

# Octagonal Flexagons

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## Abstract

Maps and cutouts for a variety of flexagons are presented, emphasizing those which can be cut out, mostly from single sheets of paper. Since TeX may not align front and back images, and in any event if cutting up the booklet is not desired, the .eps files can be printed directly to get sheets suitable for cutting. In the same spirit, only those sheets which are going to be used right away need be printed.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>First Level Octagonal Flexagon</b>	<b>3</b>
<b>3</b>	<b>Binary Octagonal Flexagon</b>	<b>7</b>
<b>4</b>	<b>Second Level Octagonal Flexagon</b>	<b>17</b>

# 1 Introduction

Flexagons can become fairly complicated. The ones based on triangles are most conveniently made from long strips of paper; a roll of adding machine or calculator tape is ideal for this purpose given its convenient width. Crooked strips can be gotten by gluing faces together, or just cutting out segments and then joining them together. Leaving one extra triangle in each segment for overlapping and later gluing leads to efficient constructions.

Coloring the triangles is another problem, which can be done with crayons or markers once it is known which colors ought to be used. Aside from copying an already existent design, this is best done by drawing the Tukey triangles and then lettering or numbering the triangles in the strip. That information is sufficient to fold up the strip, since pairs of consecutive numbers are to be hidden by folding them together. Painting can be done before folding by following a color code for the numbers, or after the folding is done, when the faces can be painted wholesale, or even embellished with designs.

Other flexagons, even the ones folded from “straight” strips, require a higher degree of preparation, although it is relatively easy to assemble a collection of primitive components which later can be glued together according to the necessities of the individual flexagon.

## 2 First Level Octagonal Flexagon

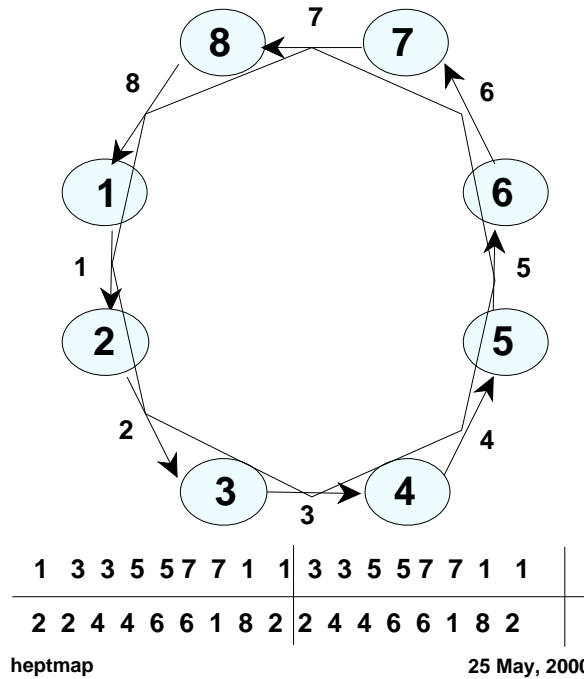


Figure 1: The first level octagonal flexagon has 8 vertices.

Although there is some overlapping of the individual octagons in a strip of octagons, it is still not severe, and is outweighed by the versatility of the resulting octagonal flexagon. In any event, it is the relationship between successive hinges which matters in a flexagon, so the remainder of the polygon can be rearranged to suit convenience or aesthetics.

If it is considered important to preserve the full symmetrical polygons, they can be cut out individually and pasted together. Once the strip is folded, overlapping will no longer be a problem. If rapidity and efficiency of assembly is preferred, corners can be cut but the flexagon will still work.

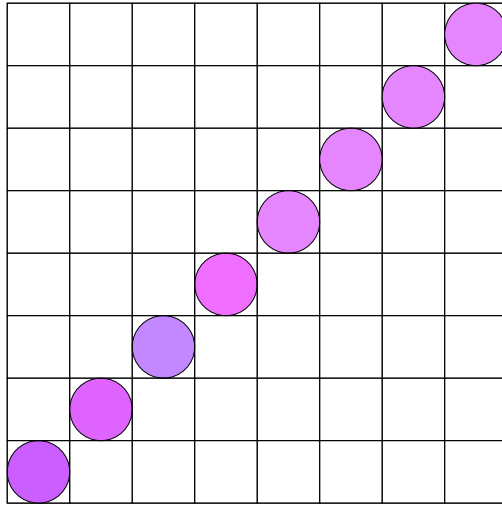
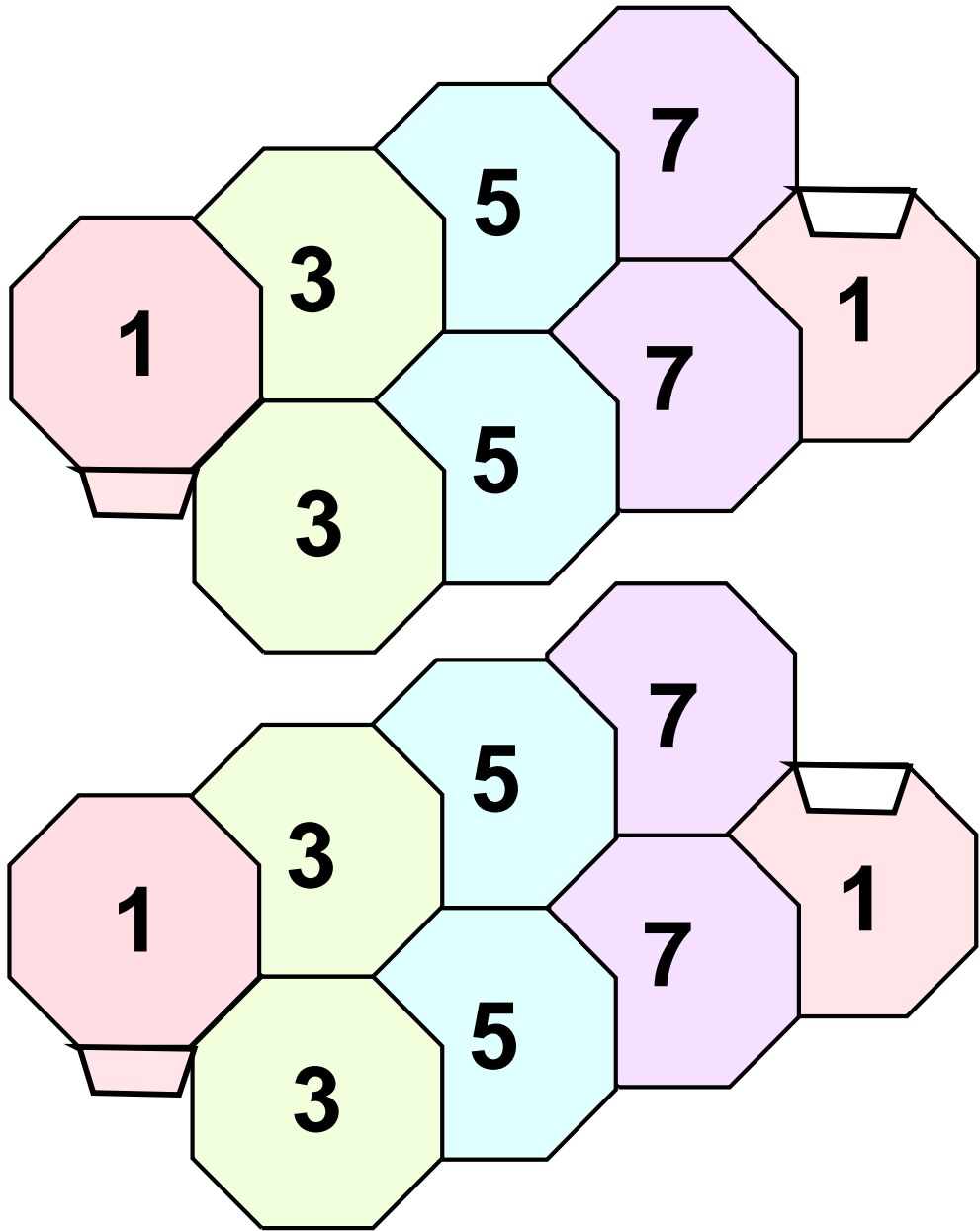


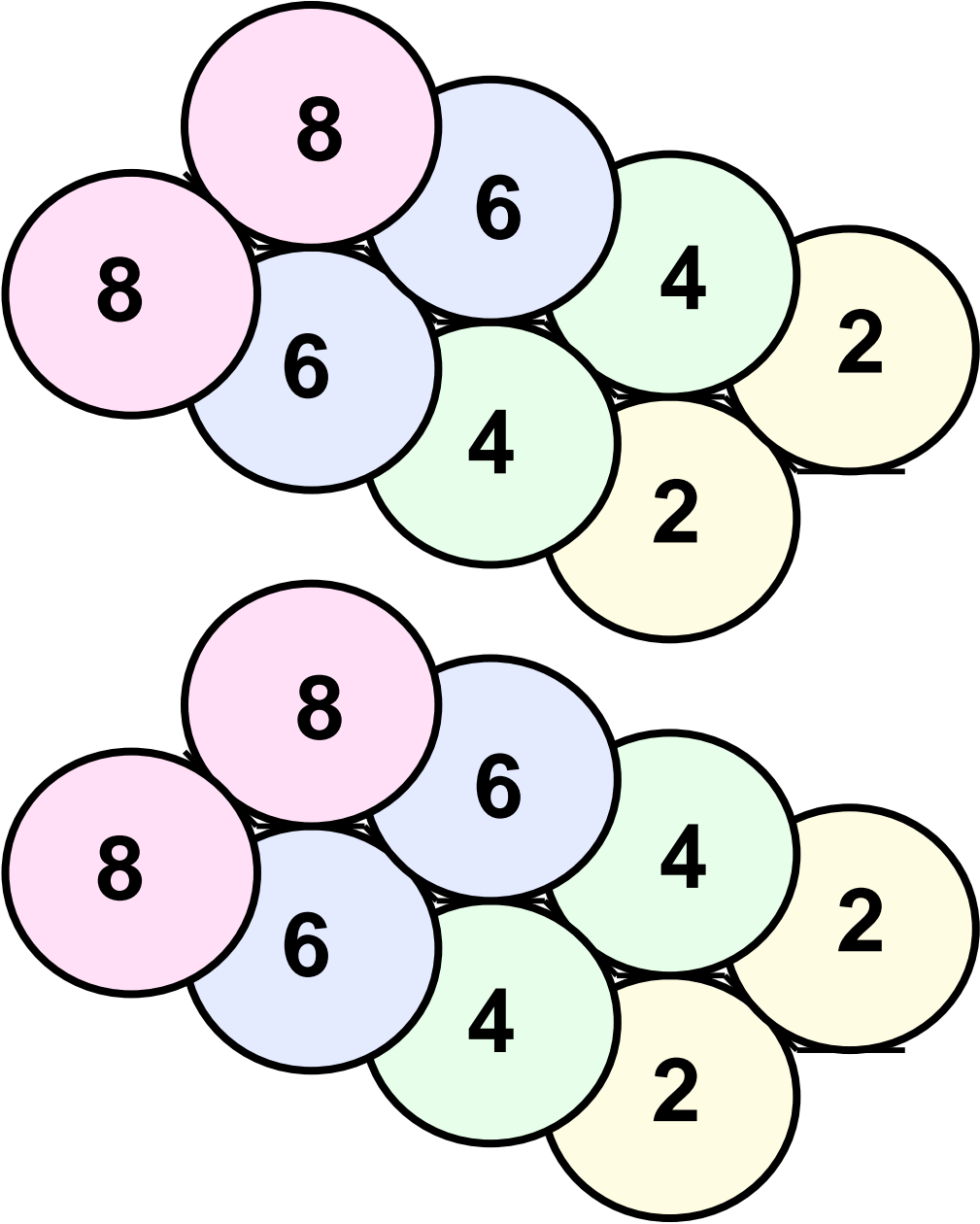
Figure 2: Permutation of the octagons along the strip for a first level octagonal flexagon.



hoctatop

May 27, 2000

Figure 3: Top side of an octagonal flexagon consisting of one single cycle.

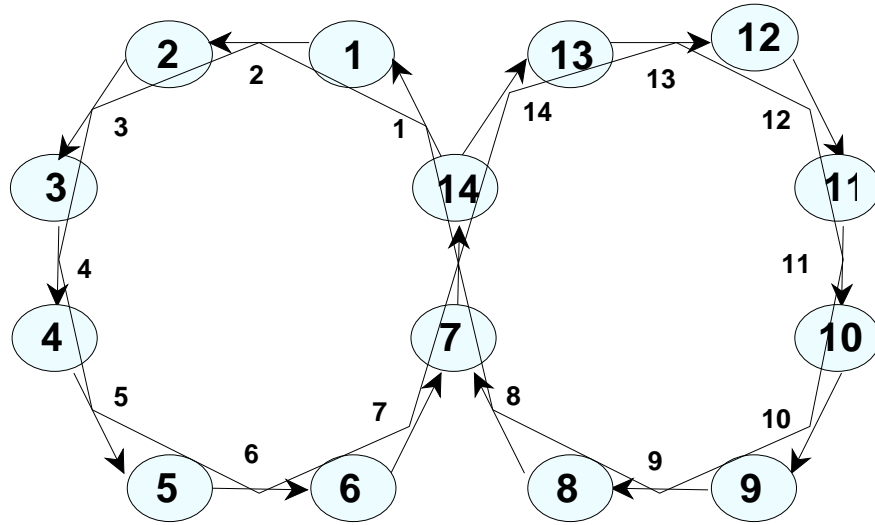


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Figure 4: Bottom side of an octagonal flexagon, consisting of one single cycle.

### 3 Binary Octagonal Flexagon



+	+	+	+	+	+	+	-	-	-	-	-	-	-	+
1	3	3	5	5	7	7	1	13	13	11	11	9	9	1
2	2	4	4	6	6	8	14	14	12	12	10	10	8	2

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Figure 5: The binary octagonal flexagon has fourteen vertices.

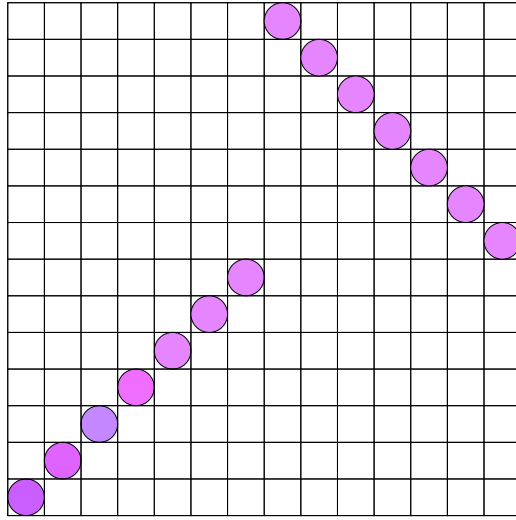


Figure 6: Permutation of the octagons along the strip for a binary octagonal flexagon.



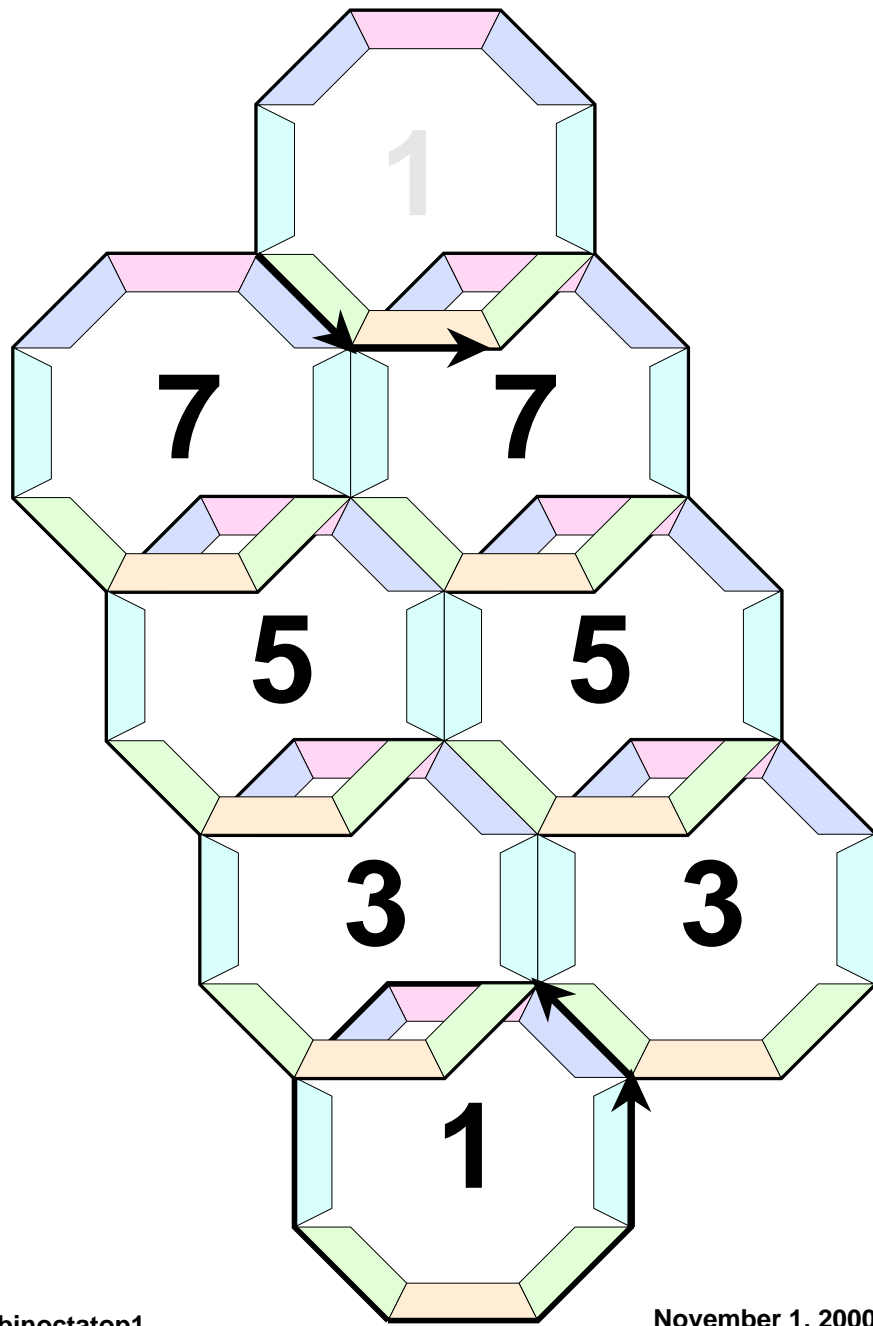


Figure 7: Top side of the first half of a binary octagonal flexagon, first sector of two.

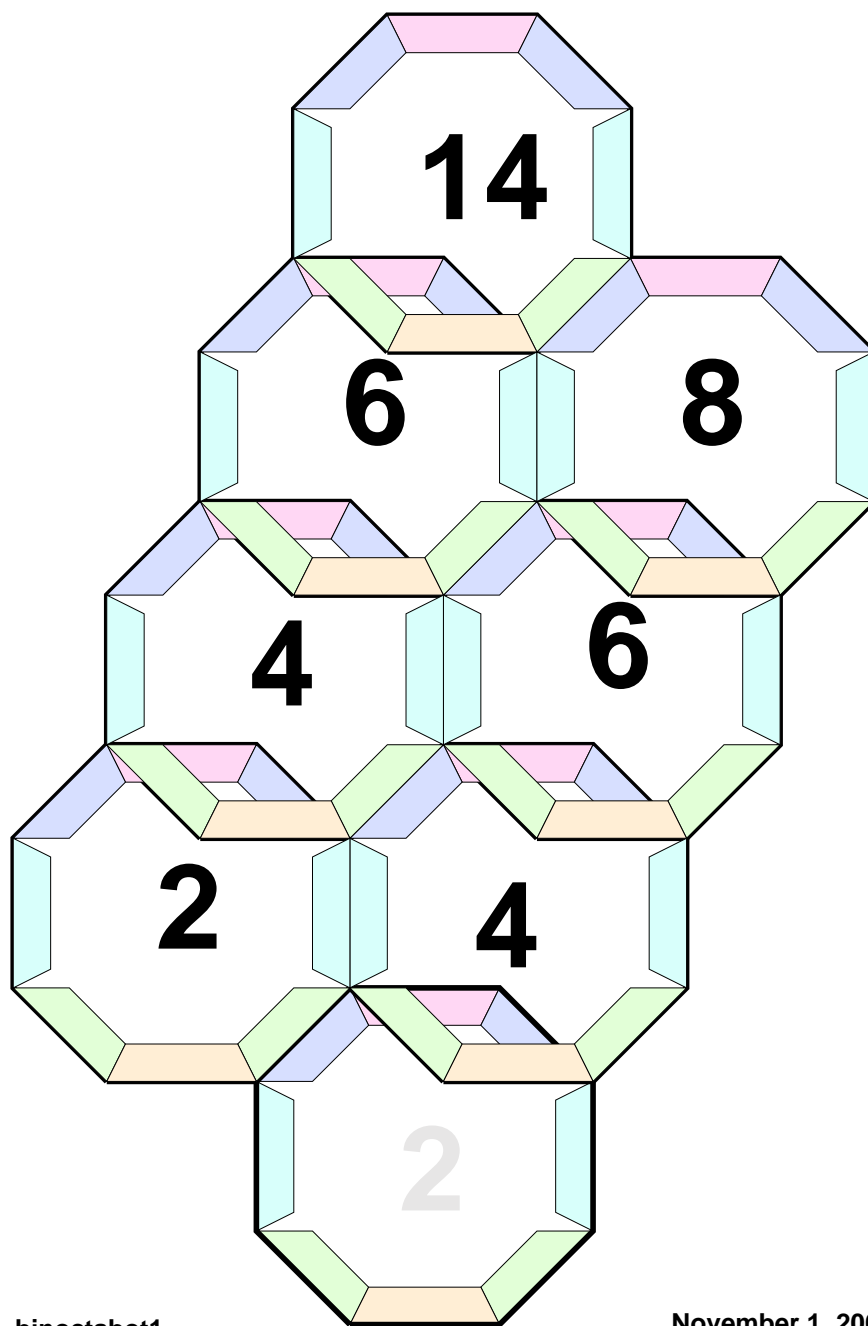


Figure 8: Bottom side of the first half of a binary octagonal flexagon, first sector of two.

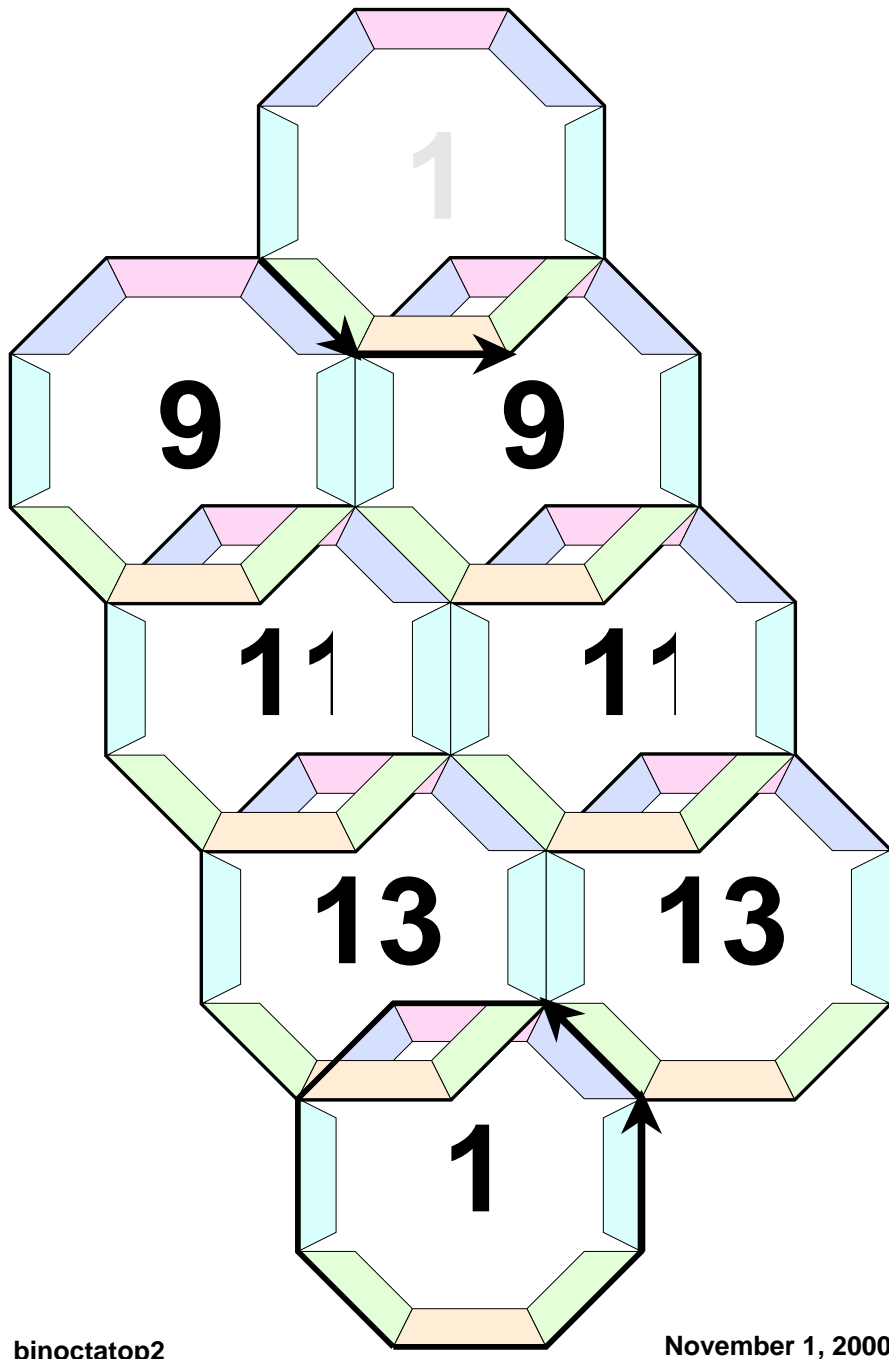
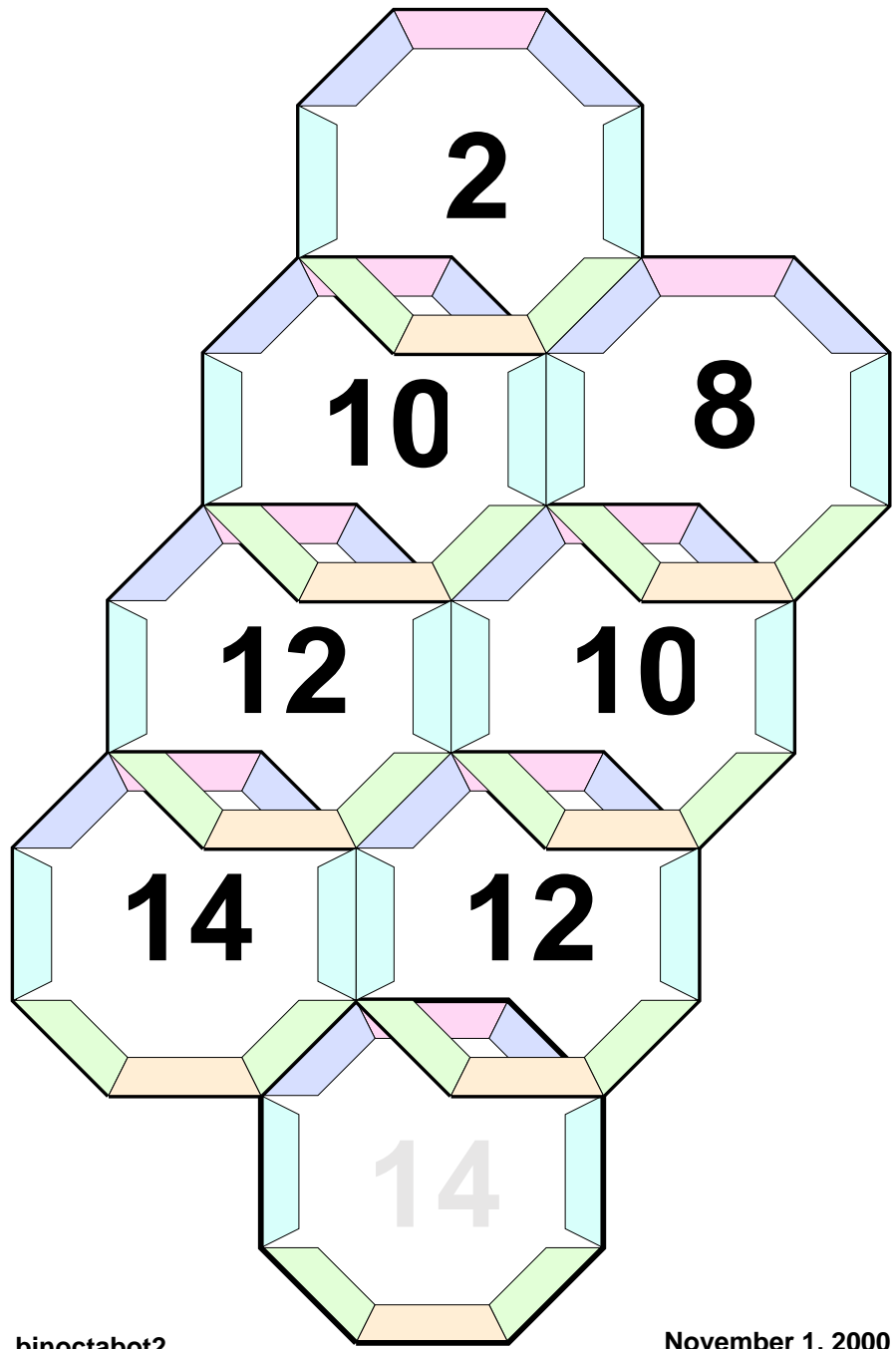


Figure 9: Top side of the second half of a binary octagonal flexagon, first sector of two.



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Figure 10: Bottom side of the second half of a binary octagonal flexagon, first sector of two.

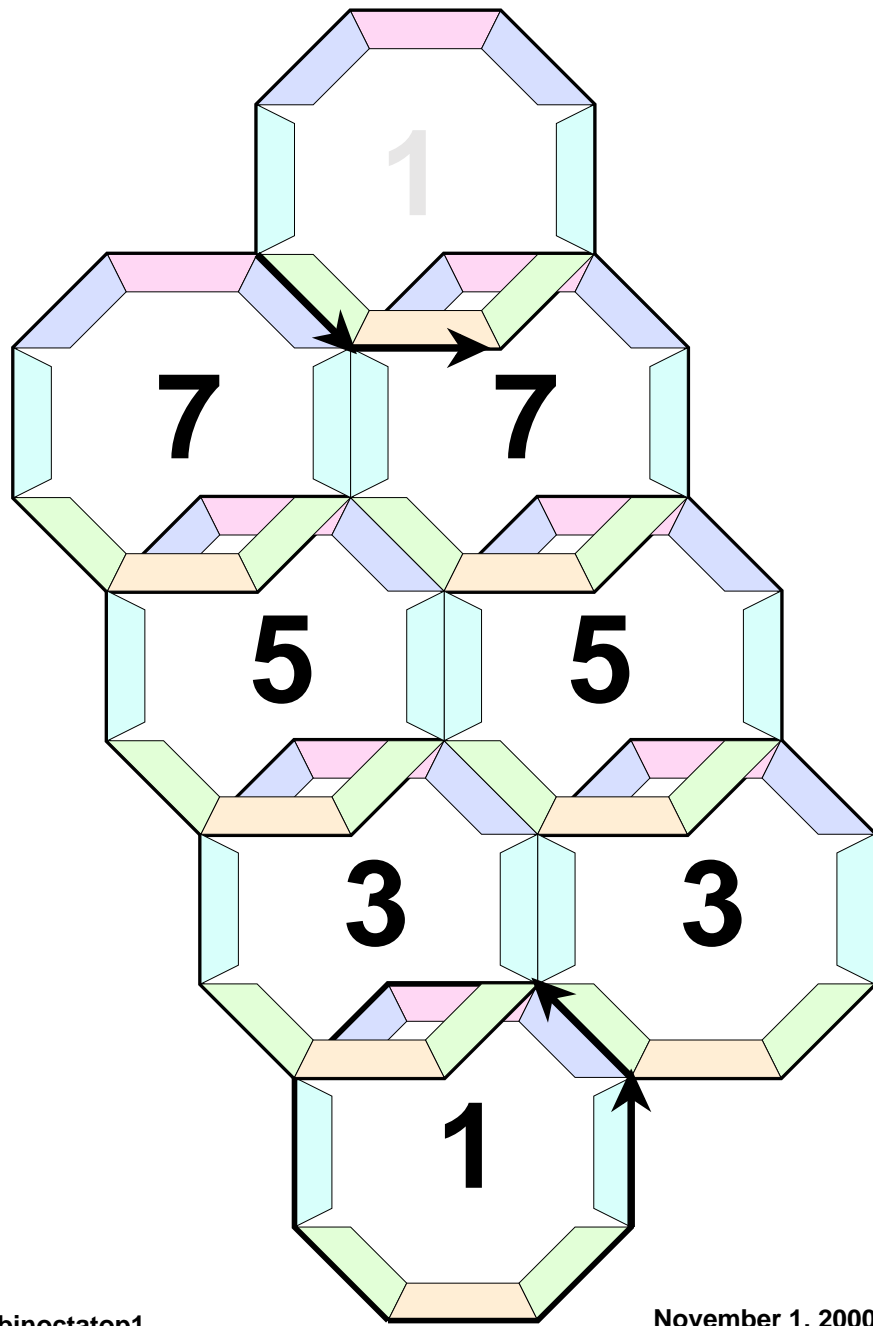


Figure 11: Top side of the first half of a binary octagonal flexagon, second sector of two.

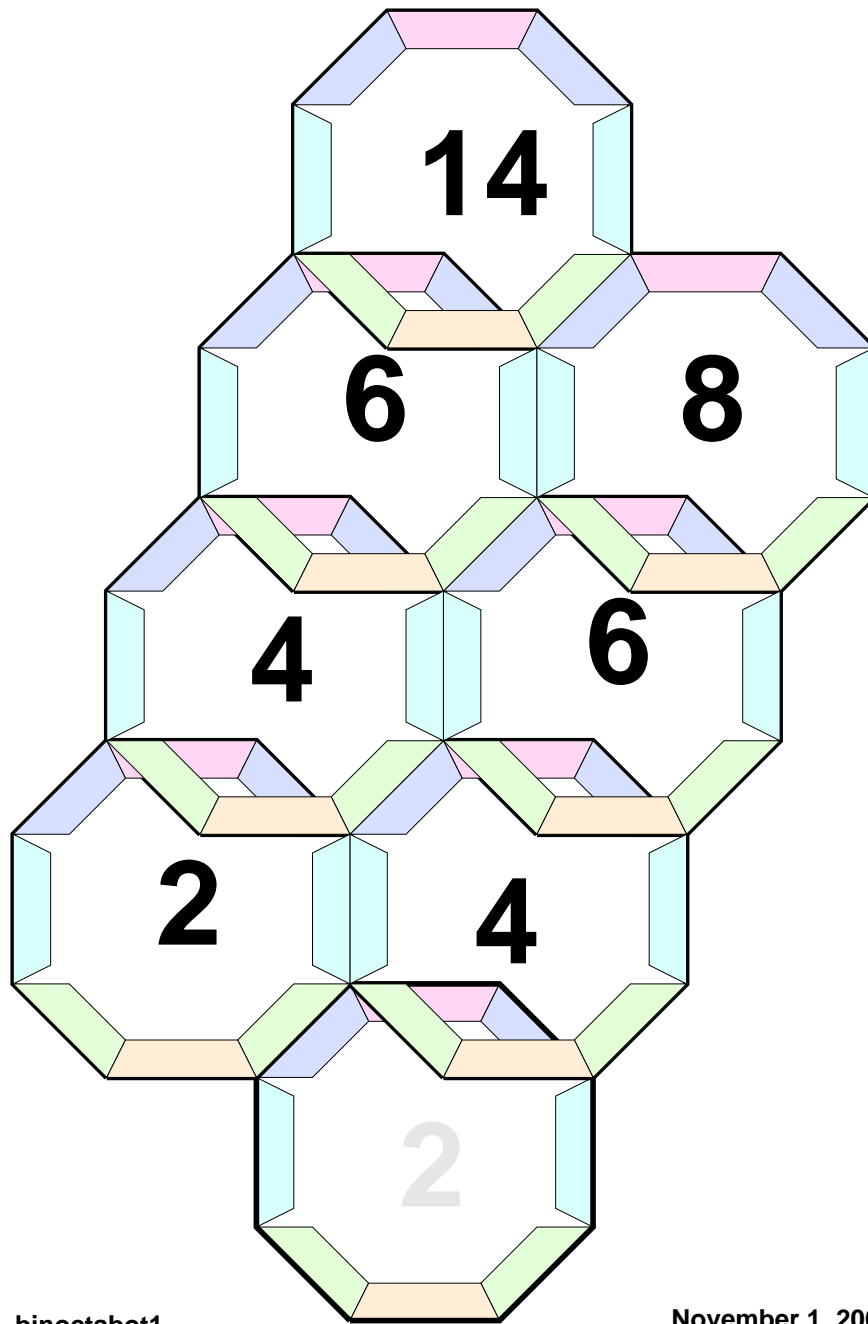


Figure 12: Bottom side of the first half of a binary octagonal flexagon, second sector of two.

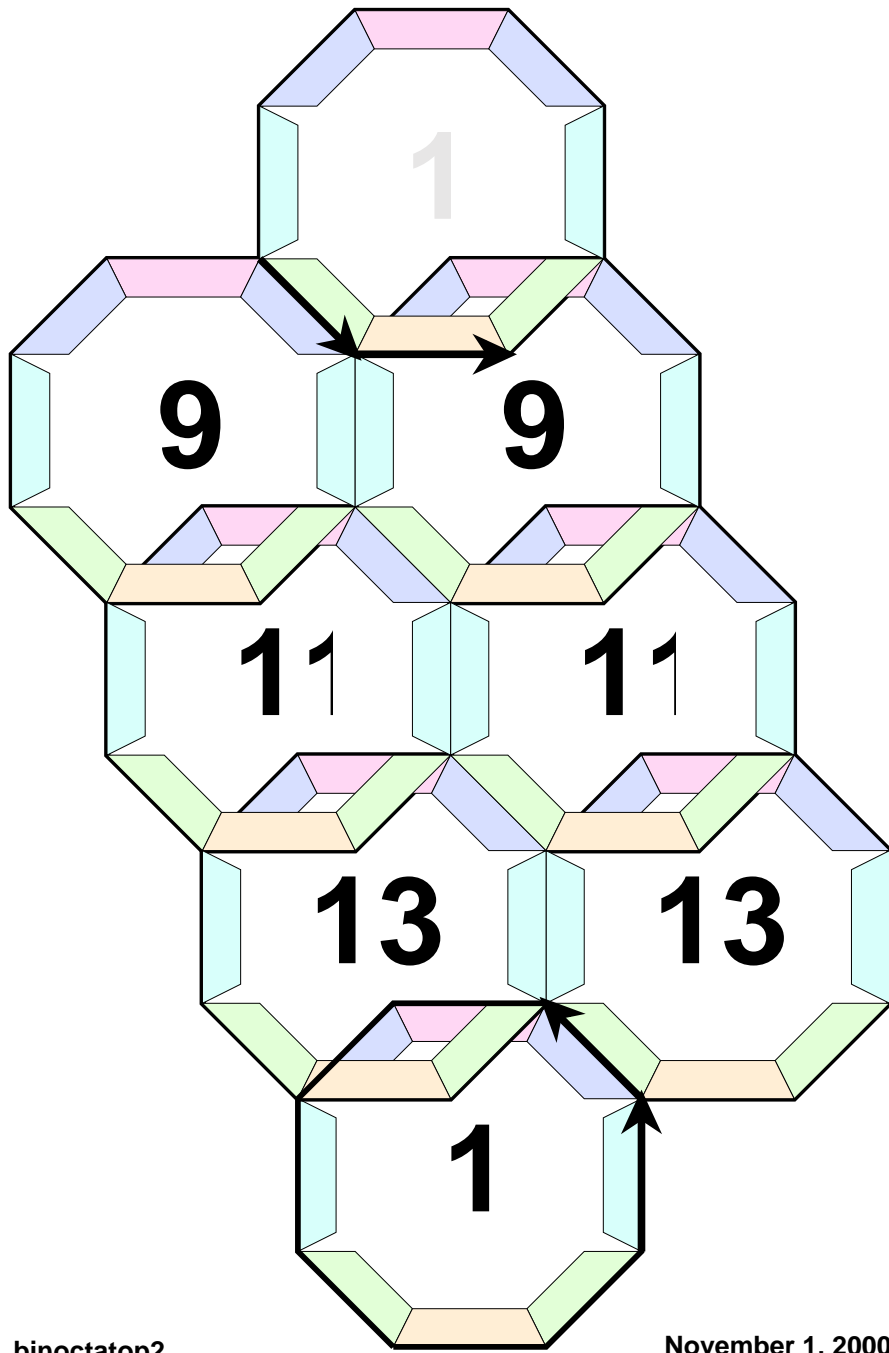


Figure 13: Top side of the second half of a binary octagonal flexagon, second sector of two.

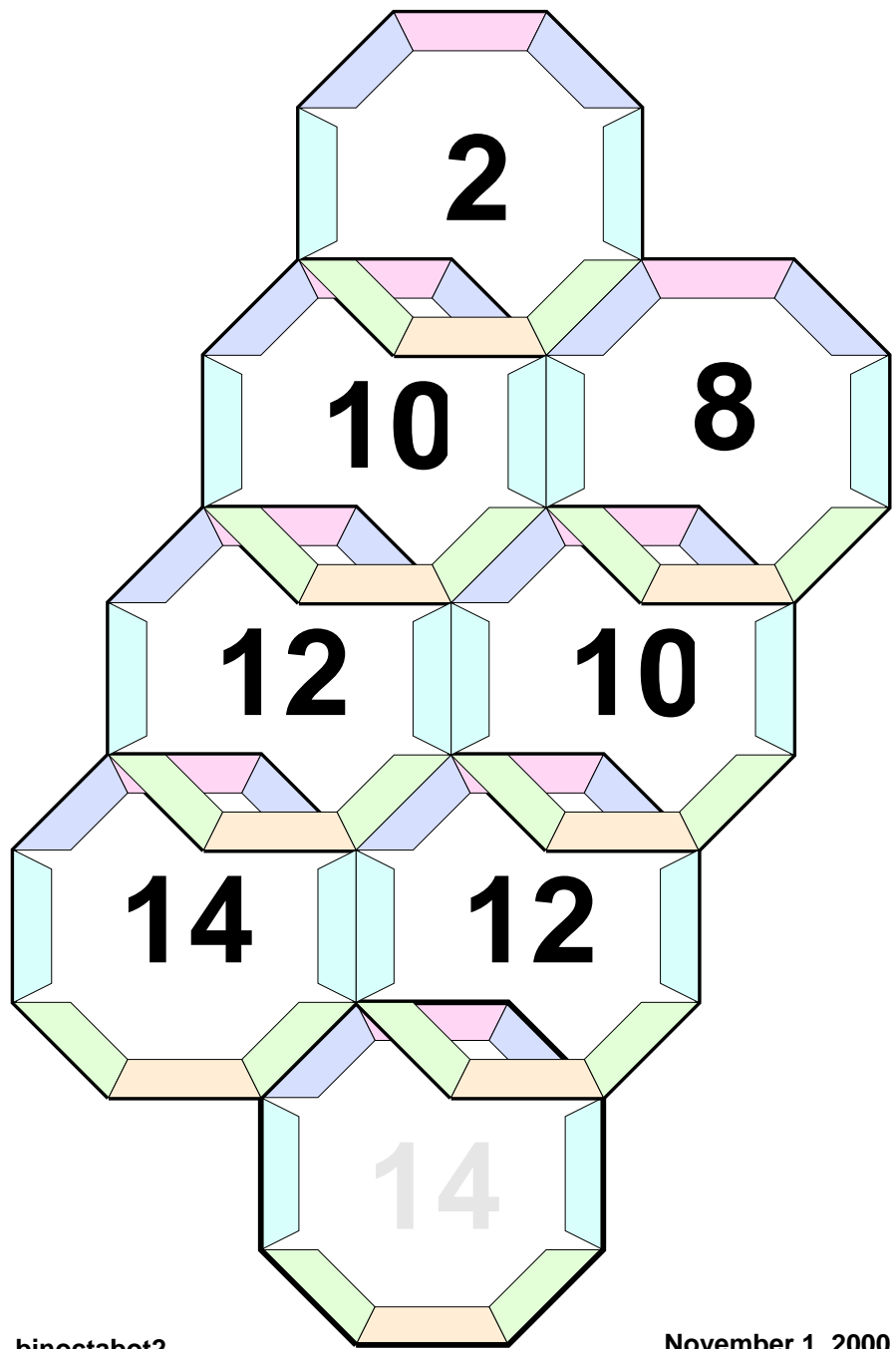
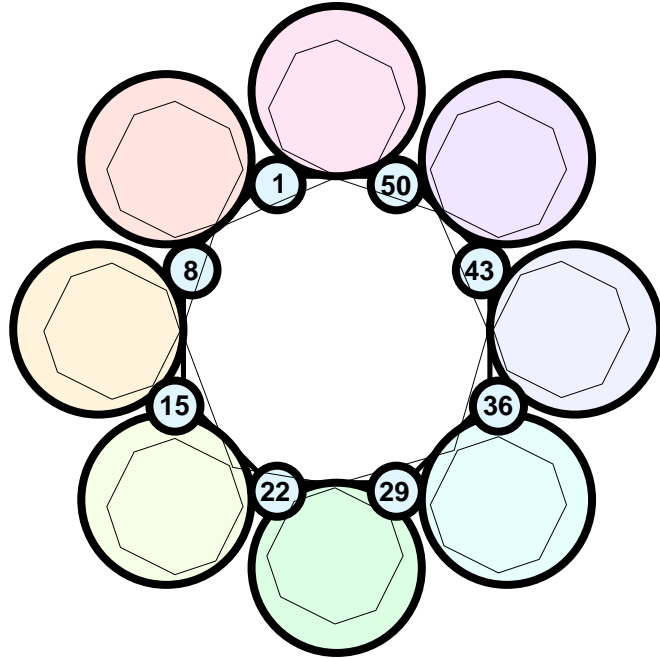


Figure 14: Bottom side of the second half of a binary octagonal flexagon, second sector of two.



## 4 Second Level Octagonal Flexagon



1	3	3	5	7	7	51	51	53	53	55	55	1	43	45	45	47	47	49	49	37	37	39	39	41	41	43	...		
2	2	4	4	6	6	8	50	52	52	54	54	56	56	44	44	46	46	48	48	50	36	38	38	40	40	42	42	...	
...	29	31	31	33	33	35	35	23	23	25	25	27	27	29	15	17	17	19	19	21	21	9	9	11	11	13	13	15	1
...	30	30	32	32	34	34	36	22	24	24	26	26	28	28	16	16	18	18	20	20	22	8	10	10	12	12	14	14	2

Figure 15: The second level octagonal flexagon has fifty six vertices.

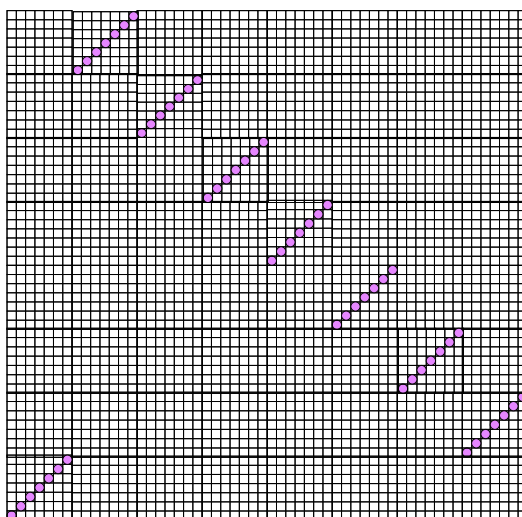


Figure 16: Permutation of the octagons along the strip for a second level octagonal flexagon.

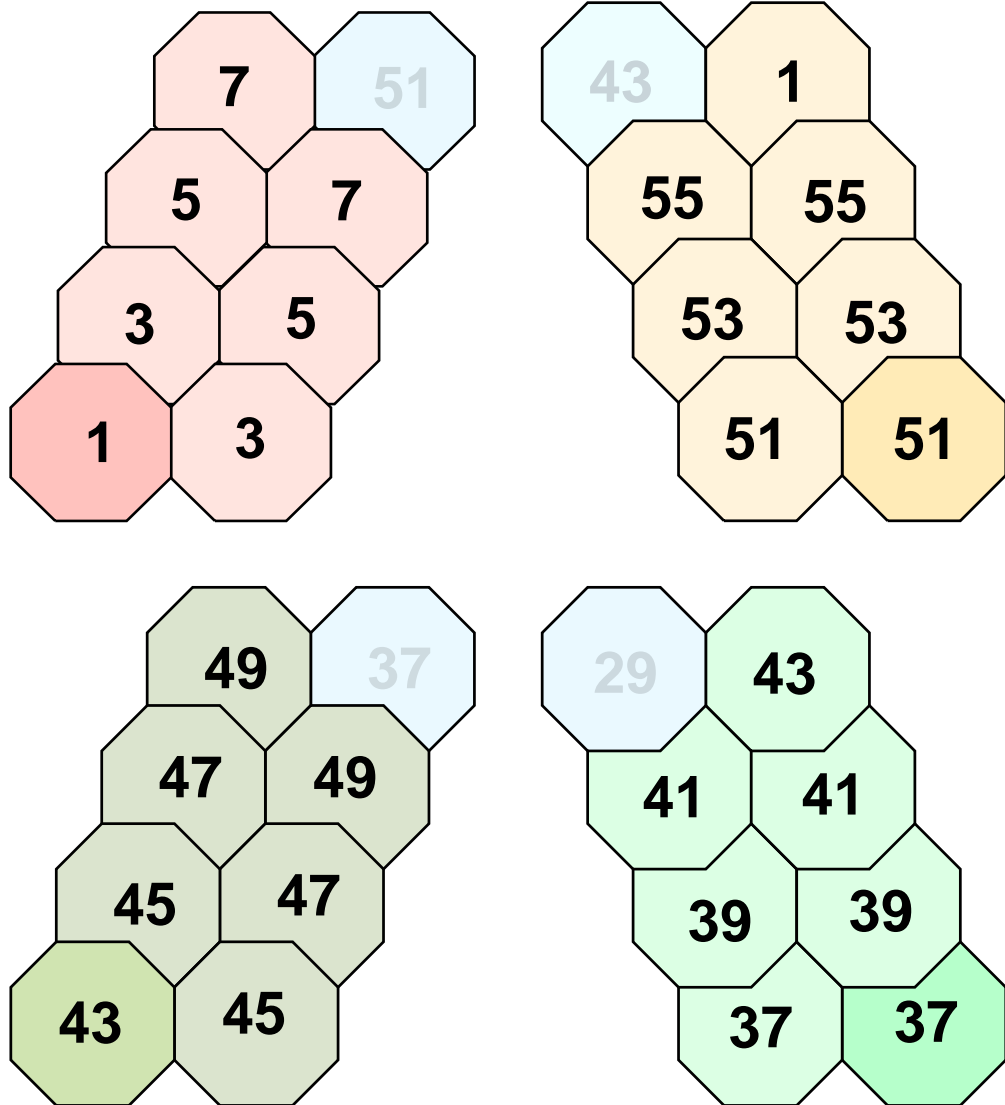


Figure 17: View of the top sides of the first four of eight segments from which the second level octagonal flexagon may be constructed.

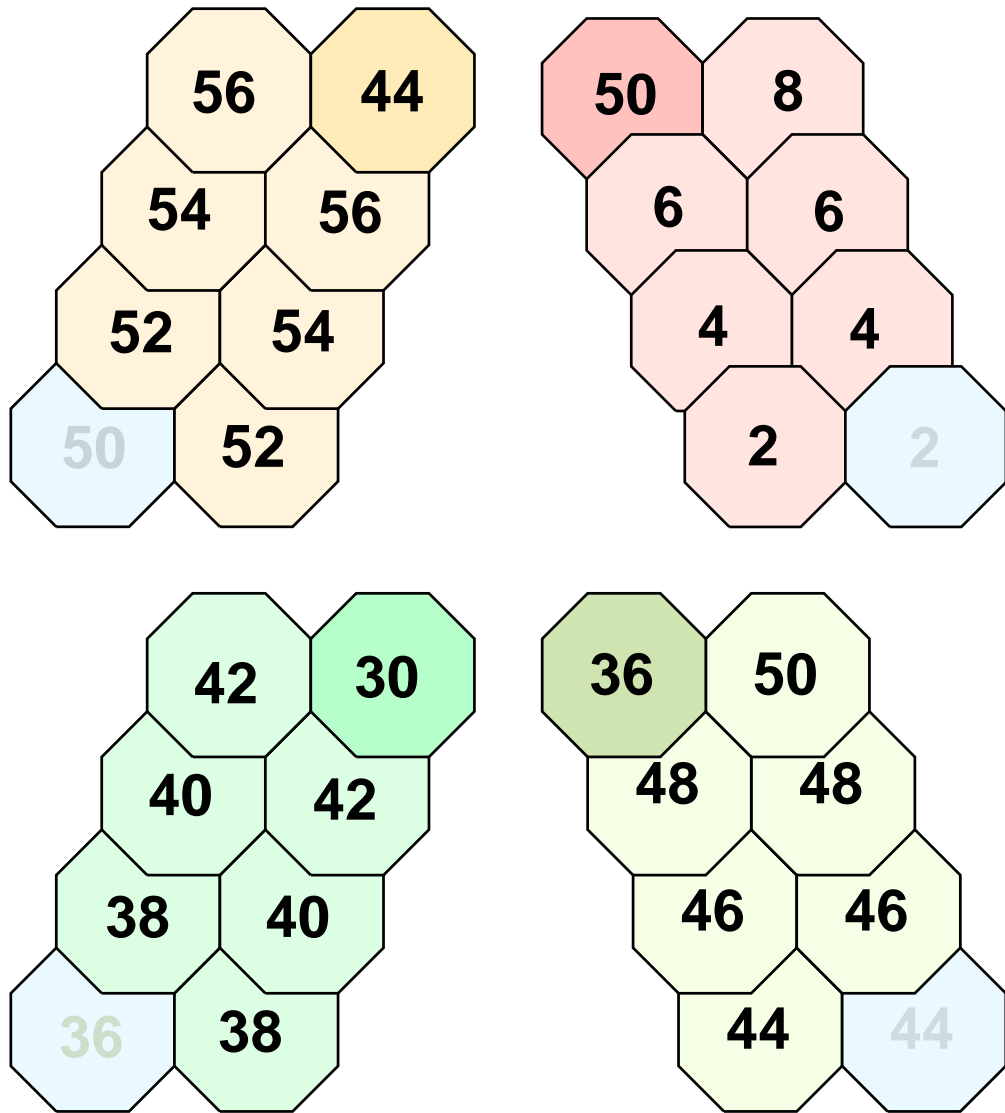


Figure 18: View of the bottom sides of the first four of eight segments from which the second level octagonal flexagon may be constructed.

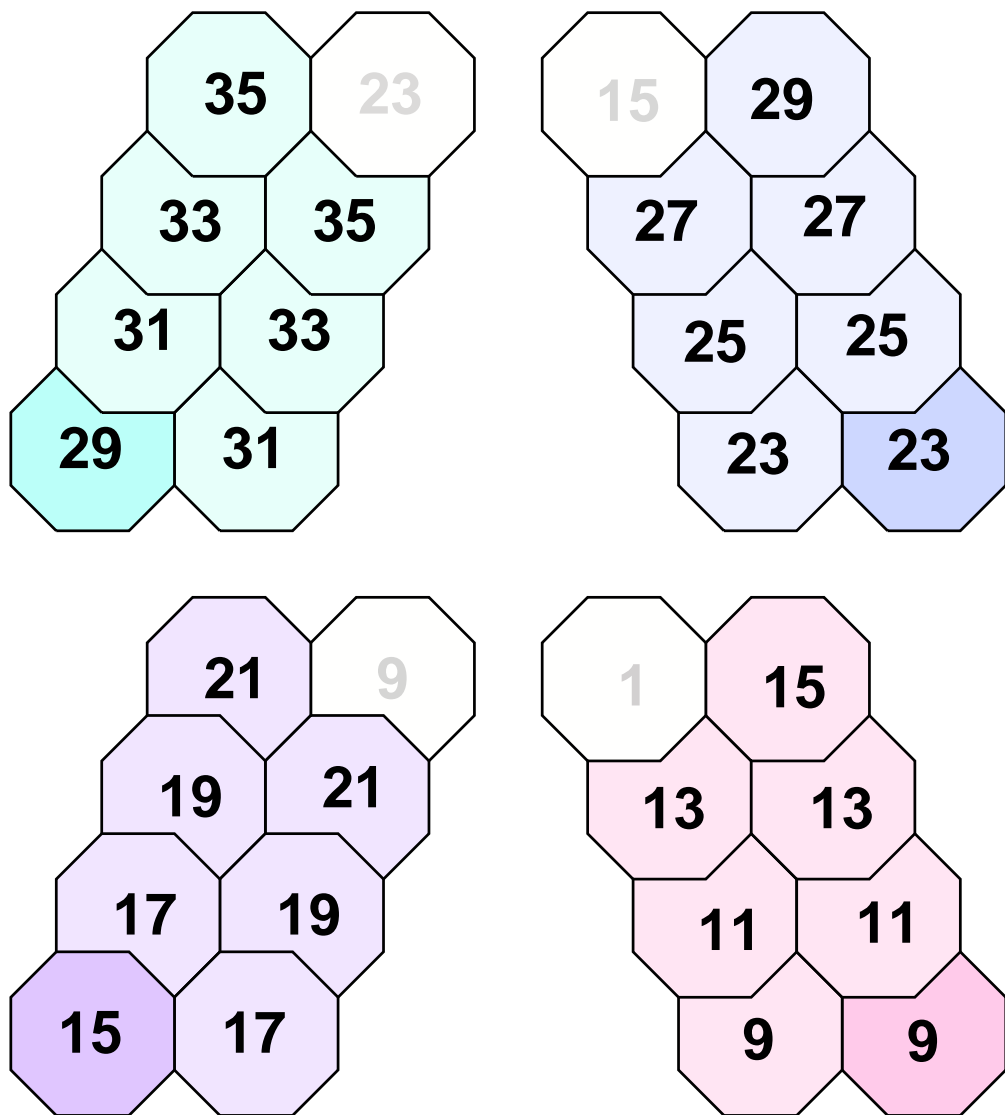


Figure 19: View of the top sides of the second four of eight segments from which the second level octagonal flexagon may be constructed.

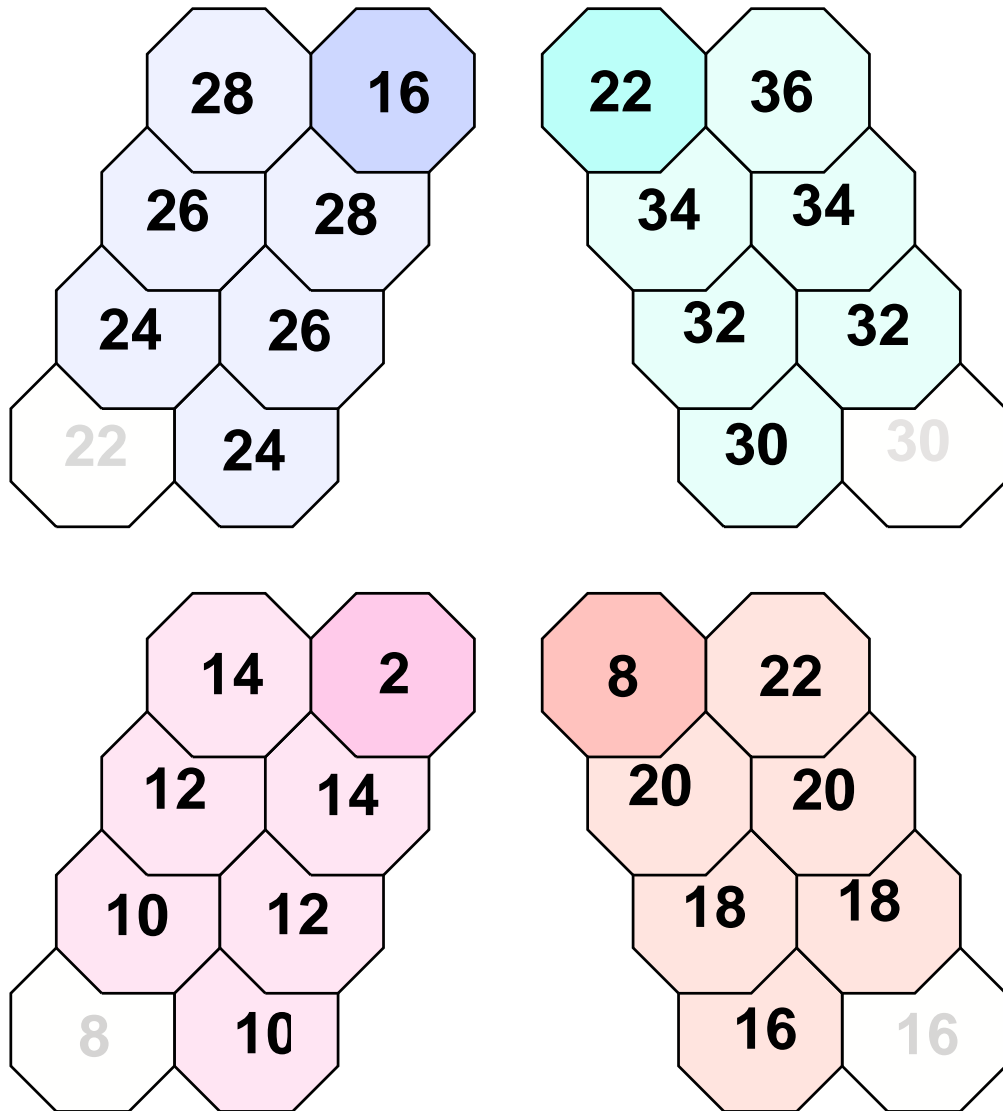


Figure 20: View of the bottom sides of the second four of eight segments from which the second level octagonal flexagon may be constructed.