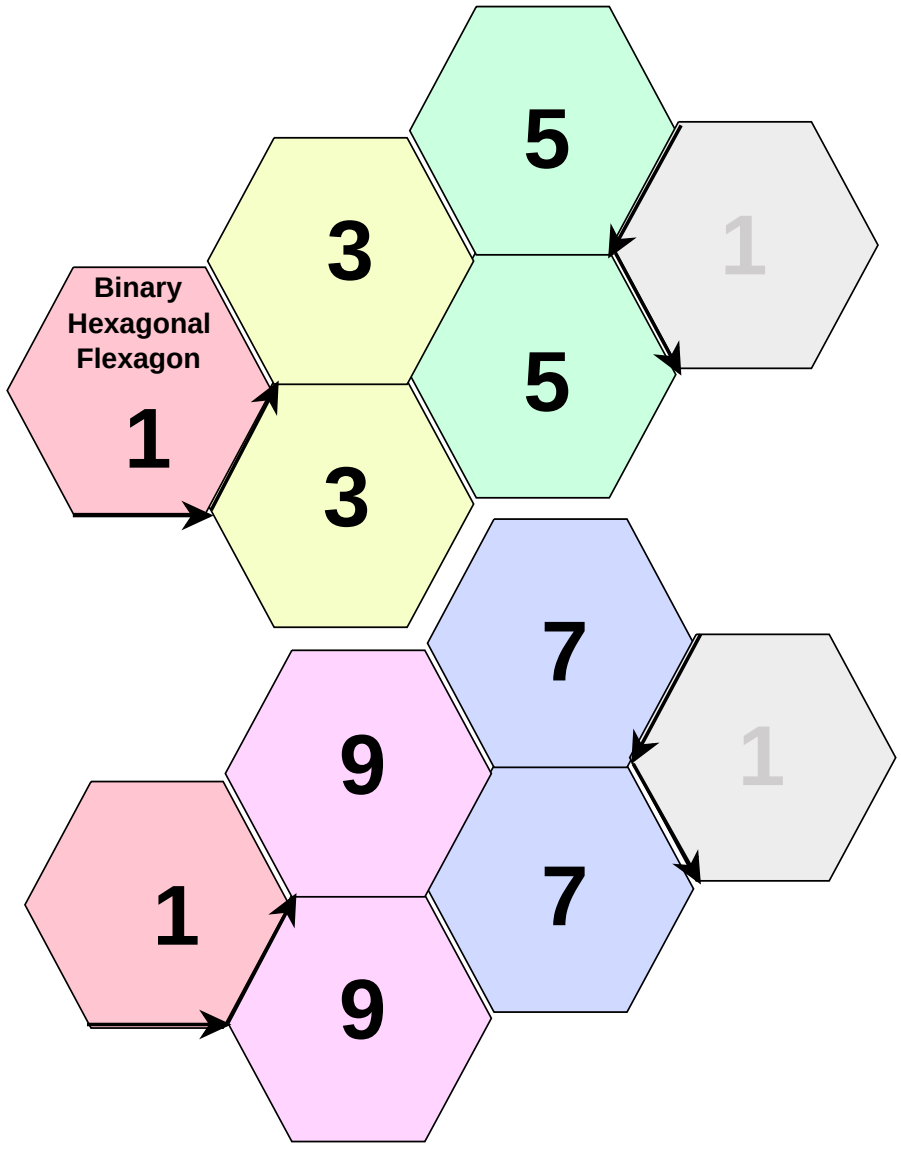


Figure 9: Top side of the binary hexagonal flexagon cutout. With ten faces, this cutout provides material for just the first sector (composed of two pats) of the two needed for the flexagon.

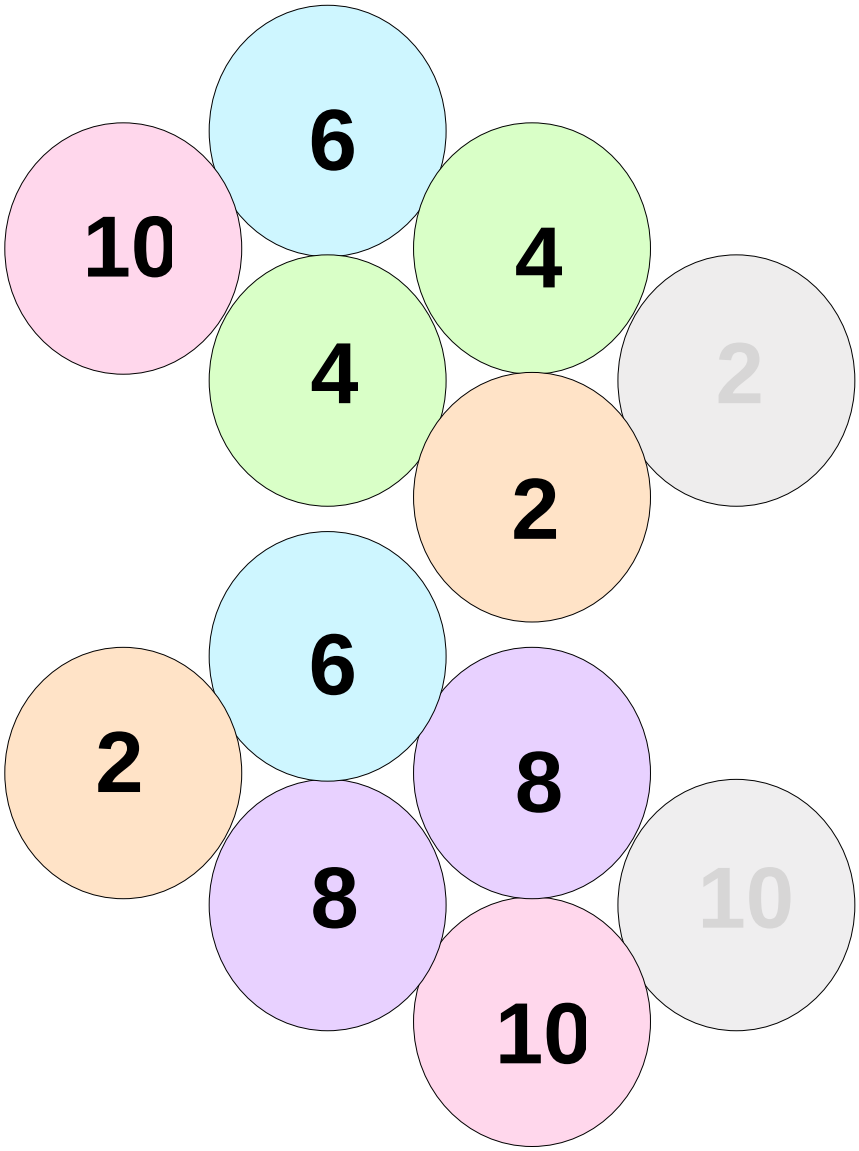


Figure 10: Bottom side of the first sector of the binary hexagonal flexagon cutout. Two sectors are required.

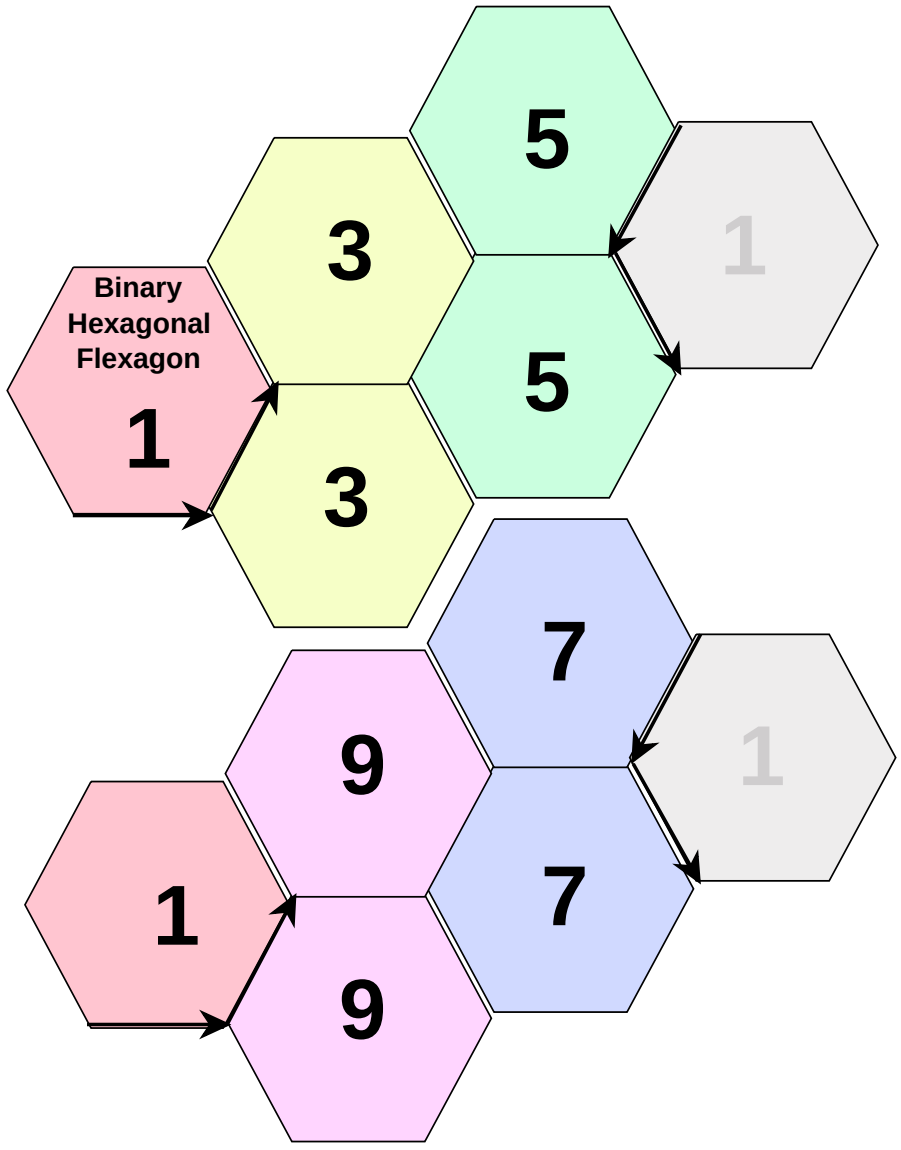


Figure 11: Top side of the binary hexagonal flexagon cutout. With ten faces, this cutout provides material for just the second sector (composed of two pats) of the two needed for the flexagon.

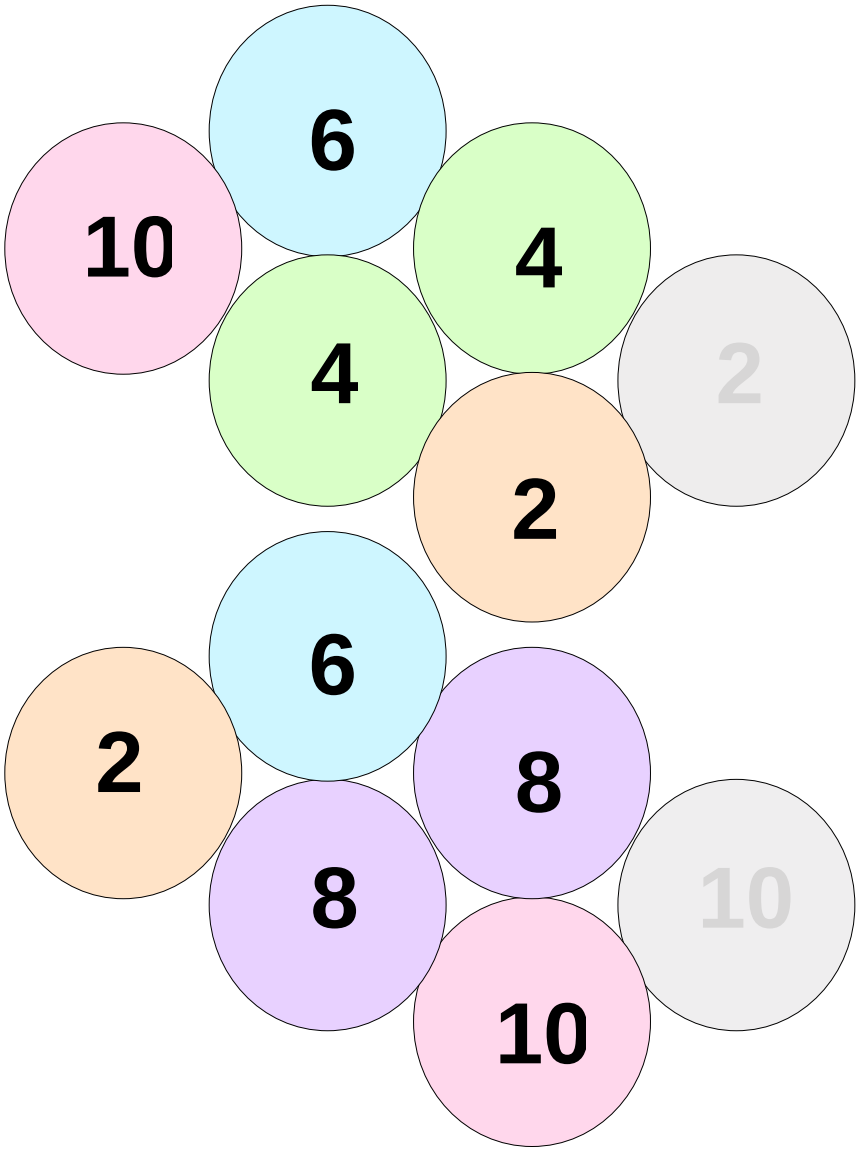


Figure 12: Bottom side of the second sector of the binary hexagonal flexagon cutout. Two sectors are required.

4 Second Level Hexagonal Flexagon

4.1 Tukey hexagons

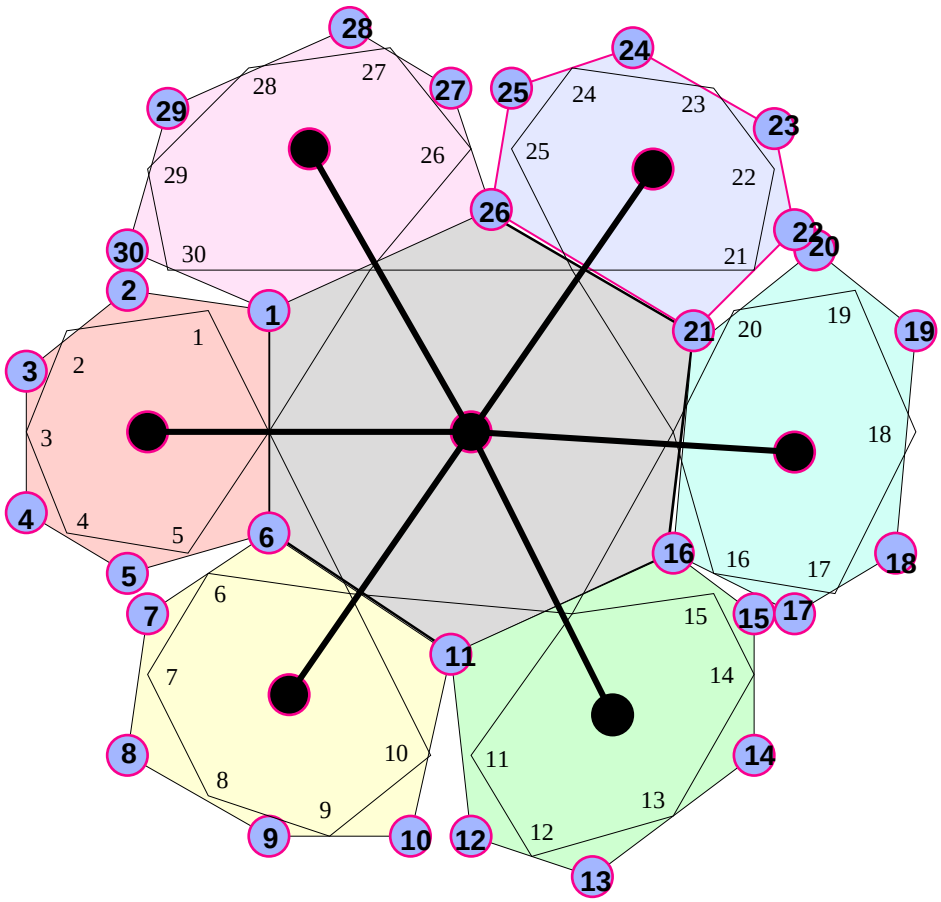


Figure 13: Since each of the six edges of the first level hexagonal flexagon spawns four new vertices, the full second level hexagonal flexagon has 30 vertices since $(4 * 6 + 6 = 30)$.

+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
1	3	3	5	5	27	27	29	29	1	21	23	23	25	25	17	17	19	19	21	11	13	13	15	15	7	7	9	9	11	1	
2	2	4	4	6	26	28	28	30	30	22	22	24	24	26	16	18	18	20	20	12	12	14	14	16	6	8	8	10	10	2	

If the strips in Figures 15 through 23 had been displayed as six pages with strings of five hexagons each, there would have been an artistic parity problem, but the structure of the second order flexagon would have been that much clearer. Anyway, once the strips have been cut out, pasted, and made ready for folding, the second order periodicity is evident enough.

The reason for this is that any n-gon in a regular flexagon can be replaced by a fanfolded strip of $n-1$ n-gons turned upside down while still making the same connections as before. When all the original n-gons have been so replaced, the next higher order of flexagon results. Treating these subpats as units, everything remains as before; but each of them can be opened up via the mountain-valley transition (pinching), to get cycles based just on the subpat.

4.2 Second level permutations

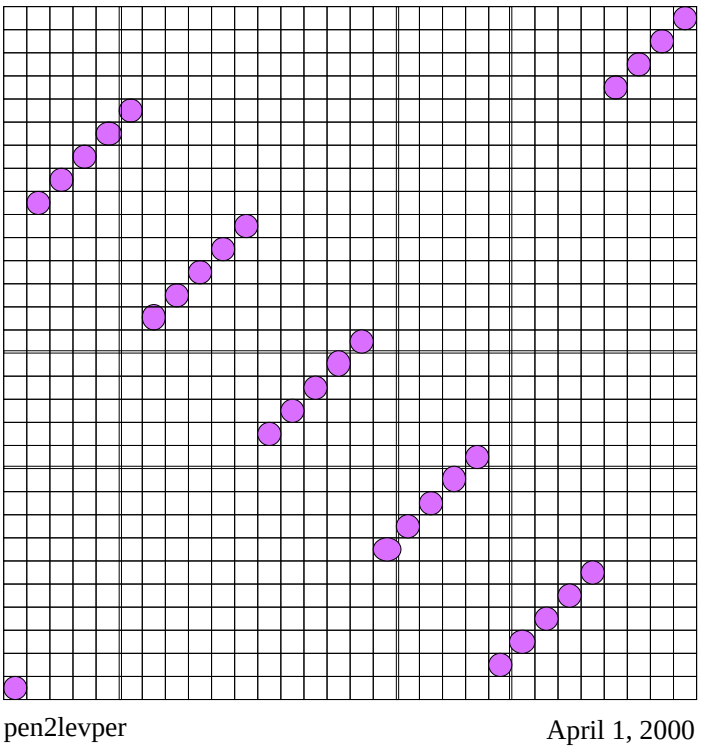
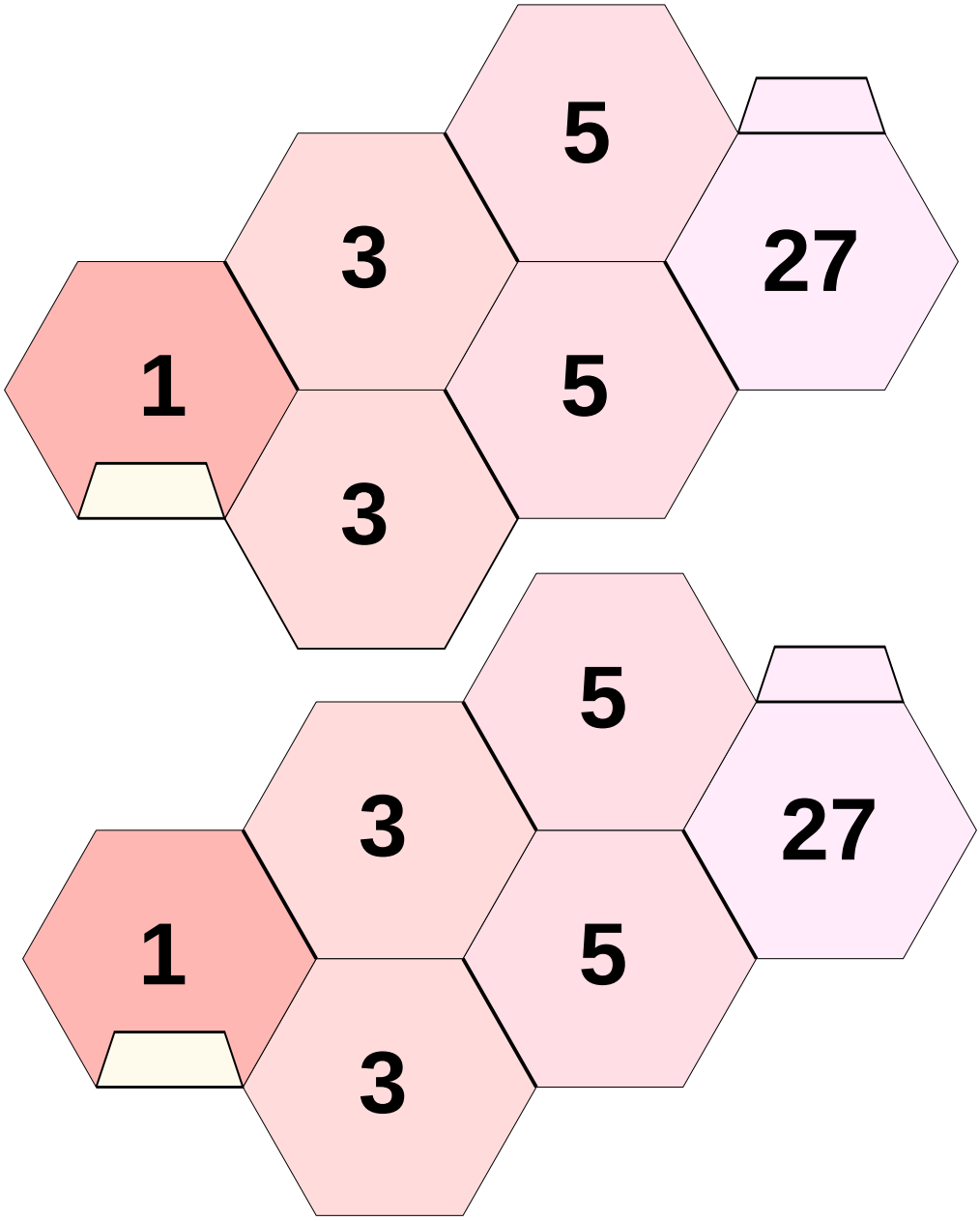


Figure 14: Permutation of the hexagons along the strip for a second level hexagonal flexagon.



hgons1top June 8, 2000

Figure 15: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level hexagonal flexagon can be constructed. This is the first copy.

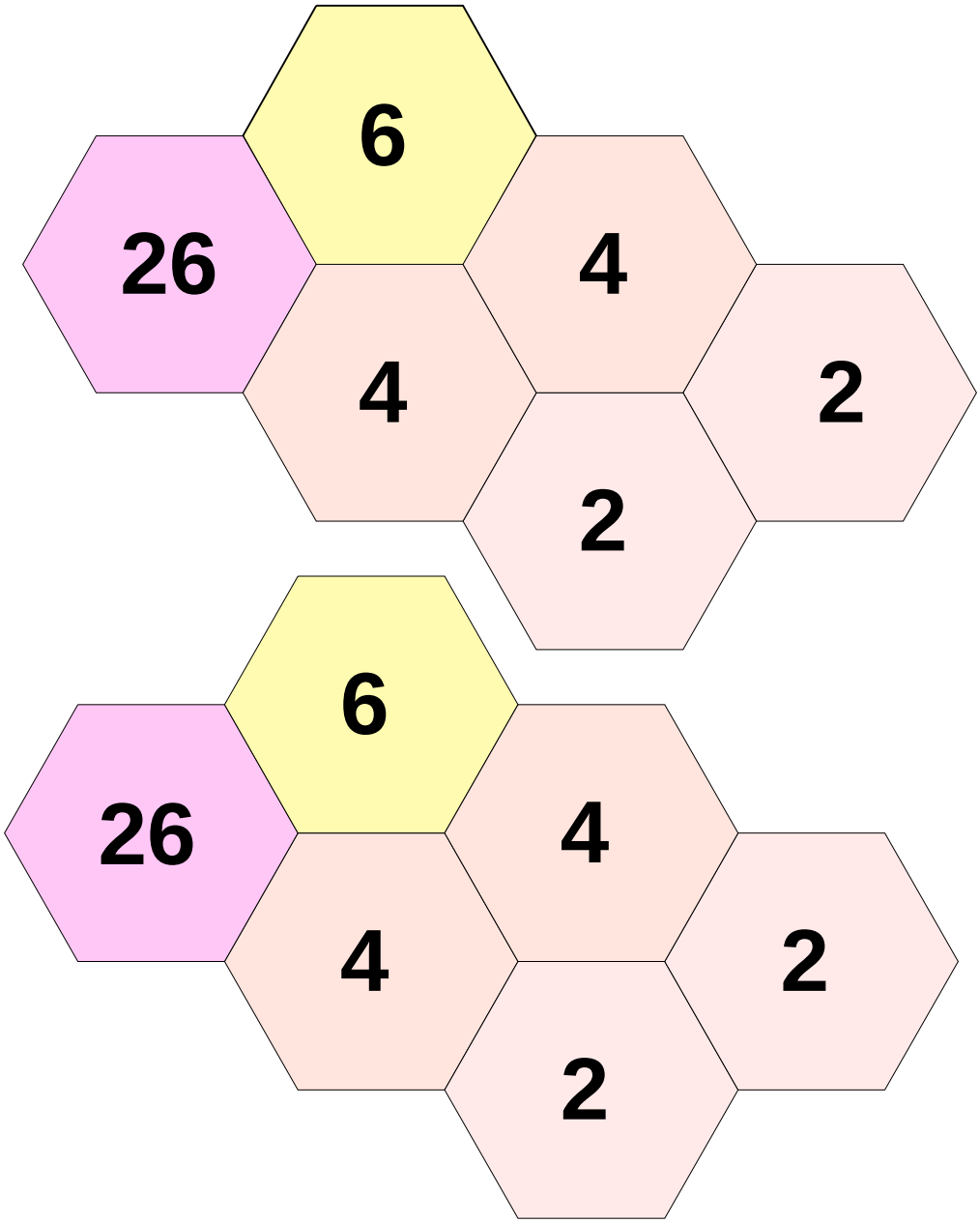
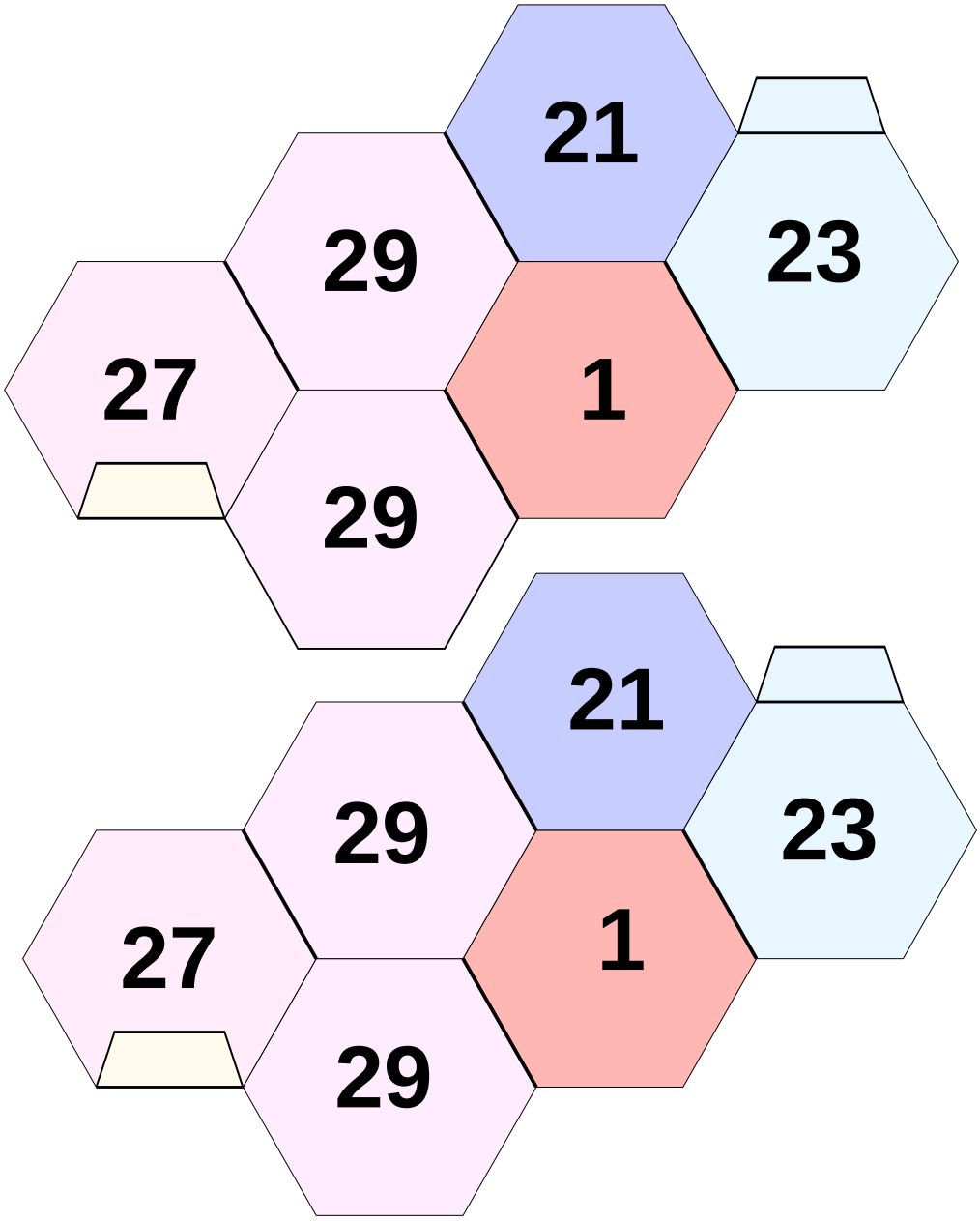
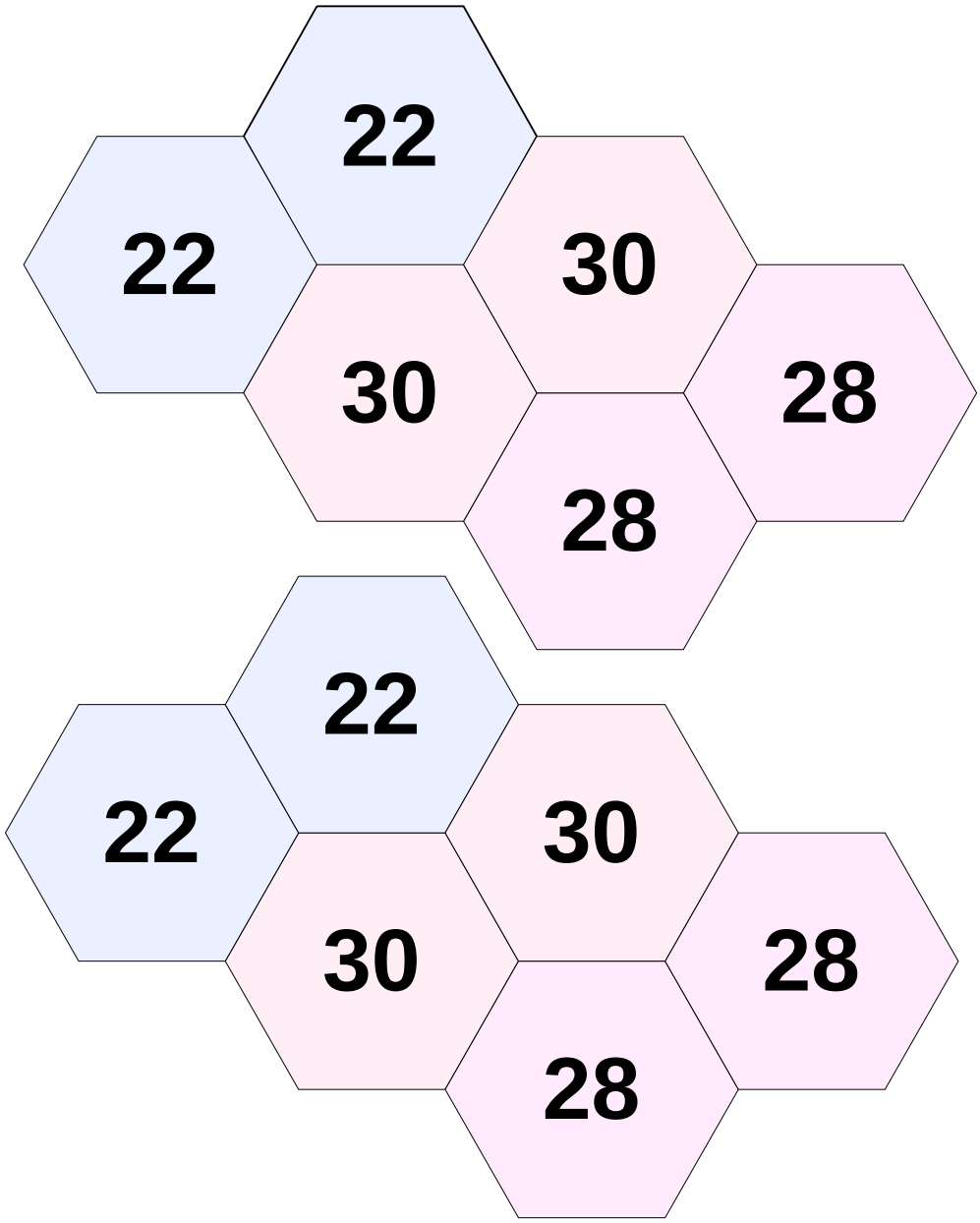


Figure 16: Bottom side of a second order hexagonal flexagon.



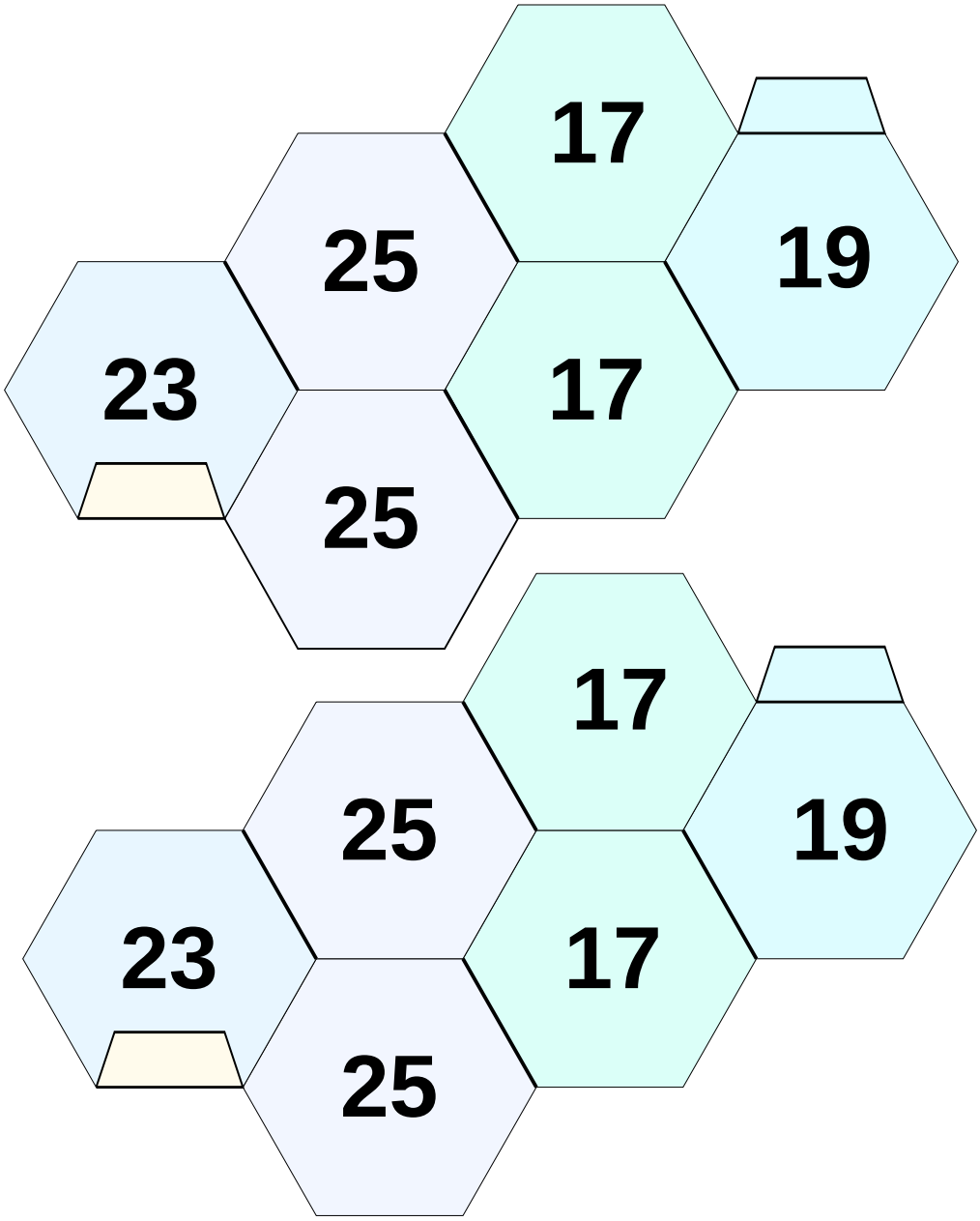
hgons2top June 8, 2000

Figure 17: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level hexagonal flexagon can be constructed. This is the second copy.



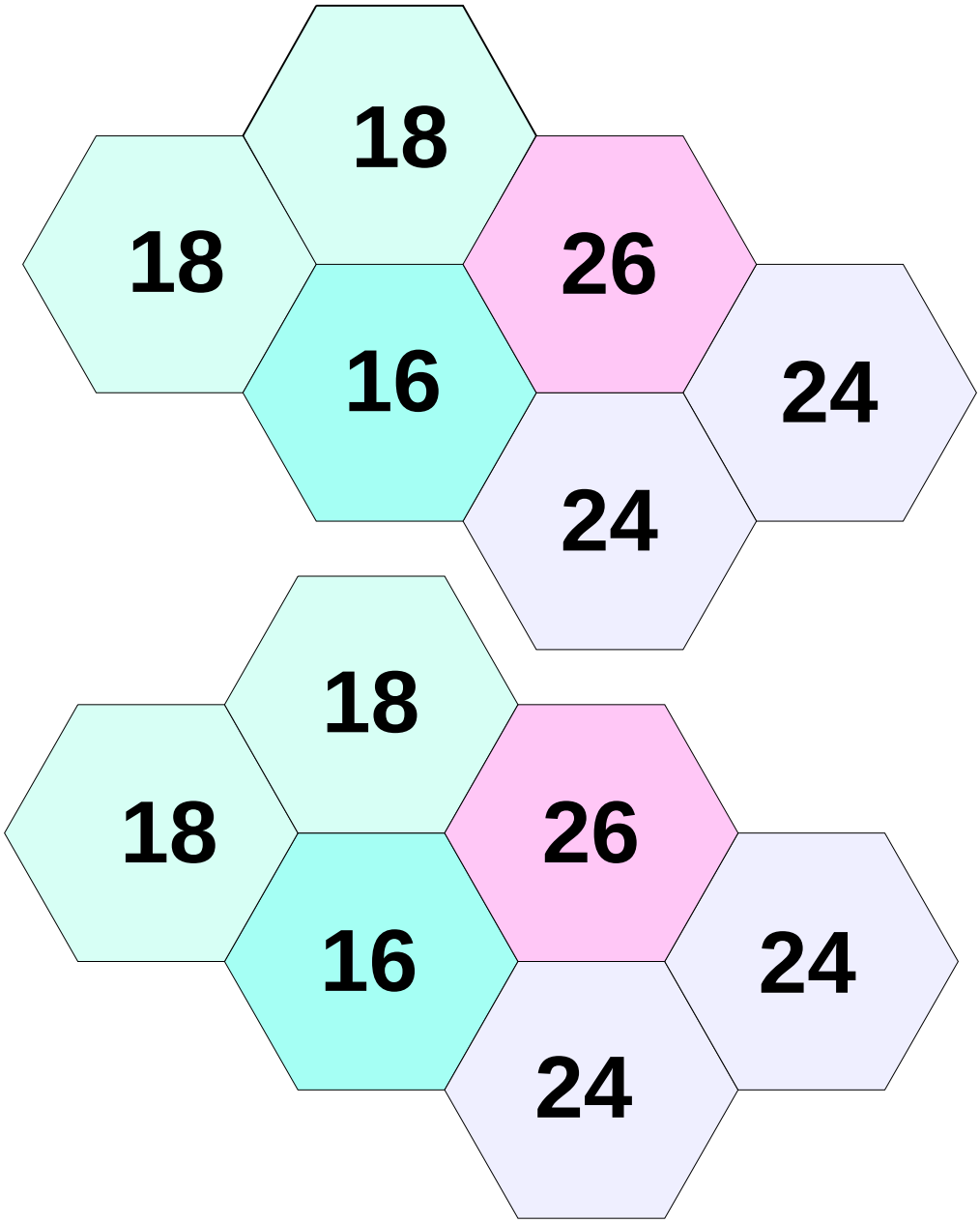
hgons2bot June 8, 2000

Figure 18: Bottom side of a second order hexagonal flexagon.



hgons3top June 8, 2000

Figure 19: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level hexagonal flexagon can be constructed. This is the third copy.



hgons3bot June 8, 2000

Figure 20: Bottom side of a second order hexagonal flexagon.

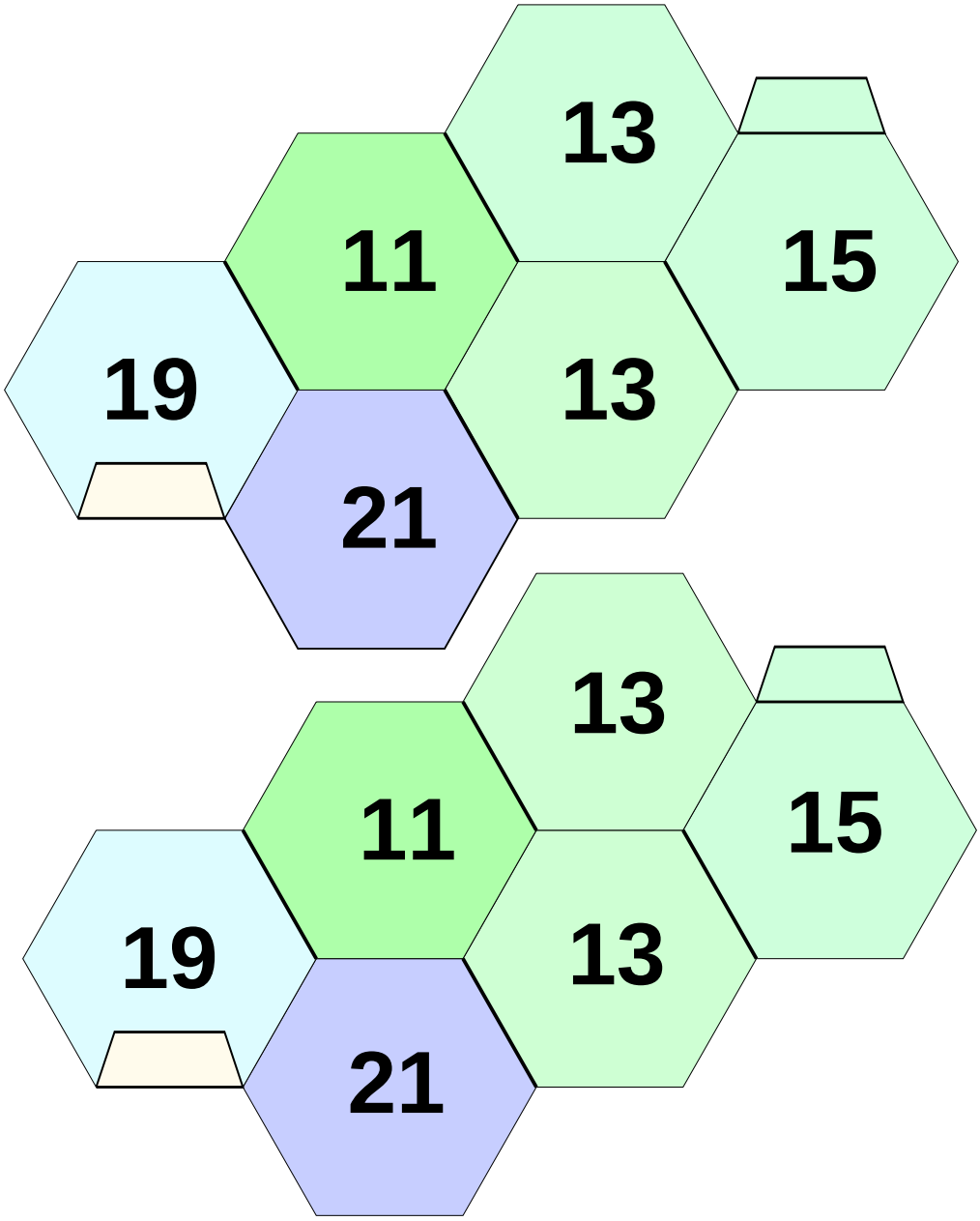


Figure 21: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level hexagonal flexagon can be constructed. This is the fourth copy.

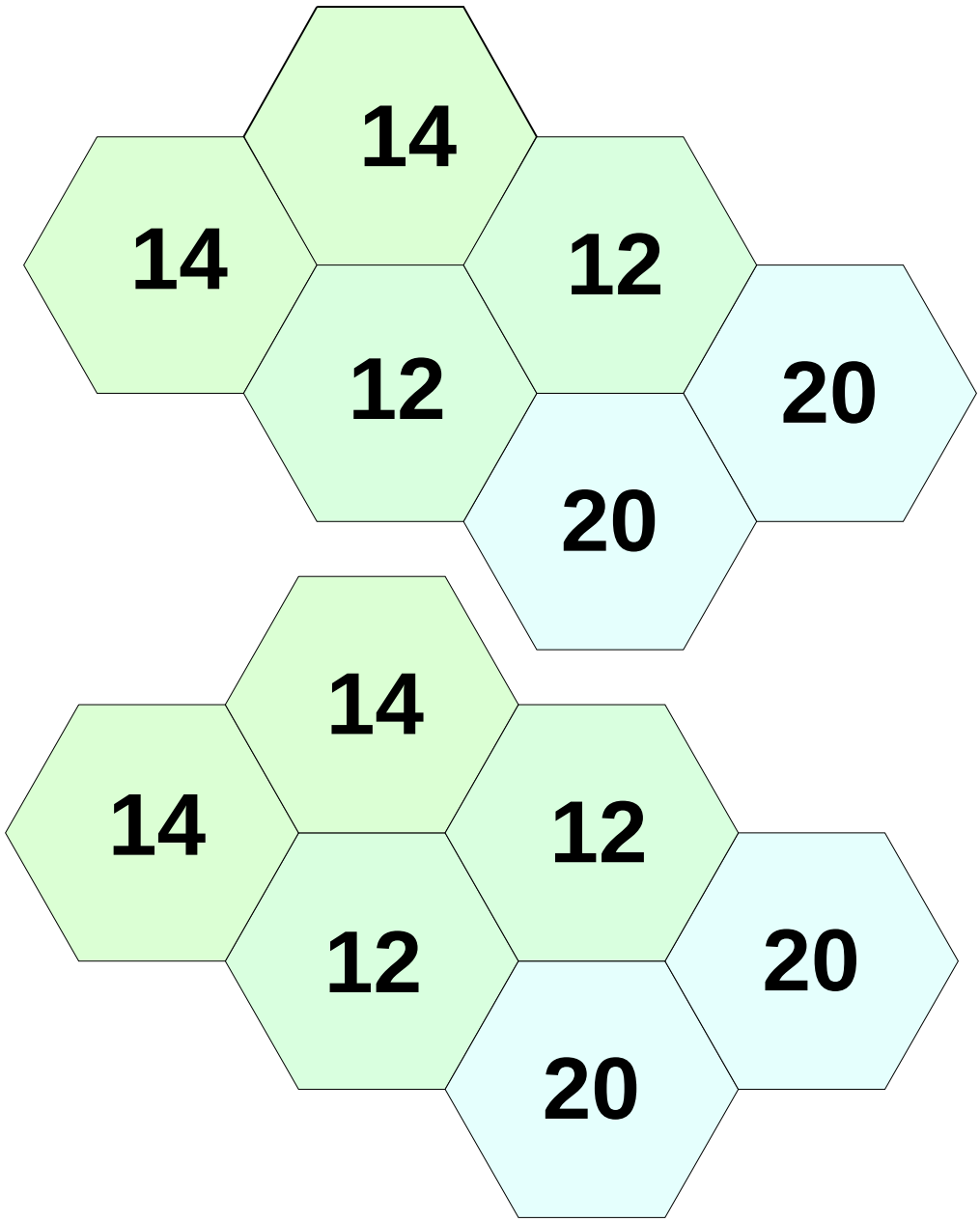
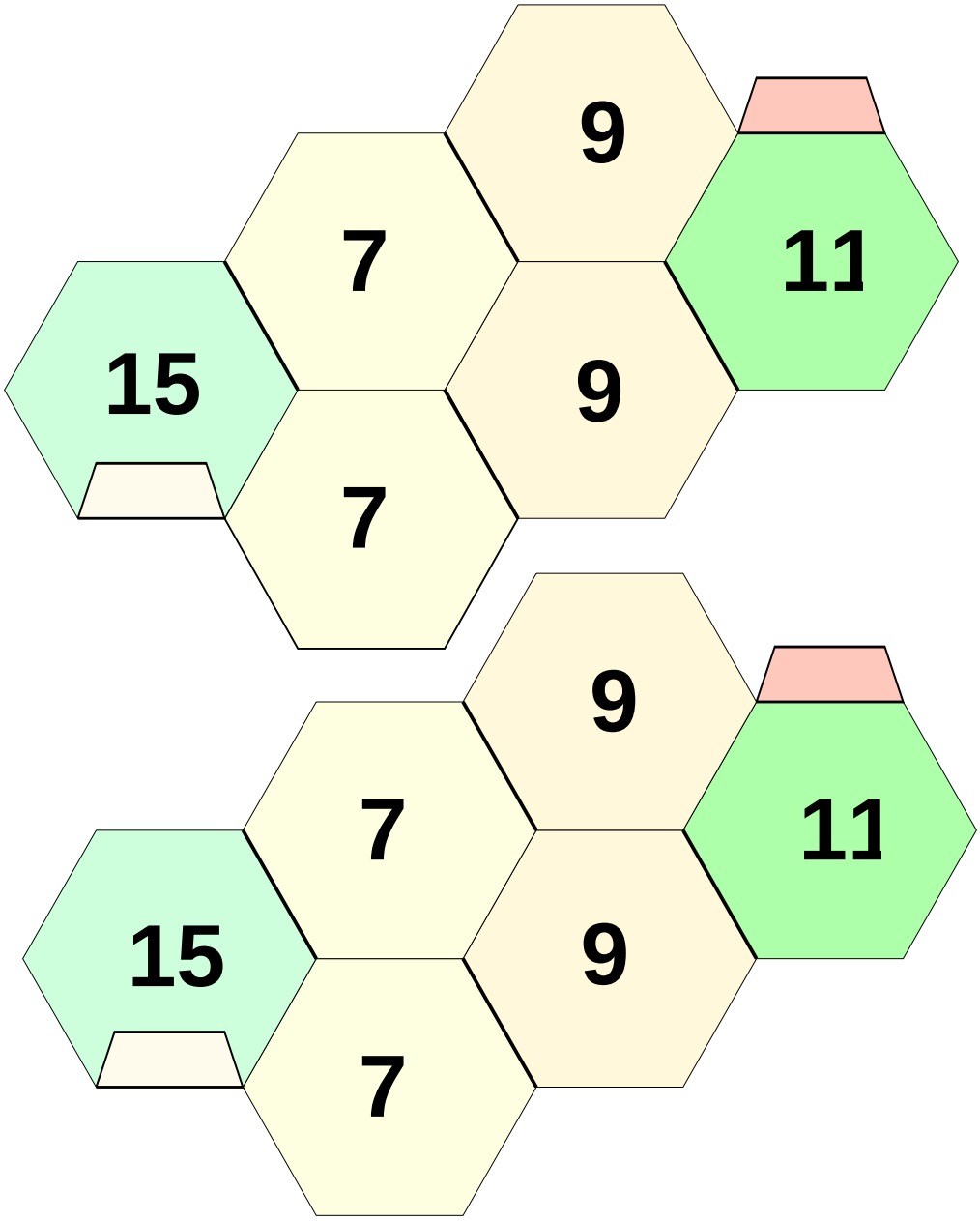
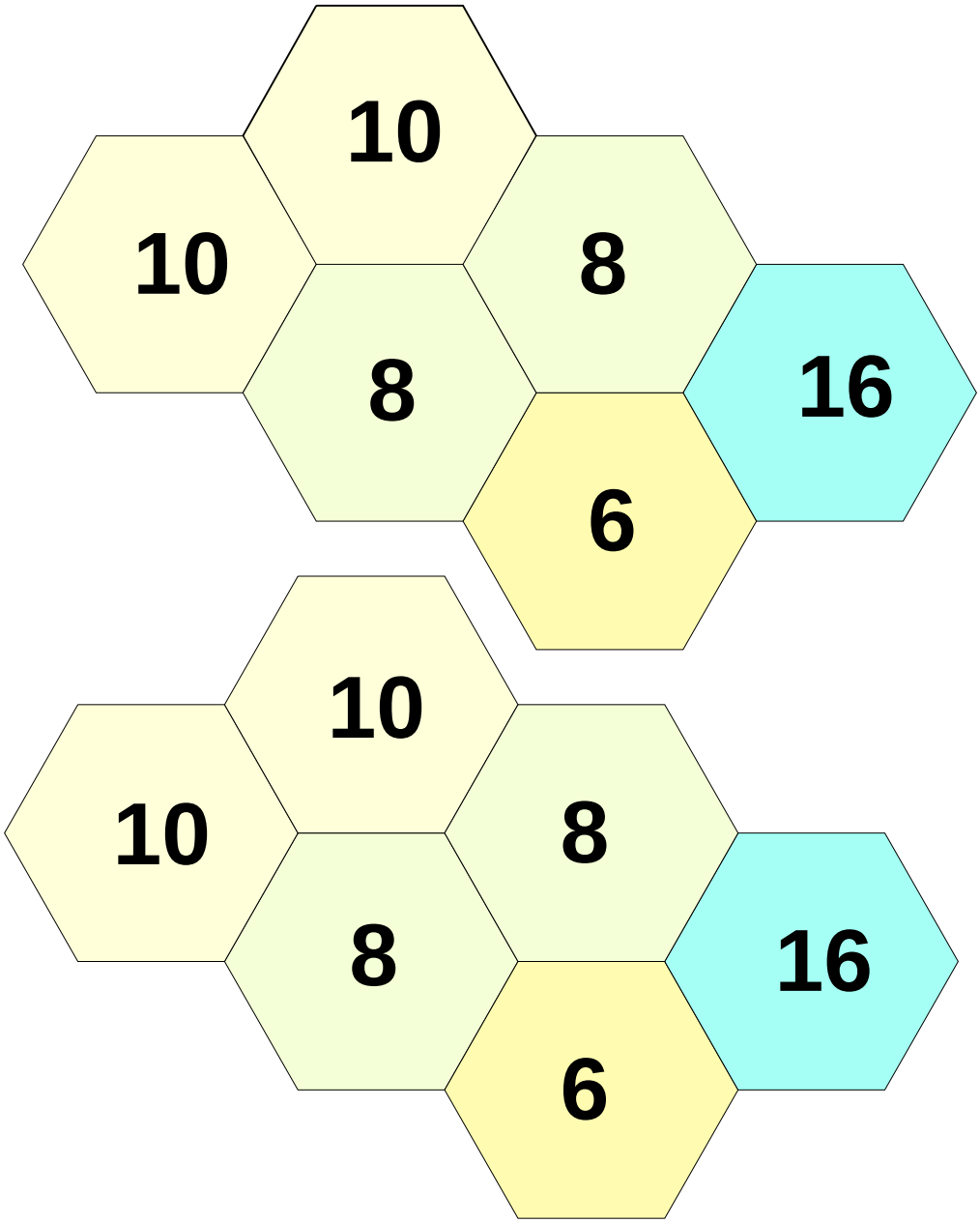


Figure 22: Bottom side of a second order hexagonal flexagon.



hgons5top June 8, 2000

Figure 23: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level hexagonal flexagon can be constructed. This is the fifth copy.



hgons5bot June 8, 2000

Figure 24: Bottom side of a second order hexagonal flexagon. - November 1, 2000-