Chapter 7

EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION: A CRITICAL REVIEW

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"We will say that members of a collectivity enjoy maximum ophelimity in a certain position when it is impossible to find a way of moving from that position very slightly in such a manner that the ophelimity enjoyed by each of the individuals in the collectivity increases or decreases. That is to say, any small displacement in departing from that position necessarily has the effect of increasing the ophelimity which certain individuals enjoy, and decreasing that which others enjoy, of being agreeable to some and disagreeable to others"

—Vilfredo Pareto, *Manual of Political Economy*, 1896

Abstract In this chapter, we will review some of the most representative research in the field of evolutionary multiobjective optimization. We will discuss the historical roots of multiobjective optimization, the motivation to use evolutionary algorithms, and the most popular techniques currently in use. Then, we will discuss some of the research currently under way, including our own. At the end, we will provide what we consider to be some of the most promising paths of future research.

Keywords: evolutionary multiobjective optimization, evolutionary algorithms, vector optimization, multiobjective optimization, genetic algorithms, multicriteria optimization

1. INTRODUCTION

Most optimization problems naturally have several objectives to be achieved (normally conflicting with each other), but in order to simplify their solution, they are treated as if they had only one (the remaining objectives are normally handled as constraints). These problems with several objectives, are called "multiobjective" or "vector" optimization problems, and were originally studied in the context of economics. However, scientists and engineers soon realized that such problems naturally arise in all areas of knowledge.

Over the years, the work of a considerable amount of operational researchers has produced an important number of techniques to deal with multiobjective optimization problems (Miettinen, 1998). However, it was until relatively recently that researchers realized of the potential of evolutionary algorithms (EAs) in this area.

This chapter will review the most important research in the area now called Evolutionary Multi-Objective Optimization, or EMOO for short. The importance of this field is reflected by a significant increment (mainly during the last five years) of technical papers in international conferences and peer-reviewed journals, special sessions in international conferences and interest groups in the Internet[1].

The organization of this chapter is the following: first, we will provide some basic concepts used in multiobjective optimization. Then, we will briefly discuss the historical roots of this discipline, and the motivation for using evolutionary algorithms. After that, we will do a critical review of the most popular EMOO techniques currently available, including some of their applications. Finally, we will discuss some of the research currently under way, including our own. We will finish this chapter with a brief discussion of what we consider to be some of the most promising paths of future research.

2. DEFINITIONS

We are interested in solving multiobjective optimization problems (MOPs) of the form:

$$\text{minimize } [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{7.1}$$

subject to the m inequality constraints:

$$g_i(\vec{x}) \geq 0 \quad i = 1, 2, \ldots, m \tag{7.2}$$

and the p equality constraints:

[1]The author maintains an EMOO repository which currently includes over 650 bibliographical entries at: `http://www.lania.mx/~ccoello/EMOO/` with a mirror at `http://www.jeo.org/emo/`

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{7.3}$$

where k is the number of objective functions $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$. We call $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ the vector of decision variables. We wish to determine from among the set $\mathcal{F}$ of all numbers which satisfy (7.2) and (7.3) the particular set $x_1^*, x_2^*, \ldots, x_n^*$ which yields the optimum values of all the objective functions.

2.1 PARETO OPTIMUM

It is rarely the case that there is a single point that simultaneously optimizes all the objective functions. Therefore, we normally look for "trade-offs", rather than single solutions when dealing with multiobjective optimization problems. The notion of "optimum" is therefore, different. The most commonly adopted notion of optimality is that originally proposed by Francis Ysidro Edgeworth (1881) and later generalized by Vilfredo Pareto (1896). Although some authors call *Edgeworth-Pareto optimum* to this notion (see for example Stadler (1988)), we will use the most commonly accepted term: *Pareto optimum*.

We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ is *Pareto optimal* if there does not exist another $\vec{x} \in \mathcal{F}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j.

In words, this definition says that $\vec{x}^*$ is Pareto optimal if there exists no feasible vector of decision variables $\vec{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always gives not a single solution, but rather a set of solutions called the *Pareto optimal set*. The vectors $\vec{x}^*$ correspoding to the solutions included in the Pareto optimal set are called *nondominated*. The plot of the objective functions whose nondominated vectors are in the Pareto optimal set is called the *Pareto front*.

3. HISTORICAL ROOTS

John von Neumann and Oskar Morgenstern (1944) were the first to recognize the existence of optimization problems in economics that were "a peculiar and disconcerting mixture of several conflicting problems". However, no real contribution to the solution of such problems was made until the 1950s.

Harold W. Kuhn and Albert W. Tucker (1951) introduced a vector-valued objective function in mathematical programming—a *vector maximum problem*, and derived the optimality conditions for efficient solutions. The so-called "proper efficiency" in the context of multiobjective optimization was also formulated in this seminal paper that many consider as the first serious attempt to derive a theory in this area. This same direction was later followed by Arrow et al. (1953) who used the term "admissible" instead of "efficient" points.

However, multiobjective optimization theory remained relatively undeveloped during the 1950s, and the subject was scarcely covered by only a few authors (see for example (Koopman, 1953) and (Karlin, 1959)).

The application of multiobjective optimization to domains outside economics began with the work of Tjalling Koopmans (1951) in production theory and with the work of Marglin (1967) in water resources planning. The first application of multiobjective optimization in engineering was in the early 1960s (Zadeh, 1963), but its use became generalized until the 1970s (Stadler, 1975; Cohon, 1978).

Good reviews of existing mathematical programming techniques for multiobjective optimization can be found in a variety of references (see for example (Cohon and Marks, 1975), (Hwang et al., 1980), and (Miettinen, 1998)).

Evolutionary algorithms have been successfully applied to a variety of optimization problems with very large (intractable) search spaces, noise, non-differentiable and even dynamic objective functions in the last few years (Goldberg, 1989; Michalewicz, 1996; Mitchell, 1996; Gen and Cheng, 1997).

The potential of evolutionary algorithms in this field was hinted in the late 1960s by Rosenberg (1967), but the first implementation was produced until the mid-1980s (Schaffer, 1985). Evolutionary algorithms seem particularly appropriate to solve multiobjective optimization problems because they deal simultaneously with a set of possible solutions (the so-called population) which allows us to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. Additionally, evolutionary algorithms are less succeptible to the shape or continuity of the Pareto front (e.g., they can easily deal with non-convex Pareto fronts), whereas these two issues are a real concern for mathematical programming techniques.

Evolutionary algorithms are not the only heuristic technique that has been used to solve multiobjective optimization problems. The good performance exhibited by some algorithms (e.g., tabu search and simulated annealing) in combinatorial optimization problems has led researchers to develop multiobjective versions of them (Hansen, 1996; Ehrgott, 2000; Czyzak and Jaszkiewicz, 1997; Gandibleux et al., 1997; Romero and Manzanares, 1999). Some researchers have also suggested hybrids between genetic algorithms and other heuristics (e.g., tabu search (Kurahashi and Terano, 2000)) for multiobjective optimization. Nevertheless, our review will only concentrate on evolutionary multiobjective optimization techniques.

4. A QUICK SURVEY OF EMOO APPROACHES

A considerable number of EMOO techniques have been proposed in the last few years and it is not our intention to enumerate them all in this chapter (interested readers should refer to (Coello, 1999) and (Veldhuizen, 1999) for more detailed surveys of EMOO approaches). Therefore, we will concentrate our discussion on those techniques that have been more popular among researchers or that are very recent (and promising according to our own personal criterion). The techniques discussed are the following: Aggregating functions, VEGA, MOGA, NSGA, NPGA, target vector approaches and two recent approaches: PAES and SPEA.

4.1 AGGREGATING FUNCTIONS

Knowing that an EA[2] needs scalar fitness information to work, it is almost natural to propose a combination of all the objectives into a single one using either an addition, multiplication or any other combination of arithmetical operations that we could devise. In fact, this is also the oldest mathematical programming method for multiobjective optimization, since it can be derived from the Kuhn-Tucker conditions for nondominated solutions (Kuhn and Tucker, 1951). An example of this approach is a sum of weights of the form:

$$\min \sum_{i=1}^{k} w_i f_i(\vec{x}) \tag{7.4}$$

where $w_i \geq 0$ are the weighting coefficients representing the relative importance of the k objective functions of our problem. It is usually assumed that

$$\sum_{i=1}^{k} w_i = 1 \tag{7.5}$$

4.1.1 STRENGTHS AND WEAKNESSES

The main strengths of this method are its simplicity and efficiency (computationally speaking). It can work properly in simple (convex) MOPs with few objective functions. This approach is normally used to generate a single (or a few) nondominated solution that can be used as an initial solution for other techniques. One of its main weaknesses is the difficulty to determine the set of weights that can appropriately scale the objectives when we do not have enough information about the problem. Its most serious drawback is that it cannot generate proper members of the Pareto optimal set when the Pareto front is concave regardless of the weights used (Das and Dennis, 1997).

[2] We will use the generic term *Evolutionary Algorithm* throughout this chapter, although most of the EMOO approaches discussed use genetic algorithms.

4.1.2 SOME APPLICATIONS

- Task planning (Jakob et al., 1992).
- System-level synthesis (Blickle et al., 1996).
- Truss optimization (Liu et al., 1998).

4.2 VEGA

David Schaffer (1985) proposed an approach that he called the *Vector Evaluated Genetic Algorithm* (VEGA), and that differed of the simple genetic algorithm (GA) only in the way in which selection was performed. This operator was modified so that at each generation a number of sub-populations was generated by performing proportional selection according to each objective function in turn. Thus, for a problem with k objectives and a population size of M, k sub-populations of size M/k each would be generated. These sub-populations would be shuffled together to obtain a new population of size M, on which the GA would apply the crossover and mutation operators in the usual way.

The solutions generated by VEGA are locally nondominated, but not necessarily globally nondominated. VEGA presents the so-called "speciation" problem (i.e., we could have the evolution of "species" within the population which excel on different objectives). This problem arises because this technique selects individuals who excel in one objective, without looking at the others. The potential danger doing that is that we could have individuals with what Schaffer (1985) called "middling" performance[3] in all dimensions, which could be very useful for compromise solutions, but that will not survive under this selection scheme, since they are not in the extreme for any dimension of performance (i.e., they do not produce the best value for any objective function, but only moderately good values for all of them). Speciation is undesirable because it is opposed to our goal of finding compromise solutions.

4.2.1 STRENGTHS AND WEAKNESSES

The main advantages of this technique are its simplicity and its efficiency. However, as we mentioned before, the "middling" problem prevents the technique from finding the compromise solutions that we normally aim to produce. In fact, if proportional selection is used with VEGA (as Schaffer did), the shuffling and merging of all the sub-populations corresponds to averaging the fitness components associated with each of the objectives (Richardson et al., 1989). In other words, under these conditions, VEGA behaves as an aggregating approach and therefore, it is subject to the same problems of such techniques.

[3]By "middling", Schaffer meant an individual with acceptable performance, perhaps above average, but not outstanding for any of the objective functions.

4.2.2 SOME APPLICATIONS

- Groundwater pollution containment (Ritzel et al., 1994).
- Constraint-handling (Surry et al., 1995; Coello, 2000c).
- Scheduling (Hilliard et al., 1989).

4.3 MOGA

Fonseca and Fleming (1993) proposed the *Multi-Objective Genetic Algorithm* (MOGA). The approach consists of a scheme in which the rank of a certain individual corresponds to the number of individuals in the current population by which it is dominated. All nondominated individuals are assigned rank 1, while dominated ones are penalized according to the population density of the corresponding region of the trade-off surface.

Fitness assignment is performed in the following way (Fonseca and Fleming, 1993):

1. Sort population according to rank.
2. Assign fitness to individuals by interpolating from the best (rank 1) to the worst (rank $n \leq N$) in the way proposed by Goldberg (1989) (the so-called Pareto ranking assignment process), according to some function, usually linear, but not necessarily.
3. Average the fitnesses of individuals with the same rank, so that all of them will be sampled at the same rate.

MOGA is combined with mating restrictions and sharing on the objective function values to preserve diversity (Deb and Goldberg, 1989). The authors of this method also provided some guidelines regarding the way in which niche sizes can be estimated.

4.3.1 STRENGTHS AND WEAKNESSES

MOGA has been a very popular EMOO technique (particularly within the control community), not only because it is relatively simple to implement, but also because of its good overall performance (Coello, 1996). Its main weakness is its dependence on the sharing factor (how to maintain diversity is the main issue when dealing with EMOO approaches in general). However, as indicated before, Fonseca and Fleming (1993) have provided some guidelines regarding the way to compute niche sizes.

4.3.2 SOME APPLICATIONS

- Controllers design (Tan and Li, 1997; Chipperfield and Fleming, 1995; Schroder et al., 1997)

- Co-synthesis of hardware-software embedded systems (Dick and Jha, 1998)
- Truss design (Narayanan and Azarm, 1999)

4.4 NSGA

The *Nondominated Sorting Genetic Algorithm* (NSGA) was proposed by Srinivas and Deb (1994), and is based on several layers of classifications of the individuals. Before selection is performed, the population is ranked on the basis of domination (using Pareto ranking): all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size). To maintain the diversity of the population, these classified individuals are shared (in decision variable space) with their dummy fitness values. Then this group of classified individuals is removed from the population and another layer of nondominated individuals is considered (i.e., the remainder of the population is re-classified). The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population.

4.4.1 STRENGTHS AND WEAKNESSES

Some researchers have reported that NSGA has a lower overall performance than MOGA, and it seems to be also more sensitive to the value of the sharing factor than MOGA (Coello, 1996; Veldhuizen, 1999). However, Deb et al. (2000a,2000b) have recently proposed a new version of this algorithm, called NSGA-II, which is more efficient (computationally speaking), uses elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters. The new approach has not been extensively tested yet, but it certainly looks promising.

4.4.2 SOME APPLICATIONS

- Investment portfolio optimization (Vedarajan et al., 1997).
- Optimization of low-thrust interplanetary spacecraft trajectories (Hartmann et al., 1998).
- Optimization of an industrial nylon 6 semibatch reactor (Mitra et al., 1998).

4.5 NPGA

Horn et al. (1994) proposed the *Niched Pareto Genetic Algorithm*, which uses a tournament selection scheme based on Pareto dominance. Two individuals are compared against a set of members of the population (typically, 10% of the population size). When both competitors are either dominated or nondominated (i.e., when there is a tie), the result of the tournament is decided through fitness sharing in the objective domain (a technique called *equivalent class sharing* was used in this case) (Horn et al., 1994).

4.5.1 STRENGTHS AND WEAKNESSES

Since this approach does not apply Pareto ranking to the entire population, but only to a segment of it at each run, its main strength are that it is faster than MOGA and NSGA[4]. Furthermore, it also produces good nondominated fronts that can be kept for a large number of generations (Coello, 1996). However, its main weakness is that besides requiring a sharing factor, this approach also requires an additional parameter: the size of the tournament.

4.5.2 SOME APPLICATIONS

- Design of laminated ceramic composites (Belegundu et al., 1994).

- Airfoil design (Quagliarella and Vicini, 1997).

- Manufacturing cell formation problems (Pierreval and Plaquin, 1998).

4.6 TARGET VECTOR APPROACHES

Under this name we will consider approaches in which the decision maker has to assign targets or goals that wishes to achieve for each objective[5]. The EA in this case, tries to minimize the difference between the current solution found and the vector of goals (different metrics can be used for that purpose). The most popular techniques included here are hybrids with: Goal Programming (Deb, 1999c; Wienke et al., 1992), Goal Attainment (Wilson and Macleod, 1993; Zebulum et al., 1998) and the min-max approach (Hajela and Lin, 1992; Coello and Christiansen, 1998).

4.6.1 STRENGTHS AND WEAKNESSES

The main strength of these methods is their efficiency (computationally speaking) because they do not require a Pareto ranking procedure. However, their main weakness is the definition of the desired goals which requires some extra computational effort. Furthermore, these techniques will yield a nondominated solution only if the goals are chosen in the feasible domain, and such condition may certainly limit their applicability.

[4] Pareto ranking is $O(kM^2)$, where k is the number of objectives and M is the population size

[5] Although target vector approaches can be considered as another aggregating approach, we decided to discuss them separately because these techniques can generate (under certain conditions) non-convex portions of the Pareto front, whereas approaches based on weighted sums cannot.

4.6.2 SOME APPLICATIONS

- Design of multiplierless IIR filters (Wilson and Macleod, 1993).
- Structural optimization (Sandgren, 1994; Hajela and Lin, 1992).
- Optimization of the counterweight balancing of a robot arm (Coello et al., 1998).

4.7 RECENT APPROACHES

Recently, several new EMOO approaches have been developed. We consider important to discuss briefly at least two of them: PAES and SPEA.

The *Pareto Archived Evolution Strategy* (PAES) was introduced by Knowles & Corne (2000a). This approach is very simple: it uses a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) together with a historical archive that records all the nondominated solutions previously found (such archive is used as a comparison set in a way analogous to the tournament competitors in the NPGA). PAES also uses a novel approach to keep diversity, which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives. A map of such grid is maintained, indicating the amount of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space). Furthermore, the procedure has a lower computational complexity than traditional niching methods. PAES has been used to solve the off-line routing problem (Knowles and Corne, 1999) and the adaptive distributed database management problem (Knowles and Corne, 2000).

The *Strength Pareto Evolutionary Algorithm* (SPEA) was introduced by Zitzler & Thiele (1999). This approach was conceived as a way of integrating different EMOO techniques. SPEA uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of MOGA, since it is proportional to the number of solutions to which a certain individual dominates. The fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. Additionally, a clustering technique is used to keep diversity. SPEA has been used to explore trade-offs of software implementations for DSP algorithms (Zitzler et al., 1999) and to solve 0/1 knapsack problems (Zitzler and Thiele, 1999).

Recently, we have been experimenting with a micro-GA (a GA with small population and a reinitialization mechanism (Krishnakumar, 1989)) for multiobjective optimization (Coello and Toscano, 2001). Our approach uses two memories: 1) a population memory, which is used as the source of diversity, and 2) an external memory, which is used to archive members of the Pareto optimal set found during the evolutionary process. Our micro-GA uses a population of four individuals, which undergo binary tournament selection, two point crossover and uniform mutation until nominal convergence is achieved (a small number of iterations is used in our case, but other criteria could also be used). Through the use of different forms of elitism and a reinitialization process that mixes good solutions previously found with random solutions, we gradually approach the true Pareto front of a problem. To keep diversity, we use an approach similar to the adaptive grid proposed by Knowles & Corne (2000a). The idea is that once the archive that stores nondominated vectors (i.e., the external memory) has reached its limit, we divide the search space that this archive covers, assigning a set of coordinates to each vector. Then, each newly generated nondominated vector will be accepted only if the geographical location to where it belongs is less populated than the most crowded location, or if it belongs to a location outside the previously specified boundaries (i.e., if it forms a new niche). The approach has a very low computational cost (with respecto to Pareto ranking) and we can regulate the amount of points from the Pareto front that we wish to find through the size of the external memory. Our preliminary results indicate that our micro-GA is able to generate the Pareto front of difficult test functions (i.e., disconnected and concave Pareto fronts) that have been previously adopted to evaluate EMOO techniques (Coello and Toscano, 2001).

5. CURRENT RESEARCH

Being a very active area of research, EMOO has seen a lot of changes in the last few years and the research trends are constantly changing. We will focus our discussion in this section to two main areas that currently interest us: constraint-handling for evolutionary optimization, and incorporation of preferences into an EMOO algorithm. These two areas have not been studied in enough depth and, from our particular point of view, seem very promising.

5.1 HANDLING CONSTRAINTS

An interesting application of EMOO techniques that we have recently explored is in constraint-handling (for single-objective evolutionary optimization). The most straightforward approach is to redefine the single-objective optimization of $f(\vec{x})$ as a multiobjective optimization problem in which we will have $m+1$ objectives, where m is the number of constraints[6]. Then, we can apply any EMOO technique to the new vector $\bar{v} = (f(\vec{x}), f_1(\vec{x}), \ldots, f_m(\vec{x}))$, where $f_1(\vec{x}), \ldots, f_m(\vec{x})$ are the original constraints of the problem. An ideal

[6]The assumption that we have m constraints will hold throughout this section.

solution $\vec{x}$ would thus have $f_i(\vec{x})$=0 for $1 \leq i \leq m$ and $f(\vec{x}) \leq f(\vec{y})$ for all feasible $\vec{y}$ (assuming minimization).

However, it should be clear that in single-objective optimization problems we do not want just good trade-offs; we want to find the best possible solutions that do not violate any constraints. Therefore, a mechanism such as Pareto ranking may be useful to approach the feasible region, but once we arrive to it, we will need to guide the search with a different mechanism so that we can reach the global optimum. In order to achieve this goal, we should also be able to maintain diversity in the population. These aspects are the main focus of the research briefly reviewed in this section.

Surry et al. (1997) proposed the use of Pareto ranking and VEGA to handle constraints. In their approach, called COMOGA, the population is ranked based on constraint violations (counting the number of individuals dominated by each solution). Then, one portion of the population is selected based on constraint ranking, and the rest based on real cost (fitness) of the individuals. COMOGA compared fairly with a penalty-based approach in a pipe-sizing problem, and was less sensitive to changes in the parameters, but the results achieved were not better than those found with a penalty function (Surry and Radcliffe, 1997). It should be added that COMOGA requires several extra parameters, although its authors argue that the technique is not particularly sensitive to the values of such parameters.

Parmee and Purchase (1994) implemented a version of VEGA that handled the constraints of a gas turbine problem as objectives to allow a GA to locate a feasible region within the highly constrained search space of this application. However, VEGA was not used to further explore the feasible region, and instead the authors used specialized operators that would create a variable-size hypercube around each feasible point to help the GA to remain within the feasible region at all times. It is important to notice that no real attempt to reach the global optimum was made in this case.

Camponogara & Talukdar (1997) proposed to restate a single objective optimization problem in such a way that two objectives would be considered: the first would be to optimize the original objective function and the second would be to minimize:

$$\Phi(\vec{x}) = \sum_{i=1}^{m} \max[0, g_i(\vec{x})]^{\beta} \tag{7.6}$$

where β is normally 1 or 2.

Once the problem is redefined, nondominated solutions with respect to the two new objectives are generated. The solutions found define a search direction $d = (x_i - x_j)/|x_i - xj|$, where $x_i \in S_i$, $x_j \in S_j$, and S_i and S_j are Pareto sets. The direction search d is intended to simultaneously minimize all the objectives. Line search is performed in this direction so that a solution x can be found such that x dominates x_i and x_j (i.e., x is a better compromise than the two previous solutions found). Line search takes the place of crossover in this approach, and mutation is essentially the same, where the direction d is projected onto the axis of one variable j in the solution space. Additionally, a process of eliminating half of the population is applied at regular intervals (only the less fitted solutions are replaced by randomly generated points).

This approach has obvious problems to keep diversity, as it is reflected by the need to discard the worst individuals at each generation. Also, the use of line search increases the computational cost of the approach and it is not clear what is the impact of the segment chosen to search in the overall performance of the algorithm.

Jiménez et al. (1999) proposed the use of a min-max approach (Chankong and Haimes, 1983) to handle constraints. The main idea of this technique is to apply a set of simple rules to decide the (binary tournament) selection process:

1. If the two individuals being compared are both feasible, then select based on the minimum value of the objective function.

2. If one of the two individuals being compared is feasible and the other one is infeasible, then select the feasible individual.

3. If both individuals are infeasible, then select based on the maximum constraint violation (max $g_j(\vec{x}),\ \ \text{for } j = 1, \ldots, m$). The individual with the lowest maximum violation wins.

A subtle problem with this approach is that the evolutionary process first concentrates only on the constraint satisfaction problem and therefore it samples points in the feasible region essentially at random (Surry et al., 1995). This means that in some cases (e.g., when the feasible region is disjoint) we might land in an inappropriate part of the feasible region from which we will not be able to escape. However, this approach may be a good alternative to find a feasible point in a heavily constrained search space. Deb (2000) proposed a similar approach but using tournament selection based on feasibility. However, niching was required to maintain diversity in the population.

Coello (2000c) proposed the use of a population-based multiobjective optimization technique such as VEGA to handle each of the constraints of a single-objective optimization problem as an objective. At each generation, the population is split into $m+1$ sub-populations (m is the number of constraints), so that a fraction of the population is selected using the (unconstrained) objective function as its fitness and another fraction uses the first constraint as its fitness and so on. This approach provided good results in several optimization problems (Coello, 2000c). Its main disadvantage was related to scalability issues. However, in a recent application in combinational circuit design we were able to successfully deal with up to 49 objective functions (Coello et al., 2000b). Furthermore, the approach showed an important improvement (in terms of efficiency) with respect to a previous GA-based approach developed by us for the same task (Coello et al., 2000a).

Recently, we have also explored the use of selection based on dominance (which was defined in terms of feasibility) to handle constraints (Coello, 2000a). Our approach uses stochastic universal sampling so that the selection pressure is not too high and no extra procedures are required to maintain diversity. Also, adaptive crossover and mutation rates were adopted as part of the approach.

The key for future research in this area is not only to adapt other EMOO approaches to handle constraints, but to exploit domain knowledge as much as possible. An example of this is the recent work by Ray et al. (2000) in which solutions are ranked separately based on the value of their objective functions and their constraints. Then a set of mating restrictions are applied based on the information that each individual has of its own feasibility (this idea was inspired on an earlier approach by Hinterding and Michalewicz (1998)), so that the global optimum can be reached through cooperative learning.

Other approaches are also possible. For example, we could combine an EMOO approach with a mechanism to incorporate preferences from the user (the topic discussed in the next section). Such preferences, however, could be directly derived from the problem (using the domain knowledge available), instead of requiring an active participation from the user.

5.2 INCORPORATION OF PREFERENCES

By looking at most of the EMOO papers in the literature, one gets the impression that researchers seem to forget that the solution of a MOP really involves three stages: measurement, search, and decision making. Most EMOO research tends to concentrate on issues related to the search of nondominated vectors. However, these nondominated vectors do not provide any insight into the process of decision making itself (the decision maker (DM) still has to choose manually one of the several alternatives produced), since they are really a useful generalization of a utility function under the conditions of minimum information (i.e., all attributes are considered as having equal importance; in other words, the DM does not express any preferences of the attributes). Thus, the issue is how to incorporate the DM's preferences into an EMOO approach as to guide the search only to the regions of main interest for the DM.

One way to classify techniques that incorporate preferences from the DM is based on the moment (within the search process) at which preferences are expressed. According to this criterion, preferences can be expressed (Horn, 1997): *a priori*, *a posteriori*, or in an *interactive* way when using EAs.

If preferences are expressed *a priori*, the DM has to define them in advance (before actually performing the search). An example of this are the aggregating approaches discussed in Section 4.1. Shaw & Fleming (1997), Greenwood et al. (1997), and Cvetković & Parmee (2000) have proposed *a priori* schemes to incorporate preferences into an EMOO approach.

In the second case, we search first, and decide later. Most EMOO approaches (those that use Pareto ranking) fall into this category. In this case, we use an EA to search the "best possible" alternatives, where "best possible" normally means members of the Pareto optimal set. Massebeuf et al. (1999) proposed an *a posteriori* scheme to incorporate preferences into an EMOO approach.

The third case is the less common in the EA literature: approaches that allow to guide the search of the EA using preferences from the DM, but in an interactive way (i.e., assuming that such preferences can change over time). Tanino et al. (1993) and Fonseca & Fleming (1993,1998) proposed *interactive* schemes to incorporate preferences into an EMOO approach[7].

The Operations Research (OR) literature has normally favored *interactive* approaches for several reasons (Monarchi et al., 1973):

1. It is normally the case that the DM wishes to find trade-offs that satisfy only a certain set of criteria, instead of wishing to find solutions that are the best trade-off considering all criteria at the same time.

2. The preferences of the DM can (and normally do) change over time.

3. The DM normally learns through the search process and tends to change (in consequence) his aspirations or desires.

If we analyze the literature on multi-criteria decision making (MCDM), we find another way of classifying approaches to incorporate preferences. In this case, two main lines of thought are normally considered: the so-called French School, which is based on the outranking concept (Vincke, 1995) and the American School, which is based on the Multi-Attribute Utility Theory (MAUT) (Hwang and Masud, 1979). Both outranking and MAUT can be used a priori, a posteriori or in an interactive way.

Outranking relationships are built under the form of pairwise comparisons of the objects under study (a graph representing preferences is normally used). Pairs of objects are compared to determine if there exists preference, indifference, or incomparability between them. Weights for each objective are derived from these pairwise comparisons. It is important, however, to be aware of the fact that these pairwise comparisons may lead to intransitive or incomplete relations (van Huylenbroeck, 1995). The main drawbacks of outranking approaches are their high computational cost when there is a large amount of alternatives, the high amount of parameters that they require, and the difficulties to define some of these alternatives (e.g., the "degree of credibility") (Brans

[7]The approach was also used to handle constraints.

et al., 1986). Rekiek et al. (2000) and Massebeuf et al. (1999) have proposed EMOO approaches that incorporate preferences using PROMETHEE (Brans et al., 1986) (Preference Ranking Organization METHod for Enrichment Evaluations), which is an outranking approach.

MAUT is based on the formulation of an overall utility function. Although it is normally assumed that such utility function can be obtained, when that is not possible, then nondominated solutions can be used (i.e., we assume that all objectives are given the same importance). Certain flexibility can be obtained through the concept of "weak dominance" (Loucks, 1975), which can be used to express a certain lack of conviction. It is also possible for the DM to express indifference, which means that both vectors under comparison are equivalent and that it does not matter which one is selected. It is worth mentioning that "indifference" is not the same that "incomparability" (as defined in outranking methods), because the second indicates vectors with strong opposite merits (van Huylenbroeck, 1995). MAUT does not allow intransitivities to occur. This considerably simplifies the modelling of the preferences. However, it is not very difficult to produce an example in which intransitivities "naturally" emerge (see for example (van Huylenbroeck, 1995)). Greenwood et al. (1997) and Cvetković & Parmee (2000) have proposed EMOO approaches that incorporate preferences using utility functions.

Despite this research, there is an obvious lack of models for the incorporation of preferences into an EMOO approach. Issues such as scalability and the presence of several DMs deserve special attention when devising such a model.

As we have indicated before (Coello, 2000b), we believe that there are several approaches from OR that could be easily coupled with EAs. Approaches such as PROTRADE (PRObabilistic TRAde-off DEvelopment method) (Goicoechea et al., 1979) and SEMOPS (Sequential Multi-Objective Problem Solving method) (Monarchi et al., 1973) could be easily tailored to incorporate preferences into EMOO approaches. Both approaches are interactive and assume a degree of uncertainty from the DM with respect to the trade-offs of the objectives under study. Compromise programming (Duckstein, 1984) is also promising, and it has in fact been used by some EMOO researchers (see for example (Deb, 1999a) and (Bentley and Wakefield, 1997)). However, more complex articulations of preferences are possible if the approach is used interactively (it has been normally used as an *a priori* technique).

We believe that a key issue to foster the development of this area in the future is that EMOO researchers be aware of the work done by operational researchers in MCDM. It should be clear to EMOO researchers that searching efficiently nondominated vectors is not the only important topic in multiobjective optimization.

6. FUTURE RESEARCH PATHS

As has been indicated before in some of the sections of this chapter, a lot of work remains to be done in this area. We will describe next some of the future research paths that we consider most promising in this area:

- Combinatorial Optimization: We believe that EMOO researchers can benefit from the considerable amount of work done in combinatorial optimization by relying on multiobjective versions of such problems. Such problems are not only challenging, but have also been studied in great depth (Ehrgott and Gandibleux, 2000). The benchmarks available for problems like the 0/1 knapsack can be used to test EMOO approaches. Such idea has been explored by a few EMOO researchers (for example (Zitzler and Thiele, 1999; Jaszkiewicz, 2000)), but more work in this direction is still necessary.

- Efficient data structures: EMOO researchers have paid little attention to the data structures used to store nondominated vectors. Operational researchers have used, for example, domination-free quad trees where a nondominated vector can be retrieved from the tree very efficiently. Checking if a new vector is dominated by the vectors in one of these trees can also be done very efficiently (Habenicht, 1982). Efficiency has been emphasized in EMOO research until recently (Deb et al., 2000a), mainly regarding the number of comparisons performed for ranking the population and to maintain diversity, but a lot of work is still necessary.

- Theoretical issues: There are very few theoretical studies related to EMOO, and most of them concentrate on convergence issues (Rudolph, 1998; Rudolph and Agapie, 2000; Hanne, 2000; Veldhuizen and Lamont, 1998), or on ways to compute niche sizes (Fonseca and Fleming, 1993; Horn et al., 1994). However, many other important areas have not been studied. It would be very interesting to study, for example, the structure of fitness landscapes in MOPs. Such study could provide some insights regarding the sort of problems that are particularly difficult for EAs and could also provide clues regarding the design of more powerful EMOO techniques. Also, there is a need for detailed studies of the different aspects involved in the parallelization of EMOO techniques (e.g., load balancing, impact on Pareto convergence, performance issues, etc.), including new algorithms that are more suitable for parallelization than those currently in use.

There are also several other research areas that are worth exploring. For example: development of MOP test functions (Veldhuizen and Lamont, 1999; Deb, 1999b; Deb and Meyarivan, 2000), appropriate metrics that allow us to evaluate performance in a quantitative way (Zitzler et al., 2000; Veldhuizen, 1999; Fonseca and Fleming, 1996), to study in more depth the role of local search in multiobjective optimization (Ishibuchi and Murata, 1996; Parks and Miller, 1998; Knowles and Corne, 2000; Coello and Toscano, 2001), etc. Some of these areas are actively being pursued by several researchers nowadays.

7. SUMMARY

We have tried to give a general perspective of the research that has been done and that is currently under way in evolutionary multiobjective optimization, including our own. Starting with a short discussion on the origins of a separate discipline devoted to the study of MOPs, we have led our discussion towards the main motivations to use EAs in these types of problems.

We have stressed the importance of studying the several issues involved in solving a MOP, rather than just focusing our research in the development of efficient procedures to generate nondominated vectors. Decision making is as important (or maybe more) than just generating trade-offs for a MOP, and most EMOO researchers seem to overlook this matter.

We have also indicated some promising research trends (from our personal perspective), from which the lack of theoretical studies remains as the area that requires more attention from EMOO researchers.

Finally, we have also surveyed the main EMOO approaches currently in use, indicating some of their applications reported in the literature, as well as their advantages and disadvantages.

But overall, one of the most reiterative issues that we have underlined in this chapter has been the importance of relying on the work done in OR as a basis for pursuing research in EMOO. The awareness of the important contributions to multiobjective optimization that operational researchers have made will help EMOO researchers to have a wider perspective of the field and a deeper understanding of the fundamental problems that need to be solved in this discipline.

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