Genetic Algorithms Incorporating A Pseudo-Subspace Method

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ABSTRACT

GA performance in high-dimensional optimisation problems can be enhanced by the use of a 'pseudo subspace' technique. The method works by projecting the parameter space onto a lower dimensional subspace in the first stages of the optimisation process, in order to allow the GA search to discover the most promising area of the solution space. Subsequently, the dimensionality of the model is progressively increased until a pre-determined limit is reached. Comparison between the pseudo-subspace procedure and a conventional GA, using two different GA implementations, shows the former to be more successful when applied to two geophysical problems characterised by different solution-space geometry and mathematics. This technique could be easily transferred to different image processing or pattern recognition problems where geometrical relationships between the parameters are maintained.

1. Introduction

In recent years genetic algorithms (GAs) have increasingly been used to address complex optimisation problems [1-3]. One field in which GAs are currently enjoying particular success is geophysics [4-8]. Although the underlaying physics and mathematics may strongly differ, geophysical inversion problems usually share three common features: a high degree of non-linearity, a large number of parameters to be determined and a high cost of function-evaluation. Accordingly, since the resulting high-dimensional solution space can be searched only with small populations in order to keep the computational effort acceptable, the correct reconstruction of many closely interconnected parameters may be particularly difficult. In this paper we present a technique to tackle high-dimensional problems that allows for a GA's performance to be improved by the inclusion of a procedure, here called 'pseudo-subspace' search, whereby the complexity of the model defining the solution is progressively increased during the inversion. This technique has been tested on two different GA implementations on two geophysical optimisation problems. The tests show that the use of such a technique is crucial to achieve high quality solutions at a reasonable cost.

2. Genetic Algorithm Implementation

For this study a real-coded GA has been implemented in which an individual is represented by an array of real values corresponding to the parameters to be determined. Currently no theoretical rules for the implementation of GA operators are available in the literature. Since such choices can critically affect the success of a particular GA application a number of possible implementations have been tested for each operator in order to select the most effective configuration. Details of such tests may be found in [5]. Our tests show a multi-point crossover with a rate of 0.8 and a traditional mutation operator with a rate of 0.01 is the most effective configuration for the optimisation problems described below. Two kinds of selection have been implemented: linear normalisation selection [2, 3], and parent selection [9].

In linear normalisation selection an individual is ranked according to its fitness and then it is allowed to generate a number of offspring proportional to its rank position. This selection technique pushes the population towards a single solution in a reasonably fast manner.

In parent selection all individuals are allowed to generate a single offspring regardless of their fitness. All the individuals are mated randomly and through crossover each couple creates two offspring. If the offspring fitnesses are better than those of the parents they are substituted for them in the population. With this method diversity is kept in the population. Accordingly, convergence is slower than in the linear normalisation selection but a large number of different solutions can usually be found.

These two different selection strategies have a major impact on the GA optimisation process. Consequently, two different GA schemes have been implemented to tackle the two different problems that will be described below.

3. Pseudo-Subspace Genetic Algorithms

Williamson [10] describes the inversion of seismic reflection data using of a multi-staged approach in which the resolution of the Earth model is progressively increased during the process. The method has been used in the context of a local optimisation scheme and a more detailed description of its theoretical basis may be found in [11, 12]. We have used a similar approach in the GA global search. For early generations, the total number of parameters required by the problem is 'compressed' into a much smaller number. In geophysical applications of GAs it is common for the problem to involve the distribution of some physical property within the subsurface, usually expressed as values at nodes of a grid. In this case 'compression' is achieved by using a coarse grid in the early stages of the optimisation, with 'decompression' achieved by introducing a progressively finer grid as optimisation progresses. At the start of the operation the parameter space has relatively small dimensions. Within a few generations the GA defines approximately correct values at the grid nodes. Because an exact solution very likely cannot be found with such a coarse grid spacing there is no need to run the GA for many generations at this stage and consequently only a small sample population is also sufficient. In subsequent generations the spacing of the grid nodes is halved and hence the parameters are progressively 'decompressed'. The new parameters that are introduced at this stage are linearly interpolated with respect to the parameters optimised in the previous stage. Different techniques may also be implemented depending on the physics of the problem. Again the GA is run, in what is now a higher dimensional space, for few generations with a relatively larger population. This process is repeated until the size of the grid reaches some pre-determined limit, whereupon the GA is run for a larger number of generations, and with a larger population, until an acceptable convergence is reached.

4. Examples

4.1. Inversion of Magnetic Data. In this problem we aim to reconstruct the bottom contact between two geological bodies of different magnetic susceptibility by the inversion of magnetic anomaly data measured on the earth surface, or at small altitudes above it [13]. The contrast in magnetisation of the two rock bodies causes local perturbation of the Earth's magnetic field and the form of these 'anomalies' can be used to define the geometry of the bodies. In our example the geometry of the contact between the different rocks types is defined at the nodes of a regular grid. In this example the final image is described by a 5 x 5 grid. The inversion of magnetic data is characterised by ambiguity problems, i.e. an infinite number of solutions can be found that satisfy the observed data (see for example [14]). Usually the ensemble of all the possible solutions belongs to a domain whose dimension is only slightly lower than the global parameter space. Recently this problem has been addressed by trying to obtain a number of solutions large enough to describe the shape of this ambiguity domain [15]. Despite the fact that the solutions do not define a normal distribution, we have found that their arithmetic average usually gives a satisfactory representation of the global ensemble. Accordingly, the 'parent selection' implementation has been used to tackle this problem. Through the use of such an operator, a number of different solutions are allowed to converge while the GA population is kept relatively diverse and thus able to span the large global minimum that results from the inherent ambiguity of the problem. Figure 1 shows the result of the application of this technique to the inversion of magnetic data. Figure 1a shows the synthetic image used to produce the dataset that has been inverted. Figure 1b and 1c shows the results from the inversion with and without the inclusion of the 'pseudo-subspace' technique, respectively. Clearly the initial low-dimensional search has directed the successive space sampling towards the most promising area of the solution space.

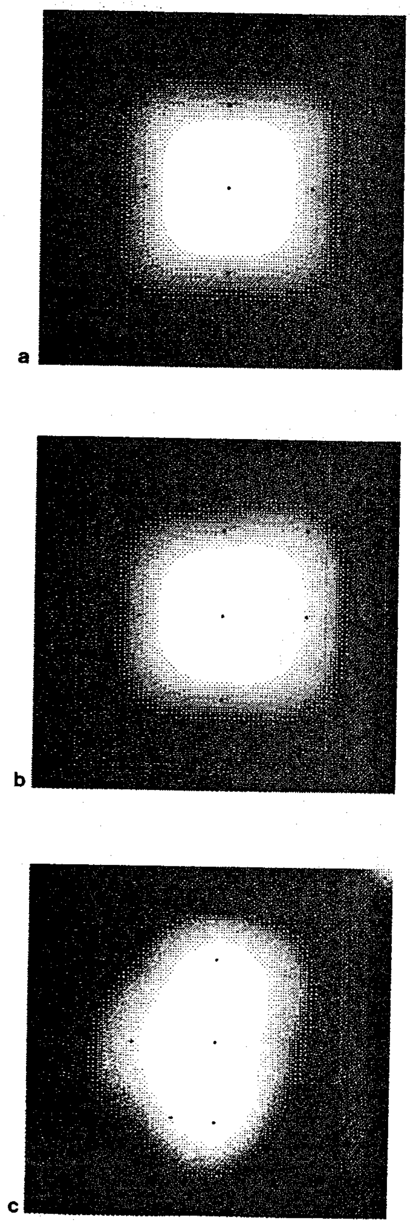

Fig. 1. Variation of the depth of the contact between two geological bodies with different magnetic properties. Synthetic model (top), results obtained

using GA with the 'pseudo-subspace' method (middle) and solution from a traditional GA (bottom).

4.2. Inversion of Seismic Refraction Data.

In seismic refraction surveying the aim is to reconstruct the seismic velocity field in a cross-section through the Earth, by the inversion of the travel times of seismic energy generated at or near the surface and detected by a line of detectors [13]. Again, this is achieved by describing the Earth in terms of some physical property at the nodes of a grid, in this case the propagation velocity of the seismic waves. Usually this kind of application requires larger grids than the ones used in the inversion of magnetic data. Here we present results obtained using a 9 x 5 grid. In another test the same technique has been used to successfully reconstruct images with up to 105 nodes [16]. In our experience, the seismic refraction problem in a fixed grid is characterised by a unique global minimum. Also, the parameters are so strongly related that a wrong parameter may affect the values of most of the remaining parameters in the domain. Accordingly, the use of the 'linear normalisation selection' is more effective because it allows for more rapid sampling of promising valleys. Figure 2 presents the results from the application of the conventional GA (Fig.2c) and the GA including the pseuo-subspace search (Fig.2b) in the inversion of a dataset produced using the model in Figure 2a. Again, the inclusion of the 'pseudo-subspace' search is clearly crucial in obtain a good quality solution.

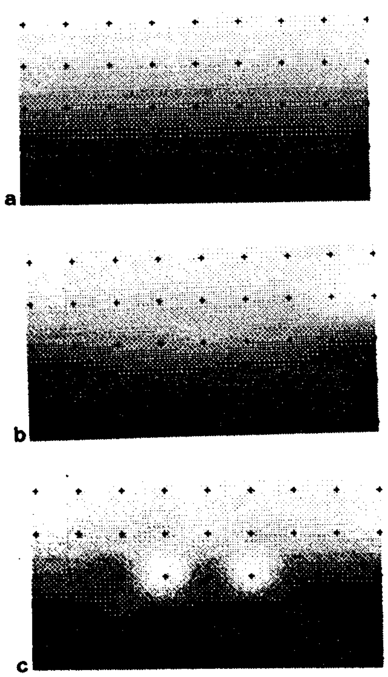

Fig. 2. Inversion of seismic refraction data. The images show the variation in the seismic waves propagation velocity. Synthetic model (a), results obtained using GA with the 'subspace-like' method (b) and solution from a traditional GA (c).

5. Discussion and Conclusions

The inclusion of a 'pseudo-subspace' technique that allows for the complexity of the model to be progressively increased during the inversion process has proved to be particularly beneficial in addressing two high-dimensional highly-non linear geophysical optimisation problems. The ability of the method to quickly and accurately detect the most promising areas of the solution space in the early low-dimensional stages of the process allows the GA to concentrate the later search in such areas and reconstruct high-quality solutions. The early low-dimensional iterations are also effecient from a computational point of view, allowing for a much faster convergence. This technique has successfully solved problems characterised by up to 105 parameters, otherwise beyond the normal computational limits for a global optimisation method in geophysical applications. The extension of the method to other image processing and pattern recognition problems where simple spatial relationships among the parameters are present is straightforward. It is important to the success of the method that the actual process of compression does not result in insufficient data such that the forward problem can not be satisfactorily computed. For instance, in the case of the seismic data, an approximate scheme is used to calculate the seismic travel times. This scheme must be able to still calculate these times to a reasonable approximation even when the grid of velocity values is coarse.

In the examples presented the parameters being 'compressed' are all of the same type, e.g. values of seismic velocity. To compress parameters of different type it is necessary to define approximate relationships between these parameters. For example, in a simultaneous inversion of seismic and gravity data, the loose relationship between density and seismic velocity can be exploited and hence these two properties compressed in a single parameter at the beginning of the process. In the later stages of the optimisation process, after 'decompression', a more accurate and independent determination of these parameters would occur.

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6. References

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