The Harmonic Decision Matrix: a subtle model of the natural neuron

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ABSTRACT

The definition of the right goal constitutes an essential part in Evolutionary Algorithms. The best EA is no use if the defined and calculated purpose does not correspond to a real need. Human needs are generally complex. That is the reason why users of EAs should be interested in utility theory and psychological mechanisms, up to the core of the single neuron. In this paper we show that the harmonic decision matrix opens a new way to model human decisions and basically, neuronal computation, that goes beyond the weighted sum and the polynomials.

Keywords

Multiobjective Optimization, convexity, model of the natural neuron, neural nets, utility function, fuzzy logic, decision making.

1. INTRODUCTION

Evolutionary Algorithms (EAs) do succeed in solving very complex problems. Indeed, many real world applications already proved the robustness, flexibility and effectiveness of this method. In many cases, the difficulty of an optimization depends on the number of variables that can be adjusted as well as the non-linearity of the relations between many variables and a certain goal. However, the first and generally most important difficulty already appears in the ability to recognize and define a problem. Consequently, at least as much attention should be devoted to setting the right goals as to the other steps in the process, namely the choice of the regulating parameters and the run of an optimization method itself, including its practical aspects.

Multi-Objective Optimizations (MOO) focus on the complexity of the possibly conflicting goals as an essential part of the whole complexity. Evolutionary algorithms have been recognized as a promising method to solve MOO problems. In this field, they can be classified in at least four groups:

First group. Objectives are divided into smaller ones and optimized separately, either in successive optimization runs (lexicographic ordering, following the users priorities) or parallel in different sub-populations (as in VEGA, Vector Evaluated Genetic Algorithms, proposed by Schaffer [26]). Of course, this way to come back to a single objective is reserved to complicated, but not complex problems.

Second group. Optimizations that exclude the obviously worst solutions and postpone a part of the difficulty by preserving diversity among the better solutions. In particular, this includes all strategies based on Pareto-ranking, see Laumanns et al. [14], Zitzler and Thiele [27]. Most of these strategies now include niching, clustering or fitness sharing to preserve diversity; these techniques however are not necessarily specific to MOO, they also apply in single objective optimizations. Unfortunately, Pareto-ranking doesn't work for a large number of objectives (figure 1), see Bagot and Pohlheim [2]. The fact that almost all the Pareto based MOO deal with two or three objectives is not the result of hazard or just a visualization problem; it is a necessity. Quite similarly, constraints exclude a part of the solutions without realizing a real MOO on the others.

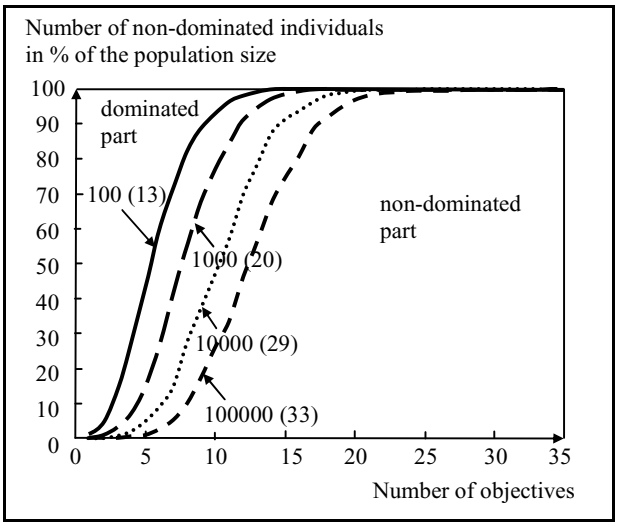

Figure 1. Proportion between dominated and non-dominated individuals depending on the number of objectives involved, for population size 100, 1000, 10000 and 100000. Each objective value has been chosen randomly. For large numbers of objectives (number in brackets), all individuals in the population are non-dominated. Therefore, Pareto ranking gets impossible.

Third group. The generalized MOO divide a problem in smooth clusters and use fuzzy relations or separations between variables and objectives in tricky functions like greedy cross-over or complementary selection and variation, see Bagot and Pohlheim [2]. Such strategies imply a knowledge of the relations between genes and objectives (see figure 2) that can be previously given or built during optimization.

Fourth group. The most common and historically oldest function (for an overview of MOO, see Coello Coello [4], Fonseca and Fleming [8], [9], Jaszkiewicz [12], Purshouse and Fleming [22]) used to deal with numerous objectives is a formula, called aggregating function or target vector function, with several inputs and a single output. The popular weighted sum allows many objectives to be reduced to a single one. The calculated distance to a target point offers another alternative. The last leads to the equation of a circle (or hyper sphere), or of an ellipse, which was the starting point for the discovery of the harmonic decision matrix. Such a reduction limits the work in EAs to the dimension of the genes (the X-axis in figure 2). If not the most effective concerning the optimization speed, compared in particular to the preceding category, it secures the observance of a compromise along the whole evolution process. Because of the limits of the Pareto-ranking, this category keeps its significance.

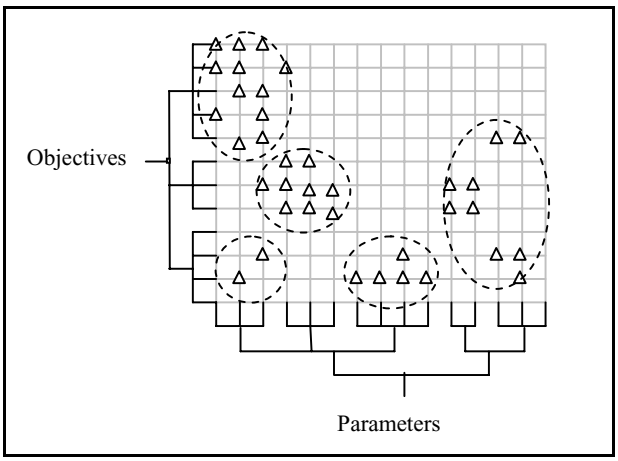

Figure 2. Association matrix showing the relations between genes and objectives. The tree views stress the structure in each of them separately, while the clusters emphasize their relational structure. The clusters represent intermediary objects between parameters and objectives and carry additional information. Δ: Parameter influences objective.

In this paper, we will present a mathematical and logical function (belonging to the above group 4) that helps to combine different, partly opposed criterions in a global assessment. Bagot [1] presented this function or group of operators from a point of view of the fuzzy logic, games and decisions, utility and economic theories. In particular, he resolved a fifty years old enigma in economics known as the Allais' paradox. Here, we will present the Harmonic Decision Matrix (HDM) as a model of the single natural neuron.

The convexity in the representation of the data sets strongly affects the way a linear combination works, regardless of the weights used, see Das [5]. A possible solution is to change the scale of the evaluations, e.g. to apply a rotation of the Pareto front or a logarithmic transformation to some of the objective values. The HDM takes another way: while keeping the same range for all the objective values, it changes the convexity of the ranking logic, visualized through the iso-curves of an operator.

At first, we will define the new function that we are introducing, building our explanation step by step on the base of the biological structure of the neuron. In the section 3, we will compare this to other existing models of the neuron. A short but important focus on the continuous character of neural computation in section 3.2 will introduce in the section 3.3 a theoretical hypothesis on the background computation mechanism. Leaving the core of the neuron, we consider in section 4.1 its interactions in a structured net. At that point, the calculations cumulate in a global decision, where our purpose is reached: make the right decision by the choice between different solutions in an evolutionary algorithm. We complete in 4.2 the details concerning the implementation in practical applications and show the possible combination with or alternative to other procedures building the evolutionary algorithm. In a last section 4.3, we describe a procedure for the practical validation of the model.

2. DEFINITION OF THE HARMONIC DECISION MATRIX

If we want to reduce several objectives to a general, single one, we will realize a logical operation or series of operations, in order to obtain a result we consider useful in the sense that it helps us to take a "good" decision: select the right option(s), eliminate others. What is "good" or "bad" generally depends on personal scales of values as well as on the environment. As an illustration, imagine several buyers who try to find the best car model. The first one wants first of all a sporty engine, the second pays more attention to the room inside, a third one would pay more for an open top for the coming summer, while another buyer wants to have a good looking design able to amaze his neighbors. Moreover, living in a hot country, they need the air-conditioner. If they buy together e.g. as members of the same family, they will find a compromise. If not, the salesman should propose different models and parameterize some options individually. In any case, they take a decision. Consciously or not, they use their brain; compute some sensory information in a net of neurons. How can we reproduce this computation in a program?

For a better understanding of the harmonic decision matrix, let us first represent in figure 3 the involved variables as they appear in a simplified model of the natural neuron. We can forget for a while that we deal with objectives and just consider quantified values to compute. On the left part of a brain cell soma with the nucleus, several dendrites vehicle inputs variables X1, X2, …Xn arriving via synapses from other neurons or sensory cells. On the right hand side, the axon forwards the result Z of the computation to other cells via synapses. Z can be seen as a compromise at this level of decision. It secures the balance inside the decision unit: F(X1, X2, …Xn, Z) = 0.

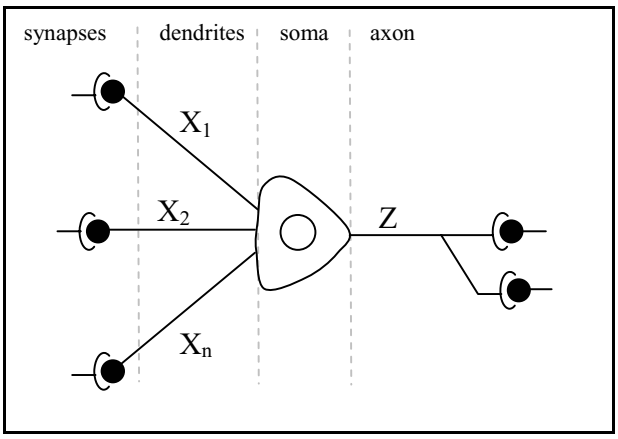

Figure 3. Simplified Model of the natural neuron.

Now, suppose that X1, X2, …Xn take continuous values. It is then more convenient in a case study to reduce the number of inputs and choose another type of representation. For the maximal reduction to a single input, we can take the frequently used representation of a sigmoid, figure 4.

In fact, other transformations are possible, sensory cells in particular may also use bell functions.

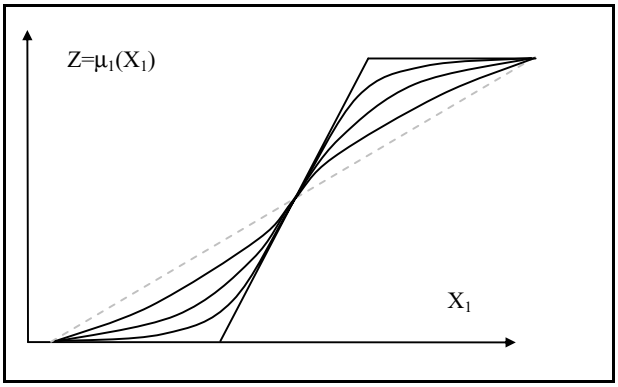

Figure 4. Sigmoides. S-like functions

Now what happens if we take two inputs? Again, the choice of an appropriate representation helps to understand what could happen. We need to graphically represent three continuous variables. In figure 5, two axes indicate the possible values for X1 and X2, while the third quantity, Z, is proportional to a color or ticked by iso-curves in the 2D diagram.

Imagine again the reflections of the car buyer: what use is the best engine in a tiny car if I can not take my girlfriend with me for a drive? Does a large boot make sense if the engine is too weak to carry any additional luggage? Between these extremes, the figure proposes - as fuzzy AND - a result for each possible combination.

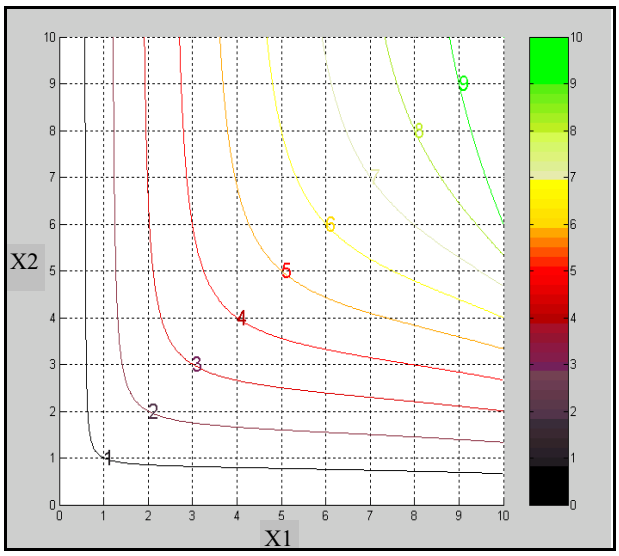

Figure 5. Fuzzy AND.

Inspired by this representation, we can express a corresponding mathematical expression:

$(X,Y) \in [0\ 10]*[0\ 10]$, Z is the smallest solution $\in [0\ 10]$ of

$(X+f1(X)-Z)(Y+f2(Y)-Z)-f1(X)*f2(Y)=0$

In a first step, the weights $w_1=f_1(X)$ and $w_2=f_2(Y)$ can be taken as constants. Then, other continuous functions can sophisticate the model. We can study different parameterizations of the weights (figure 6) and extend the equation to higher dimensions.

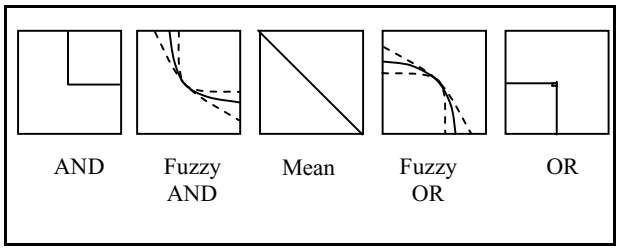

Figure 6. From AND to OR in small steps. In the middle: the mean value.

For N input values ∈ [0 10], the HDM is written:

$$\prod_{n=1}^{N}(\mu_n(x_n) + f_n(\mu_n(x_n))-Z) - \prod_{n=1}^{N} f_n(\mu_n(x_n))=0$$

μ_n are sigmoid functions. f_n are weight functions that are possibly constants $f_n=c_n$ or linear $f_n=c_n * x_n+\varepsilon$ in case of Fuzzy AND. For Fuzzy OR, $f_n(x_n)=f_n'(x_n-10)$.

3. COMPARISON TO EXISTING MODELS

Numerous models of the brain cell, beginning with the representations given by Ramon Cajal at the beginning of the twentieth century, already exist. We give here a short review and, in comparison, point out the main specificities of the newcomer.

3.1 Descriptive, Electric, Chemical and Mathematical models

Descriptive models

The first representations of a neuron show the structure of the cell that appears by observation under a microscope, helped by different techniques of coloration, see Dudel, Menzel and Schmidt [7], Nicholls, Robert Martin and Wallace [20]. Such studies underline the role of the single neuron anatomy, taxonomy, localization and connections within the brain network, see Rolls and Treves [25].

Electric models

Another kind of representation results from electrical measurements on the neuron or axon and takes its inspiration from cable theory, see Koch [13]. Here, dissipation on long distances, transmission velocity and summation of currents play a predominant role.

Chemical models

Molecular biologists explain how the neuron functions with models of the ionic currents at the synapses, in the dendrites or axons, at the cell membrane, see Koch [13].

Mathematical models

Among the mathematical models, the weighted sum and the sigmoid are found very frequently in the specific literature. Moreover their combination in the integrate and fire model, is commonly used in artificial neural nets. However, the sigma-pi unit or polynomial models, see McKenna, Davis and Zornetzer [15] and Koch [13], make supplementary use of multiplication

Most of the proposed models can be seen from several points of views. For example, the famous Hodgkin-Huxley model of action potential generation accounts with a mathematical expression for ionic concentrations in the axon and its membrane electrical potential. For an overview, see Gerstner and Kistler [10], Rieke et al. [24], Koch [13].

We do not have sufficient chemical and neurological knowledge to complete the harmonic decision matrix with its equivalent molecular description. Does the harmonic decision matrix, as McKenna et al. [15] asks, "capture the essential computations of real neurons in models that will provide processing elements for the next generation of biologically inspired neural networks?"

An important particularity of the HDM is the move of the sigmoid to the input side. The polynomial expression z=P(x1,x2,..xn) is abandoned in favor of a more general F(z,x1,x2…xn)=0.

3.2 The frequency modulated nature of neuronal signals

Today, many authors recognize the modulated nature of neuronal signals. Strangely, the most common representation of the transmitted information remains a succession of spikes figure 7 top.

This kind of representation concentrates on the question as to how information is transmitted between neurons. But it does not visualize properly the essential: how much? In fact, a modulated signal is wrongly assimilated to a digital code that transmits only zeros and ones. This confusion let the pioneer John von Neumann [19] to qualify the nervous system as "prima facie digital". The spikes used for the long distance transmissions just set a frame to the real information unit that is frequency, or the time between two of them.

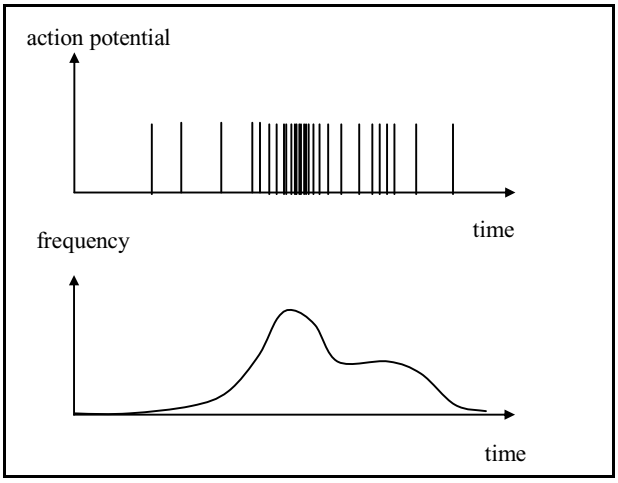

Figure 7. Spikes and Time.

Time is generally considered as a continuous value. Activation thresholds and the minimal response time between two spikes just limit the values between a>0 and b<∞, they do not suppress their continuous characteristic in this range. Recognizing this, a growing number of researchers consider that the brain computes prima facie continuously and uses different mechanisms from analogue, chemical transmission in the synapses, to modulated (rather than digital), electric transmission in the axon or dendrites.

Of course, this does not exclude all-or-nothing answers seen as particular cases of a general rule.

The following remark to the minimal level of precision in synapses is to be noted: according to J. Dudel, Menzel and Schmidt [7], the delivery of 500 quanta contained in small vesicles or 1 Million Ach-molecules manage the transmission of excitatory impulses at the synapse.

However interesting the question of the neural code may be (see Gerstner [10]), we will not discuss that point more in detail. Let us just point out for the further argumentation the role of time.

3.3 Computation algorithms in the natural neuron

Even without detailed molecular understanding of the computation mechanisms in single neurons, we can test some assumptions from a theoretical point of view.

The preceding chapter already gives us a clue. A calculation process using a simple operation and in which time plays an essential role is very likely iterative.

Each time the neuron sends a spike, it performs an iteration with the purpose to find a balance in an equation like the HDM. The spike – or, on a basic level, ionic movements - carries out a calculation step towards the solution to the equation. This idea revealed itself from the experience of programming: coding the function for two entrances, it was possible to use a direct formulation of the solution. Trying to generalize the function to greater dimensions, we chose an iterative zero search available in every mathematical library.

The principal consequence of such a supposition is that the neuron can compute advanced solutions to complex problems, far away from the simple sum or product.

It is probably too early to confirm or reject this theory. The most important work towards an answer concerning the validity of the mathematical model probably consists in realizing artificial synapses. With them, it will be possible to draw a precise map of the answer intensity by different combinations of input values. On the other side, a more and more accurate understanding of the ionic movements in the brain cell, linked to mathematical hypothesis, will deliver new clues.

4. APPLICATION IN EVOLUTIONARY ALGORITHMS

4.1 Single neuron and neural net

After half a century of computer development and decades of progresses in artificial neural networks, the single neuron remains “at the eye of the hurricane”, to quote Churchland in his foreword to [19]. However, beyond the single neuron computation, specific knowledge (e.g. about the Helmholtz machine in Dayan and Abbott [6]) increases around the neural nets. We could not explore yet all the possible applications in this field. We used the harmonic decision matrix in a supervised, feed-forward neural net, embedded (for goal definition) in an evolutionary algorithm. The number and structure of the intermediary layers have been chosen according to specific knowledge. The purpose of our program is to deliver a specific product assessment (decision function) and product improvements (EA) that can both compete with the judgment and action of an expert. The objective function performs in real time an intensive signal analysis, calculates basic decision variables and combines them with the HDM. The relevant topics include vehicle comfort, spontaneity and load limits by shifts with an automatic gear box, figure 8; they are measured and optimized automatically in an online procedure on a test bench. We achieved qualitatively better results with this method, compared to the manual optimization by an expert, however with a significantly greater number of tests. Concerning the optimization speed, see the remark in the introduction, fourth item.

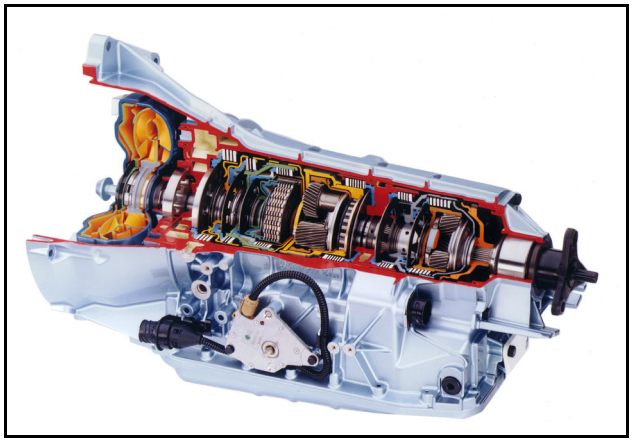

Figure 8. Automatic gear Box. The optimization of software parameters in the control unit is performed by an online Evolutionary Algorithm on a test bench

Intermediary neurons deliver non-requested, but meaningful information. They usually work like a black box, but it is every time possible to look inside and understand what happens. Therefore, modifications or fine tuning can be performed easily. Furthermore, the subtle tuning of the logics in the single neurons already brings good results by a simple (understandable) global structure of the net. Here is the key advantage of the new model. As Bishop [3] judiciously reports, it has been proved that “feed-forward neural networks with threshold units can generate arbitrarily complex decision boundaries. The proof is of little practical interest, however, since it requires the decision boundary to be specified in advance, and also it will typically lead to very large networks.” The question of the theoretical order of the error, depending on the number of intermediate neurons in a neural network and for a finite and randomly

created dataset, is not essential here. The purpose is rather to model datasets derived from real human decisions, without asking (or at the most afterwards) why these decisions are good or effective. Therefore, the advantages of the HDM will be primarily experienced in practice.

Theoretically, it will be possible in further developments to introduce the HDM in different architectures of neural nets, including back-propagation, simulated annealing or different learning methods. The use of the HDM as radial basis function, even if not obvious, should be considered as well. For a good overview of all these possibilities, see Nauck, Klawonn and Kruse [18].

4.2 MOO

From this experience in a multi-objective optimization, we retain the following observations. With the HDM, we have a rational method to recognize, structure and ponder our real needs in a complex problem. This way, we reduce the dimensionality of the Pareto front (see number of objectives figure 1) to hierarchically important criteria. We spend more time in the target definition before the optimization. The final result and its overall assessment are considered in advance as the best result and right decision. Thus, unnecessary repetitions of the optimization process are avoided. We can still keep a representation of the Pareto front for two particularly important criteria. One sometimes changes his mind and his priorities. Then it is still possible to find new good solutions among other Pareto-optimal ones that were created during the optimization. Generally, the priorities are just slightly modified, and the new choices will be localized near the first output. Therefore, even in this case, the new solution picked out of the previous test is considered as good enough and the optimization is not repeated.

The figure 9 compares for real test data (a Pareto front extracted from a bigger measurement) different ranking methods that all use a function F with several inputs x_1, x_2 and a single output z. The last function in the figure, rescaling and weighted sum, can be expressed as

$$z=F(x_1,x_2) = ((k_1x_1^{\alpha}+k_2x_2^{\alpha} / (k_1+k_2))^{1/\alpha}.$$

z is considered as a kind of radius. We want and have the idempotency property: $x_1=x_2 \Rightarrow z=x_1=x_2$. If we define f_1 as the distance between a point A: $x_{10}=x_{20}=z_0$ and B: x_1, $x_2=10$, $z=z_0$, we can prove (with l'Hospital's rule) that $f_1(\alpha\rightarrow 0) = z\text{-}\exp((k_1+k_2)\ln(z)-k_2\ln(10))/k_1)$ for given k_1, k_2 and z. This means, there is a limit in the convexity that can be obtained with this method. Or we have to abandon the idempotency (the term $^{1/\alpha}$ in the previous equation) to get any desired convexity. In fact, among all these methods, the HDM is the only one that both can match the expert's choice and verify the idempotency property, which is important when the function is used several times successively in a net.

There is only one best solution of the HDM in the example figure 9. However, four other solutions (on the Pareto front) get a very similar ranking value and may be reconsidered for an in depth analysis of the results.

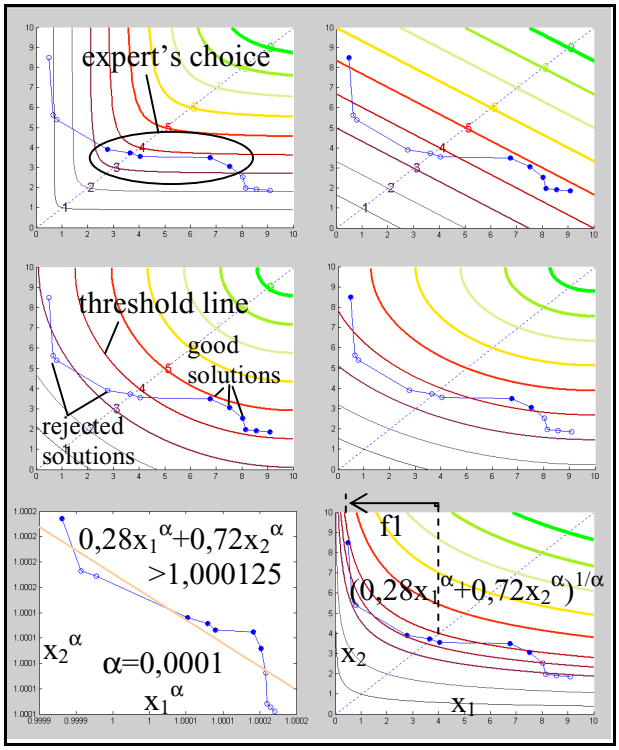

Figure 9. Comparison of ranking methods: HDM parametrized to match the expert's choice (up left), weighted sum (up right), distance to a target (middle left), weighted distance (middle right) rescaling and weighted sum (bottom). The iso-lines separate ten ranking levels. The plain points show the best ranked values, above a chosen threshold.

In a large and complex problem, an evolutionary algorithm that takes into account only the global assessment of each individual works too slowly. With the HDM, a larger number of criteria are calculated: basic values, intermediary objectives and global objective can be all considered in multi-objective optimization techniques mentioned in the introduction in the first, second and third item.

<u>Supervised learning.</u>

Take your own problem; focus, in the beginning, on only two criteria (e.g. physical measurements) about which you feel very concerned. Define a scale of scores to assert them individually by different imaginable or already experienced occurrences. Then, take different combinations of these criteria and assert these combinations globally in an intuitive manner. Now, report all these values in a diagram and draw matching iso-curves. With a little experience, it is

possible to find very quickly manually approximated parameters for the mathematical expression of these curves.

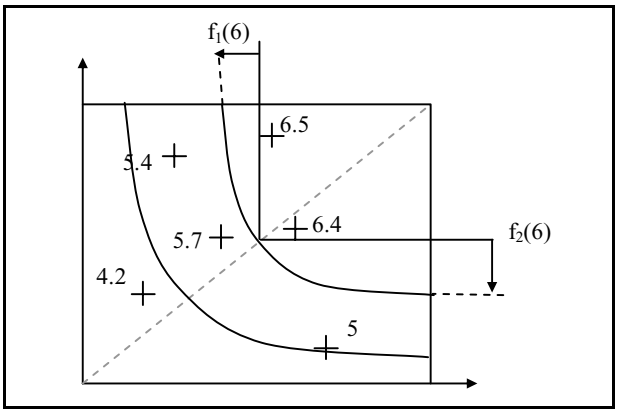

Figure 10. Graphical determination of the HDM weights

If your problem appears more complicated, structure the items in a decision tree and repeat the same principles at the nodes. More generally, a net structure with the HDM as nodes can be parameterized in a procedure called supervised learning in the same way as any other neural network. For given inputs, the outputs of the net are compared to corresponding desired outputs. The error between them (e. g. sum-of-squares error) is taken as goal in optimization techniques (inclusive EAs!) to determine the adaptive parameters of the net more precisely.

Verification of the predictive power.

To control the validity of the model, we proceeded in the reverse sense. We submitted some events and the results of the calculation to several users and asked them if they could agree the judgments or at least consider them as an expert opinion. They did in 94% of our tests. Of course, the HDM applies to the real world… no more than an arithmetic or any other utility function. It cannot force people to behave rationally according to its particular logic. Therefore, the new results will certainly arouse ardent discussion upon the kind of problems concerned or not.

Alan Turing proposed a similar and famous test. In a hypothetical situation, an observer asks two partners, the one a human, the other a computer. If the observer can not determine who the human is, who the computer is, without asking this question directly, the test succeeded. The same test can be realized by limiting the subject to a very specific field and does not need any particular equipment.

5. CONCLUSION

This work has grown from and in an industrial context. Our success in a specific problem encouraged us to propose them to the scientific community. We thank Manfred Bek who made this presentation possible. Readers acquainted with the vast subjects of neural computation, neurobiology, neural circuits and networks, will recognize in the second part of this paper an extreme short resume of established knowledge. In this state of the art context, the harmonic decision matrix that we introduced in the first section clearly appears to be a decisive improvement. This formula and its practical application support the reflection about the iterative computation in the single neuron. We also would like to insist in conclusion on the practical advantages of the function, which we could implement in many situations: it is easy to understand thanks to the proposed visualization, general enough to cover a very broad (and even infinite) spectrum of logical decisions, flexible through simple parameterization, saving many lines of code in a program. Many functions developed for MOO, like weighted sum, optimization under constraints, Pareto ranking, can be completed by using the harmonic decision matrix in a unique decision function structured as a neural network. Simultaneously, the HDM can compute sub-criteria building a complex structure of objectives in generalized MOO mentioned in the introduction.

The HDM provides one among many other ranking strategies. After three years of use, it appears to be a good model for human decisions, at least in the specific area where it has been tested, namely the quality assessment of gear shifts with an automatic gearbox. It is quite intuitive to think that decision patterns statistically confirmed by simple psychological tests should correlate with basic computations of the natural neuron. Psychological experiments inspired by this new concept will show its potential and its limits. Further research will reveal if really a realistic model of the natural counterpart, the brain cell, has been discovered. Today, it seems that this will arouse a passionate debate.

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