

Phoenix

A Portfolio of Period 2, c/2,

Space Ships of Width

5, 6, 7, 8, 9, 10, 11, 12

and



Harold V.
McIntosh

Phoenix

A Phoenix is a Life configuration which each an every member of a given generation disappears; nevertheless they succeed in bringing forth a new generation, so preserving the population indefinitely.

Phoenix de Bruijn diagrams (introduction)

width 4

width 5

width 6

width 7

width 8

width 9

width 10

width 11

width 12

Phoenix de Bruijn diagrams

This current de Bruijn analysis began some time ago when someone asked about that huge blinker which appeared on the front cover of Scientific American along with one of Martin Gardner's columns about Life. The immediate reaction was that they were something of a triviality, given that the two tiles (shall we call them a and b?)

```

1 .   1 .
2 2   2 2
1 1 and 1 1
2 .   . 2

```

could be strung out into chains, and could easily turn corners. Such a configuration is called a Phoenix, for the obvious reason that each generation disappears completely as it generates its successor. There are some other things which fulfill the requirement; many of them connect to figures which remain constant from generation to generation. The barber pole is an example of one of them.

The phoenix condition is restrictive enough that the de Bruijn diagrams for some narrow strips and low periods could be calculated on a PC. When Gerardo Cisneros worked out the spaceships, he also extended the phoenix results, but their analysis was left until after the examination of the spaceships. Now it is their turn; here are some results for freestanding figures. Quiescence at infinity in one direction is imposed from the outset, but in the orthogonal direction it depends on whether the zero tile is an isolated component or not.

width of nonzero column	number of nodes	number of links	consolidated nodes	consolidated components
-----	-----	-----	-----	-----
1,2,3	1	1	1	1
4	17	17	3	3
5	33	33	5	5
6	71	79	14	6
7	145	171	55	4
8	337	411	137	10
9	818	970	..	44
10	1845	2187	..	98
11	4556	5640	..	91
12	11229	14607	..	50

Having analyzed the spaceship diagrams in considerable detail, it is interesting to look at the similarities and differences between the two systems.

In both cases, the minimum width is 4, but one of the first points for conjecture concerns its value in general. Unless the diagram is forbidden - for example, by the first level having no links to zero, or there exists an independent proof that the figures do not exist - such a width is to be expected. That it is not too small is reasonable, and there is a (generally) extravagant upper limit (the maximum cycle length in the first level diagram).

For spaceships, the smallest figure had three components; that zero was one of them implied that there was no configuration of finite vertical extent. The same is true here; as with the spaceship, each generation belongs to its own component. Unlike the spaceship, for which each component is symmetric by vertical reflection, the two generations, strings of a tiles and their vertical reflections, are mutual reflections, as is evident from examining the tiles displayed above.

At width 5, the only variation possible is the centering of the filaments of width 5, since they cannot be bridged.

Two possibilities arise at width 6 --- the b tile can be used to shift the filament over by TWO cells, and two new tiles can be used to line up a tiles against the outer walls of the strip. There are three centerings for the a- and a'- filaments, but the center one admits of no variation. Consequently there are six components: zero is still isolated, alpha and beta respectively hold the filaments hugging the walls and crossing over, two for the generations of the central filament, and a gamma component.

At width 7 there are only four components - zero (1 consolidated node), alpha and beta, (with 14 consolidated nodes each) and gamma (with 26 consolidated nodes).

Width 8 is the significant width, of course, because it is the smallest width for which the zero tile is no longer isolated; this is a consequence of the formation of a ring of four a-tiles, but unfortunately there is still not enough wriggle room for attaching anything else to the ring - for example, to extend its vertical edges. Such rings do have a compressed version, already visible at width 7, which fits sideways into the strips, since it can be stabilized by connecting it to filaments, which can eventually terminate on a mirror image or something else suitable. But nothing in the assemblage is yet sufficient to make it freestanding.

However, it requires width 10 to get a zero-component with any appreciable versatility.

Width 8 also accomodates parallel avatar filaments, if they are packed just right; there are three packings of two generations each, for a total of 6 isolated components; together with zero, alpha, beta, and gamma (which are enlargements of their respective themes) making a total of 10.

Giving the possibility of twisting and turning, notably of the a-tiles abetted by the b-tiles, the phoenix avatar structure is not so pronounced as for the spaceships. Nevertheless it is readily visible and exerts a strong influence on the structure of the diagrams and the physical appearance of a phoenix.

At widths of eight and greater, the packing of multiple filaments, and the arbitrariness of their placement with respect to one another generates a myriad of components. In particular, filaments 4 wide can be stacked in 3 different ways without interference. Thus width 8 has 6 isolated components, 3 each for the two generations, which are pure cycles of the 8 joined pairs of a-tiles and their reflection. For width 14, there will be 18, 9 each for the generations is which three filements stretch alongside each other, and so on.

Slightly wider strips allow more relative displacements, up to all 8 possible shifts; but at the same time shifting and joining become possible, resulting that the components no longer consist of a single cycle, and thus become less and less isolated.

Doubtless the same phenomonon occurs for spaceships, but for them it was only beginning to manifest itself at the widest strips which it was possible to examine.

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Width 11

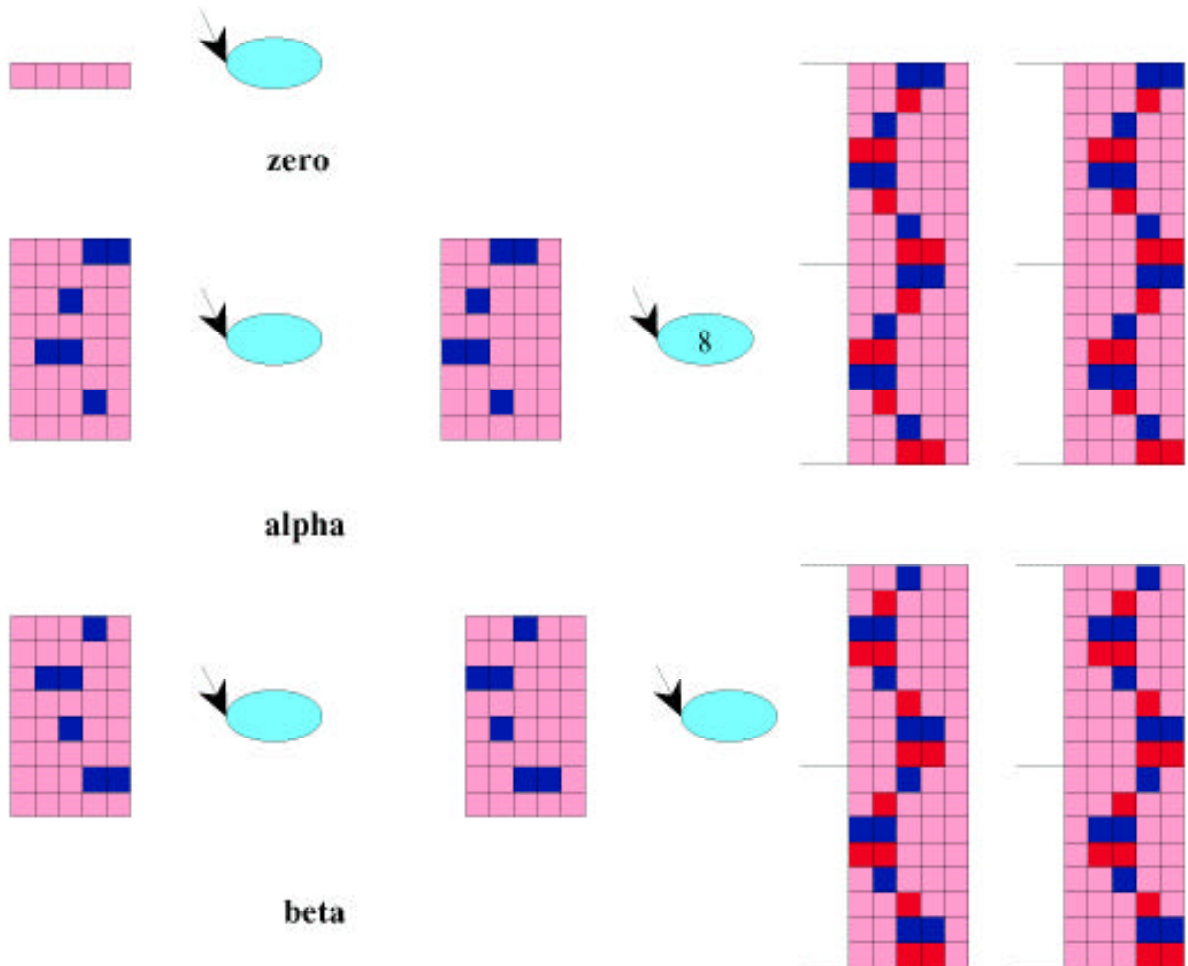
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Life phoenix de Bruijn diagram, width 5

5 tiles

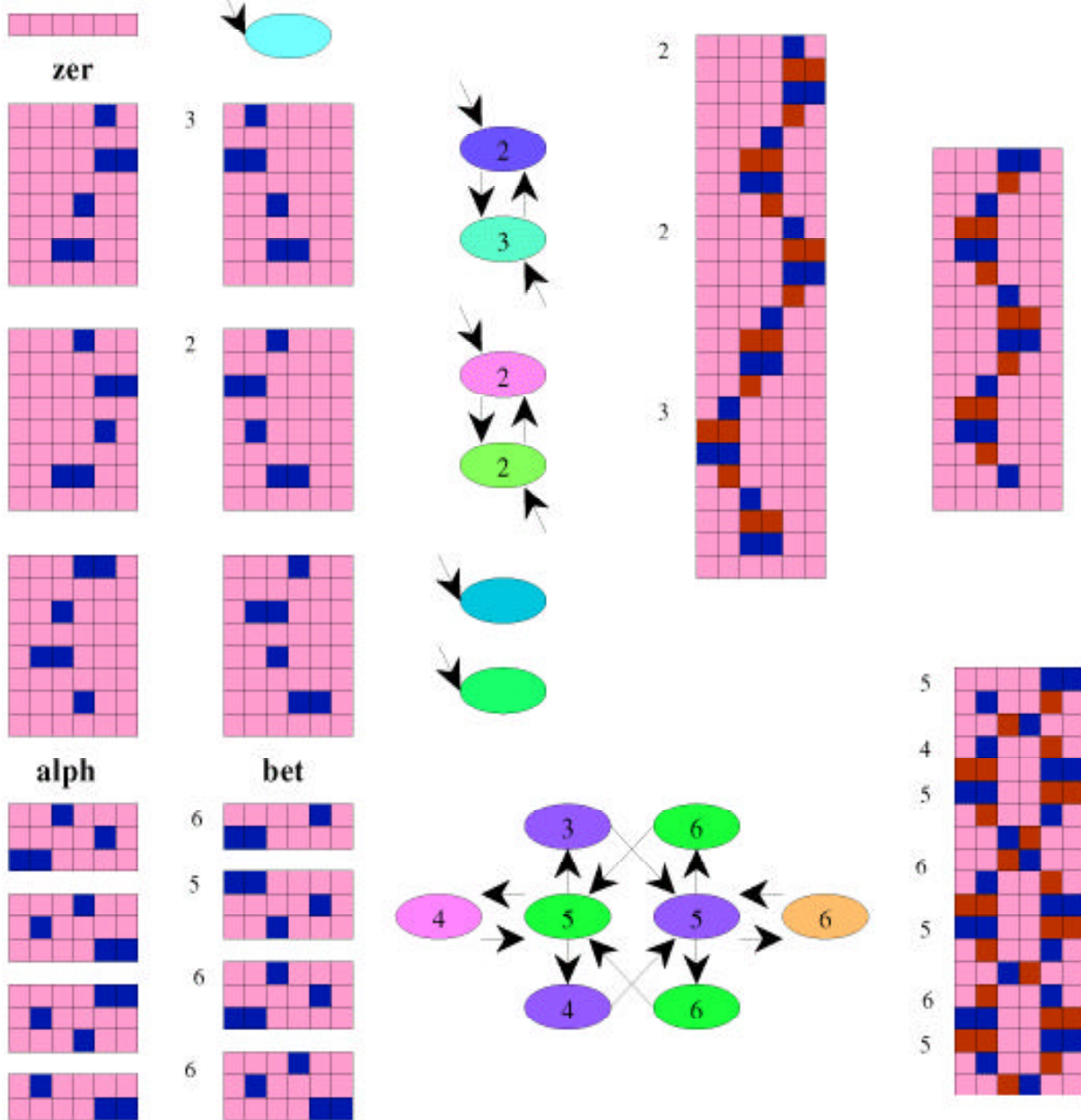


NODES	33
LINKS	33
COMPONENTS	5

Maximum Linkage	1
Consolidated nodes	5
Consolidated links	5

COMPONENT	NODES	LINKS
ZERO	1	1
ALPHA	1	1

Life phoenix de Bruijn diagram, width 6 15 tiles

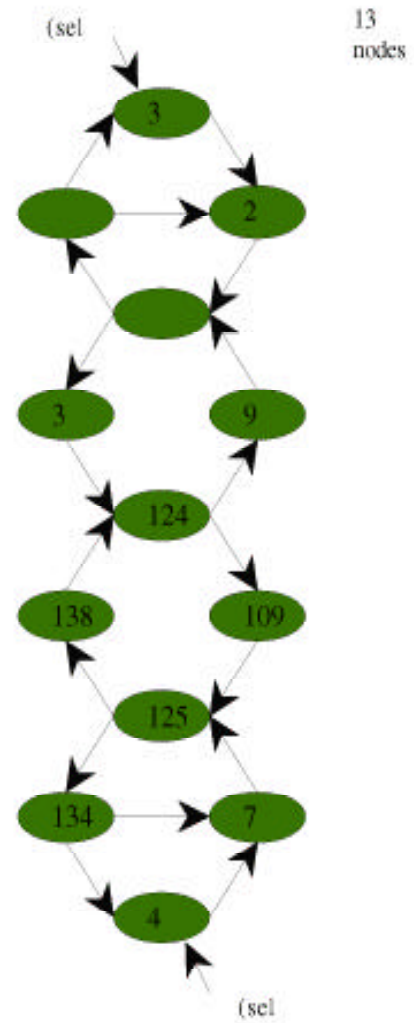
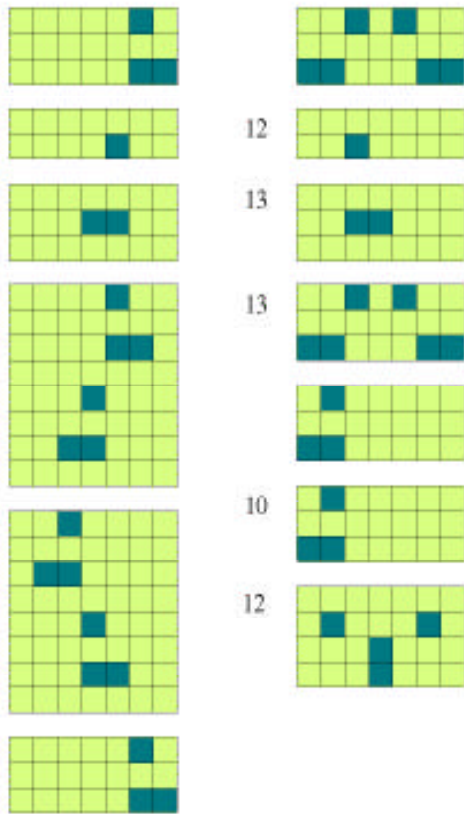


NODES	71
LINKS	79
COMPONENTS	6

Maximum Linkage	3
Consolidated nodes	15
Consolidated links	19

COMPONENT	NODES	LINKS
ZERO	1	1
ALPHA	1	1

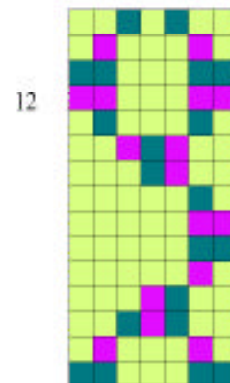
Life phoenix de Bruijn diagram, width 7 alpha component



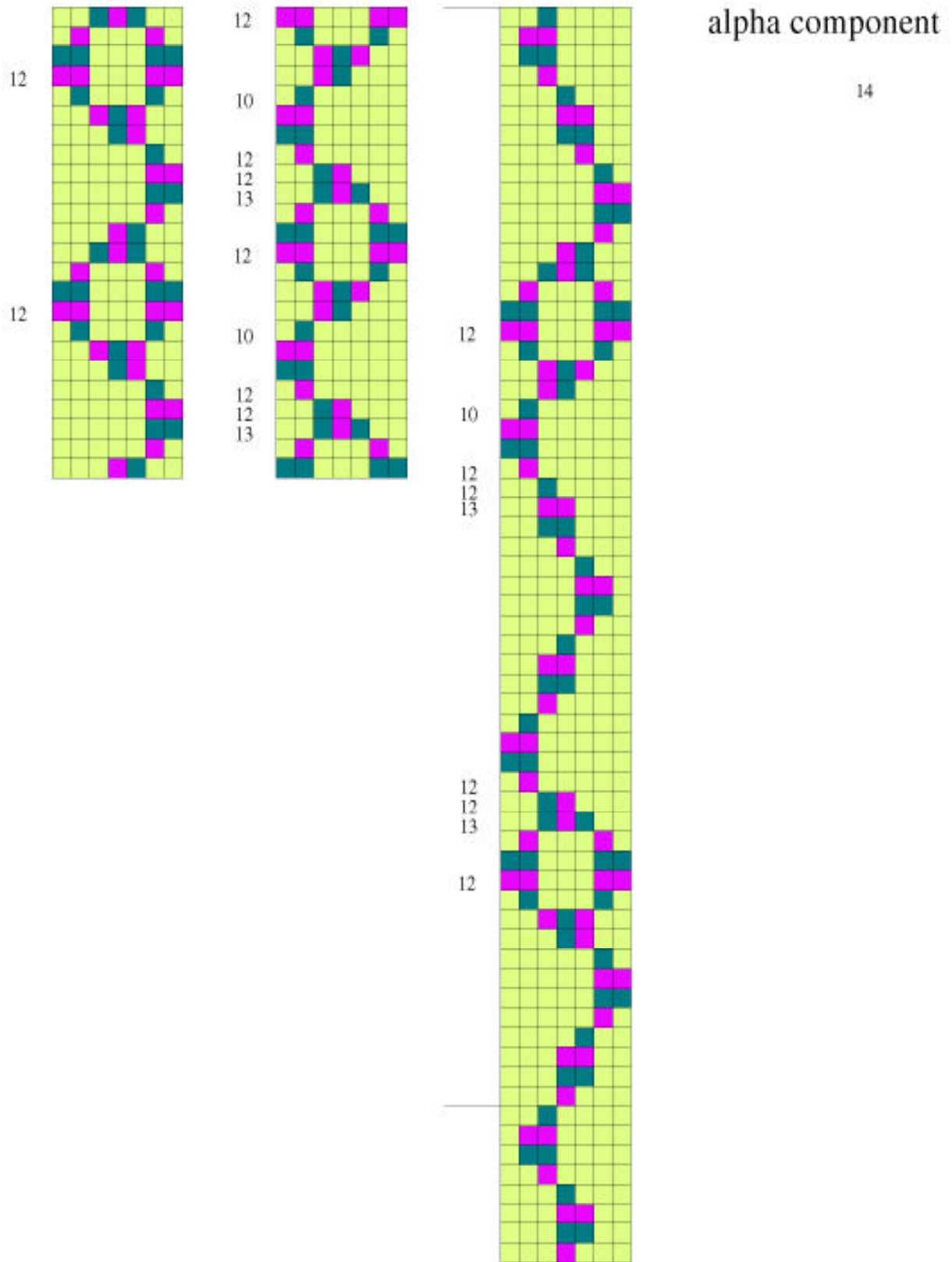
NODES	145
LINKS	171
COMPONENTS	4

Maximum Linkage	3
Consolidated nodes	5
Consolidated links	78

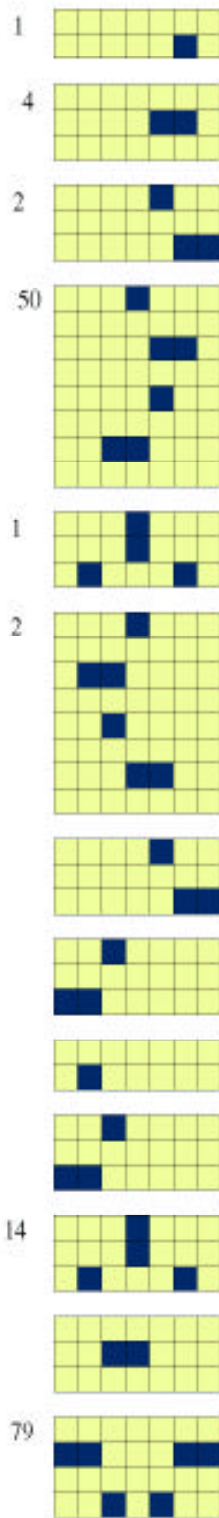
COMPONENT	NODES	LINKS
ALPHA	13	20



Life phoenix de Bruijn diagram, width 7

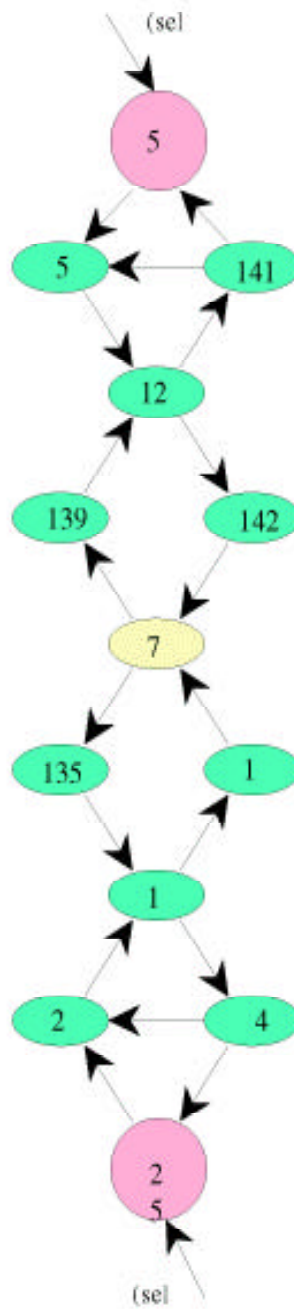


Life phoenix de Bruijn diagram, width 7



beta component

13 tiles
20 links



The beta component is the phase alternate of the alpha component.

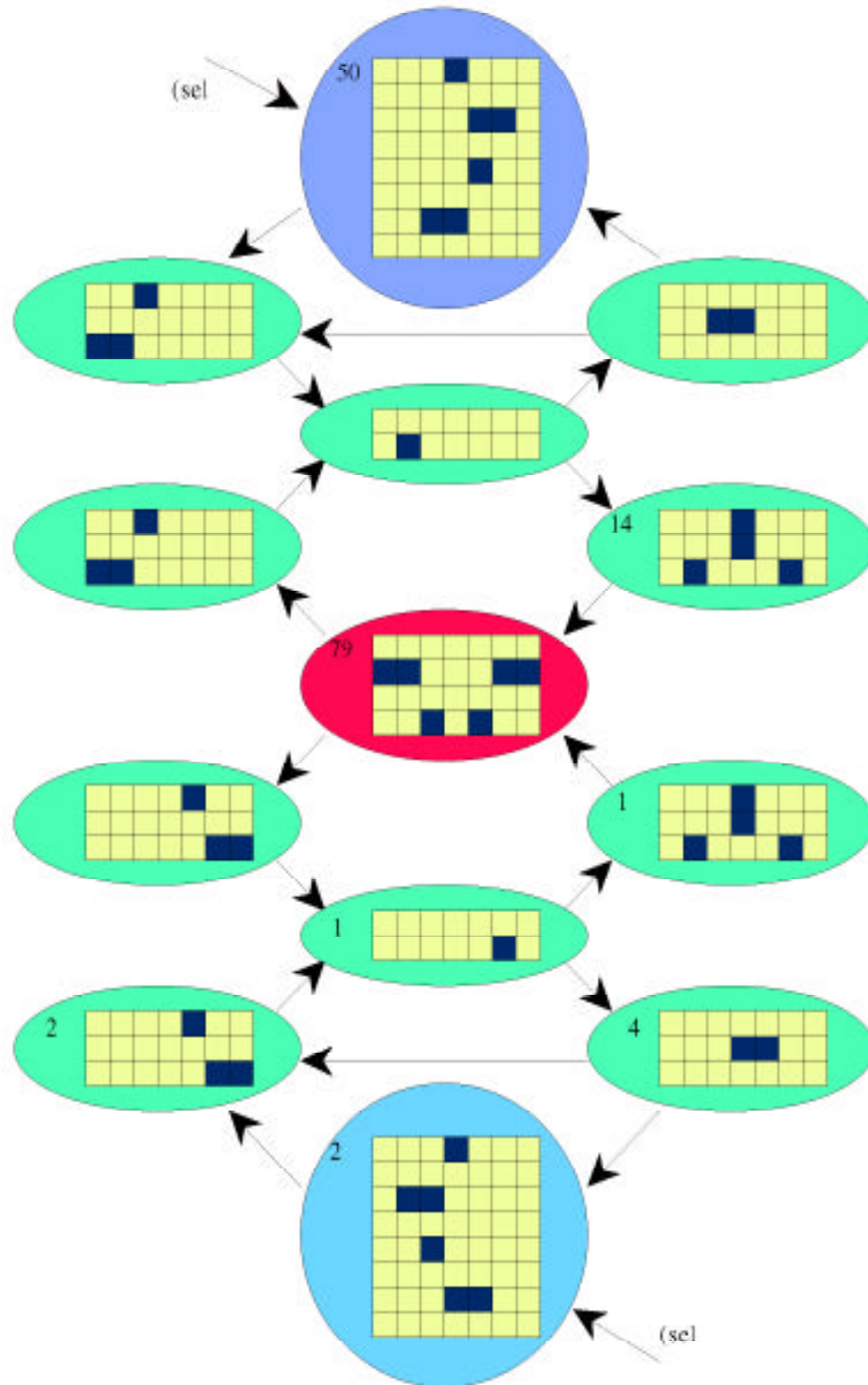
NODES	145
LINKS	171
COMPONENTS	4

Maximum Linkage	3
Consolidated nodes	5
Consolidated links	78

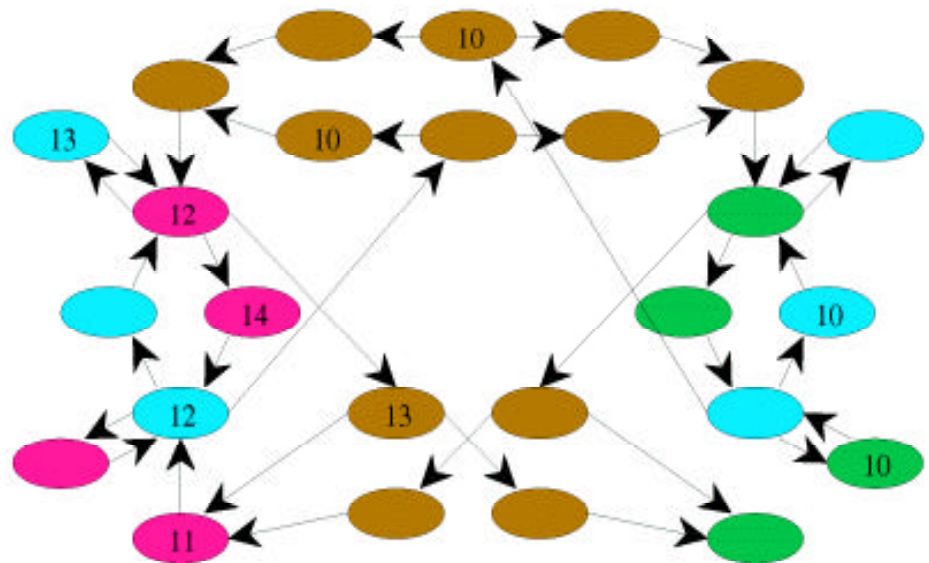
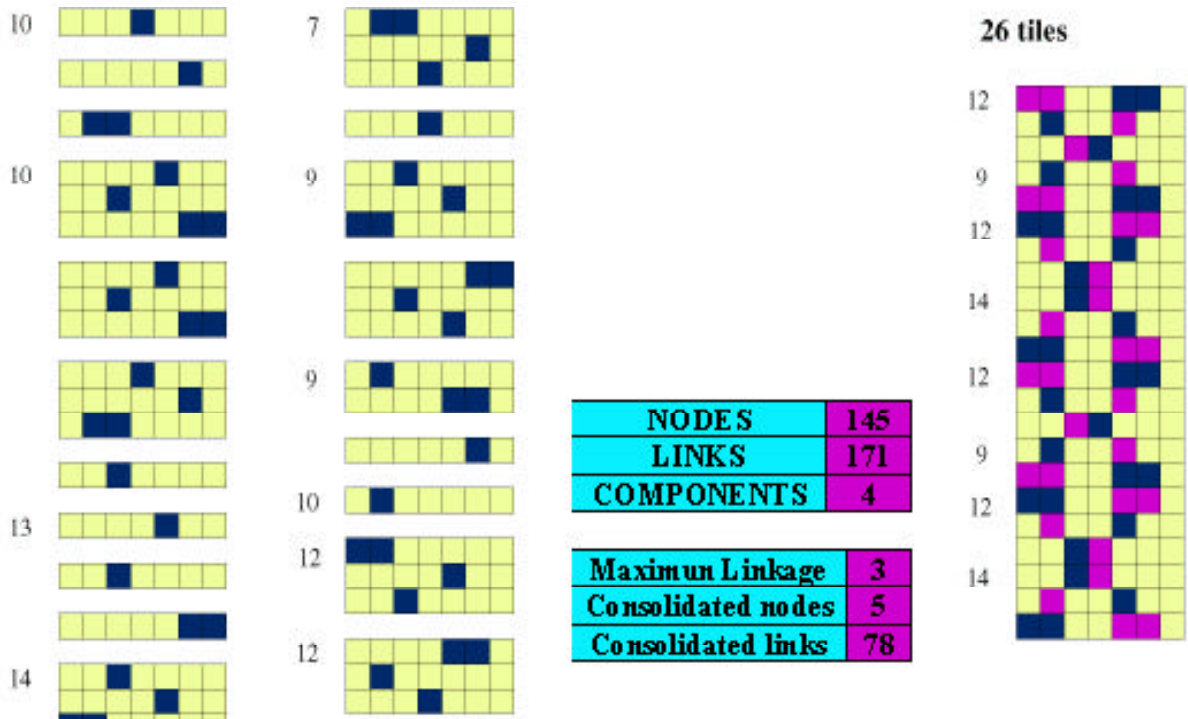
COMPONENT	NODES	LINKS
BETA	13	20

Life phoenix de Bruijn diagram, width 7 beta component

13 tiles
20 links



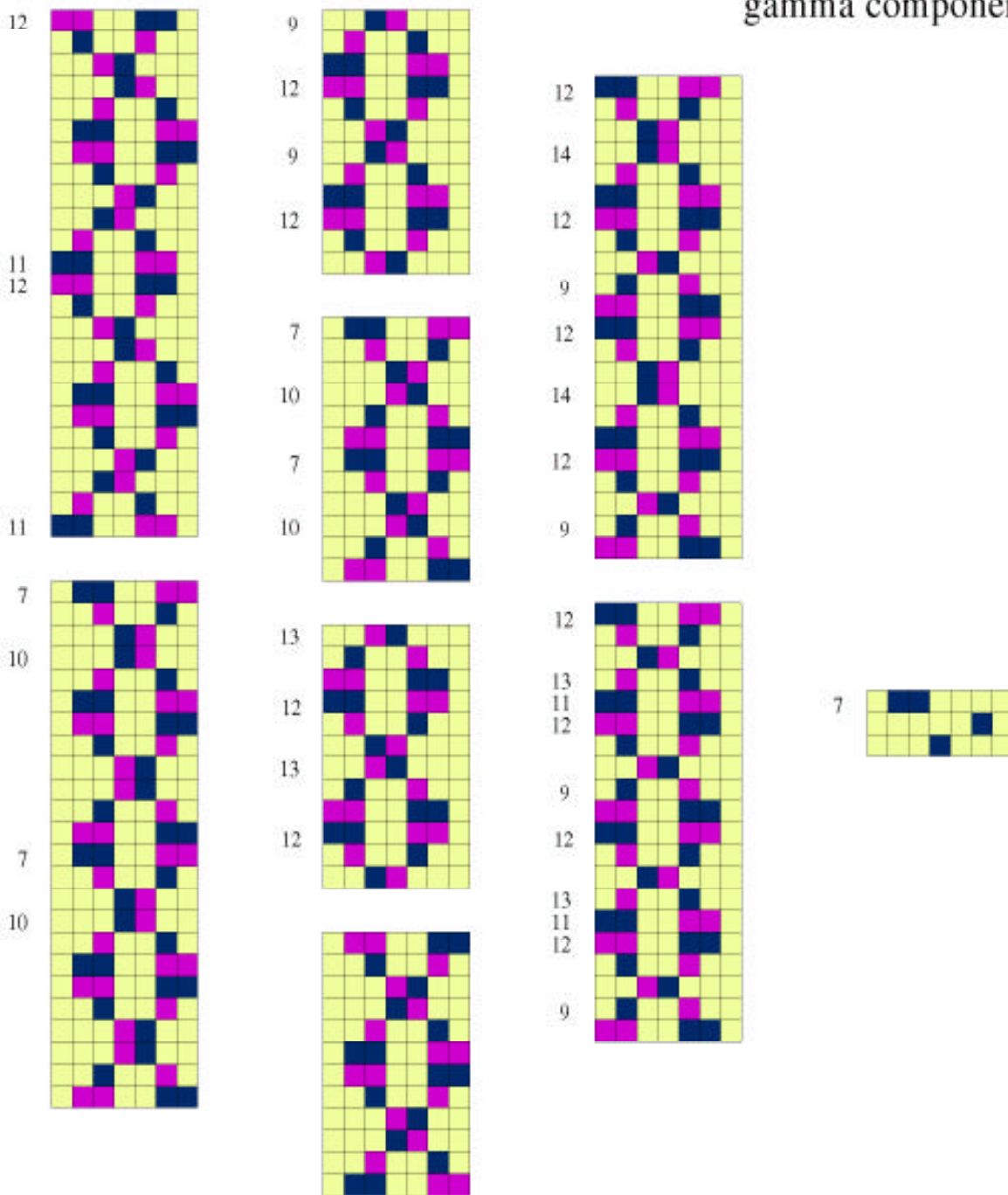
Life phoenix de Bruijn diagram, width 7 gamma component



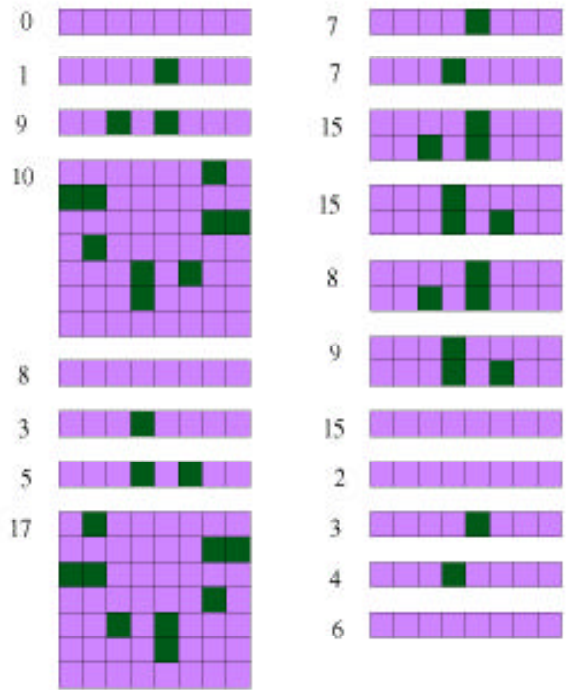
COMPONENT	NODES	LINKS
GAMMA	26	37

Life phoenix de Bruijn diagram, width 7

gamma component



Life Phoenix de Bruijn diagram, width 8



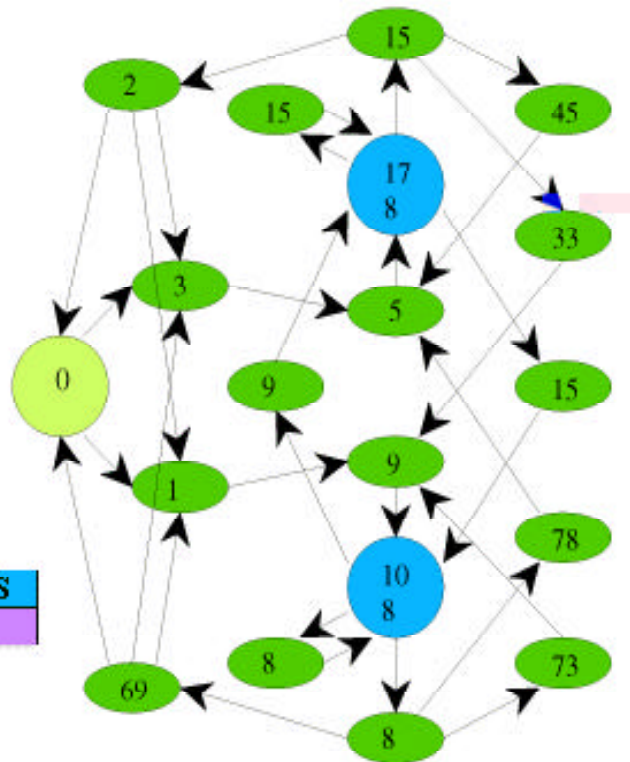
zero component

19
nodes

NODES	337
LINKS	411
COMPONENTS	10

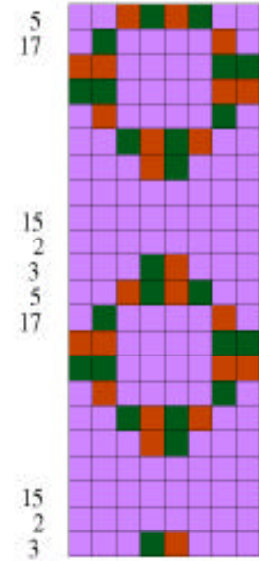
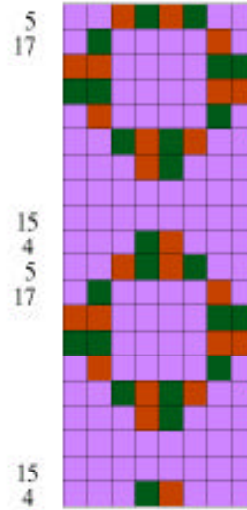
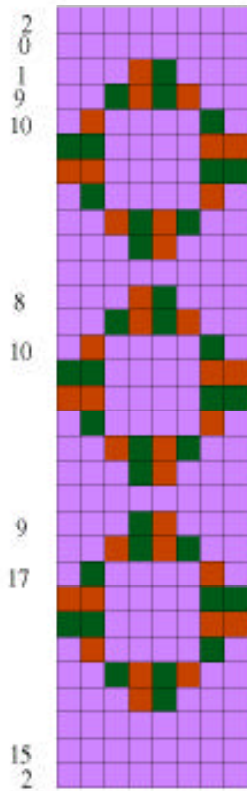
Maximum Linkage	3
Consolidated nodes	141
Consolidated links	211

COMPONENT	NODES	LINKS
ZERO	19	33



Life Phoenix de Bruijn diagram, width 8

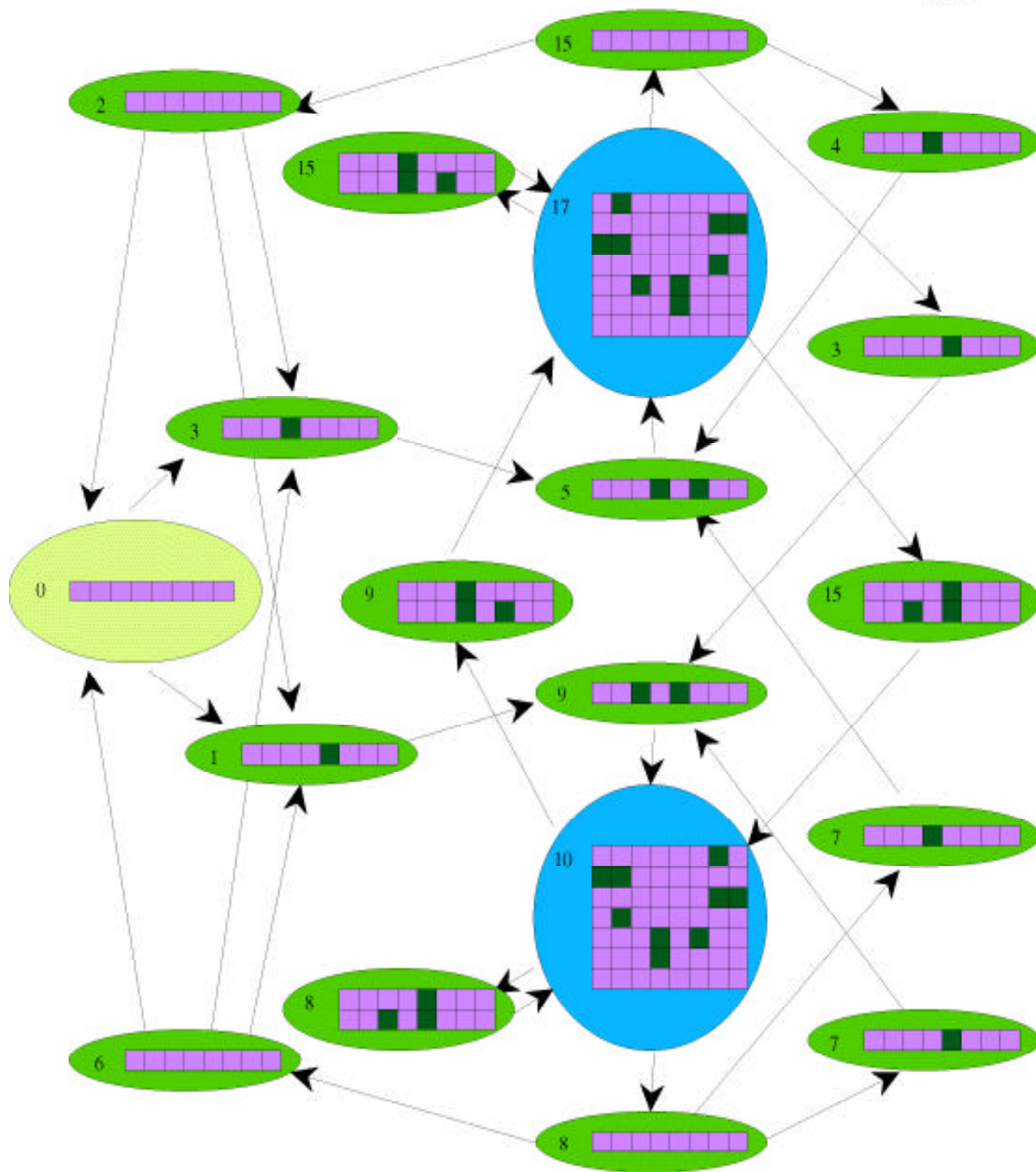
zero component



Life Phoenix de Bruijn diagram, width 8

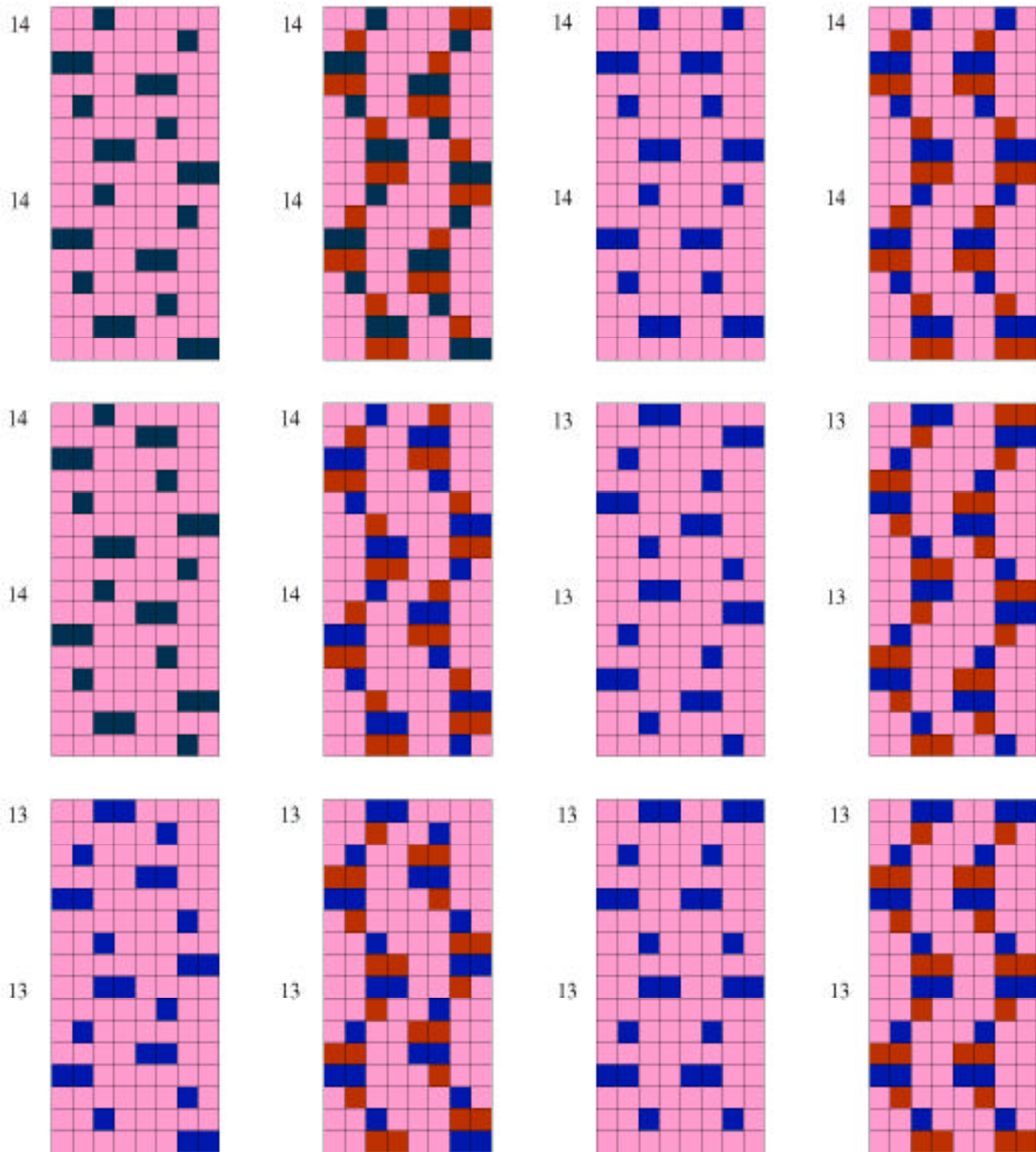
zero component

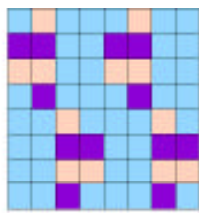
19
nodes



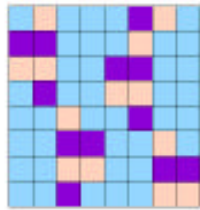
Life phoenix de Bruijn diagram, width 8

6 isolated components

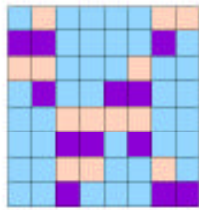




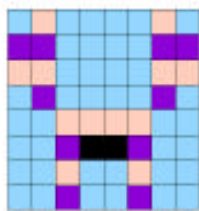
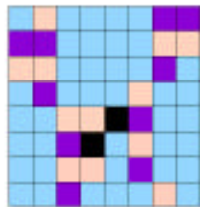
(parallel)



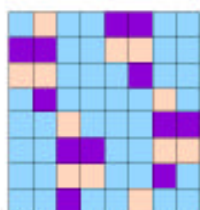
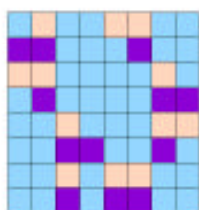
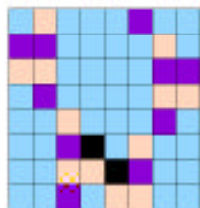
(displaced)



Only three alignments of pairs of alpha strands can

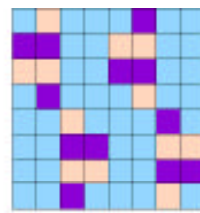


make a phoenix; they constitute isolated

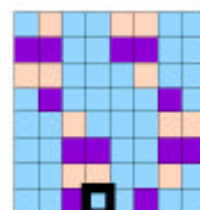
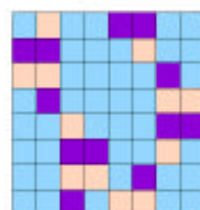
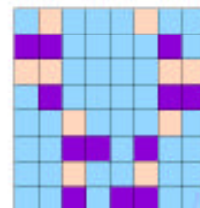
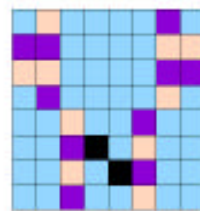
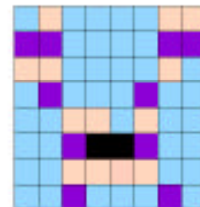
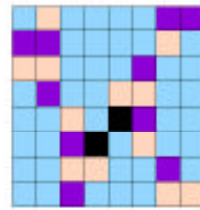
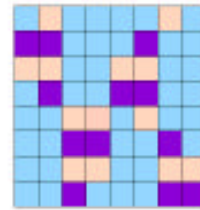


(displaced upward)

phh8.draw



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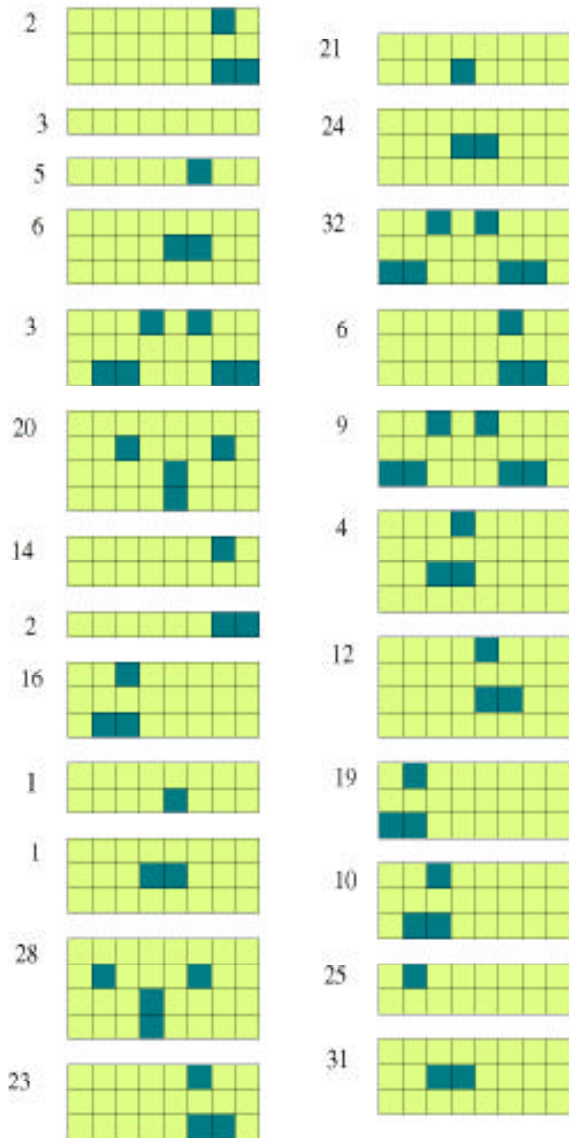
Life phoenix de Bruijn diagram, width 8

Only one alignment of an alpha strand with a beta strand can make a phoenix; such a pair becomes one of the gamma strands

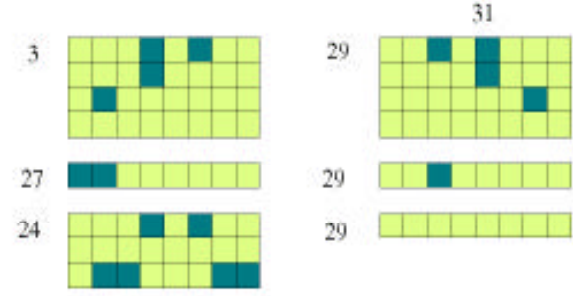
November 25, 1996

Life Phoenix de Bruijn diagram, width 8

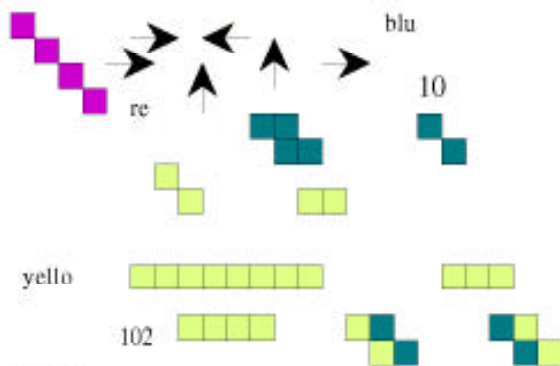
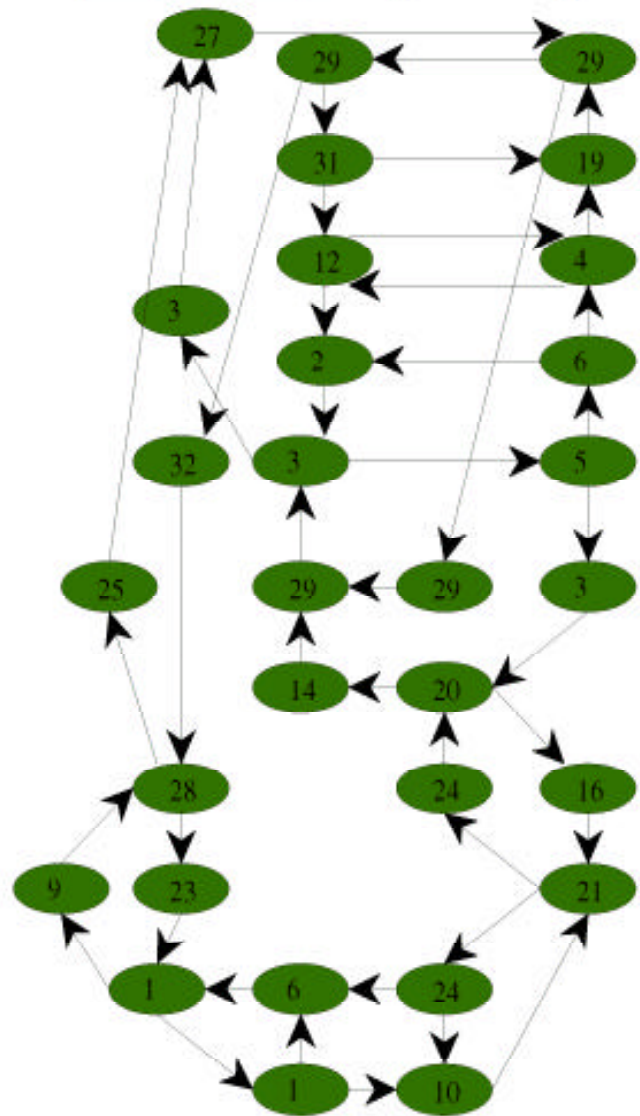
alpha component



NODES	337	Maximum Linkage	3
LINKS	411	Consolidated nodes	141
COMPONENTS	10	Consolidated links	211



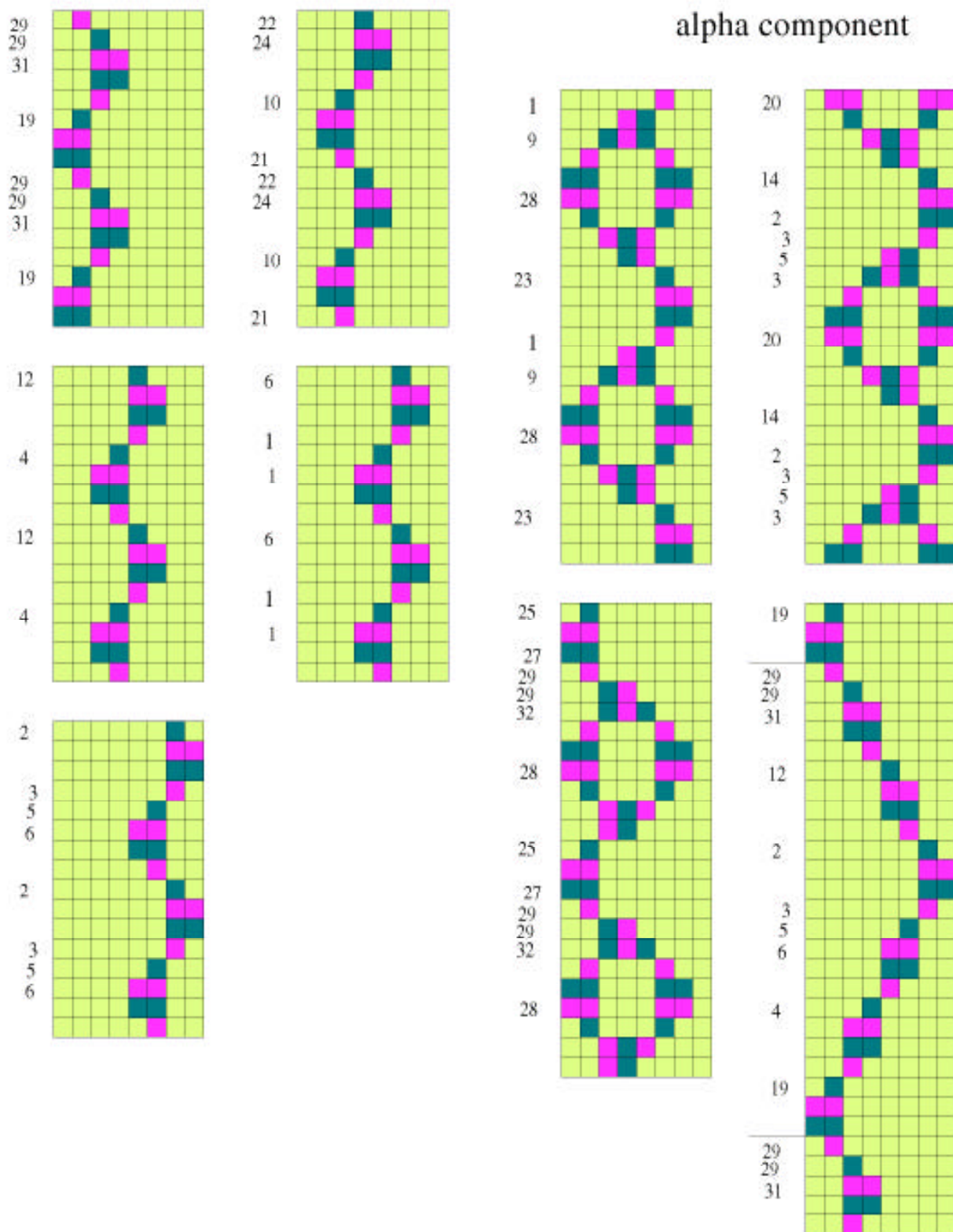
COMPONENT	NODES	LINKS
ALPHA	30	43



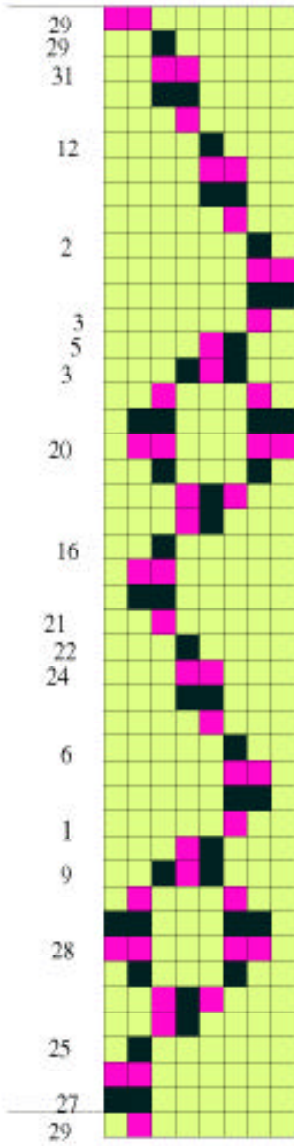
pi8.draw

February 11, 1994

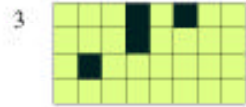
Life Phoenix de Bruijn diagram, width 8



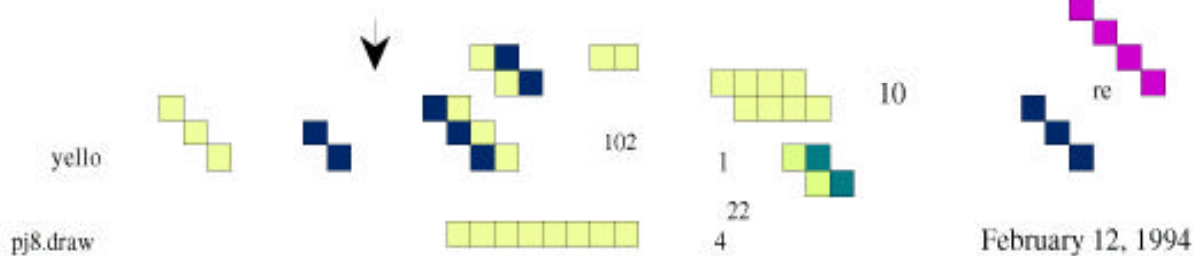
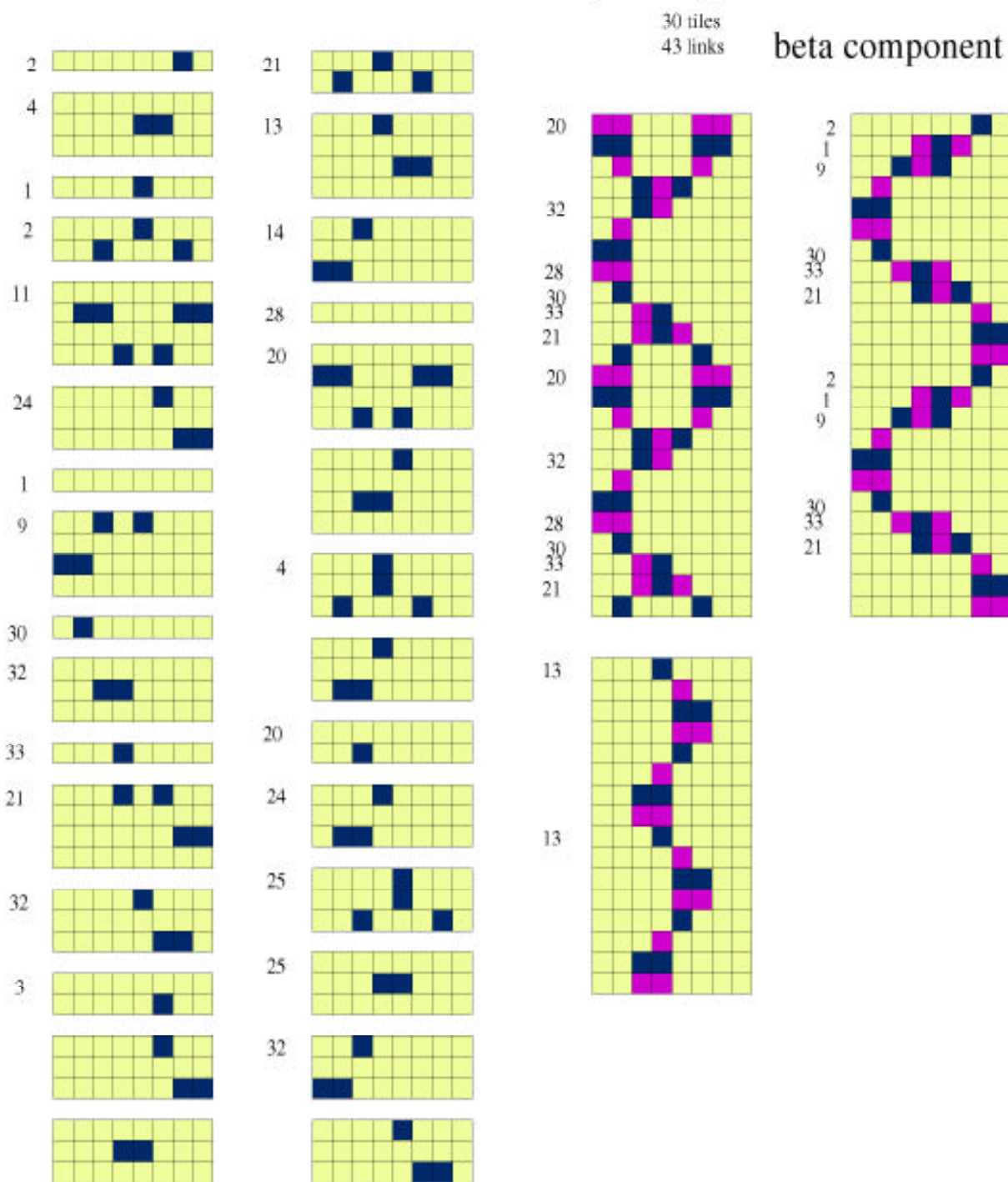
Life Phoenix de Bruijn diagram, width 8



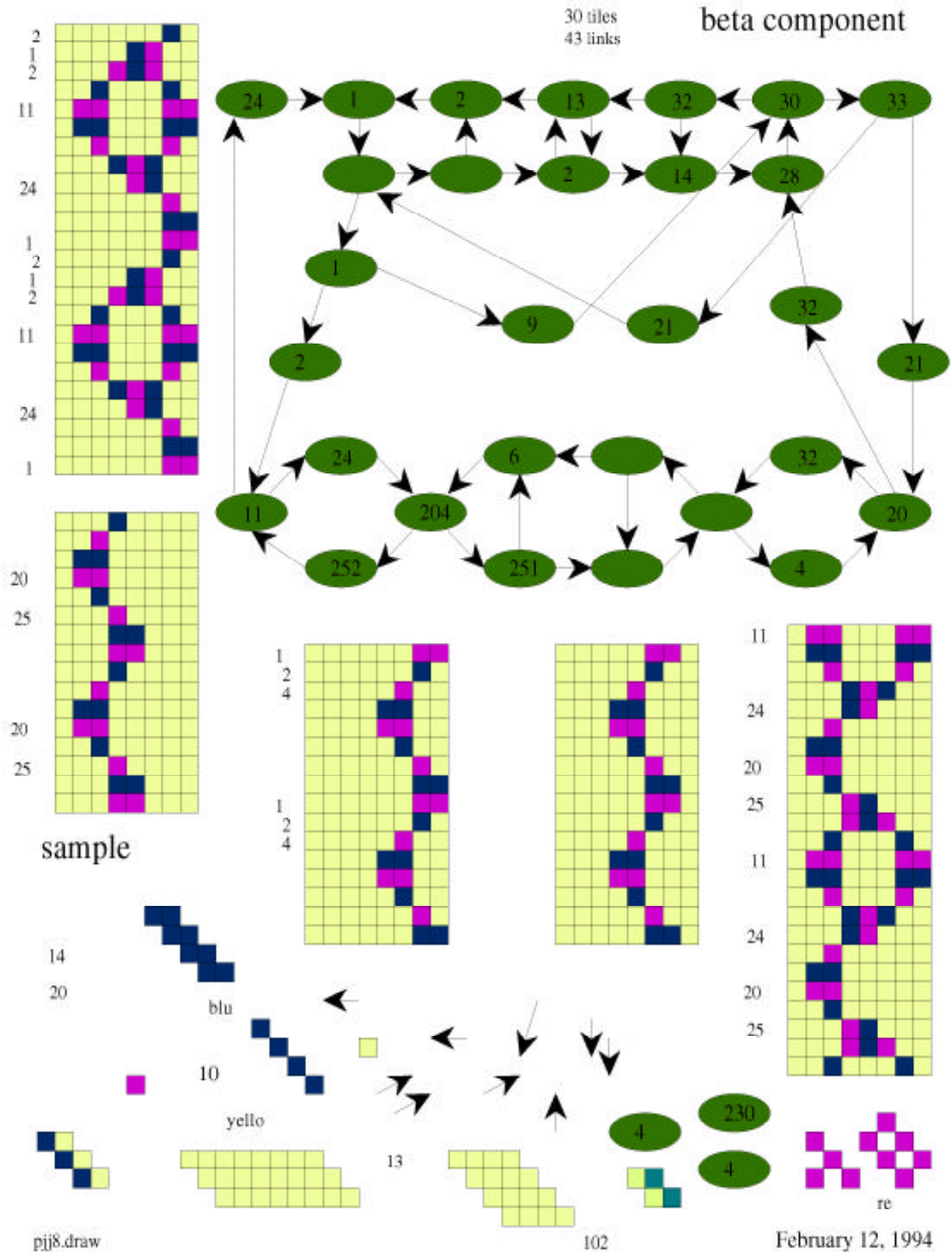
alpha component



Life Phoenix de Bruijn diagram, width 8

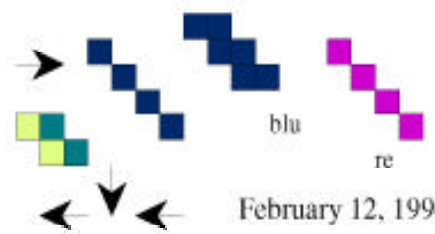
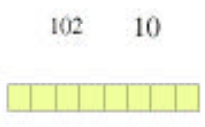
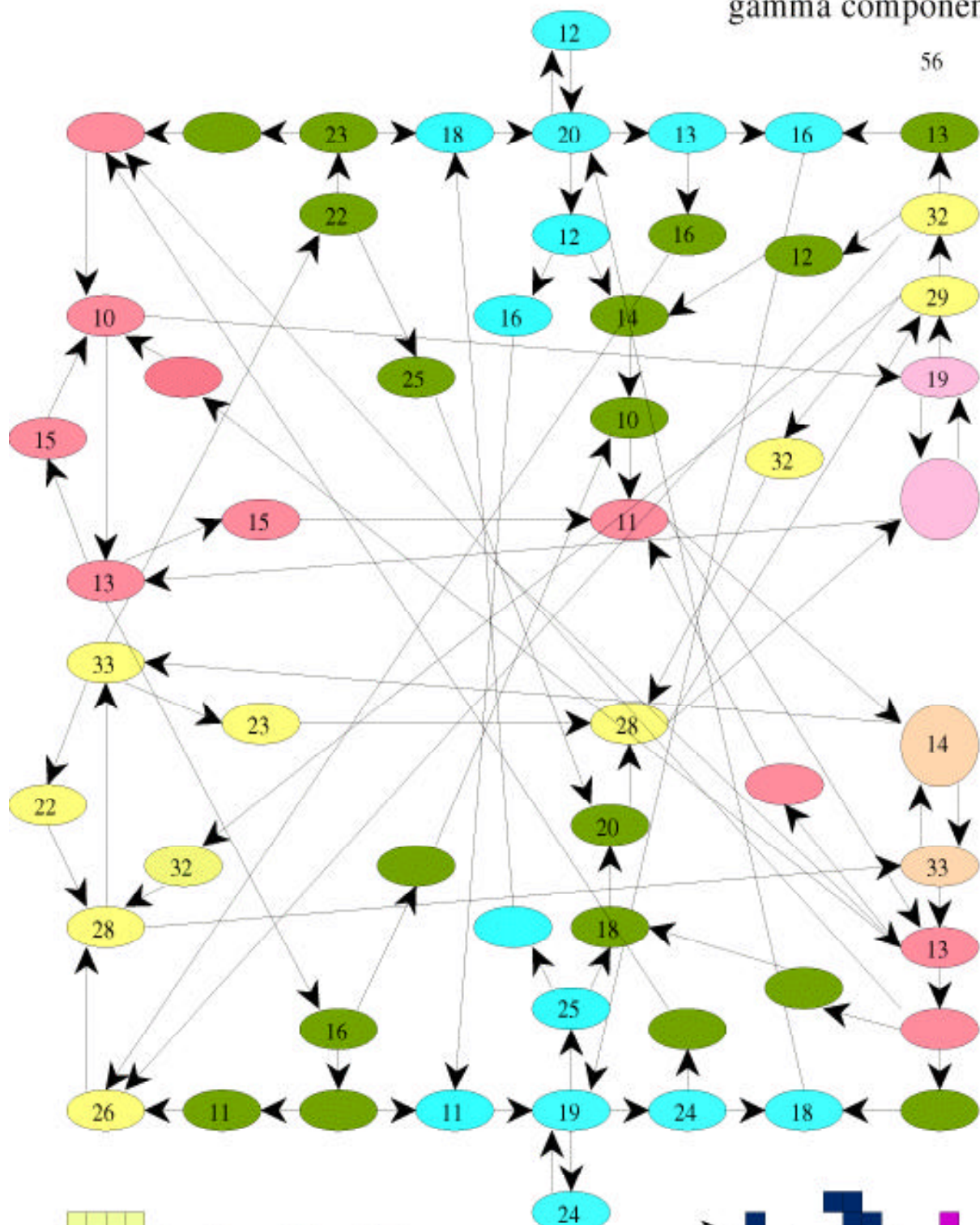


Life Phoenix de Bruijn diagram, width 8



Life Phoenix de Bruijn diagram, width 8

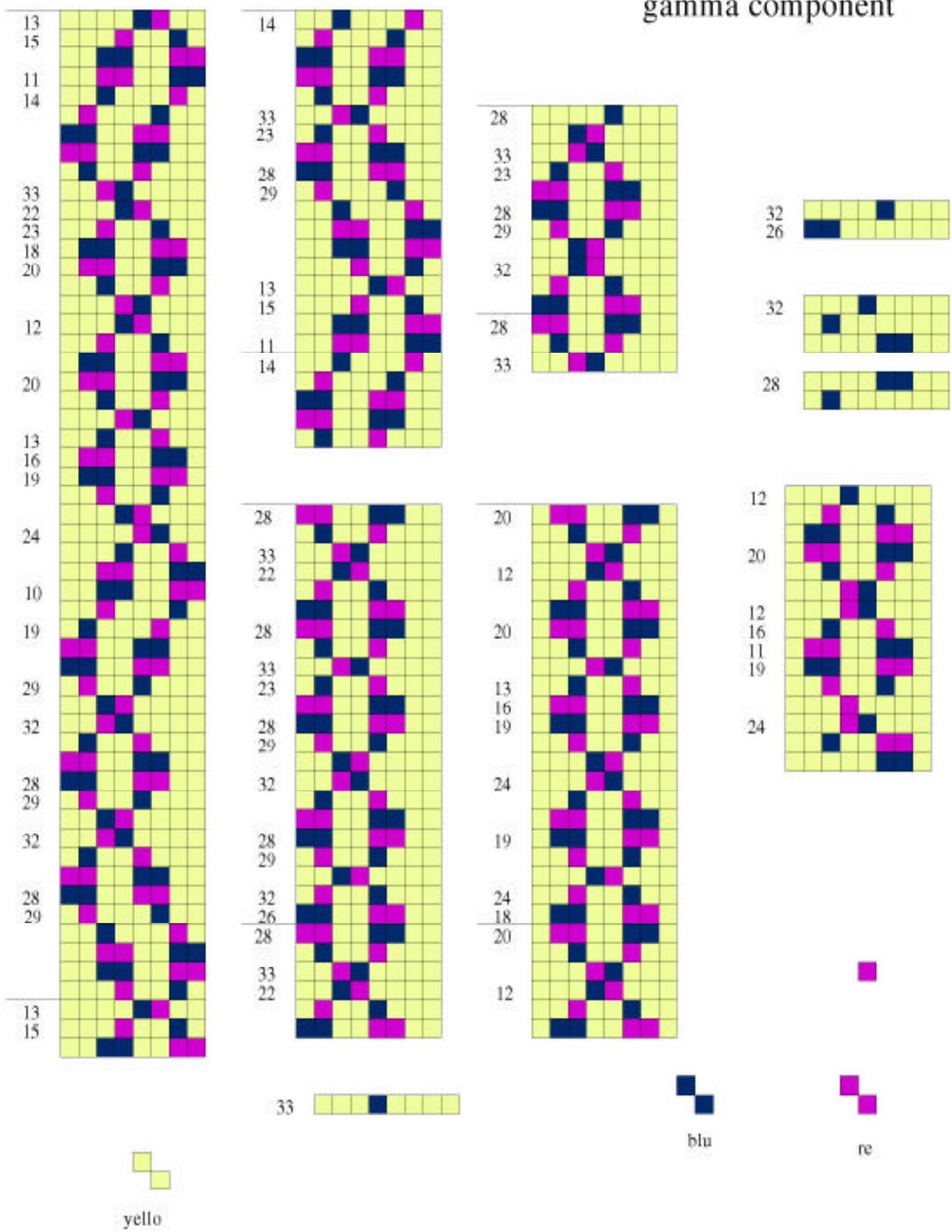
gamma component



February 12, 1994

Life Phoenix de Bruijn diagram, width 8

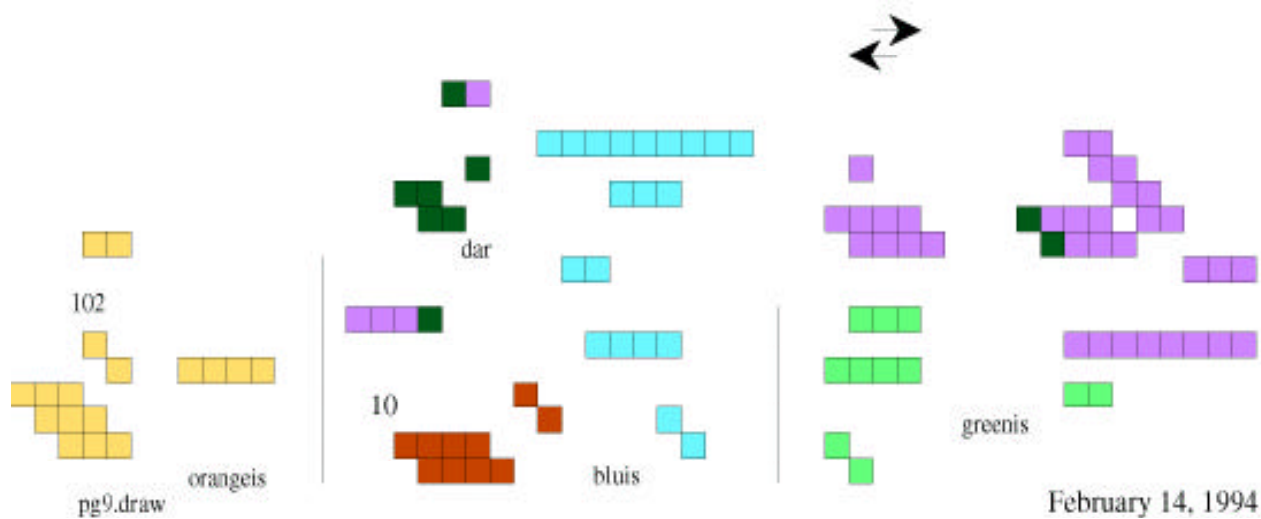
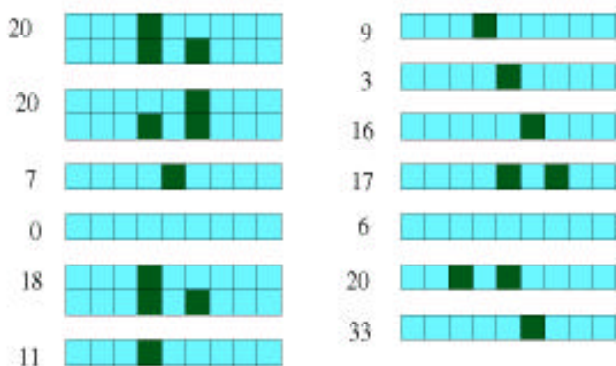
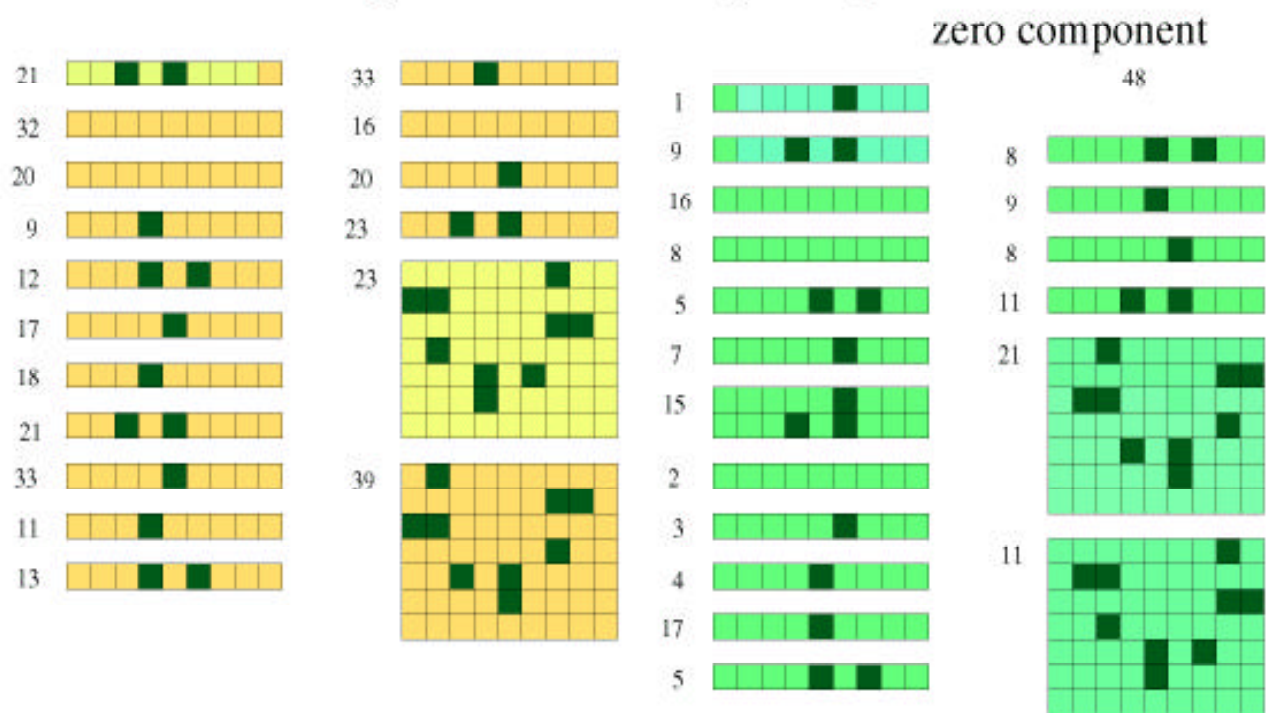
gamma component



pkkk8.dra

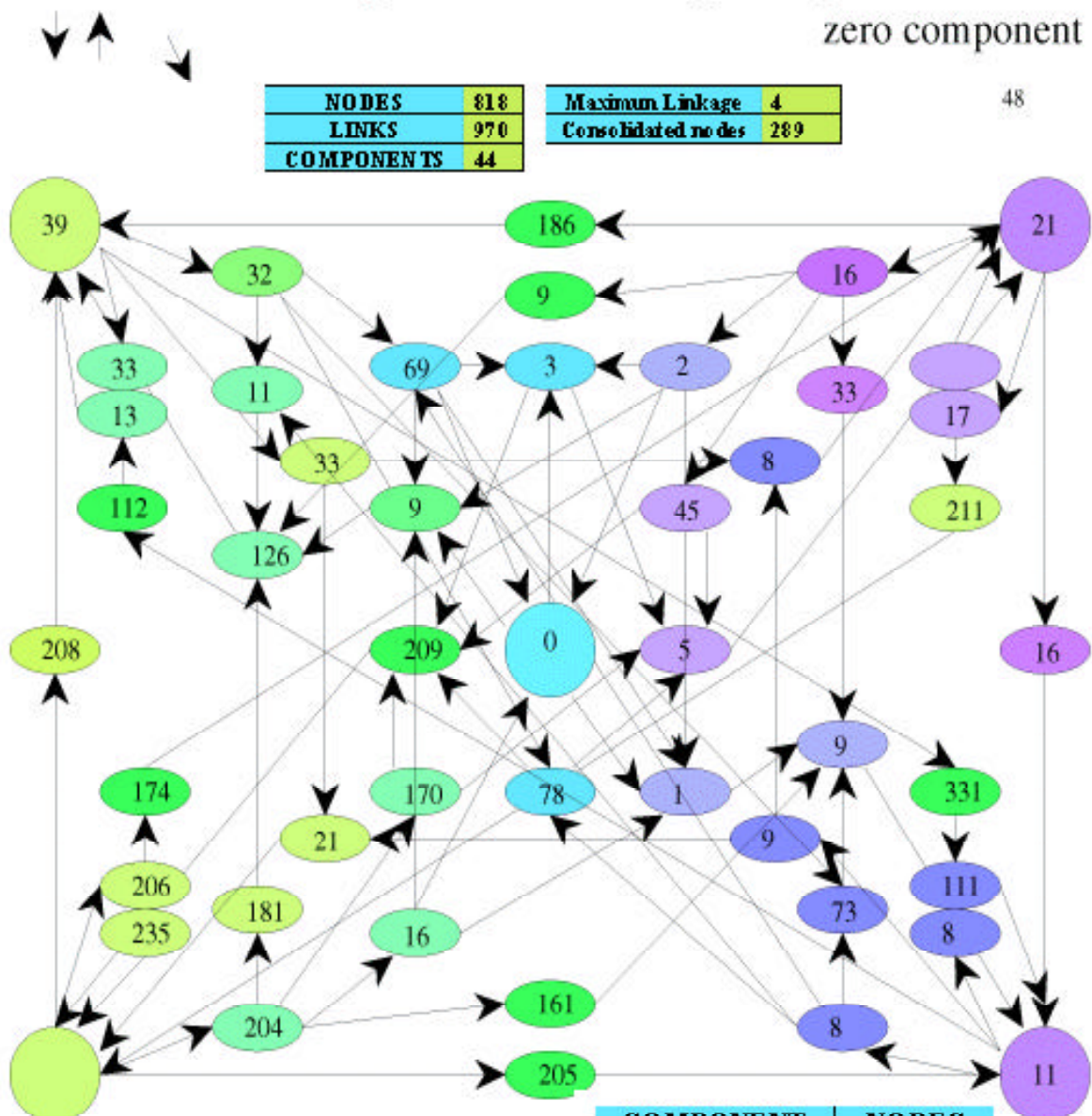
February 13, 1994

Life phoenix de Bruijn diagram, width 9



Life phoenix de Bruijn diagram, width 9

zero component

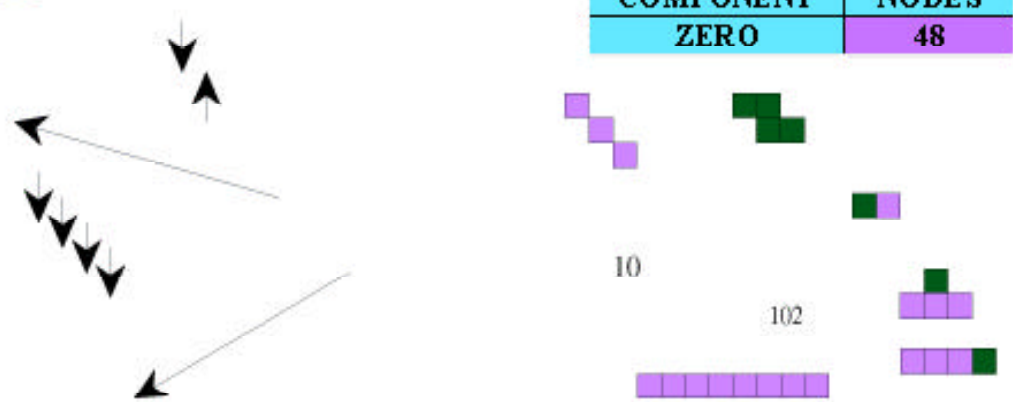


NODES	818
LINKS	970
COMPONENTS	44

Maximum Linkage	4
Consolidated nodes	289

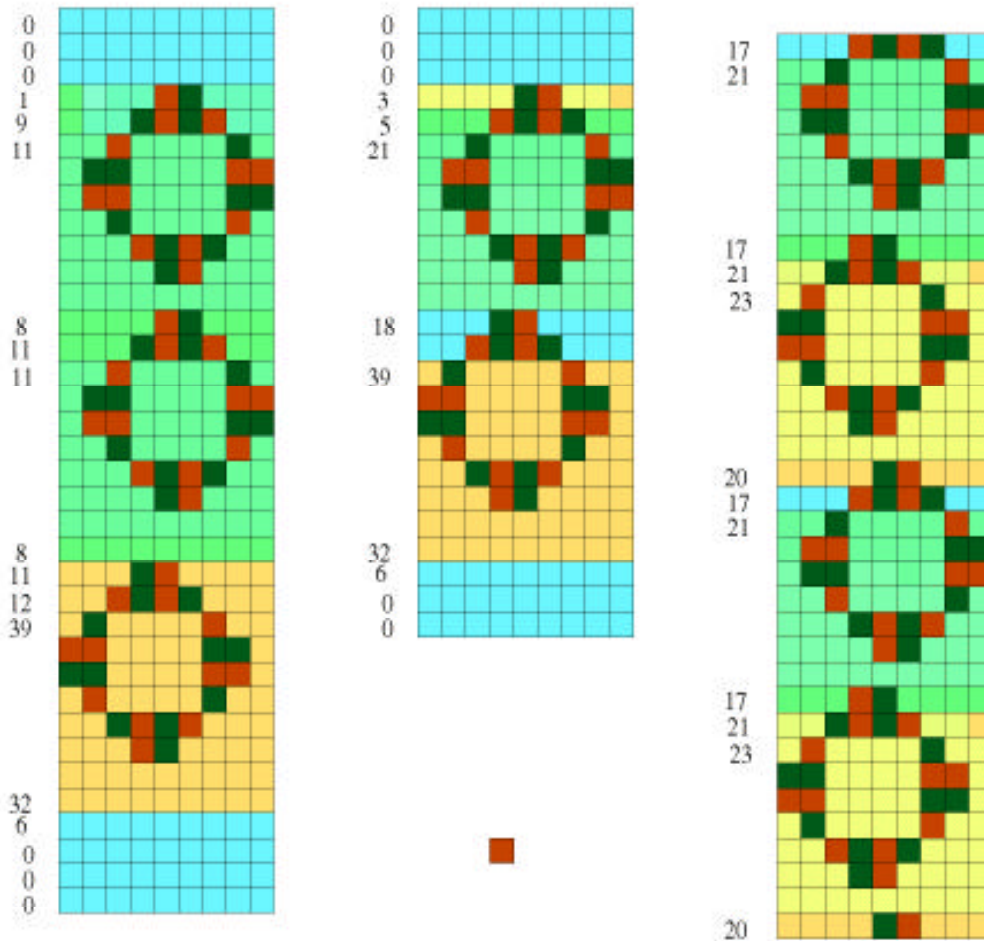
48

COMPONENT	NODES
ZERO	48



Life phoenix de Bruijn diagram, width 9

zero component



pggg9.dra

orangeis

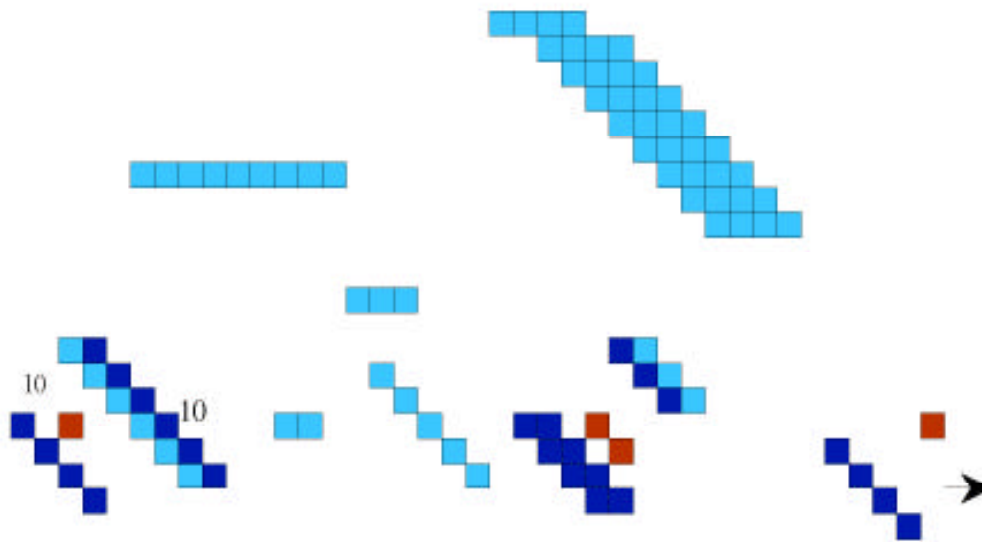
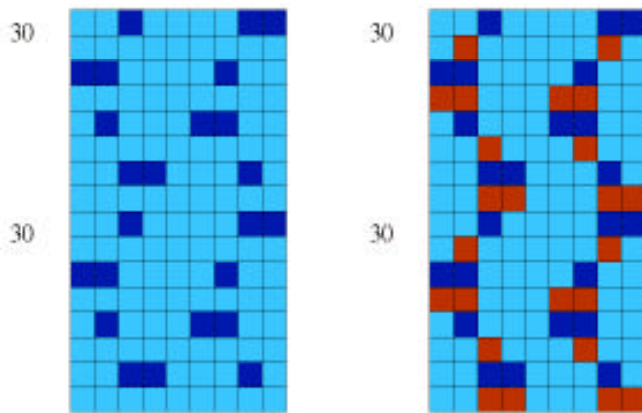
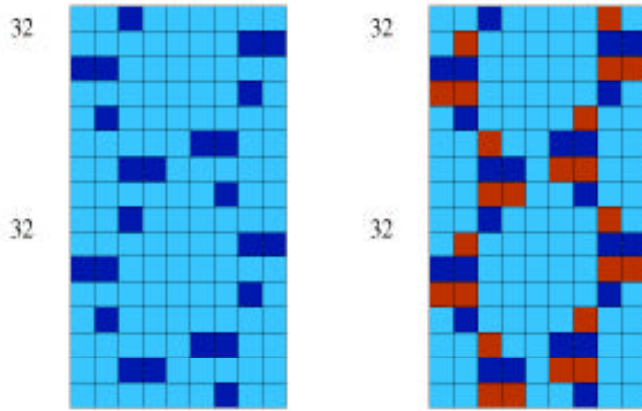
reddis
bluis

February 23, 1994

Life phoenix de Bruijn diagram, width 9

isolated cycles

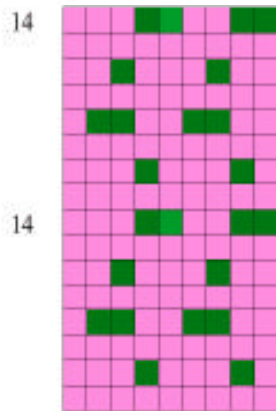
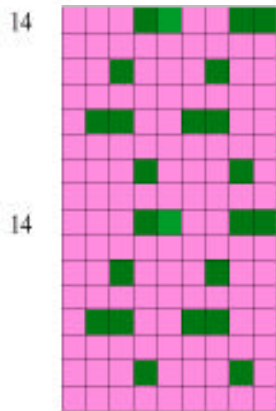
(38)



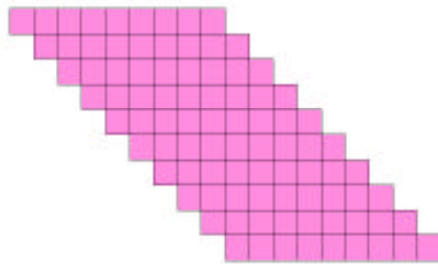
ph9.draw

February 14, 1994

Life phoenix de Bruijn diagram, width 9 isolated cycles



(38)



10
10
10

Life phoenix de Bruijn diagram, width 9

38 isolated cycles

The smallest phoenix avatars occupy 2x4 rectangles which can be strung out into strands whose central glide axis makes for a 4x8 unit cell. Given wider space, they can twist and turn, but there will always be a family of phoenixes comprised of perfectly straight parallel • laments. Individual strings can be paired according to their phase, given that alternate generations are reflections in a mirror perpendicular to the glide axis, and thus distinct from one another.

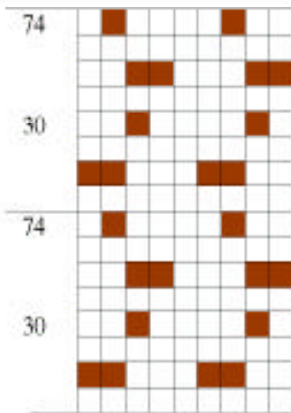
The closest packing is to crowd a pair of • laments in a column of width 8, wherein just three relative displacements of the same phase are free from interference. With opposite phases, only one possibility arises, but it takes its place in the gamma component.

Wider spacing allows any of eight alignments, of either relative phase.

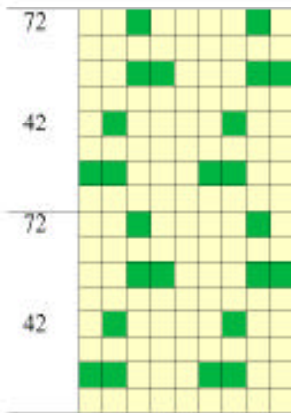
Besides the two subdiagrams of width 8, one left clinging, the other right clinging, further possibilities arise in the strip of width 9 when the two strands cling to opposite walls, for which there are no packing problems and all eight relative displacements can occur, moreover each for both relative phases between the strands.

This would all seem to add up to 44 different arrangements, 32 of full width 9, 6 of embedded width 8 on the left, and 6 more on the right. Six of this total, precisely the most symmetric ones, are lost to enlarged alpha (1), beta (1), and middle (4) components, leaving the remaining 38 in isolated cycles.

Life phoenix de Bruijn diagram, width 9

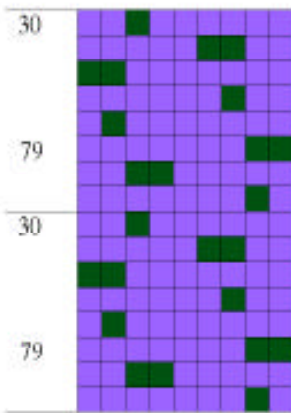


alph

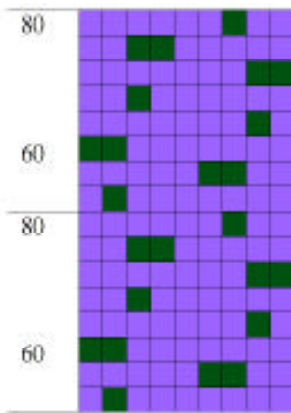


bet

Parallel • lament pairs which incorporate into other components rather than just isolated cycles.



middl

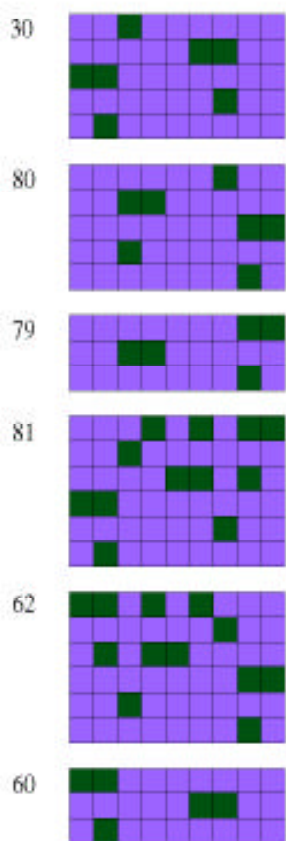


middl

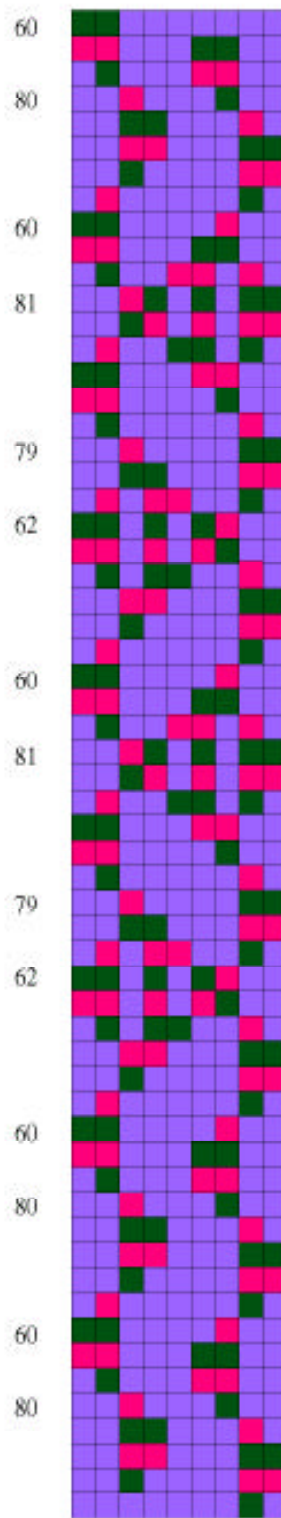
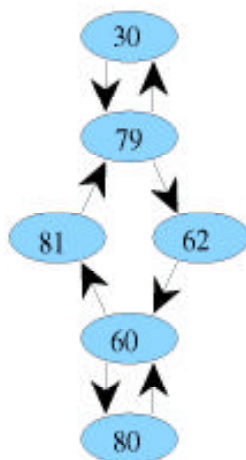
Life phoenix de Bruijn diagram, width 9

(2)

middle components

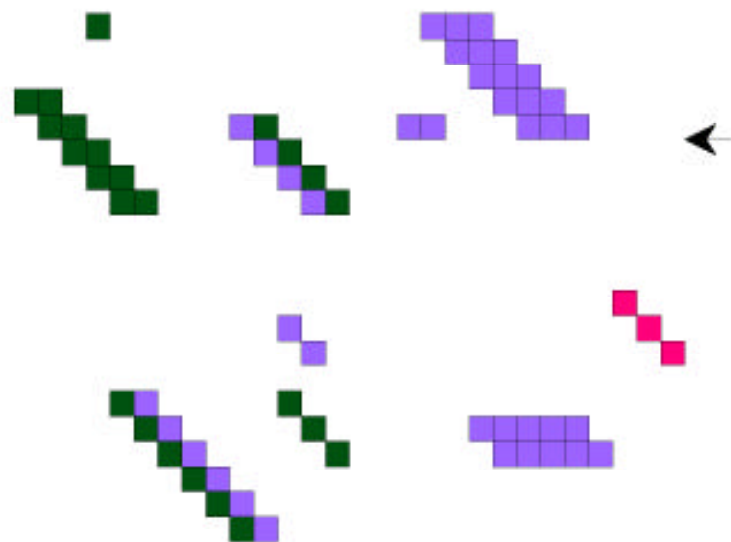


COMPONENT	NODES
MEDIUM	6



NODES	818
LINKS	970
COMPONENTS	44

Maximum Linkage	4
Consolidated nodes	289



33

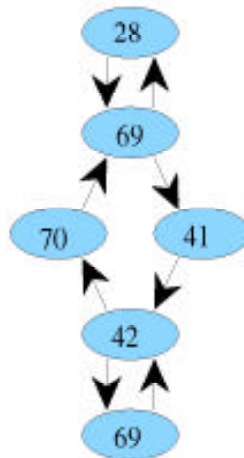
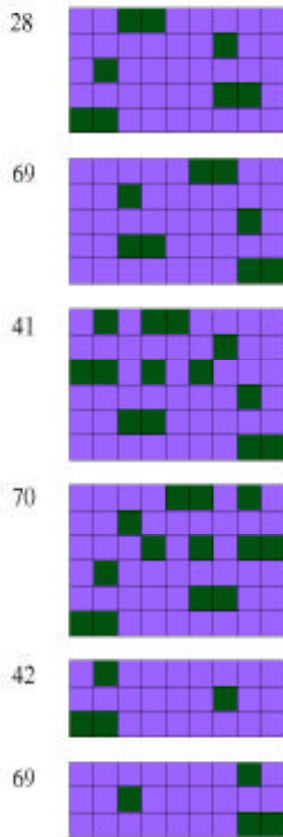
10

February 24, 1994

Life phoenix de Bruijn diagram, width 9

(2)

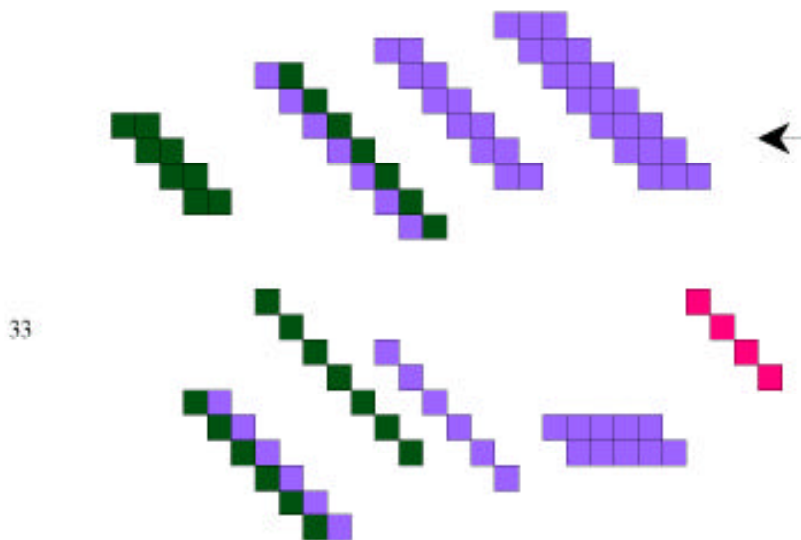
middle components



COMPONENT	NODES
MEDIUM	6

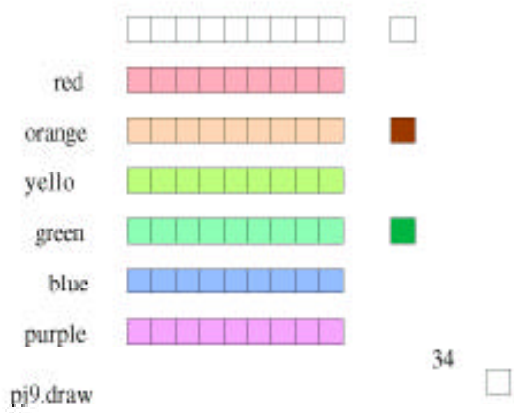
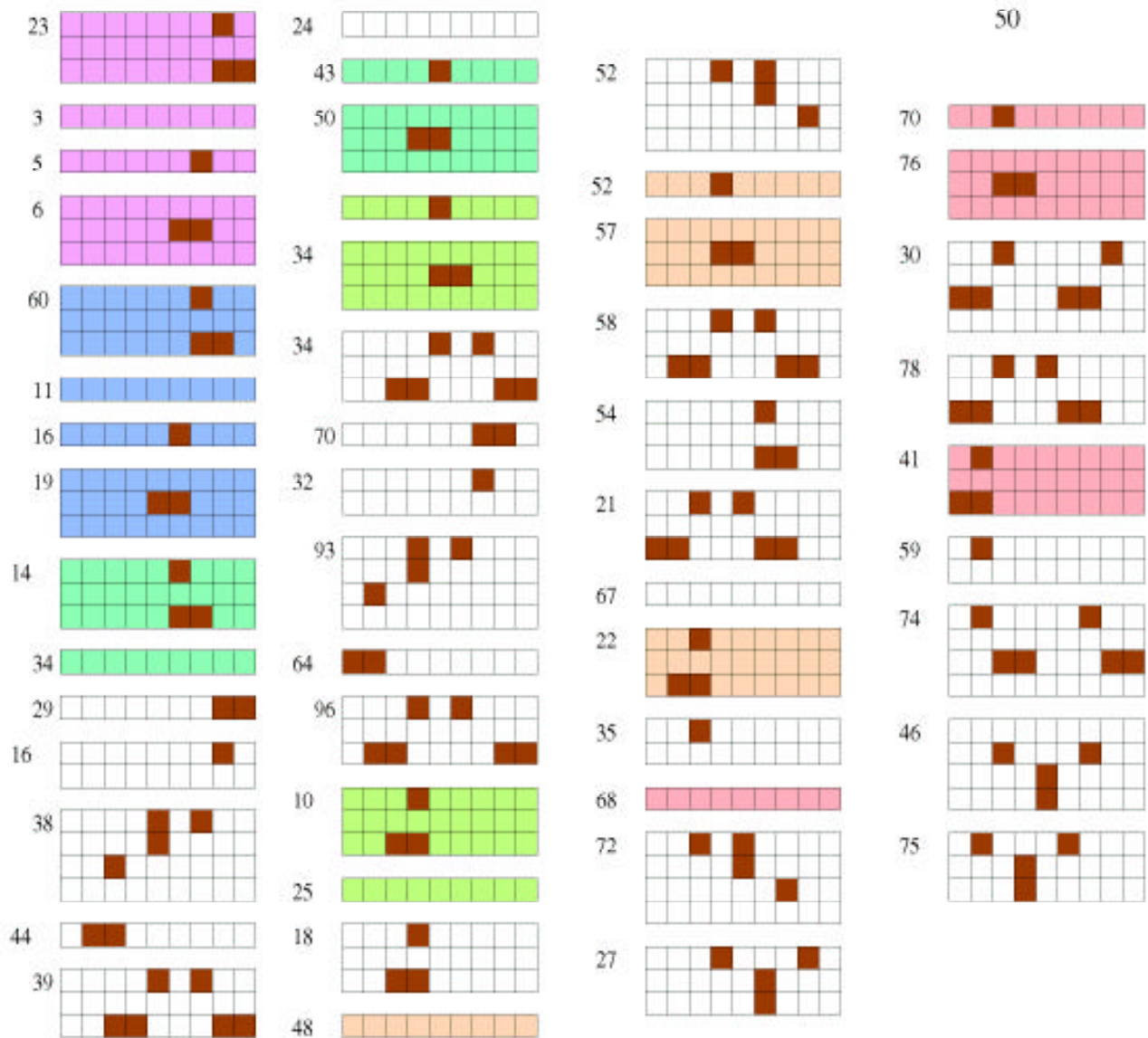
NODES	818
LINKS	970
COMPONENTS	44

Maximum Linkage	4
Consolidated nodes	289



Life phoenix de Bruijn diagram, width 9

alpha component

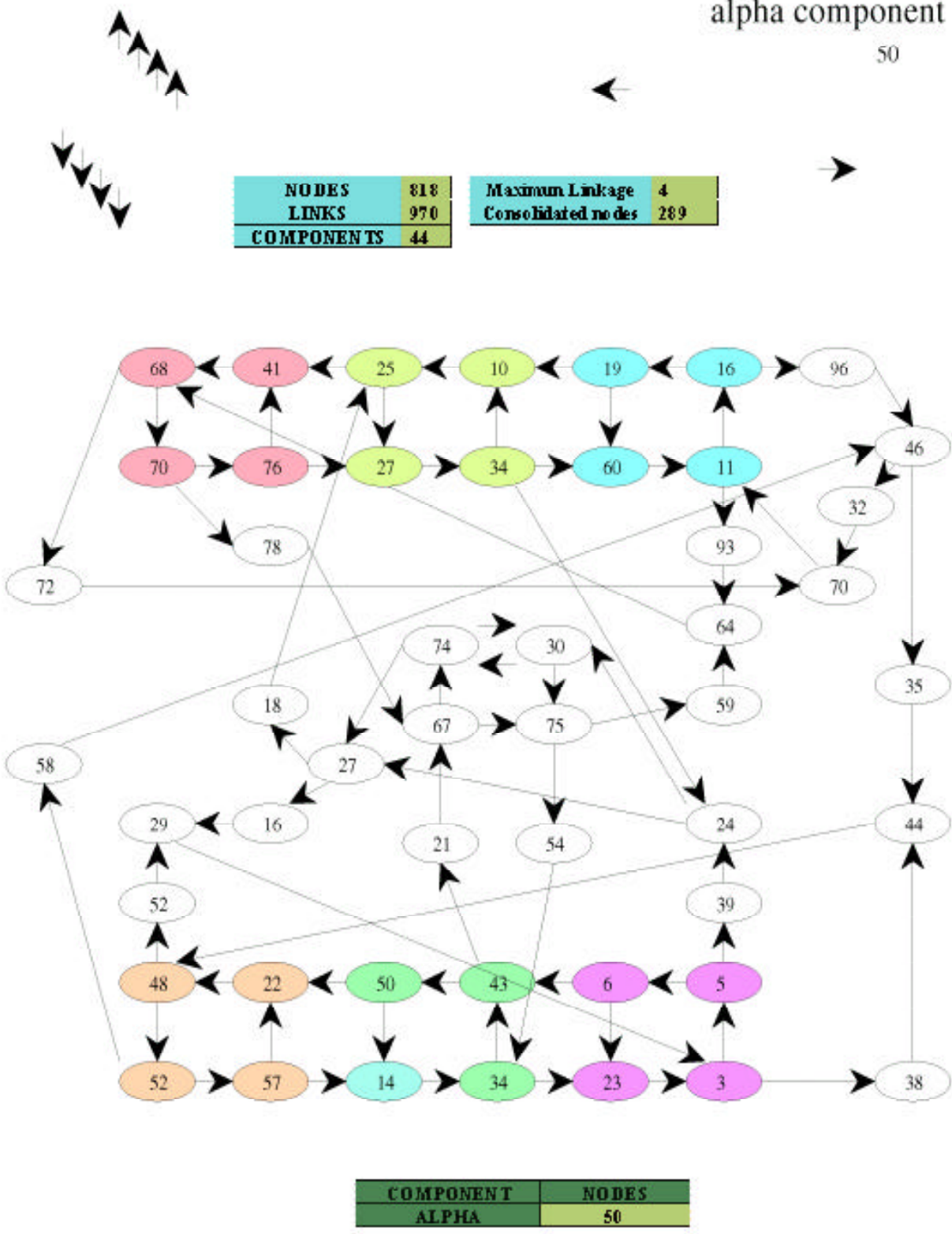


November 19, 1996

Life phoenix de Bruijn diagram, width 9

alpha component

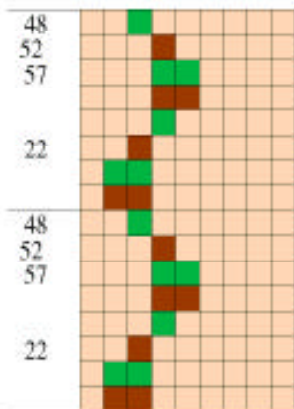
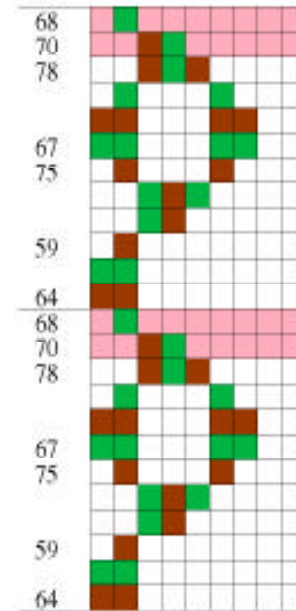
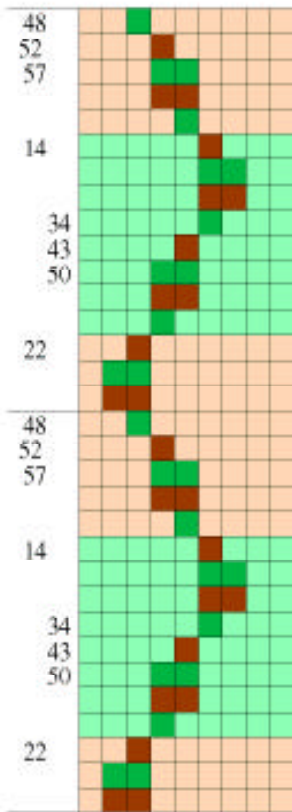
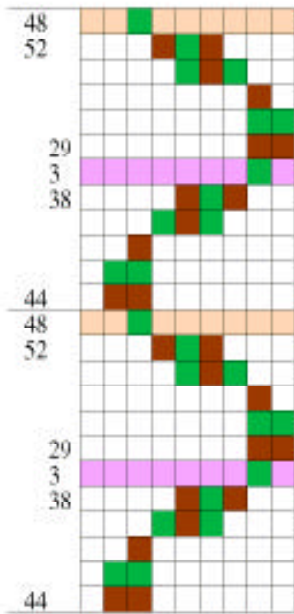
50



Life phoenix de Bruijn diagram, width 9

alpha component

50



Life phoenix de Bruijn diagram, width 9

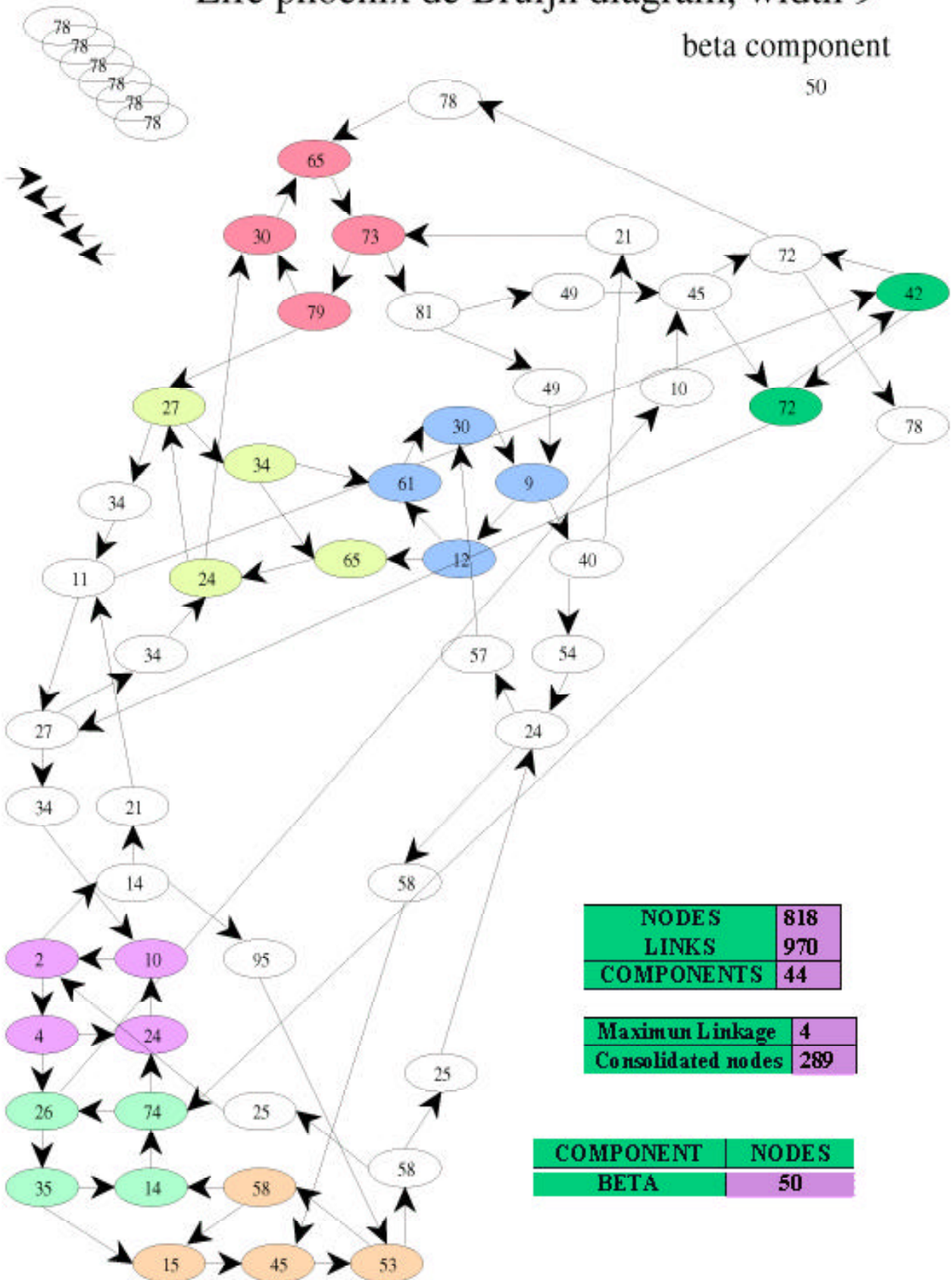


November 19.

Life phoenix de Bruijn diagram, width 9

beta component

50



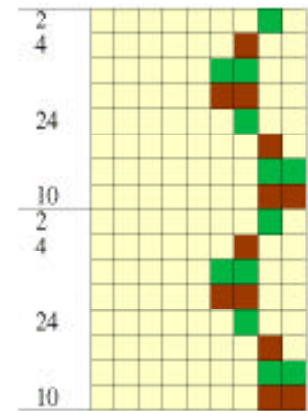
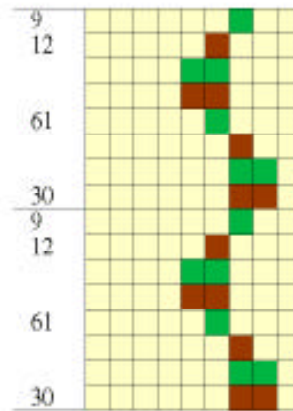
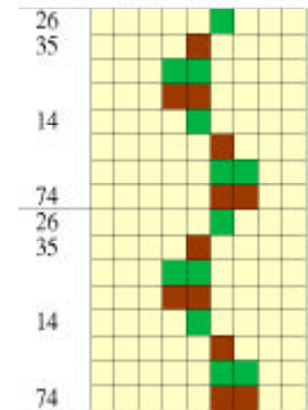
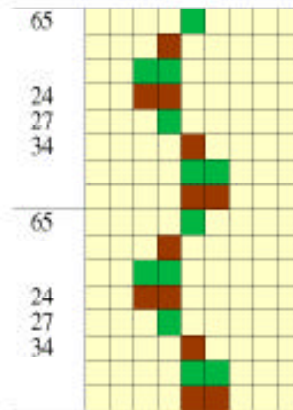
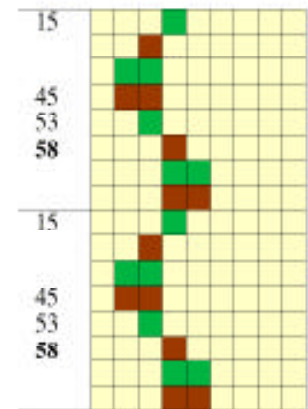
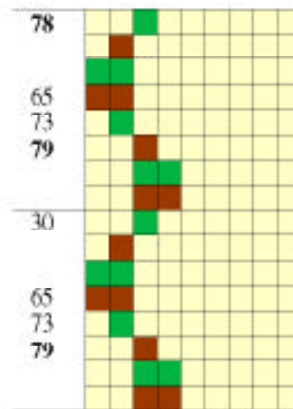
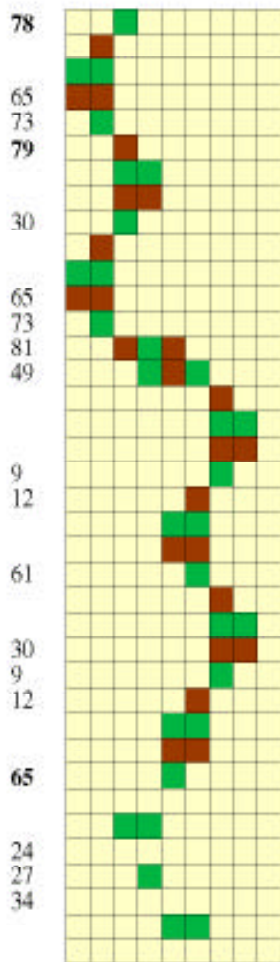
NODES	818
LINKS	970
COMPONENTS	44

Maximum Linkage	4
Consolidated nodes	289

COMPONENT	NODES
BETA	50

pkk9.draw

November 19.



50



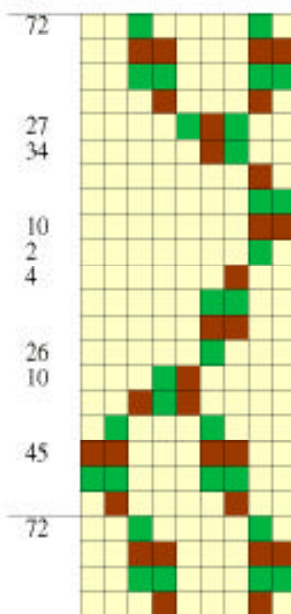
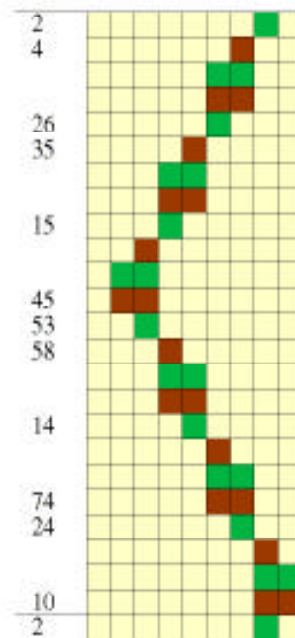
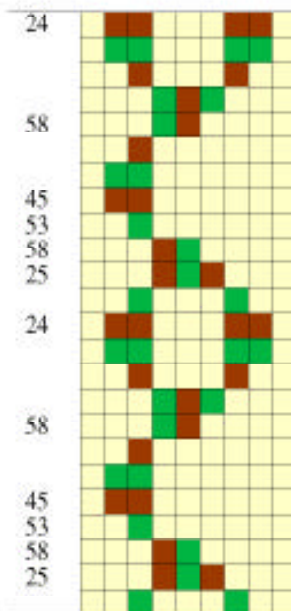
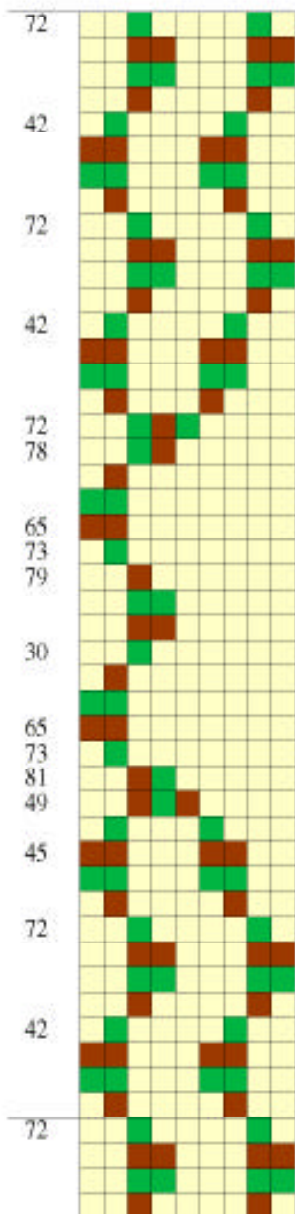
beta component

Life phoenix de Bruijn diagram, width 9

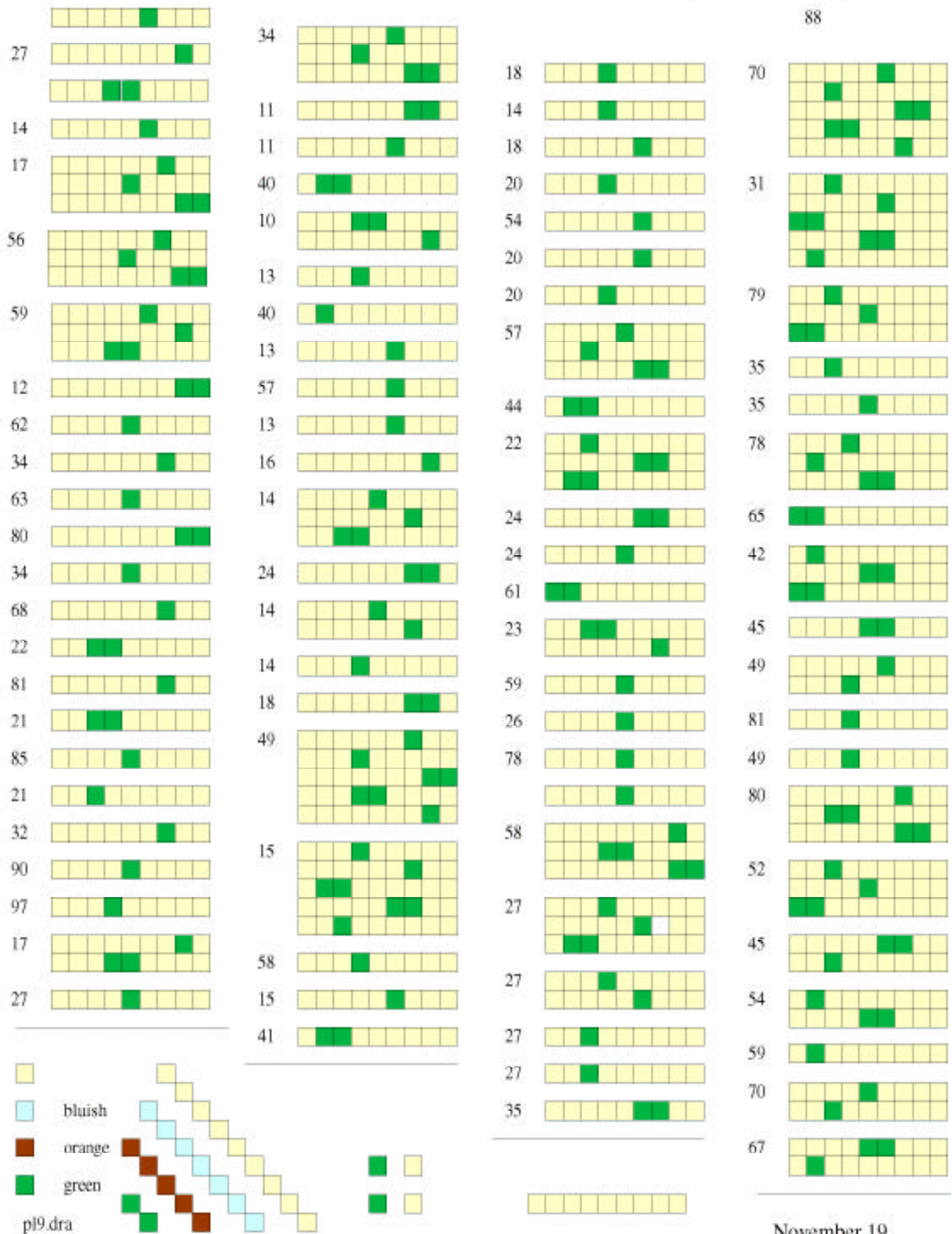
Life phoenix de Bruijn diagram, width 9

50

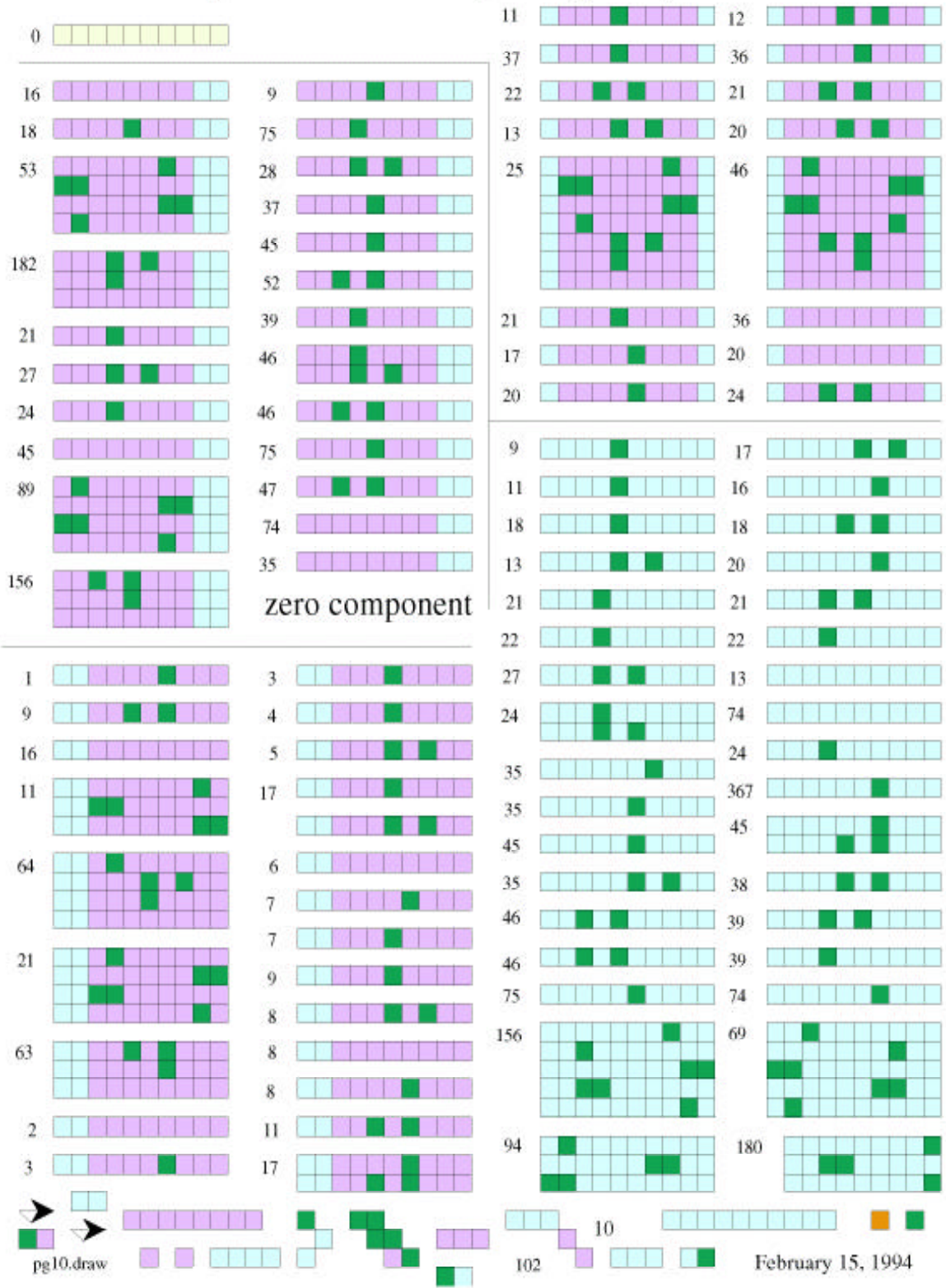
beta component



Life phoenix de Bruijn diagram, width 9 gamma component



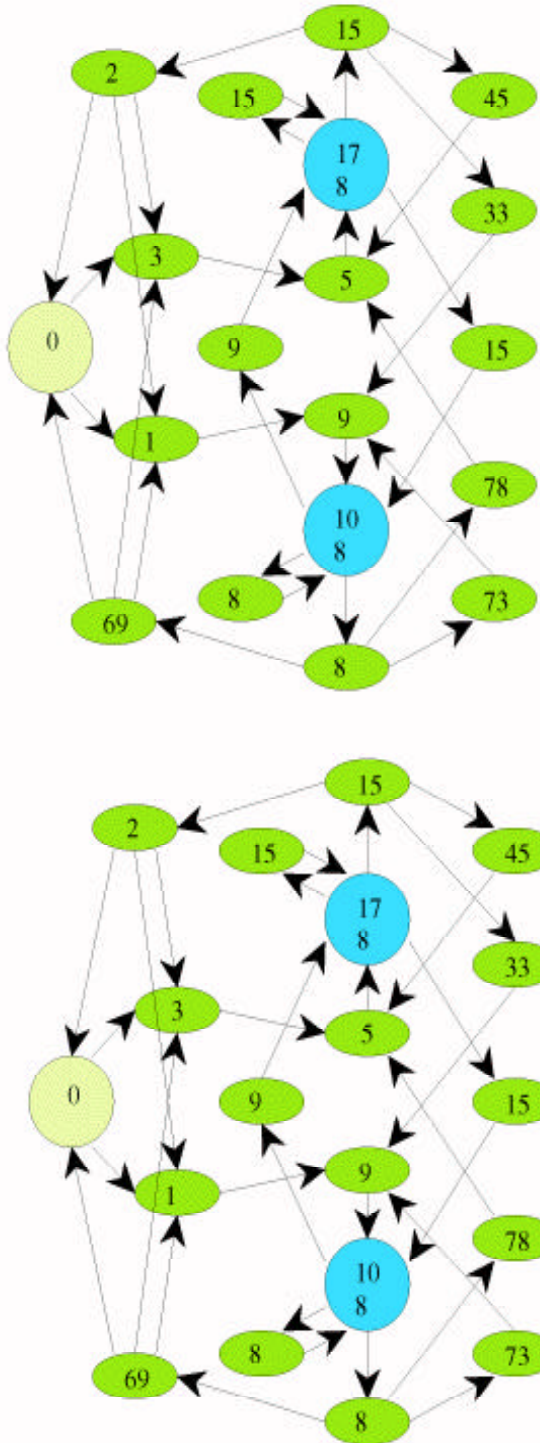
Life phoenix de Bruijn diagram, width 10 97



Life phoenix de Bruijn diagram, width 10

zero component

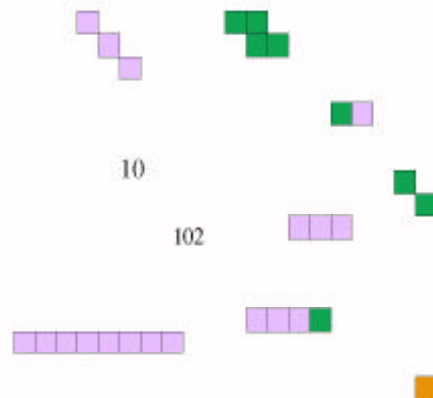
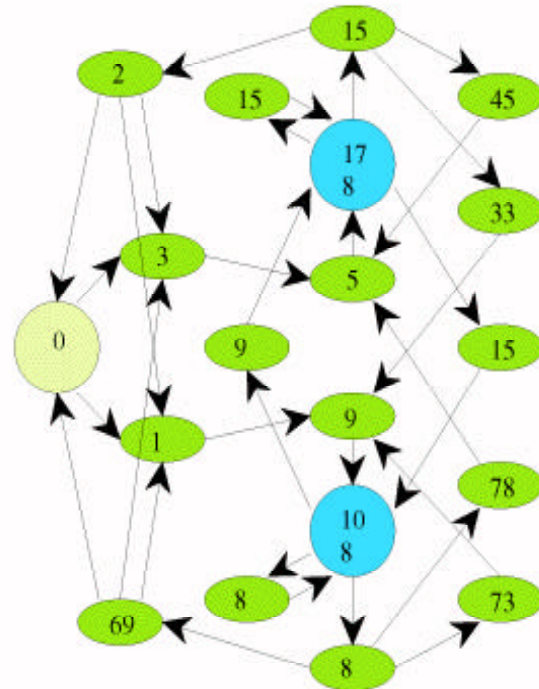
97



NODES	1845
LINKS	2187

Maximum Linkage	5
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COMPONENT	NODES
ZERO	97

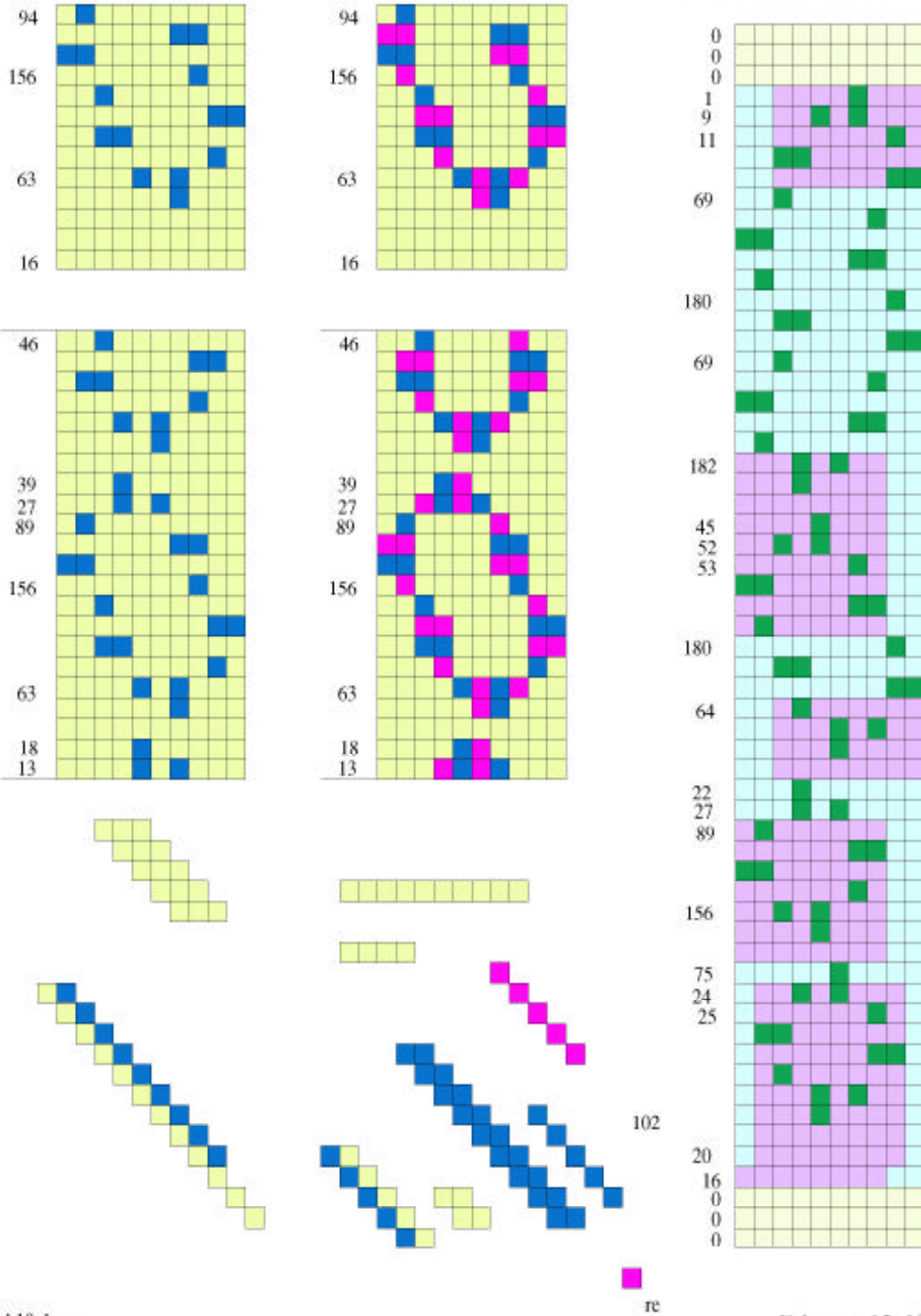


pgg10.dra

February 14, 1994

Life phoenix de Bruijn diagram, width 10

zero component

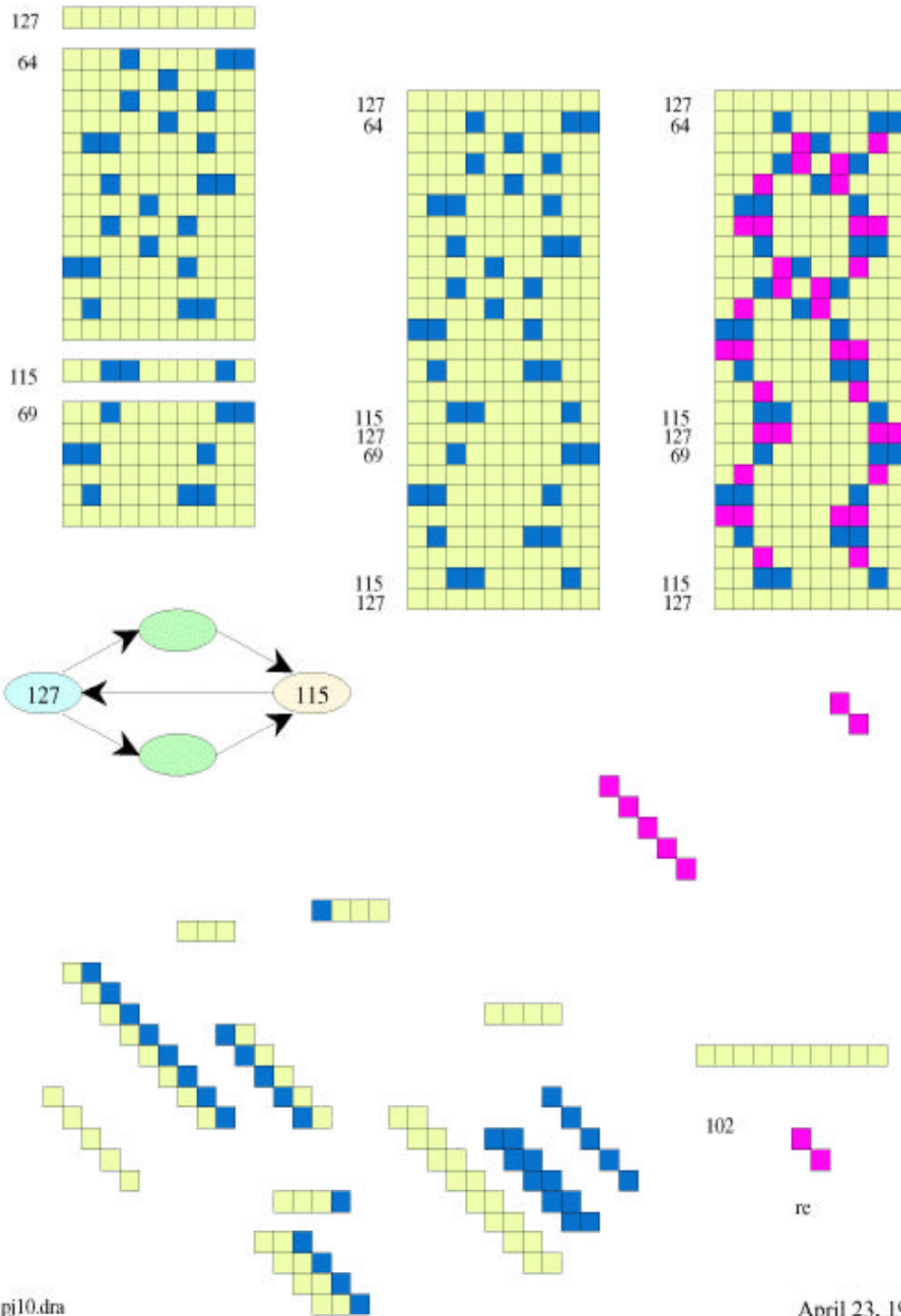


ph10.draw

February 15, 1994

Life phoenix de Bruijn diagram, width 10

small components

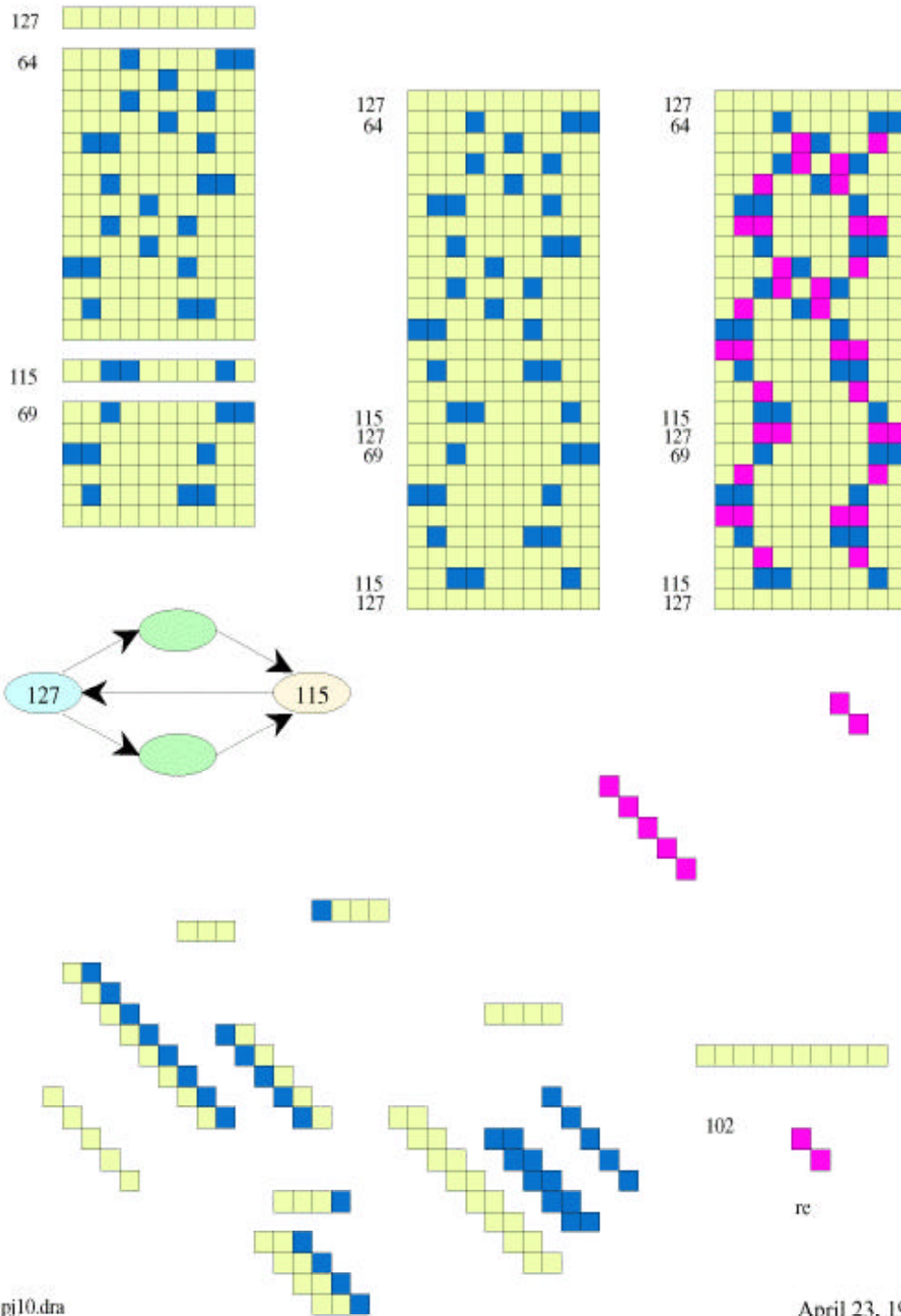


pj10.dra

April 23, 1994

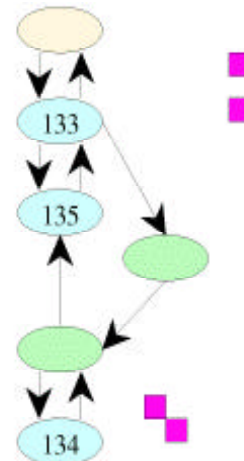
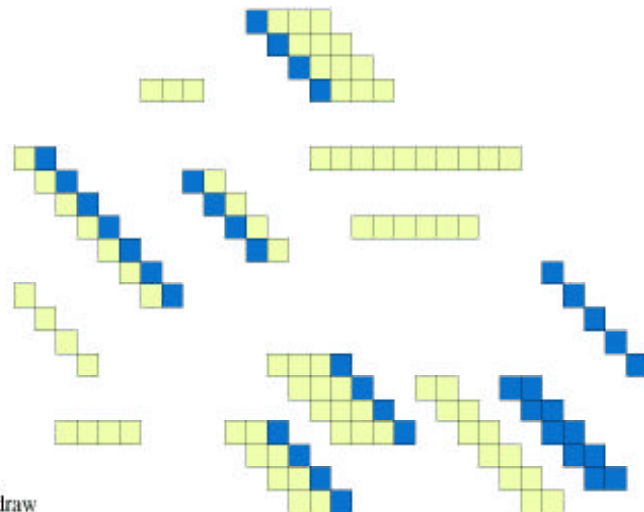
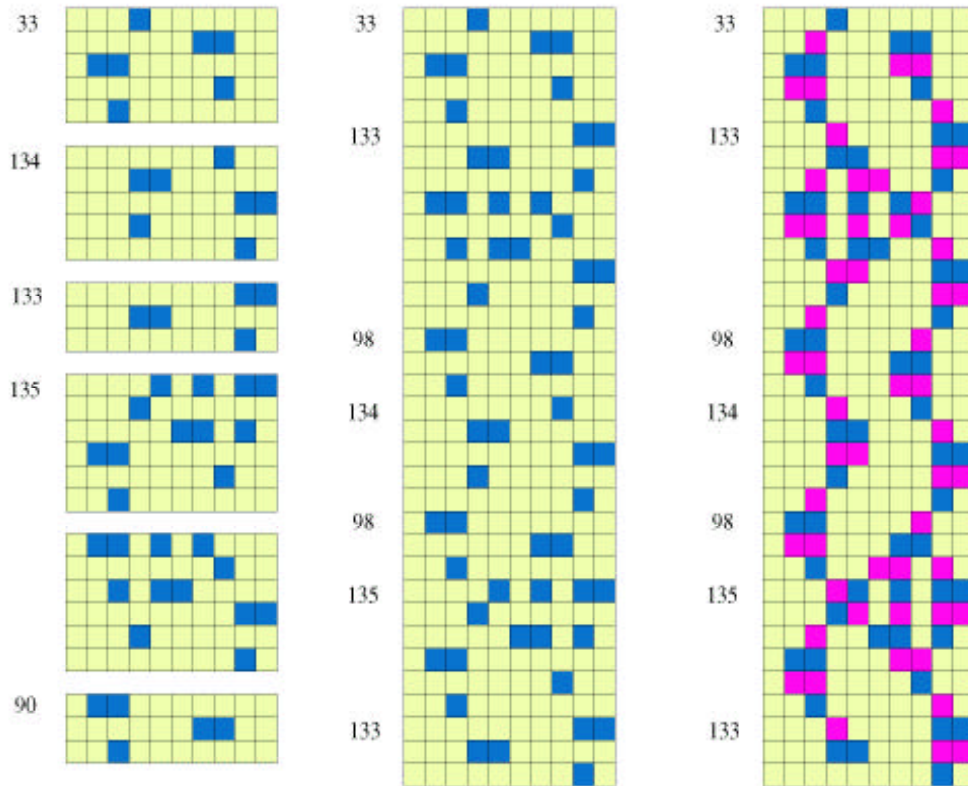
Life phoenix de Bruijn diagram, width 10

small components



Life phoenix de Bruijn diagram, width 10

small components

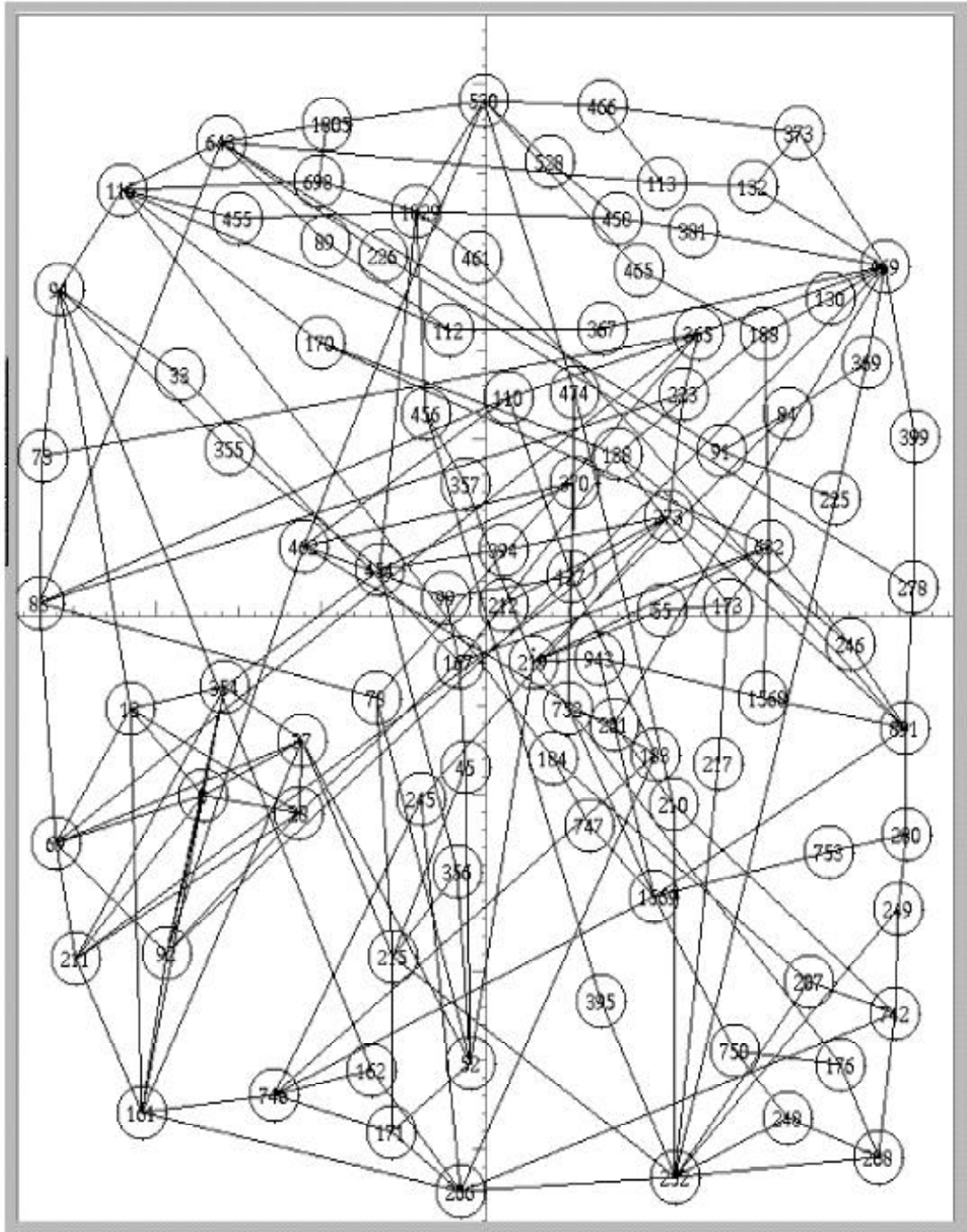


re

102

pk10.draw

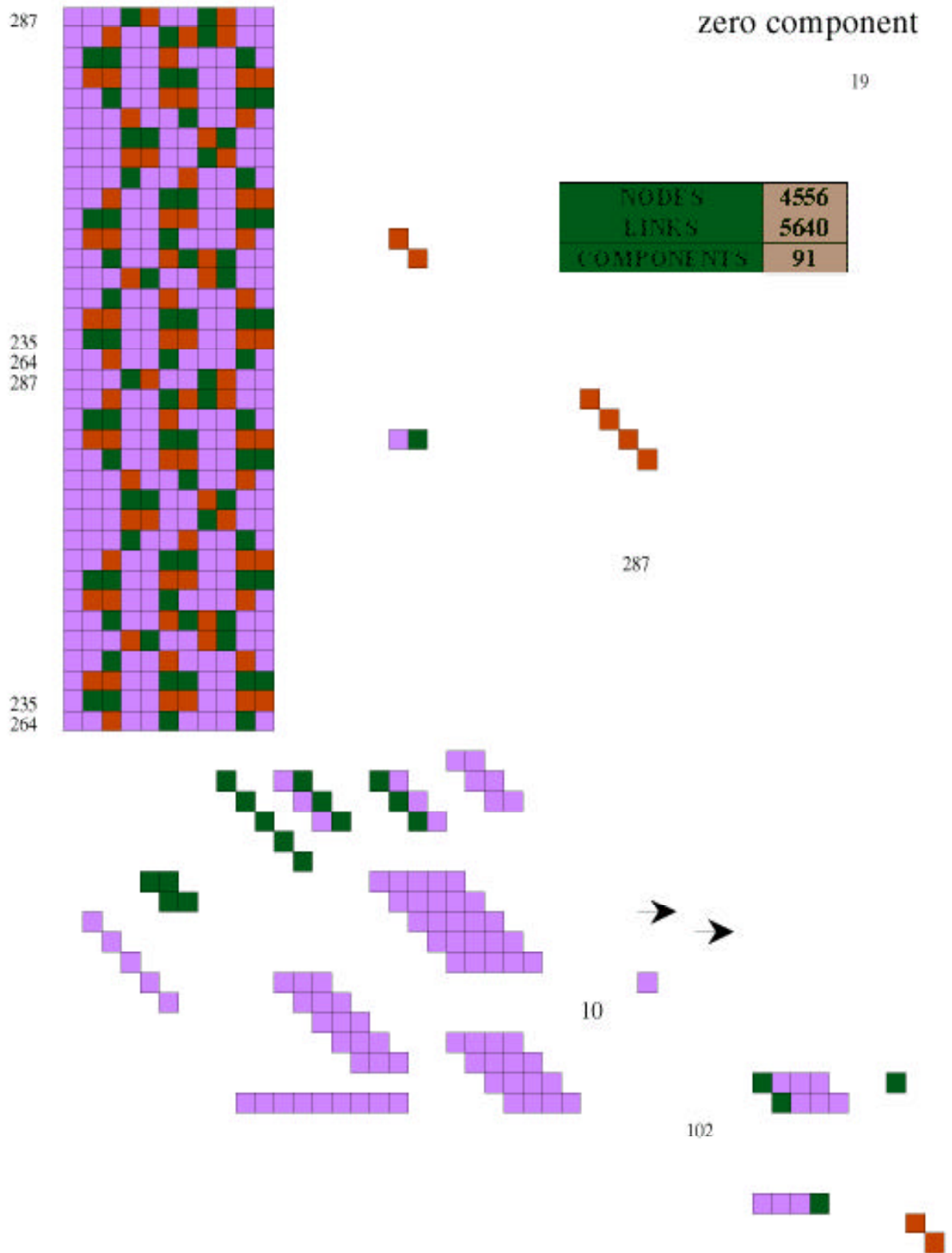
April 24, 1994



ps10.draw

February 28, 1994

Life phoenix de Bruijn diagram, width 11

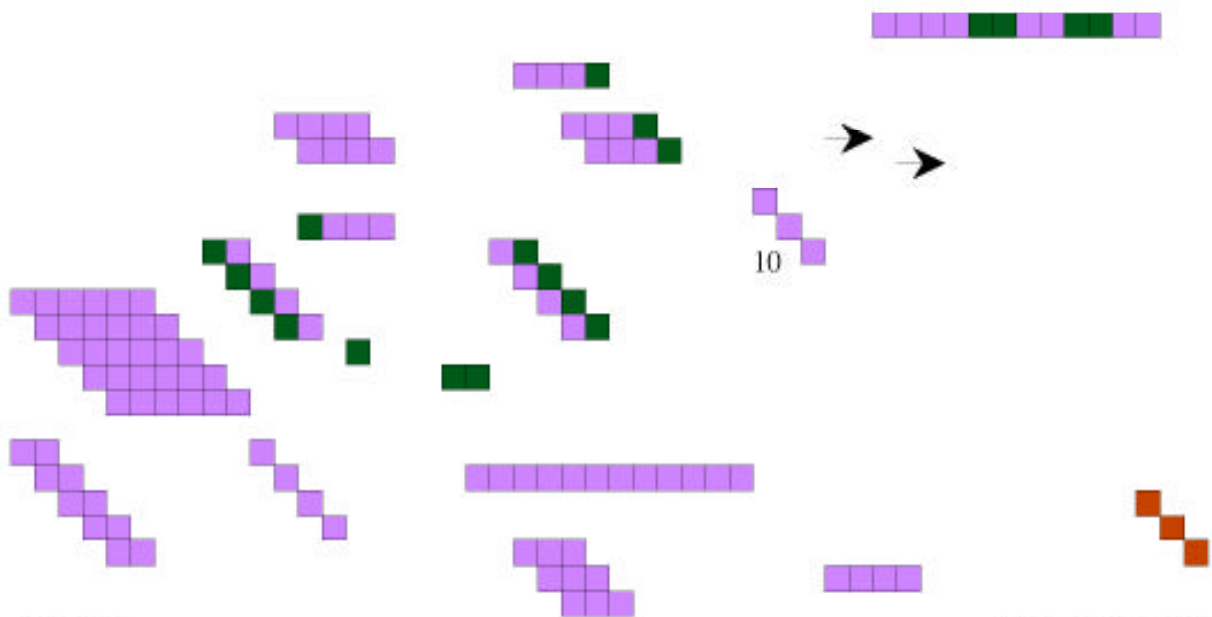
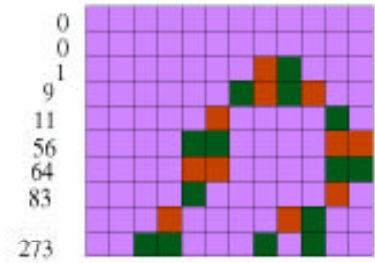


Life phoenix de Bruijn diagram, width 12

19



zero component

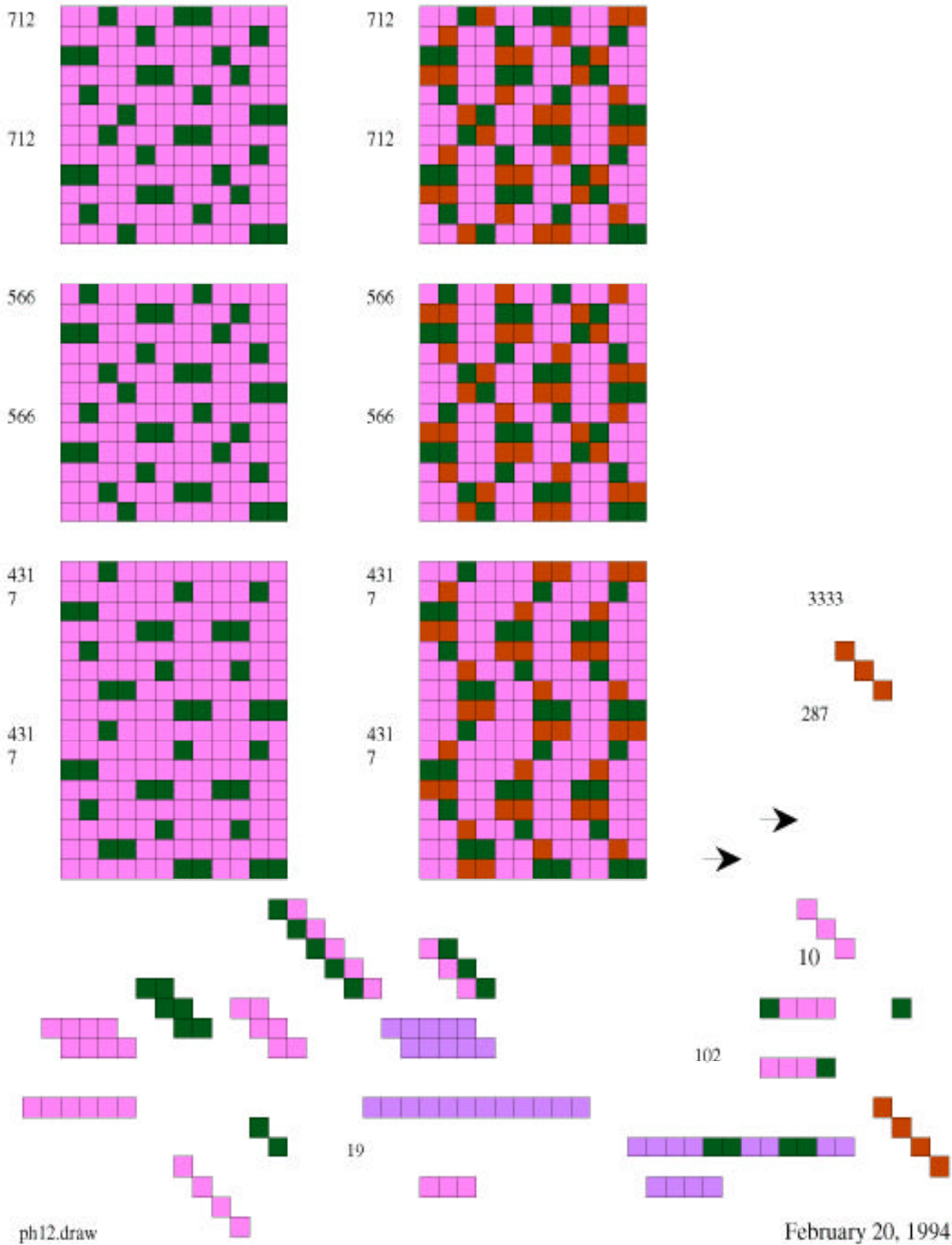


pgg12.dra

February 20, 1994

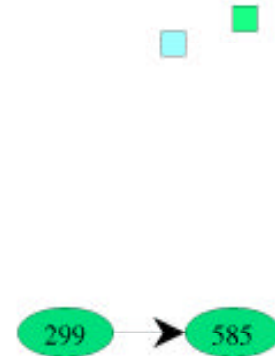
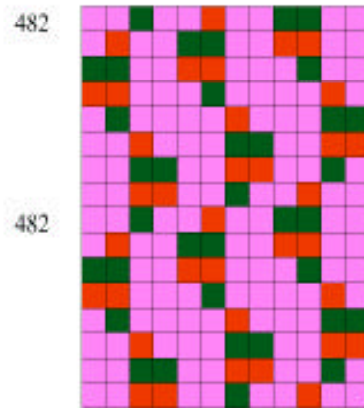
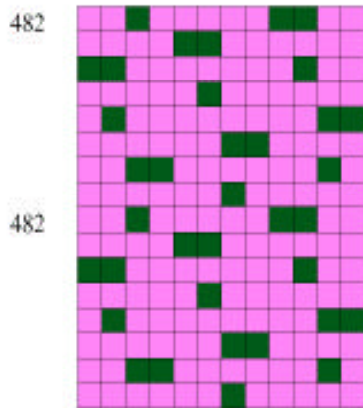
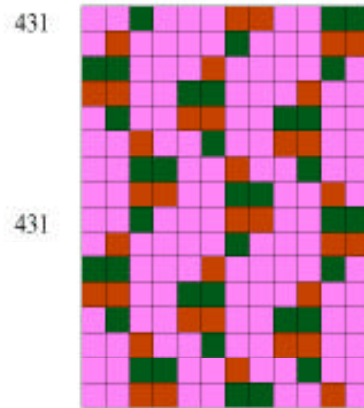
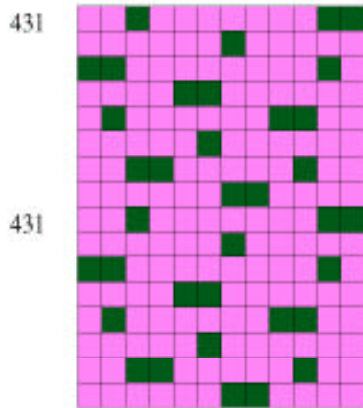
Life phoenix de Bruijn diagram, width 12

isolated

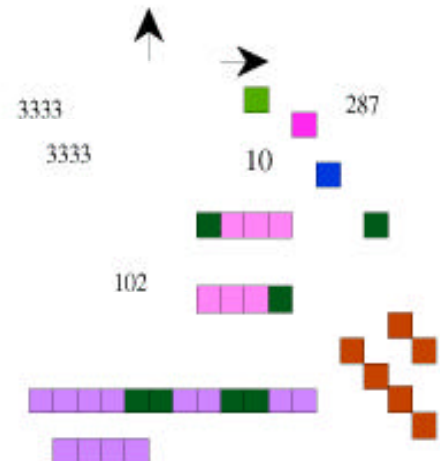
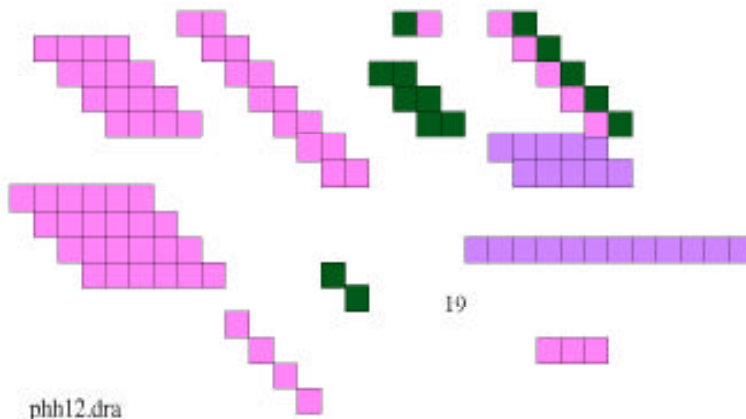


Life phoenix de Bruijn diagram, width 12

isolated



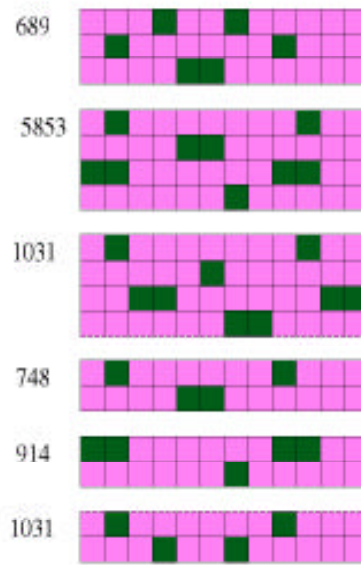
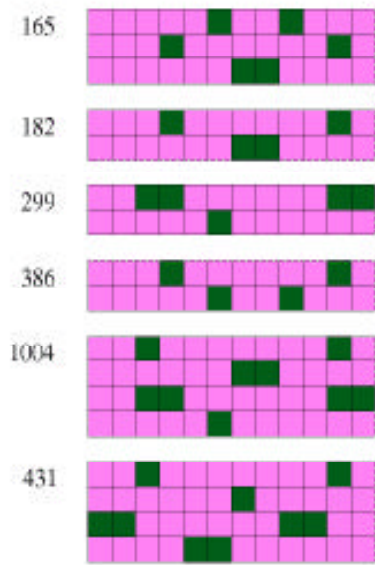
NODES	1129
LINKS	14607
COMPONENTS	50



phh12.dra

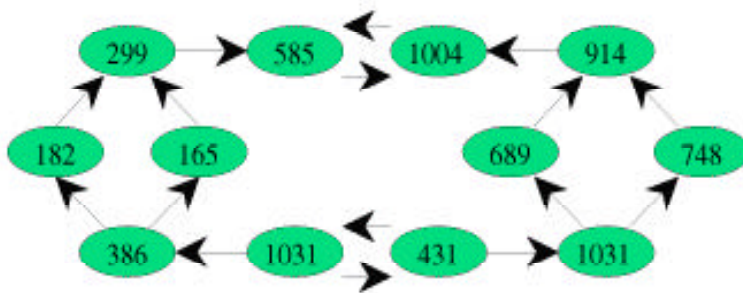
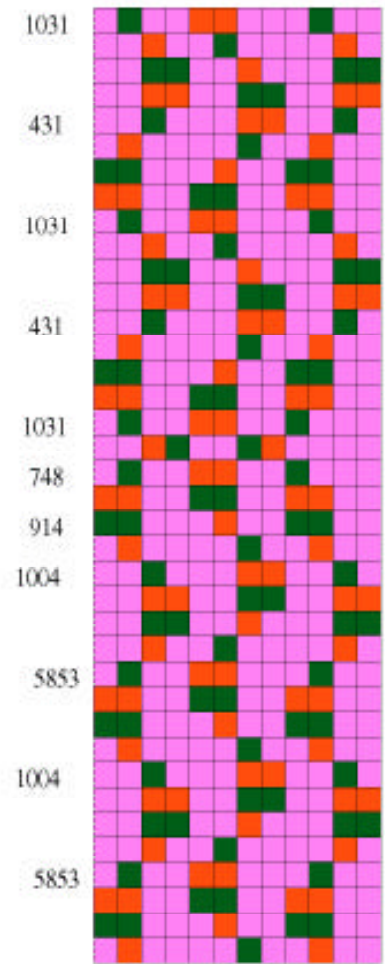
February 20, 1994

Life phoenix de Bruijn diagram, width 12



isolated

12 nodes



NODES	1129
LINKS	14607
COMPONENTS	50

