A multi-objective approach to the design of low thrust space trajectories using optimal control

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Abstract

In this article, we apply a particular composition of recently developed optimization methods to the problem of sending a group of spacecraft to a specified Earth L_2 Halo orbit. We face the optimal control task with a three-step approach: We select first a restricted class of control functions for a global multi-objective optimization which is performed in a set-oriented way. The solutions of this problem are (Pareto-)optimal with respect to ΔV and flight time. In the second step the obtained compromise trajectories serve as initial guesses for a direct local optimization method called DMOC (Discrete Mechanics and Optimal Control). By means of DMOC we construct trajectories using a more flexible control law and hence, the obtained solutions are improved with respect to control effort. Finally, we compute trajectories for all spacecraft using the previous results and – with some additional constraints – these spacecraft trajectories end at the Halo orbit in a prescribed relative formation.

Keywords: space mission design, multi-objective optimization, Pareto set, optimal control, formation flight.

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1 Introduction

Over the last years dynamical system techniques have been developed for the design of energy-efficient spacecraft trajectories. These methods exploit the natural dynamics of the system and often result in trajectories that are close to invariant manifolds. One of the

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most famous space missions using dynamical system techniques was the *Genesis Discovery Mission*[1] which was a solar-wind sample return mission between 2001 and 2004. The delicate heteroclinic dynamics employed by this mission illustrates the need to study the three body problem (3BP) using dynamical systems theory (see e.g. [Howell et al. (1997), Koon et al. (1999), Dellnitz et al. (2001), Gómez et al. (2001)]). Other missions also used those concepts like for instance the *Hiten mission* (see e.g. [Belbruno and Miller (1993)] for more mathematical details).

One major drawback, however, is that the high efficiency in fuel consumption is relativized by long flight times. Thus, a typical trade-off in space mission design is to apply more thrust (and, in turn, increase the cost of the resulting mission) to obtain shorter mission times (see e.g. [Coverstone-Caroll et al. (2000), Vasile et al. (2006), Schütze et al. (2008b)]). A natural mathematical framework to deal with such trade-offs is to describe the task as a (global) *multi-objective optimization problem (MOP).* Global optimization methods are well suited for more or less detailed preselections of promising trajectories as a *first guess.* Typically, in a second step this first guess is then subject to a local optimization method with a more detailed model and a more flexible control law.

Global multi-objective optimization While in the single-objective case the solution is typically – i.e., under mild regularity assumptions – given by *one* point, the solution set of an MOP consisting of k objectives typically forms a $(k-1)$–dimensional object [Hillermeier (2001)]. This set is called the *Pareto set* (see [Pareto (1971)]) or the set of optimal compromises. Over the last years quite a few methods from different fields have been suggested for the solution of MOPs (see e.g. [Miettinen (1999), Coello Coello et al. (2007), Deb (2001), Hillermeier (2001)] and references therein). Within this study we will use multi-objective subdivision techniques [Dellnitz et al. (2005)] which are state of the art for the numerical treatment of moderate dimensional MOPs such as the one considered in Section 3.1.

In the context of this article we want to stress that the knowledge of the *entire* Pareto set of the bi-objective problem (minimization of flight time and fuel consumption) can help a mission designer to decide which optimal compromise to take. Questions like 'how much time do we save if we use a bit more fuel during the mission' can be quickly answered by looking at a graphical representation of the Pareto front, i.e. the image of the Pareto set.

Local single-objective optimal control Although the benefits of a global multi-objective optimization for decision making are sketched above, the computational burden of solving such optimization problems is relatively high. Hence, the underlying models are typically somehow restricted. As mentioned before, a standard approach is to select a compromise by means of the restricted model and then locally (re-)optimize the corresponding trajectory by a single-objective local method with a more detailed model.

[1] `http://genesismission.jpl.nasa.gov/`

We will give a short overview of local optimal control methods. In principle, there exist indirect and direct local optimal control methods, here we will focus on direct methods. Some well-known direct methods are shooting techniques (see e.g. [Hicks and Ray (1971), Stoer and Bulirsch (1993), Kraft (1985)]), multiple shooting (see e.g. [Deuflhard (1974), Bock and Plitt (1984), Leineweber et al. (2003)]), and collocation methods (see e.g. [Biegler (1984), von Stryk (1993)]). These methods rely on a direct integration of the associated ordinary differential equations, see also [Betts (1998)] and [Binder et al. (2001)] for an overview of the current state of the art. In contrast to these tools the recently developed DMOC (*Discrete Mechanics and Optimal Control*) [Junge et al. (2005), Ober-Blöbaum (2008)] is based on the discretization of the variational structure of the mechanical system directly. In the context of variational integrators (see for instance [Marsden and West (2001)]), the discretization of the Lagrange-d'Alembert principle leads to structure preserving time stepping equations which serve as equality constraints for the resulting finite dimensional nonlinear optimization problem. This problem can be solved by standard nonlinear optimization techniques such as *Sequential Quadratic Programming (SQP)* (see e.g. [Gill et al. (1997), Gill et al. (2000), Powell (1978), Han (1976)]).

Formation flight With the described methods on-hand, we are able to compute fuel- and time-efficient reconfiguration maneuvers of formation flying spacecraft in the following way. After having identified promising reference trajectories by the composition of global multi-objective optimization and local optimal control methods, we determine a reconfiguration maneuver of a group of spacecraft along these trajectories. Work on the reconfiguration of formation flying spacecraft (see e.g. [Junge and Ober-Blöbaum (2005)] and references therein) has been motivated by the ESA mission DARWIN[2]. The aim of DARWIN is to detect Earth-like planets by interferometric measurements ('nulling interferometry') using a formation of spacecraft as a high performance telescope. That means, the huge size of a single telescope necessary for managing such a task is overcome by using accurately light collectors which will redirect light to the central hub spacecraft. Important mission goals are the reconfiguration of the spacecraft as well as its long operation time. The latter is dealt with by using low thrust engines with their high specific impulse. For the reconfiguration the local optimization method DMOC can be used for precise computations.

A numerical example In our example, we compute energy- and time-efficient low thrust trajectories for a formation of spacecraft which arrives at an Earth L_2 Halo orbit. For this purpose, we consider a **c**ontrolled version of the **C**ircular **R**estricted **T**hree **B**ody **P**roblem (CCRTBP) with Sun and Earth as the primaries.

Organization of the article The remainder of this article is organized as follows: In Section 2, we state the background required for the understanding of the sequel. That

[2] http://www.esa.int/esaSC/120382_index_0_m.html

is, we present some basic facts on the standard three body problem and introduce the restricted control term which we will utilize for the global optimization. Further, we state the required background on multi-objective optimization and one state of the art algorithm for the numerical treatment of such problems. Finally, we will present DMOC, the local optimization method we will use to compute the refined trajectories. In Section 3, we present the three-step optimization process and demonstrate its applicability to a particular example, the CCRTBP. To be more precise, we will compute a set of 'optimal' trajectories – according to the simplified system – from Earth to an Earth L_2 Halo orbit (Section 3.1), and we will refine them in the next step using DMOC (Section 3.2). To complete the example, we will compute reconfiguration maneuvers where we will choose the obtained solutions as reference trajectories (Section 3.3). Finally, we draw some conclusions and state possible directions for future work in Section 4.

2 Model description and optimization methods

In this section we briefly summarize the background required for this work. Thereby, we start with a description of the **C**ircular **R**estricted **T**hree **B**ody **P**roblem (CRTBP) and emphasize the extension to a controlled problem (CCRTBP). Using the CCRTBP we are able to compute reachable sets which will be of use for the upcoming optimization. Afterwards, the basic definition of the concept of multi-objective optimization are given followed by the description of the subdivision techniques we are using. At the end of this section we introduce the concept of the local optimal control method DMOC.

2.1 The Controlled Circular Restricted Three Body Problem

The Spatial Circular Restricted Three Body Problem As alluded to in the Introduction, we consider the *(spatial) circular restricted three body problem (CR3BP)* with Sun and Earth as main bodies, with masses m_1 and m_2, respectively. The mass of the third body – typically an asteroid, a comet, a spacecraft, or just a particle – is assumed to be negligible. For instance [McGehee (1969)] and [Koon et al. (2000)] noticed the importance of this model for modelling the dynamics of a space mission between different planets.

Let us briefly recall the basics of this model – for a more detailed exposition we refer to [Szebehely (1967), Meyer and Hall (1992), Belbruno (2004), Gómez et al. (2001), Abraham and Marsden (1978)]. The CR3BP models the motion of a particle in the gravitational field of two bodies like e.g. Sun and Earth. These two primaries move in a plane counterclockwise on circles about their common center of mass with the same constant angular velocity. The third body does not influence the motion of the primaries while it is only influenced by their gravitational forces. In a normalized rotating coordinate system the origin is the center of mass and the two primaries are fixed on the x_1-axis at $(-\mu, 0, 0)$ and $(1 - \mu, 0, 0)$, respectively, where $\mu = m_1/(m_1 + m_2)$. For the three body problem Sun-Earth-spacecraft we use $\mu_{SE} = 3.04041307864 \cdot 10^{-6}$.

The equations of motion for the spacecraft with position (x_1, x_2, x_3) in rotating coordinates are given by

$$\begin{aligned}
\ddot{x}_1 - 2\dot{x}_2 &= \Omega_{x_1}(x_1, x_2, x_3), \\
\ddot{x}_2 + 2\dot{x}_1 &= \Omega_{x_2}(x_1, x_2, x_3), \\
\ddot{x}_3 &= \Omega_{x_3}(x_1, x_2, x_3),
\end{aligned} \tag{1}$$

where

$$\Omega(x_1, x_2, x_3) = \frac{x_1^2 + x_2^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}$$

and

$$r_1 = \sqrt{(x_1+\mu)^2 + x_2^2 + x_3^2}, \quad r_2\sqrt{(x_1 - 1 + \mu)^2 + x_2^2 + x_3^2}.$$

Here, the subscripts of Ω denote partial differentiation in the respective variable, and r_1, r_2 are the distances from the particle to the Sun and Earth, respectively.

The equations (1) have a first integral, the *Jacobi integral*, which is given by

$$C(x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3) = -(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2) + 2\Omega(x_1, x_2, x_3). \tag{2}$$

The five-dimensional manifold of constant values of the Jacobi constant are an indicator of the type of global dynamics possible for a particle in the CRTBP and are invariant under the flow of (1). Their projection onto position space, *Hill's region*, determines the allowed region for the motion of the spacecraft (cf. Figure 1). In our example we consider

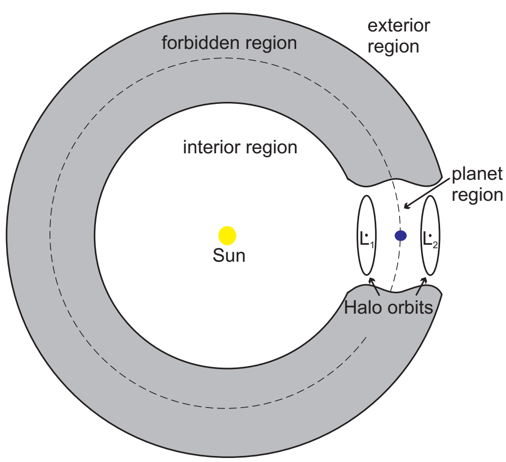


Figure 1: Projection of an energy surface onto the x_1x_2-plane (schematic) for a value of the Jacobi integral for which the spacecraft is able to transit between the exterior and the interior region.

a target with a Jacobi constant given by $C = 3.0005$.

The system posesses five equilibrium points (the *Lagrange points*) – the collinear points L_1, L_2 and L_3 on the x_1-axis and the equilateral points L_4 and L_5. It is known (see e.g. [Barden et al. (1997)] and [Dichmann et al. (2003)]) that there exist families of periodic orbits – the so called *Halo orbits* around the Lagrange points L_1 and L_2. Those Halo orbits of the spatial problem are of interest to us. In [Junge et al. (2002)] further references are given that these orbits are especially suited for space missions for single and multiple spacecraft. For multiple spacecraft missions, key aspects are for instance the movement along a trajectory in a special formation or the arrival at the destination with a prescribed relative position. Whereas in [Junge et al. (2002)] a heuristic suboptimal control law is developed for keeping a formation near the Halo orbit we tackle the problem in such a way that the spacecraft build a given formation at arrival time. Moreover, we employ a sophisticated global and local optimization method to determine an optimal control law.

A heuristic control For current mission concepts, like the ESA interplanetary mission *BepiColombo*[3] or the DARWIN mission ion propulsion systems are being considered that continuously exert a small force on the spacecraft ("low-thrust propulsion"). This new engine type was already successfully applied in previous missions such as *Smart I*.[4] However, the previously introduced circular restricted three body problem (1) does not model this continuous thrusting capability and thus the model needs to be enhanced by a suitably defined control term. As in [Dellnitz et al. (2006b)], we restrict our considerations to the special case of a control force whose direction is defined by the velocity of the spacecraft. The reason is that an acceleration parallel to the velocity vector yields a maximum instantaneous impact[5] onto the kinetic energy of the spacecraft. This can be seen by the time derivative of kinetic energy which solely depends on the dot product of the spacecraft's velocity and its acceleration if the mass is assumed to be constant (see [Gerthsen and Vogel (1993)]).

The control term which has to be included into the model is therefore parametrized by a single real value u, determining the magnitude of the control acceleration. We do not take into account that the mass of the spacecraft changes during its flight since in our modelling the spacecraft has neglegible mass. The only effect would be that the same acceleration u can be achieved by less driving force if the mass decreases over time. This would allow to employ a higher upper bound for u at a later time which means that we maintain a conservative estimate for this upper bound.

The velocity vector of the spacecraft has to be viewed with respect to the inertial coordinate system and not the rotating one. In view of this, one is lead to the following control system, modeling the motion of the spacecraft under the influence of its low thrust

[3] `http://sci.esa.int/science-e/www/area/index.cfm?fareaid=30`

[4] `http://sci.esa.int/science-e/www/area/index.cfm?fareaid=10`

[5] This does not imply a globally optimal impact, see [Tang and Conway (1995)], but serves as heuristic to reduce the space of control functions in the global part of the latter optimization procedure.

propulsion engines in rotating coordinates (cf. Figure 2):

$$\ddot{x} + 2\dot{x}^{\perp} = \nabla\Omega(x) + u\,\frac{\dot{x} + \omega x^{\perp}}{|\dot{x} + \omega x^{\perp}|}. \tag{3}$$

Here, $u = u(t) \in [u_{\min}, u_{\max}] \subset \mathbb{R}$ denotes the magnitude of the control force, $x = (x_1, x_2, x_3)$, $x^{\perp} = (-x_2, x_1, 0)$ and $\omega = 1$ is the common angular velocity of the primaries in the x_1x_2-plane.

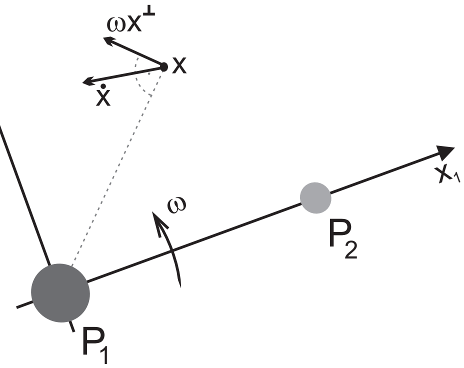


Figure 2: The velocity of the spacecraft with respect to the inertial frame is given by $\dot{x} + \omega x^{\perp}$. Since the rotation of the primaries takes places in the x_1x_2-plane, only the projection onto that plane is illustrated.

Reachable sets Obviously, every solution of (1) is also a solution of (3) for the control function $u \equiv 0$. We are going to exploit this fact in order to generalize the standard manifold approach to the case of controlled 3-body problems. Instead of computing the relevant time-backward invariant manifolds of the periodic Halo orbit, we compute certain time-backward *reachable sets* (see e.g. [Colonius and Kliemann (2000)]), i.e. sets in phase space that can be accessed by the spacecraft when employing a certain control function.

We denote by $\phi(t, z, u)$ the solution of the control system (3) for a given initial point $z = (x, \dot{x})$ in the phase space Z at $t_0 = 0$ and a given admissible control function $u \in \mathcal{U} = \{u : \mathbb{R} \to [u_{\min}, u_{\max}], u \text{ admissible}\}$. Here $u_{\min}, u_{\max} \in \mathbb{R}$ are predetermined bounds on the magnitude of the control force, and the attribute "admissible" alludes to the fact that only a certain subset of functions is allowed. Both, the bounds and the set of admissible control functions will be determined by the design of the thrusters. For example, the set of admissible control functions could be the set of piecewise constant functions, where the minimal length of an interval on which the function is constant is determined by how fast the magnitude of the accelerating force can be changed within the thrusters.

For a set S in phase space Z (S being an element of the power set $\mathbb{P}(Z)$) and a given function $\tau : S \times \mathcal{U} \to \mathbb{R}$, we call $\mathcal{R} : \mathbb{P}(Z) \times (S \times \mathcal{U} \mapsto \mathbb{R}) \mapsto \mathbb{P}(Z)$ with

$$\mathcal{R}(S, \tau) = \{\phi(\tau(z, u), z, u) \mid u \in \mathcal{U}, z \in S\}$$

the set which is (τ-)*reachable from* S. Later on, we choose $\tau(x,u)$ in such a way that the reachable sets are contained in an intersection plane close to Earth.

2.2 Multi-objective optimization

For the global multi-objective optimization process we consider continuous MOPs of the form

$$\min_x G(x), \tag{4}$$

where $G : Q \subset \mathbb{R}^n \to \mathbb{R}^k$ is a vector of objective functions

$$G(x) = (g_1(x), \ldots, g_k(x)),$$

and each objective $g_i : Q \to \mathbb{R}$ is continuous. Further, we assume that each parameter x_i, $i = 1, \ldots, n$, is restricted to a certain range $a_i \le x_i \le b_i$ leading to the domain Q given by

$$Q = \{x \in \mathbb{R}^n \,:\, a_i \le x_i \le b_i,\, i = 1, \ldots, n\}\,. \tag{5}$$

The optimality of an MOP is defined by the concept of *dominance* which dates back over one century and was first introduced by Pareto [Pareto (1971)]. For the test of dominance the following definitions are necessary.

Definition 2.1 (a) Let $v, w \in \mathbb{R}^k$. Then the vector v is *less than* w ($v <_p w$), if $v_i < w_i$ for all $i \in \{1, \ldots, k\}$. The relation $\le_p$ is defined analogously.

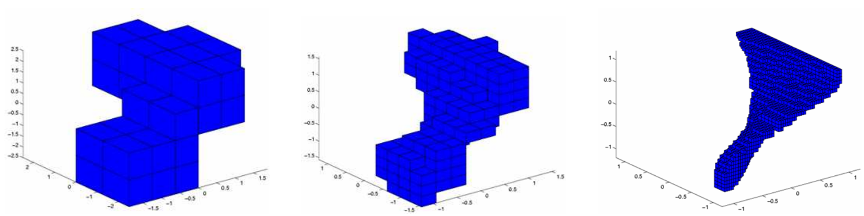
(b) A vector $y \in \mathbb{R}^n$ is *dominated* by a vector $x \in \mathbb{R}^n$ ($x \prec y$) with respect to (4) if $G(x) \le_p G(y)$ and $G(x) \ne G(y)$, else y is called non-dominated by x.

(c) A point $x \in Q$ is called *(Pareto) optimal* or a *Pareto point* if there is no $y \in Q$ which dominates x.

The set of all Pareto optimal solutions is called the *Pareto set*, denoted by P_Q. The image $G(P_Q)$ of the Pareto set is called the *Pareto front*. P_Q typically – i.e. under mild regularity assumptions on the MOP – forms a $(k-1)$-dimensional object.

2.3 Multi-objective subdivision techniques

The subdivision techniques described in [Schütze et al. (2003)] and [Dellnitz et al. (2005)] are set-oriented methods and have been primarily designed for the numerical treatment of MOPs without equality constraints. Algorithms of this type start with the domain Q (see (5)), which constitutes an n-dimensional *box*. This box gets subdivided into a set of smaller boxes, and according to certain conditions it is decided which box could contain a part of the Pareto set and is thus suited for further investigation. The other, unpromising boxes, are discarded from the collection. This process, i.e., subdivision and selection, is performed on the current box collection until the desired granularity of the boxes is reached (which is problem dependent). The approach is of global nature, that is, in principle capable of detecting the entire set of Pareto points, see Figure 3 for an example. The convergence of the underlying abstract algorithm is analyzed in [Dellnitz et al. (2002), Dellnitz et al. (2005)].



(10 Iterations) (15 Iterations) (20 Iterations)

Figure 3: Application of the subdivision algorithm on an MOP $G : Q \subset \mathbb{R}^3 \to \mathbb{R}^3$, where Q is defined by box-constraints (see [Dellnitz et al. (2005)]). The box collections show different coverings of the Pareto set.

Subdivision algorithms are very effective for the numerical treatment of moderate dimensional models, and have been used successfully in several applications (see e.g. [Dellnitz et al. (2005), Schütze et al. (2008a), Schütze et al. (2008b)]).

At the end of this section we introduce a local optimization method which is used to optimize the initial guess obtained by the MOP and allows to take a more general control law into account. Furthermore, we are able to design trajectories for all formation flying spacecraft which move close to the locally optimized favorite barycentric trajectory.

2.4 Discrete Mechanics and Optimal Control (DMOC)

In order to solve optimal control problems, we use DMOC, a technique that relies on a direct discretization of the variational formulation of the dynamics of the system (see [Junge et al. (2005), Ober-Blöbaum (2008)]). For convenience, we briefly summarize the basic idea.

A mechanical system with configuration space M is to be moved on a curve $x(t) \in M$, $t \in [0, T]$, from a state $(x^0, \dot{x}^0)$ to a state $(x^T, \dot{x}^T)$ under the influence of a force $f(x(t), \dot{x}(t), u(t)) \in T^*M$, where $u(t) \in U$ is a control parameter. The curves x and u shall minimize a given objective functional

$$J(x, \dot{x}, u) = \int_0^T C(x(t), \dot{x}(t), f(x(t), \dot{x}(t), u(t)))\, dt. \tag{6}$$

If $L : TM \to \mathbb{R}$ denotes the Lagrangian of the system, its motion $x(t)$ satisfies the *Lagrange-d'Alembert principle*, which requires that

$$\delta \int_0^T L(x(t), \dot{x}(t))\, dt + \int_0^T f(x(t), \dot{x}(t), u(t)))\, dt = 0 \tag{7}$$

for all variations δx with $\delta x(0) = \delta x(T) = 0$.

Using a global discretization of the states and the controls one obtains the *discrete Lagrange-d'Alembert principle* which specifies equality constraints for the resulting finite dimensional nonlinear optimization problem as described next.

Discretization We replace the state space TM by $M \times M$ and consider the grid $\Delta t = \{t_k = kh \mid k = 0, \dots, N\}$, $Nh = T$, where N is a positive integer and h the stepsize. We replace a path $x : [0, T] \to M$ by a *discrete path* $x_d : \{t_k\}_{k=0}^N \to M$, where we view $x_k = x_d(kh)$ as an approximation to $x(kh)$ [Marsden and West (2001), Ober-Blöbaum (2008)]. Similar, we replace the control path $u : [0, T] \to U$ by a discrete one. To this end we consider a refined grid $\Delta\tilde{t}$, generated via a set of control points $0 \leq c_1 < \cdots < c_s \leq 1$ as $\Delta\tilde{t} = \{t_{k\ell} = t_k + c_\ell h \mid k = 0, \dots, N-1; \ell = 1, \dots, s\}$. With this notation the *discrete control path* is defined to be $u_d : \Delta\tilde{t} \to U$. We define the *intermediate control samples* u_k on $[t_k, t_{k+1}]$ as $u_k = (u_{k1}, \dots, u_{ks}) \in U^s$ to be the values of the control parameters guiding the system from $x_k = x_d(t_k)$ to $x_{k+1} = x_d(t_{k+1})$, where $u_{kl} = u_d(t_{kl})$ for $l \in \{1, \dots, s\}$.

Via an approximation of the action integral in (7) by a *discrete Lagrangian* $L_d : M \times M \to \mathbb{R}$,

$$L_d(x_k, x_{k+1}) \approx \int_{kh}^{(k+1)h} L(x(t), \dot{x}(t))\, dt,$$

and *discrete forces*

$$f_k^- \cdot \delta x_k + f_k^+ \cdot \delta x_{k-1} \approx \int_{kh}^{(k+1)h} f(x(t), \dot{x}(t), u(t)) \cdot \delta x(t)\, dt,$$

where the left and discrete forces $f_k^\pm$ now depend on (x_k, x_{k+1}, u_k) we obtain the *discrete Lagrange-d'Alembert principle* (8). This requires to find discrete paths $\{x_k\}_{k=0}^N$ such that for all variations $\{\delta x_k\}_{k=0}^N$ with $\delta x_0 = \delta x_N = 0$, one has

$$\delta \sum_{k=0}^{N-1} L_d(x_k, x_{k+1}) + \sum_{k=0}^{N-1} \left(f_k^- \cdot \delta x_k + f_k^+ \cdot \delta x_{k+1} \right) = 0, \tag{8}$$

which is equivalent to the *forced discrete Euler-Lagrange equations*

$$D_2 L_d(x_{k-1}, x_k) + D_1 L_d(x_k, x_{k+1}) + f_{k-1}^+ + f_k^- = 0, \quad k = 1, \dots, N-1. \tag{9}$$

In the same manner we obtain via an approximation of the objective functional (6) the *discrete objective functional* $J_d(x_d, u_d)$, such that we can formulate the *Discrete Constrained Optimization Problem* as

$$\min_{x_d, u_d} J_d(x_d, u_d) = \sum_{k=0}^{N-1} C_d(x_k, x_{k+1}, u_k) \tag{10}$$

subject to the discretized boundary constraints and the discrete Euler-Lagrange equations (9). This is a nonlinear optimization problem with equality constraints, which can be solved by standard optimization methods like SQP. Optionally, we can also include inequality constraints on states and controls.

Due to the discretization of the variational principle structural properties of the continuous solution are preserved for the discrete solution provided by DMOC, e.g. in the presence of symmetry groups in the continuous dynamical system, also along the discrete trajectory the change in momentum maps is consistent with the control forces (see [Ober-Blöbaum (2008)]). DMOC has been successfully applied to problems in space mission design (see e.g. [Junge and Ober-Blöbaum (2005), Dellnitz et al. (2006a), Junge et al. (2006)]) and robotics (e.g. [Kobilarov et al.(2007), Leyendecker et al. (2007), Kanso and Marsden (2005)]).

3 Problem formulation and numerical solution

3.1 Multi-objective optimization and transfer to L_2

The basic idea is to compute an approximation of a time-backward reachable set (see Section 2.1) of a selected Halo orbit and perform the multi-objective optimization on all trajectories which end up in this set. An important issue is how to represent the reachable set: We choose a parametric representation to make the optimization procedure straightforward. Finally, we compute the best compromises (Pareto set) of the parametrized reachable set with respect to the objectives ΔV and flight time (TOF).

Computing the parametrized reachable set Now we describe how to parametrize the reachable set in a natural way by two parameters such that it is possible to calculate the corresponding trajectories by only knowing those parameters. Note that, since we consider time-backward reachable sets, each point of the reachable set is the *starting* point for a corresponding trajectory.

For the purpose of global optimization we restrict ourselves to constant control functions with control values $u \in [u_{\min}, u_{\max}]$ as described in (3). The second parameter θ characterizes the target point $P(\theta)$ on the periodic Halo orbit. A natural choice for θ is to select an arbitrary point $P(0) \in \mathbb{R}^6$ on the periodic Halo orbit and to parametrize it by the flow of the CRTBP. That means, $P(\theta) = \phi(\theta, P(0), 0)$ where we denote by $\phi(t, z, u)$ the solution of the control system (3) as before. In this way, the range of θ is between 0 and the period of the orbit (see also Figure 4).

We numerically proceed to associate trajectories with these two parameters as follows. We shift each point of the Halo orbit slightly to the orbit's stable manifold and numerically integrate the control system (3) for a fixed u backwards in time.[6] After each integration step, we check whether the computed trajectory has crossed the plane $\{x_1 = 1 - \mu_{SE}\}$ and, if so, we start Newton's method to determine a point in that plane.

[6] For the integration we use an embedded Runge-Kutta scheme with adaptive stepsize control as implemented in the code DOP853 by Hairer, Nørsett and Wanner, see [Hairer et al. (1993)].

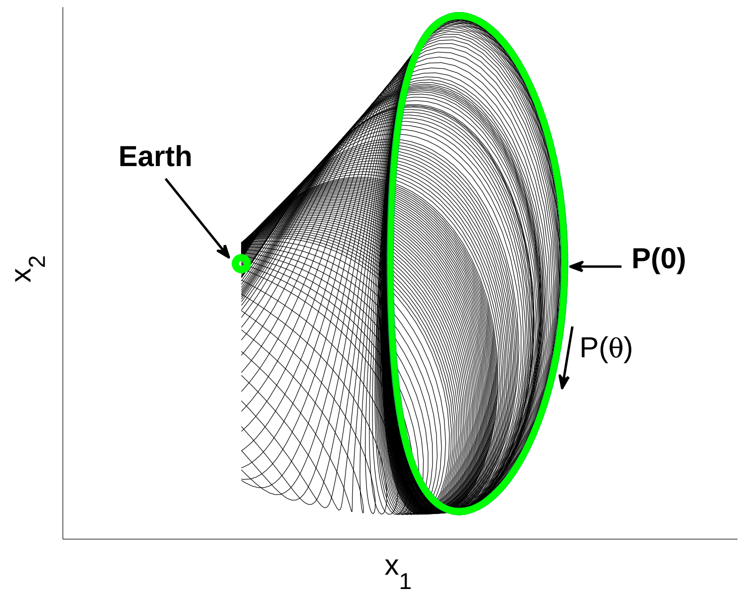


Figure 4: Illustration of the parameter θ of the global MOP for constant control $u = 0$. For any arbitrary selected point $P(0)$ on the periodic Halo orbit the parameter θ uniquely specifies a point $P(\theta)$ on the same orbit with θ between 0 and the period of the orbit. This point serves as target for the corresponding trajectory. For $u = 0$ the trajectories are on the stable manifold of the Halo orbit (schematically shown as black lines).

It is important to note that crossing the plane $\{x_1 = 1 - \mu_{SE}\}$ *only* means that the x_1 coordinate of the spacecraft is the same as that of the Earth. All the other coordinates, particularly x_2 and the 3d-velocity, can (theoretically) differ arbitrarily. Although the consideration of special manifolds restricts the set of trajectories obtained, we are aware that the very next step is to include more realistic constraints on the start points of the spacecraft. However, we believe that this is a technical issue which in principle can be dealt with within our methodical framework presented in this article.

In our example, we choose $u_{\min} = 0\,\mathrm{mN}$, $u_{\max} = 800\,\mathrm{mN}$, and the mass of the spacecraft to be $m_{\mathrm{SC}} = 4000\,\mathrm{kg}$ which only affects the ratio between control force and acceleration in our model. The Halo orbit we consider is determined by its Jacobi constant $C = 3.0005$ and has a period of roughly 0.4778 years, i.e. $\theta \in [0, 0.4778]\,\mathrm{a}$.

The resulting global MOP All in all, we obtain a family of solutions of the controlled dynamical system which are parametrized by the target point of the specified Halo orbit and by the magnitude of the applied continuous control. These two parameters are then globally optimized to account for a minimum of flight time (TOF) *and* control effort

($\Delta V \int |u(t)|\, \mathrm{d}t$ with $|\cdot|$ being the 2-norm). This bi-objective MOP

$$\begin{aligned} \text{minimize:} \quad & \begin{cases} \text{TOF} \\ \Delta V \end{cases} \\ \text{subject to:} \quad & u \in [0, 800]\,\mathrm{mN},\ \theta \in [0, 0.4778]\,\mathrm{a} \end{aligned} \tag{11}$$

is solved by the subdivision techniques desribed in Section 2.3. Since merely two parameters are involved the numerical treatment of such a problem does not represent a challenge for the method, see Section 3.2 for a numerical result.

3.2 Optimal control for a fully actuated transfer to L_2

At this stage we consider a three-dimensional system that is controlled via three time-dependent translational control forces, one for each degree of freedom of the spacecraft.

The Lagrangian corresponding to a spacecraft in the CRTBP reads as

$$L(x, \dot{x}) = \frac{1}{2} m (\dot{x}_1 - \omega x_2)^2 + \frac{1}{2} m (\dot{x}_2 + \omega x_1)^2 + \frac{1}{2} m \dot{x}_3^2 - m \frac{1-\mu}{r_1} - m \frac{\mu}{r_2} \tag{12}$$

with $r_1 = \sqrt{(x_1 + \mu)^2 + x_2^2 + x_3^2}$ and $r_2 \sqrt{(x_1 - 1 + \mu)^2 + x_2^2 + x_3^2}$. For the angular velocity of the rotating coordinate frame $\omega = 1$ this Lagrangian provides the equations of motions defined in (1) multiplied with the spacecraft mass m. Since the spacecraft is fully actuated, the control forces are simply

$$f(u(t)) = u(t) = (u_x(t), u_y(t), u_z(t)) \in \mathbb{R}^3. \tag{13}$$

According to Section 2.4 the application of the discrete Lagrange-d'Alembert principle provides the forced discrete Euler-Lagrange equations. These serve as equality conditions for minimizing a discrete version of the objective functional according to minimal control effort

$$J(u) = \frac{1}{2} \int_0^T |u(t)|^2\, dt,$$

with the final flight time T, and initial and final states $(x(0), \dot{x}(0), x(T), \dot{x}(T))$ as determined in the MOP (11). For simplicity we choose the above measure of control effort rather than ΔV.

As a balance between accuracy and efficiency we employ the midpoint rule for approximating the relevant integrals. Correspondingly, we choose a single intermediate control sample for the interval $[t_k, t_{k+1}]$ to be $u_d(t_k + \frac{1}{2}h)$. We also make use of the stepsize sequence resulting from the embedded Runge-Kutta scheme, i.e. rather than using the same timestep h for each discrete time interval, our discretization of the Lagrange-d'Alembert principle bases on a sequence of stepsizes $\{h_k\}_{k=0}^{N-1}$.

Results We apply the optimal control scheme to *all* Pareto solutions for the purpose of illustration and solve the resulting constrained optimization problem by the SQP routine `nag_opt_nlp_sparse` of the NAG library[7]. Note, that such a computation may be time consuming and, in fact, a strong point of our approach is that the local optimization in principle only must be carried out for a few promising Pareto solutions.

Figure 5 illustrates the solutions obtained from the global multi-objective optimization given in (11) applied to the simplified system with constant control (dashed) as well as the solutions resulting from the local single-objective reoptimization with the more flexible control model (denoted by the *full system*) described above (solid). For the local optimization the trajectories corresponding to the points on the dashed curve served as initial guesses. These initial guesses consist of the entire state and control trajectories as well as the boundary constraints, the flight time, the number of discretization points, and the stepsize lengths as resulting from the solutions of (11). Depending on the flight time, the number of discretization points varies between $N = 40$ and $N = 80$ points for one trajectory. As the solid curve lies below the dashed one, the ΔV is slightly improved. However, the multi-objective optimization for the restricted model already seems to provide very good initial guesses for the optimal control scheme applied to the more detailed model.

In order to obtain an estimate on how useful the information from the global Pareto optimization is we repeat the DMOC computations using more naive initial guesses. Here, we still make use of information about the boundary constraints, the flight time, the number of discretization points, and the stepsize lengths as resulting from (11), but rather than initializing DMOC with the solution trajectories of (11), we choose a straight line connecting the bounds as initial guess and assume the controls to be zero.

The results are illustrated in Figure 6. Although all solutions are feasible, not all trajectories are optimal in the sense that the tolerances for optimality have not been satisfied of the SQP solver. Therefore, the points, where each point represents one solution trajectory, spread out (cf. left of Figure 6). On the right of Figure 6 (zoom of the lower part) we observe that for the trajectories with low flight time a good initial guess is less important since the local optimal control method provides solutions which are as good as those with the better initial guess. For larger flight times, however, the optimal trajectories generated with a naive initial guess require a much higher ΔV, i.e. the SQP solver got stuck in a different local minimum. This example indicates that only an entire exploitation of the knowledge gained from a global multi-objective optimization leads to a smooth connected Pareto set and confirms the importance of good initial guesses for the local optimal control scheme.

In Figure 7 three particular optimal trajectories and controls corresponding to the points in Figure 5 are illustrated. The trajectories start close to Earth ($\circ$) and end on the Halo orbit ($\times$). The first trajectory is a relatively time-efficient optimum with a flight time of 0.1042 years and $\Delta V = 560.5924\,\mathrm{m/s}$. On the contrary, the third trajectory is

[7] `www.nag.com`

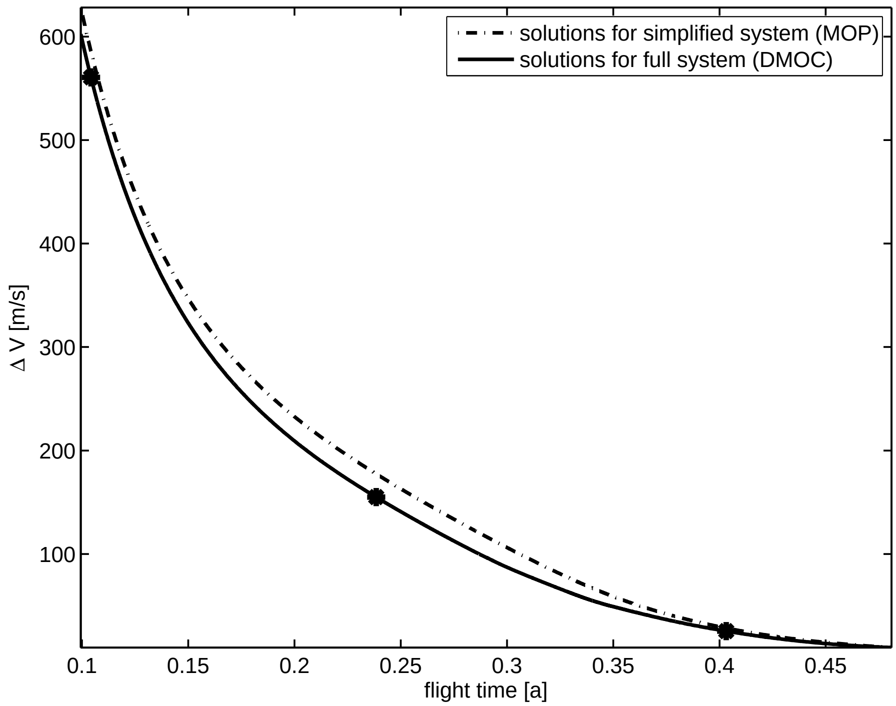


Figure 5: Optimized solutions for the transfer from Earth to a Halo orbit around L_2. Dashed: Solutions of the global multi-objective optimization problem for the simplified system with objective functions ΔV and flight time T. Solid: Local reoptimization of control effort by DMOC with the full system and fixed flight time. Points: Selected reference trajectories for the formation flight (cf. Figure 7).

relatively fuel-efficient as indicated by a ΔV of 25.5464 m/s but requires a flight time of 0.4031 years. Finally, the second trajectory can be considered as a compromise between short flight time (0.2385 years) and low control effort ($\Delta V = 154.9627$ m/s).

3.3 Reconfiguration of a formation flight

Our next aim is the computation of a formation flight of a group of n spacecraft along the precomputed reference trajectories. For the reconfiguration we compute the optimal control force $u^{(i)}$, $i = 1, \ldots, n$, for each spacecraft, such that the group moves from a given initial state $(x^{(i)}, \dot{x}^{(i)})_{i=1}^{n}$ into a prescribed target manifold within a given time interval $[0, T]$. Similar computations have been done in [Dellnitz et al. (2006a), Junge et al. (2006), Junge and Ober-Blöbaum (2005)] where a reconfiguration of a group of spacecraft modeled by rigid bodies along the L_2 Halo orbit was performed. For the purpose of this contribution, we model the spacecraft as point masses. However, the same computations can be performed with the more complex rigid body model. Each spacecraft is described by the same Lagrangian (12) and steered by the same control function (13) assuming that each spacecraft is affected by the same gravitational potential but there is no interaction between the vehicles. We consider $n = 4$ spacecraft as planned for the

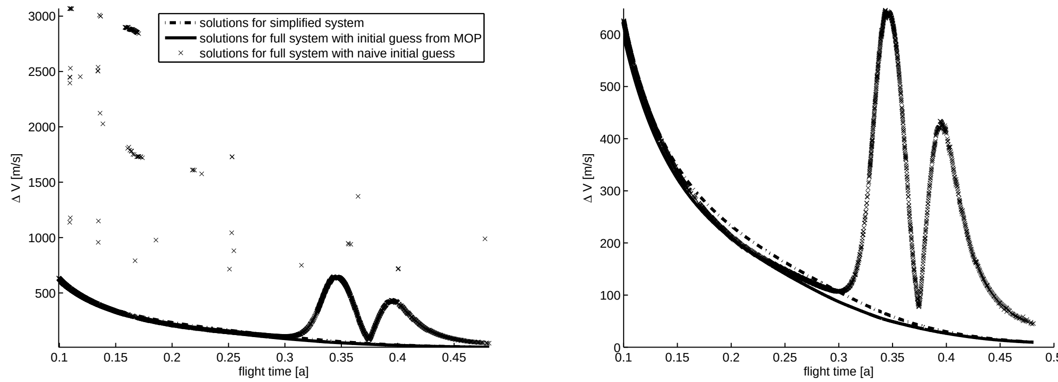


Figure 6: Optimized solutions for the transfer from Earth to an L_2 Halo orbit (Right: Zoom of the lower part). Dashed: Solutions of the multi-objective optimization problem for the simplified system with objective functions ΔV and flight time T. Solid: Solutions of the optimal control problem for the system model with fixed flight time, minimal control effort and initial guesses provided by the MOP. Crosses: Solutions of the optimal control problem for the full system with fixed flight time, minimal control effort and naive initial guesses.

DARWIN mission and the target manifold is defined by prescribing the relative position of the spacecraft and their common velocity. We additionally require the resulting trajectory to minimize a given objective functional related to the associated fuel consumption of the spacecraft.

More precisely, for the target state, we require the spacecraft to be located on the corners of a square with center on a Halo orbit, where this square is assumed to span a prescribed plane given by its normal $\nu \in \mathbb{R}^3$. The target manifold $Y \subset \mathbb{R}^{24}$ is the set of all states $(x, \dot{x})$ with $x = (x^{(i)})_{i=1}^4$ and $\dot{x} = (\dot{x}^{(i)})_{i=1}^4$ such that the following conditions are satisfied:

1. The spacecraft are located on the corners of a square of prescribed size and prescribed center M_0 on a fixed Halo orbit, where M_0 is the final point of the reference trajectory computed in Section 3.2. Let $r_0 \in \mathbb{R}$ be a given side length. To ensure the common center of mass to be M_0 we require that

$$h(x) = \frac{1}{4} \sum_{i=1}^{4} x^{(i)} - M_0 = 0, \tag{14}$$

for a function $h : \mathbb{R}^{12} \to \mathbb{R}^3$. In addition, we have the constraints

$$k_1(x) = \|x^{(i)} - x^{(j)}\| - r_0 = 0, \quad (i,j) \in \{(1,2),(2,3),(3,4),(4,1)\}, \tag{15}$$

$$k_2(x) = \|x^{(i)} - x^{(i+2)}\| - \sqrt{2} r_0 = 0, \quad i = 1, 2, \tag{16}$$

with functions $k_j : \mathbb{R}^{12} \to \mathbb{R}$, $j = 1, 2$, which guarantee an equidistant arrangement to a square with side length r_0.

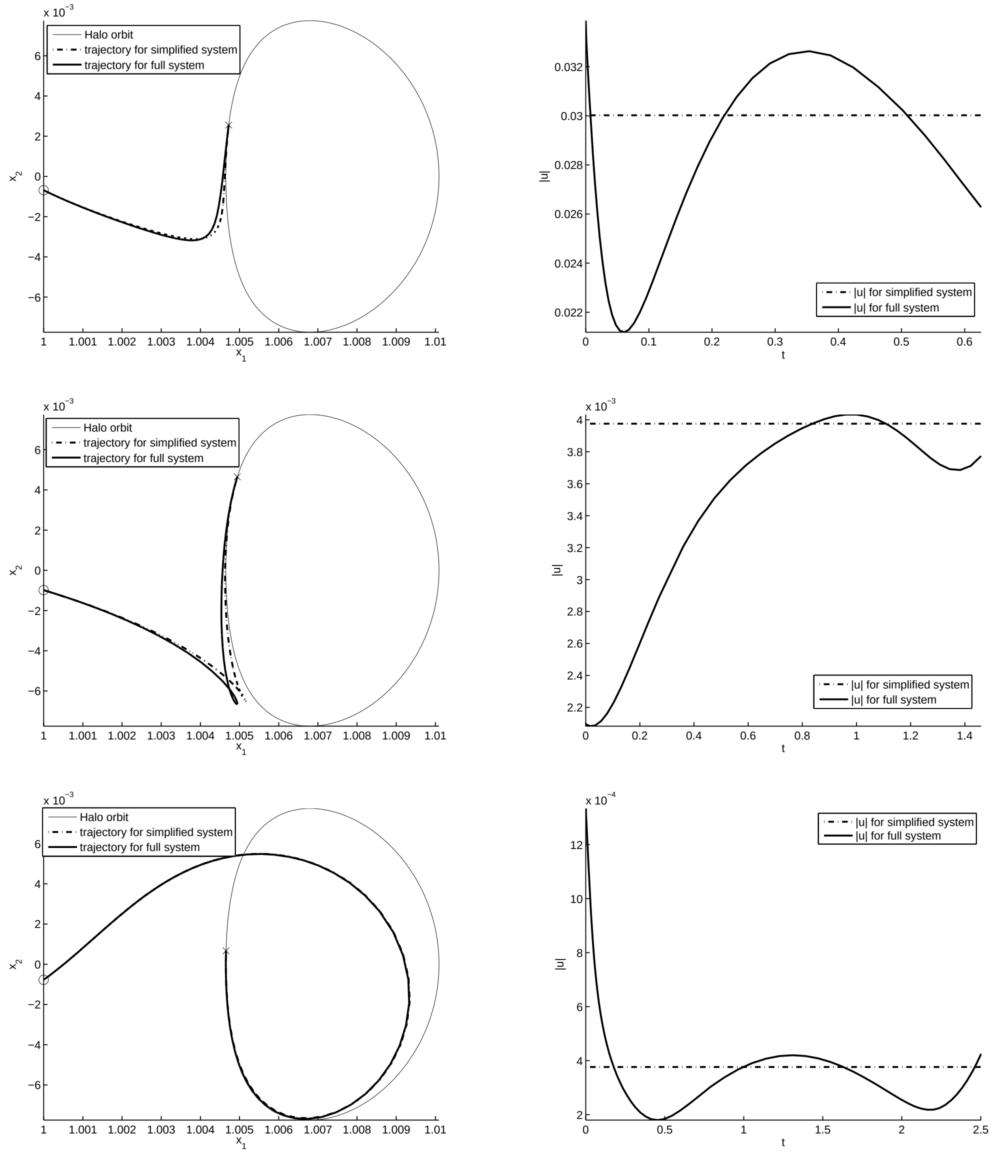


Figure 7: Optimized trajectories and controls for the transfer from a region close to Earth to an L_2 Halo orbit with objective functions ΔV and flight time T. Left: The trajectories start at $\circ$ and end at $\times$. Right: Time evolution of the 2-norm of the controls. Top: Time-efficient solution ($\Delta V = 560.59\,\mathrm{m/s}$ and $T = 0.1042\,\mathrm{a}$). Center: Compromise of time and control efficiency ($\Delta V = 154.96\,\mathrm{m/s}$ and $T = 0.2385\,\mathrm{a}$). Bottom: Fuel-efficient solution ($\Delta V = 25.55\,\mathrm{m/s}$ and $T = 0.1042\,\mathrm{a}$).

2. All spacecraft are in a plane with normal ν. Due to the constraints (14)–(16), the condition

$$l(x) = \langle x^{(i)} - x^{(i+2)}, \nu \rangle = 0, \quad i = 1, 2,$$

for a function $l : \mathbb{R}^{12} \to \mathbb{R}$ ensures the square formation to span the prescribed plane.
3. All spacecraft have the same prescribed linear velocity, $\dot{x}^{(i)} = \dot{x}_0,\ i = 1, \ldots, 4$, where $\dot{x}_0$ is determined on basis of the Halo orbit under consideration.

Thus, the target manifold is defined by

$$\begin{aligned} Y \;=\; & \{(x, \dot{x}) \in \mathbb{R}^{24} \,|\, h(x) = 0,\ k_j(x) = 0,\ l(x) = 0,\ \dot{x}^{(i)} = \dot{x}_0 \\ & \text{with } j = 1, 2 \text{ and } i = 1, \ldots, 4\}. \end{aligned}$$

As mentioned, in addition to controlling to the target manifold, we would like to minimize the fuel consumption of the spacecraft. Here we consider the objective function

$$J(u) = \sum_{i=1}^{4} J_i(u^{(i)}) = \sum_{i=1}^{4} \int_0^T |u^{(i)}(t)|^2 \, dt,$$

where J_i is the objective functional for spacecraft i and $u(t) = (u^{(1)}(t), \ldots, u^{(4)}(t))$ denote the control functions for the system.

Collision avoidance For communication reasons the spacecraft are required to keep a prescribed maximal distance d_{max} to each other during the formation flight. Additionally, collisions between the spacecraft have to be avoided during the reconfiguration which leads to inequality constraints for each time step t_k as

$$d_{min} \leq \|x^{(i)}(t_k) - x^{(j)}(t_k)\| \leq d_{max},\ i, j = 1, \ldots, 4,\ i \neq j,\ k = 0, \ldots, N.$$

Linearization Since we are interested in the relative positions of the spacecraft with respect to each other and the scales of interest differ by a factor of around 10^{11}, the computations are performed in a local coordinate system (y_1, y_2, y_3) by linearizing the system around the reference trajectory under consideration to avoid rounding errors.

Results For each of the three re-optimized trajectories depicted in Figure 7 we compute a formation flight along these trajectories. For our computations we choose a minimal spacecraft distance $d_{min} = 5\,\text{m}$ and a maximal distance $d_{max} = 50\,\text{m}$. All spacecraft start in a line on the y_1-axis with a distance of 15 m from each other. They form up a square with side length of 30 m. In Figure 8 the relative trajectories of each spacecraft for the different reference trajectories are illustrated. Here, the reference trajectories are located in the coordinate origin $(0, 0, 0)$. For the time-efficient trajectory and the trajectory representing a compromise between time- and fuel-efficiency the solutions for each spacecraft look similar (see Figure 8 (a) and (b)), anyhow the time-efficient formation flight requires a higher ΔV of $2.1203 \cdot 10^{-3}$ m/s compared to the $\Delta V = 7.6457 \cdot 10^{-4}$ m/s required for the compromise trajectory. However, for the formation flight along the fuel-efficient trajectory we have an even higher $\Delta V = 2.6849 \cdot 10^{-3}$ m/s as indicated by Figure 8 (c). The single spacecraft trajectories spiral around the reference trajectory before they form up the final desired configuration.

The ΔV required additionally for the formation control is almost negligible compared to the ΔV for the transfer from Earth to L_2. Thus, a space mission designer may base his or her first decision on suitable trajectories only on the data of the reference trajectories.

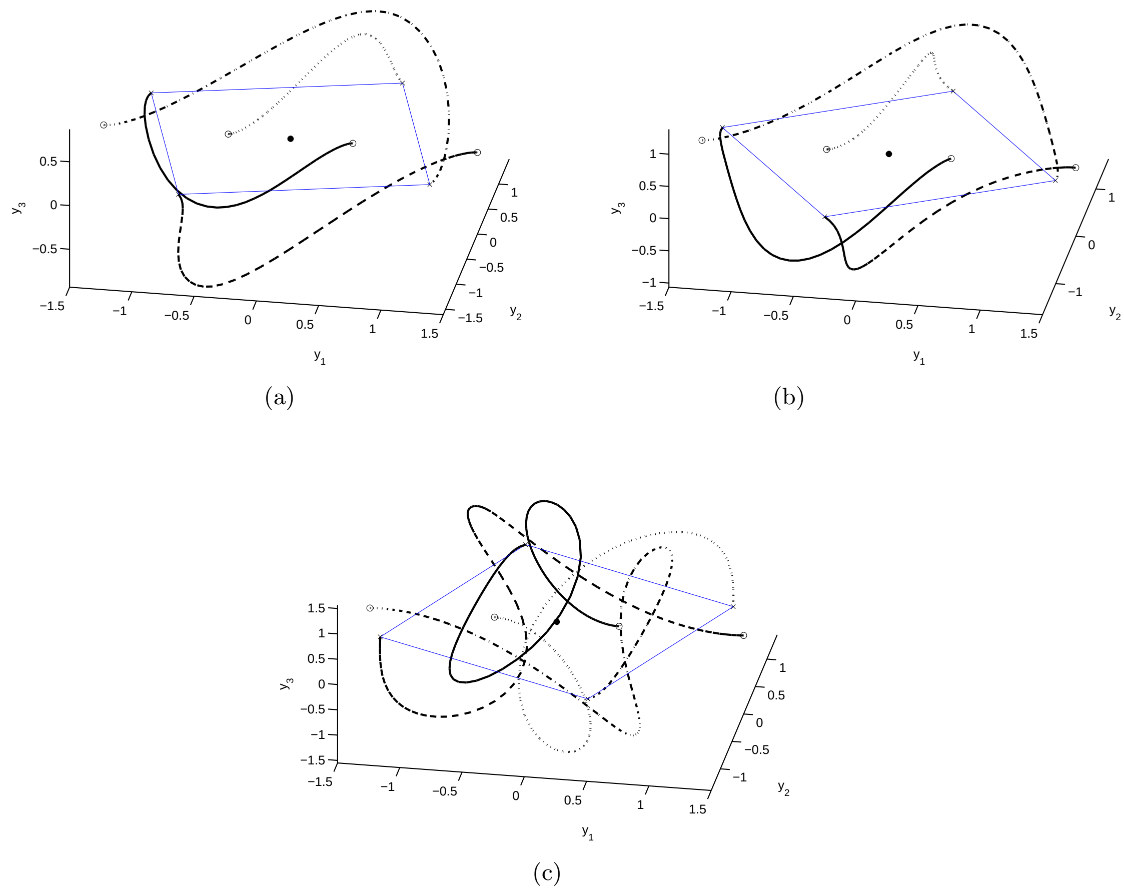


Figure 8: Relative trajectories of four spacecraft flying in formation along the three reference trajectories (located in $(0, 0, 0)$) depicted in Figure 7. The formation starts in a line with a distance of 15 m from each other ($\circ$) and ends in a square with side length of 30 m ($\times$). (a) Formation flight along a time-efficient transfer trajectory (additional $\Delta V = 2.1203 \cdot 10^{-3}$ m/s). (b) Formation flight along a time- and fuel-efficient transfer trajectory (additional $\Delta V = 7.6457 \cdot 10^{-4}$ m/s). (c) Formation flight along a fuel-efficient transfer trajectory (additional $\Delta V = 2.6849 \cdot 10^{-3}$ m/s).

4 Conclusion and future work

Conclusion In this article we have described an efficient strategy of finding low thrust spacecraft trajectories which are optimal with respect to flight time *and* ΔV. We have also presented how to utilize these optimal compromises as reference trajectories for a formation flight. Particularly, we want to stress that not only fixed end points but also manifold constraints on the formation can be solved by employing the recently developed method DMOC. In summary, we applied the following three-stage approach: We performed a multi-objective optimization by a set-oriented subdivision technique, then we refined the resulting trajectories by the local optimal control method DMOC, and finally, we obtained several trajectories for the formation using DMOC again.

Our detailed numerical study documents that the preselection of trajectories by a global method with a yet simple control law substantially contributes to the quality of solutions obtained by the local method (Sections 3.2 and 3.3). In particular, the comparison of trajectories resulting from local optimization with different initial guesses indicate that only the exploitation of the full knowledge gained from the global multi-objective optimization leads to satisfactory results. A further observation is that the changes in ΔV and flight time introduced by the local reoptimization do not influence the qualitative picture of the Pareto set too much and that the additional ΔV for the formation flight is almost negligible with respect to the total one. This legitimates a posteriori the division into three separate steps and illustrates that the knowledge of the *entire* Pareto set in the first step can already help mission designers to decide which optimal compromise to take.

Future work A goal for future work is to even more improve the solution trajectories with respect to a lower ΔV and shorter flight times at the first stage. The issue here is to find a compromise between the fast calculation of the global Pareto set and obtaining trajectories closer to the locally optimized ones. The trade-off is directly reflected in the number of parameters in the global multi-objective optimization which can easily be enlarged by considering more complex control laws, i.e. control functions which are parametrized by a higher number of parameters. In addition, rather than fixing one Halo orbit for the destination of the spacecraft, a further parameter would let the entire family of Halo orbits be subject to optimization.

Methodically, it is of great interest to combine both approaches in a more intertwined way to directly compute global Pareto-optimal solutions for optimal control problems within the *detailed* model. The key point is here that the use of two different models for the global and local optimization would not be necessary any longer. In this way, the mission designer could base his or her decision on the final solution instead of an intermediate result.

Acknowledgements

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