Monografías del Seminario Matemático García de Galdeano **nn**, 1–20 (2009)

A Tutorial on Multi-Objective Optimization using Metaheuristics

Carlos A. Coello Coello

Abstract. This paper provides an overview of the use of metaheuristics for solving multi-objective optimization problems. The metaheuristics discussed include multi-objective evolutionary algorithms (going from the early approches to the most recent research trends in that area), multi-objective particle swarm optimizers, multi-objective artificial immune systems, multi-objective ant colony systems and multi-objective scatter search. In the final part of the paper, we provide a review of sample applications of multi-objective metaheuristics, and a discussion of some of the topics in which more research is required.

Keywords: multi-objective optimization, metaheuristics.

AMS classification: 90C29, 65K10.

§1. Introduction

A wide variety of problems in engineering and other disciplines have two or more objectives which we wish to minimize simultaneously. Such objectives are normally in conflict with each other (at least partially) and tend to be expressed in different units. These problems are called multi-objective and their solution requires a different notion of optimality that aims for the best possible trade-offs among the objectives (i.e., solutions for which no objective can be improved without worsening another one). For dealing with these problems, it is common to rely on the so-called *Pareto optimality* [92]. This definition gives rise to several compromise solutions, called the *Pareto optimal set*. The objective function values corresponding to the elements of the Pareto optimal set constitute the so-called *Pareto front*.

The algorithms for solving multi-objective optimization problems which are currently available in the mathematical programming literature [87] have a number of limitations, including the facts that some of them have a fairly limited applicability and that others need, in many cases, of problem specific information (e.g., derivatives). Additionally, some of those methods can be easily trapped in local Pareto optimal solutions when dealing with complex search spaces. This has motivated the use of alternative approaches, from which metaheuristics have gained an increasing popularity in the last few years. The main reasons for this popularity are their ease of use and their effectiveness to deal with a wide variety of problems, requiring little or no problem-specific information. Within the many types of metaheuristics currently available, evolutionary algorithms are, with no doubt, the most popular choice [19].[1]

[1] The author maintains the EMOO repository, which currently contains over 4800 bibliographic references on this topic. The EMOO repository is available at: `http://delta.cs.cinvestav.mx/~ccoello/EMOO/`

The remainder of this paper is organized as follows. In Section 2, we provide some basic concepts necessary to understand the rest of the paper. Section 3 is devoted to multi-objective evolutionary algorithms which are, the most popular multi-objective metaheuristic (MOMH) in current use. In Section 4, we talk about four more MOMHs that are relatively popular in the specialized literature. Section 5 summarizes some of the main applications of MOMHs. Two research topics that deserve further exploration are briefly discussed in Section 6. Finally, our conclusions are provided in Section 7.

§2. Basic Concepts

We are interested in solving problems of the type[2]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

Therefore, our aim is to obtain the Pareto optimal set from $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). It is worth indicating, however, that in practice, to obtain all

[2]Without loss of generality, we will assume only minimization problems.

the elements of the Pareto optimal set is normally undesirable and could also be impossible. Thus, our main goal when using a MOMH is to generate a good approximation of the Pareto optimal set (i.e., containing solutions that, when mapped in objective function space, are as close as possible from the true Pareto front of the problem) and having a good distribution (i.e., also in objective function space).

§3. Multi-Objective Evolutionary Algorithms

Since evolutionary algorithms are, by far, the most popular metaheuristic that has been used for solving multi-objective optimization problems, we will devote this entire section to them.

3.1. The Early Days

Evolutionary algorithm (EA) is a generic term used to denote several metaheuristics inspired on the "survival of the fittest" principle from Darwin's evolutionary theory. Their origins can be traced back to the 1960s [68, 100, 49], and have been found to be quite effective in solving a wide variety of complex search, classification and optimization problems [41].

EAs are particularly suitable for solving multi-objective optimization problems because of their capability to operate on a set of solutions (the *population*) at each iteration, which allows them to generate several trade-off solutions in a single run. They have also become popular because of their ease of use and generality (i.e., EAs are less susceptible to the shape and continuity of the Pareto front of a problem than mathematical programming techniques) [19].

It is worth indicating than traditional EAs require some modifications in order to deal with multi-objective optimization problems. The main two are the following:

1. All the nondominated solutions should be considered equally good by the selection mechanism. This means that a different notion of fitness is required for dealing with multi-objective optimization problems. The most popular mechanism to deal with this problem is called Pareto ranking and was introduced by Goldberg [59]. This approach assigns a rank to each solution based on its Pareto dominance, such that nondominated solutions are all sampled at the same rate.

2. EAs tend to converge to a single solution if run long enough, because of stochastic noise [59]. Therefore, a mechanism to maintain diversity is required. This component is known as the *density estimator*. Fitness sharing [60] was the earliest density estimator, but many others have been proposed over time, including clusters [122], entropy [47], adaptive grids [80] and crowding [31], among others.

The first actual implementation of a multi-objective evolutionary algorithm (MOEA) was David Schaffer's *Vector Evaluation Genetic Algorithm* (VEGA), which was introduced in the mid-1980s, mainly aimed for solving problems in machine learning [99].

In the period that goes from the second half of the 1980s to the first half of the 1990s, a few relatively simple and naive MOEAs were introduced. Most of them relied on aggregating functions (mostly linear) [104], lexicographic ordering [51], and target-vector approaches

[113]. Most of these MOEAs did not modify their selection mechanism or any other component, except for the definition of the fitness function. Most of these MOEAs would soon be forgotten.

As indicated before, Pareto ranking was proposed in Goldberg's famous book on genetic algorithms [59]. However, he only provided an informal description of this new selection mechanism but no actual implementation. This gave rise to several MOEAs based on Goldberg's proposal. The three most representative of the early days of MOEAs are:

1. **Nondominated Sorting Genetic Algorithm** (NSGA): This algorithm was proposed by Srinivas and Deb [103] and was the first MOEA to be published in a specialized journal (*Evolutionary Computation*). The NSGA is based on several layers of classification of the individuals as suggested by Goldberg. Before selection takes place, the population is ranked on the basis of nondominance: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. Fitness sharing is used to distribute the population along the Pareto front of the problem.

2. **Niched-Pareto Genetic Algorithm** (NPGA): Proposed by Horn et al. [69]. It uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is the following: Two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. When both competitors are either dominated or nondominated (i.e., there is a tie), the result of the tournament is decided through fitness sharing [60]. In [45], a revised version of this algorithm, called NPGA 2 was proposed. This algorithm relies on a traditional Pareto ranking approach (similar to Fonseca and Fleming's MOGA [50]), but it keeps its tournament selection scheme. Ties are solved through fitness sharing as in its predecessor. However, the niche count of the NPGA 2 is computed using individuals from the next partially filled generation using a technique called "continuously updated fitness sharing" [91].

3. **Multi-Objective Genetic Algorithm** (MOGA): Proposed by Fonseca and Fleming [50]. Here, the rank of a certain individual corresponds to the number of individuals in the current population by which it is dominated. Consider, for example, an individual x_i at generation t, which is dominated by $p_i^{(t)}$ individuals in the current generation. The rank of an individual is given by [50]:

$$\text{rank}(x_i, t) = 1 + p_i^{(t)} \tag{4}$$

All nondominated individuals are assigned rank 1, while dominated ones are penalized according to the population density of the corresponding region of the trade-off surface.

Fitness assignment is performed in the following way [50]:

(a) Sort population according to rank.

(b) Assign fitness to individuals by interpolating from the best to the worst in the way proposed by Goldberg [59], according to some function, usually linear, but not necessarily.

(c) Average the fitnesses of individuals with the same rank, so that all of them are sampled at the same rate. This procedure keeps the global population fitness constant while maintaining appropriate selective pressure, as defined by the function used.

From these 3 algorithms, a few comparative studies undertaken during the mid and late 1990s, indicated that MOGA was the most effective and efficient approach, followed by the NPGA and by the NSGA [22, 108]. It is worth indicating that during the early days (up to the end of the 1990s), most papers would compare MOEAs without using performance measures, but only in a graphical way (plotting the Pareto fronts generated by each MOEA).

3.2. Elitist MOEAs

Towards the end of the 1990s, elitism became a standard mechanism to be provided into any MOEA. The Strengh Pareto Evolutionary Algorithm (SPEA) [118] played a key role in popularizing elitism, since it adopted an external population, and its publication in a specialized journal (the *IEEE Transactions on Evolutionary Computation*), quickly made it a landmark in the field. Consequently, many researchers started to incorporate external populations in their MOEAs, popularizing this mechanism. There are, however, also theoretical reasons for which elitism is a required mechanism in MOEAs (see [97]). Elitism consists of retaining the best solutions found during the search so that they are subject to crossover or mutation. In the context of multi-objective optimization, elitism usually (although not necessarily) refers to the use of an external population (also called secondary population) to retain the nondominated individuals found during the search. External archives can be unbounded but, mainly because of practical reasons, they are normally bounded. Another mechanism that can be used instead of an external archive is the so-called $(\mu + \lambda)$-selection in which parents compete with their children and those which are nondominated (and possibly comply with some additional criterion such as providing a better distribution of solutions) are selected for the following generation. This sort of selection is implicitly elitist, because it will retain the best half of the individuals under consideration.

With the advent of elitist MOEAs, performance measures started to become popular in the specialized literature [27, 119, 109]. It has been found, however, that some of these performance measures are not Pareto-compliant and can provide no reliable assessment [124]. There are also several benchmarks for testing new MOEAs, from which the most popular are: the Zitzler-Deb-Thiele (ZDT) test suite [119], the Deb-Thiele-Laumanns-Zitzler (DTLZ) test suite [32] and the Walkig-Fish-Group (WFG) test suite [70].

The three following approaches are representative of the elitist MOEAs in common use nowadays:

1. **Strength Pareto Evolutionary Algorithm** (SPEA): This algorithm was introduced in [118], and was conceived as a way of integrating different MOEAs. It uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set. For each individual in this external set, a *strength* value is computed. This strength is similar to the ranking value of MOGA [50], since it is proportional to the number of solutions to which a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. The fitness assignment process of SPEA considers both closeness to the true Pareto front and even distribution of solutions at the same time. Thus, instead of using niches based on distance, Pareto dominance is used to ensure that the solutions are properly distributed along the Pareto front. Although this approach does not require a niche radius, its effectiveness relies on the size of the external nondominated set. In fact, since the external nondominated set participates in the selection process of SPEA, if its size grows too large, it might reduce the selection pressure, thus slowing down the search. Because of this, the authors decided to adopt a technique that prunes the contents of the external nondominated set so that its size remains below a certain threshold (a clustering technique called "average linkage method" [88] was used for that sake). There is a revised version of SPEA, called SPEA2, which has three main differences with respect to its predecessor [121]: (1) it incorporates a fine-grained fitness assignment strategy which takes into account for each individual the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.

2. **Pareto Archived Evolution Strategy** (PAES): This algorithm was introduced in [81]. It consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive (the elitist mechanism) that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. An interesting aspect of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).

3. **Nondominated Sorting Genetic Algorithm II** (NSGA-II): This approach was introduced in [29, 31] as an improved version of the NSGA [103]. In the NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, the NSGA-II uses a crowded-

comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The NSGA-II combines the best parents with the best offspring obtained (i.e., a $(\mu+\lambda)$-selection), instead of using an external archive. Due to its clever mechanisms, the NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good, that it has become very popular in the last few years, becoming a landmark against which other MOEAs have to be compared.

Although many other MOEAs exist (see for example [20, 101, 115]), it is not the intention of this paper to be comprehensive. The interested reader may refer to [19, 28, 105] for more information on this topic.

3.3. Current Trends in MOEAs

During some time, the use of relaxed forms of Pareto dominance became popular as a mechanism to regulate convergence of a MOEA. From these mechanisms, ϵ-dominance was, with no doubt, the most popular [85]. ϵ-dominance allows to control the granularity of the approximation of the Pareto front obtained. As a consequence, it is possible to accelerate convergence using this mechanism (if we are satisfied with a very coarse approximation of the Pareto front). Several MOEAs incorporated ϵ-dominance in their external archives (see for example [33, 65]), and there was even one MOEA fully developed around this concept (see [30]).

However, the main current research trend regarding algorithmic development is to adopt a performance measure in the selection scheme of a MOEA (hypervolume[3] has been the most popular). See for example:

- **Evolution Strategy with Probability Mutation (ESP)**: This approach uses a hypervolume-based, scaling independent, parameterless measure, to truncate overpopulated external archives [71].
- **Indicator-Based Evolutionary Algorithm (IBEA)**: This is a framework that allows any performance indicator to be incorporated into the selection mechanism of a MOEA [120]. Its authors tested it with the hypervolume and with the binary ϵ indicator.
- **S Metric Selection Evolutionary Multiobjective Algorithm (SMS-EMOA)**: This approach is based on the hypervolume performance measure [42, 9].
- **Set Preference Algorithm for Multiobjective optimization (SPAM)**: This can be seen as a generalization of IBEA which allows the use of any sort of set preference relation [123].

The use of hypervolume has some advantages, from which the main one is that it has been proved that the maximization of this performance measure is equivalent to finding the

[3]The hypervolume (also known as the S metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively.

Pareto optimal set [48]. Additionally, empirical studies have shown that (for a certain number of points previously determined) the maximization of the hypervolume does indeed produce subsets of the Pareto front which are well-distributed [82, 42]. Furthermore, hypervolume measures convergence and, to a certain extent, also the spread of solutions along the Pareto front. Finally, it has been shown that hypervolume-selection is less sensitive to scalability in objective function space, which makes it promising to deal with problems having many objectives [75].

Hypervolume has, however, some problems of its own. First, the computation of this performance measure depends on a reference point, which can influence the results in a significant manner. Some people have proposed to use the worst objective function values in the current population, but this requires scaling of the objectives. Its main drawback, however, is that the best algorithms known to compute hypervolume have a polynomial complexity on the number of points used, but such complexity grows exponentially on the number of objectives. This has triggered an important amount of efforts aimed to produce more efficient algorithms to approximate the hypervolume [112, 10, 8, 7, 13].

§4. Other Metaheuristics

Several other metaheuristics have also been used as multi-objective optimizers [19, 23]. Next, we will discuss four of the most popular of them in more detail:

- **Particle Swarm Optimization**: This metaheuristic was inspired on the choreography of a bird flock which aim to find food [78, 79]. The implementation of this algorithm employs a population of particles, whose behavior is affected by either the best local (i.e., within a certain neighborhood) or the best global individual (i.e., with respect to the entire swarm). Particle swarm optimization (PSO) has been successfully used for both continuous nonlinear and discrete binary optimization [43, 44]. An important number of multi-objective versions of PSO currently exist (see for example [21, 94, 95, 46]). However, until relatively recently, most of the research had concentrated on producing new variations of existing algorithms, rather than on studying other (more interesting) topics, such as the role of the main components of a multi-objective particle swarm optimizer. Some recent research in that direction has shown that certain components that had been traditionally disregarded (e.g., the leader selection mechanism and the parameters of the flight formula) play a key role in the performance of a multi-objective particle swarm optimizer [12, 107]. There are also other interesting comparative studies aimed to identify their advantages and limitations [39]. It is expected that more research of this sort will be conducted in the next few years.

- **Artificial Immune Systems**: If considered from a computational point of view, our natural immune system can be considered a distributed intelligent system, which is able to learn and retrieve knowledge previously acquired, in order to solve several (highly complex) recognition and classification tasks [89]. These features make our immune has motivated researchers to develop computational models of our immune system which have been used for a variety of tasks, including classification, pattern recognition, and optimization [26, 89]. Several multi-objective extensions of artificial immune systems have been proposed in the specialized literature (see for example

[17, 53, 14, 52]). Also, several hybrid approaches have been proposed to solve specific tasks (see for example [2], where the authors use a multi-objective immune system hybridized with evolutionary operators and local search, in order to solve a rule extraction problem). More hybrid approaches are still to come, but until now, the high potential of multi-objective artificial immune systems in classification and pattern recognition tasks has not been fully exploited yet [117].

- **Ant Colony Optimization**: This metaheuristic was inspired on the foraging behavior of real ants. It is a distributed, stochastic search procedure based on the indirect communication of a set (called "colony") of artificial ants, which mediate using artificial pheromone trails. These pheromone trails can be seen as distributed information which is used by the ants to construct their solutions to the problem at hand. Such pheromone trails are modified during the algorithm's execution, such that they reflect the search experience acquired by the ants. This metaheuristic is intended for solving difficult (both static and dynamic) combinatorial optimization problems, in which solutions can be generated through the use of a construction procedure [36, 37]. There are several multi-objective extensions of ant colony optimization (ACO) algorithms (see for example [66, 35, 55, 1]), but as multi-objective combinatorial optimization becomes more attractive for researchers [40, 54], it is expected that more multi-objective ant colony optimization approaches (and hybrids of ACO algorithms with other metaheuristics) are proposed within the next few years.

- **Scatter Search**: This approach was originally conceived as an extension of a heuristic called surrogate constraint relaxation, which was designed for solving integer programming problems [56]. Its core idea is to adopt a series of different initializations to generate solutions. A reference set of solutions (the best found so far) is adopted, and then such solutions are "diversified" in order to generate new solutions within the neighborhood of the contents of the reference set. This sort of simple procedure is repeated until no further improvements to the contents of the reference set are detected. In the mid-1990s, some further mechanisms were added to the original scatter search algorithm, which allowed its extension to solve nonlinear, binary and permutation optimization problems [57]. These new applications triggered an important amount of research in the following years [83, 86]. Multi-objective extensions of scatter search are relatively recent, but have been steadily increasing [4, 90]. Scatter search has a lot of potential for hybrid approaches, such as memetic MOEAs [58], since it can act as a powerful local search engine for tasks such as generating missing parts of a Pareto front [98]. Because of its flexibility and ease of use, scatter search is expected to become more commonly adopted in the near future, particularly when designing hybrid MOEAs that rely heavily on good local search engines.

§5. Applications

MOEAs have been applied to a wide variety of domains (see for example [18]). However, and for the purposes of this paper, we can roughly classify the applications of MOEAs into three large groups: engineering, industrial and scientific. Some specific areas within each of

these groups are indicated next.

We will start with the engineering applications, which are, by far, the most popular in the literature. This should not be surprising, since engineering disciplines normally have problems with better understood mathematical models, which makes them more suitable for the use of MOEAs. Some sample applications of MOEAs in engineering are the following:

- Electrical engineering [111]
- Hydraulic engineering [5]
- Structural engineering [15]
- Aeronautical engineering [93]
- Robotics [106]
- Control [114]
- Telecommunications [24]
- Civil engineering [38]
- Transport engineering [61]

Now, we will provide some applications of MOEAs in industry:

- Design and manufacture [62]
- Scheduling [64]
- Management [72]

Finally, we have a variety of scientific applications of MOEAs:

- Chemistry [34]
- Physics [96]
- Medicine [125]
- Geography [6]
- Bioinformatics [3]
- Computer science [102]

Although small, this sample should give a good idea of the type of work being done with MOEAs these days. Nevertheless, many other applications exist. The interested reader should refer to the EMOO repository [16] for more information on this topic.

§6. Future Research Paths

In spite of the high volume of research done around multi-objective metaheuristics, several interesting topics remain to be explored in greater depth. Next, we briefly discuss two of them:

1. **Hybridization**: The hybridization of MOMHs with other metaheuristics and with local search mechanisms (either gradient-based or not) aimed to improve their performance is a topic that is currently being explored by many researchers (see for example [63, 67, 76, 84]), because of its high potential. Hybrid MOMHs could be viable alternatives for solving some of the great challenges of today, such as many-objective optimization problems (i.e., problems having 4 or more objective functions) [73, 74]. The use of scalarization methods combined with MOMHs is also another type of interesting hybridization that has a lot of potential for solving highly complex problems (see for example [116]).

2. **Incorporation of user's preferences**: In most real-world applications, users are not interested in the entire Pareto front, but only in a portion of it. Several mechanisms to incorporate user's preferences into a MOMH have been reported in the specialized literature (see for example [25, 77, 110, 11]), but this topic has only been scarcely explored and certainly deserves more attention.

§7. Conclusions

In this paper, we have provided a short (and highly compact) tutorial on the use of metaheuristics for solving multi-objective optimization problems. As such, this tutorial provides a very general overview of the field and is intended to serve as a quick reference for those interested in in this area. The author hopes that, in spite of favoring breadth over depth, this tutorial can be useful for those wishing to do research in multi-objective optimization using metaheuristics, since that has been the purpose of this work.

Acknowledgements

The author acknowledges support from CONACyT project no. 103570.

References

[1] A. Afshar, F. Sharifi, and M.R. Jalali. Non-dominated archiving multi-colony ant algorithm for multi-objective optimization: Application to multi-purpose reservoir operation. *Engineering Optimization*, 41(4):313–325, April 2009.

[2] J. H. Ang, K. C. Tan, and A. A. Mamum. An evolutionary memetic algorithm for rule extraction. *Expert Systems with Applications*, 37(2):1302–1315, March 2010.

[3] R. Dilão, D. Muraro, M. Nicolau, and M. Schoenauer. Validation of a Morphogenesis Model of Drosophila Early Development by a Multi-objective Evolutionary Optimization Algorithm. In C. Pizzuti, M. D. Ritchie, and M. Giacobini, editors, *Evolutionary Computation, Machine Learning*

and Data Mining in Bioinformatics (EvoBIO'2009), pages 176–190. Springer, Lecture Notes in Computer Science, Vol. 5483, Tübingen, Germany, 2009. ISBN 978-3-642-01183-2.

[4] R. P. Beausoleil. "MOSS" multiobjective scatter search applied to non-linear multiple criteria optimization. *European Journal of Operational Research*, 169(2):426–449, March 2006.

[5] E. G. Bekele and J. W. Nicklow. Multi-objective automatic calibration of SWAT using NSGA-II. *Journal of Hydrology*, 341(3-4):165–176, August 2007.

[6] D. A. Bennett, N. Xiao, and M. P. Armstrong. Exploring the Geographic Consequences of Public Policies Using Evolutionary Algorithms. *Annals of the Association of American Geographers*, 94(4):827–847, 2004.

[7] N. Beume. S-Metric Calculation by Considering Dominated Hypervolume as Klee's Measure Problem. *Evolutionary Computation*, 17(4):477–492, Winter 2009.

[8] N. Beume, C. M. Fonseca, M. Lopez-Ibanez, L. Paquete, and J. Vahrenhold. On the Complexity of Computing the Hypervolume Indicator. *IEEE Transactions on Evolutionary Computation*, 13(5):1075–1082, October 2009.

[9] N. Beume, B. Naujoks, and M. Emmerich. SMS-EMOA: Multiobjective selection based on dominated hypervolume. *European Journal of Operational Research*, 181(3):1653–1669, 16 September 2007.

[10] L. Bradstreet, L. While, and L. Barone. A Fast Incremental Hypervolume Algorithm. *IEEE Transactions on Evolutionary Computation*, 12(6):714–723, December 2008.

[11] J. Branke and K. Deb. Integrating User Preferences into Evolutionary Multi-Objective Optimization. In Yaochu Jin, editor, *Knowledge Incorporation in Evolutionary Computation*, pages 461–477. Springer, Berlin Heidelberg, 2005. ISBN 3-540-22902-7.

[12] J. Branke and S. Mostaghim. About Selecting the Personal Best in Multi-Objective Particle Swarm Optimization. In T. P. Runarsson et al., editors, *Parallel Problem Solving from Nature - PPSN IX, 9th International Conference*, pages 523–532. Springer. Lecture Notes in Computer Science Vol. 4193, Reykjavik, Iceland, September 2006.

[13] K. Bringmann and T. Friedrich. Approximating the Least Hypervolume Contributor: NP-Hard in General, But Fast in Practice. In M. Ehrgott, C. M. Fonseca, X. Gandibleux, J.-K. Hao, and M. Sevaux, editors, *Evolutionary Multi-Criterion Optimization. 5th International Conference, EMO 2009*, pages 6–20. Springer. Lecture Notes in Computer Science Vol. 5467, Nantes, France, April 2009.

[14] F. Campelo, F. G. Guimarães, and H. Igarashi. Overview of Artificial Immune Systems for Multi-Objective Optimization. In S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 937–951, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.

[15] T.Y. Chen and H.C. Chen. Mixed-discrete structural optimization using a rank-niche evolution strategy. *Engineering Optimization*, 41(1):39–58, January 2009.

[16] C. A. Coello Coello. The EMOO repository: a resource for doing research in evolutionary multiobjective optimization. *IEEE Computational Intelligence Magazine*, 1(1):37–45, February 2006.

[17] C. A. Coello Coello and N. Cruz Cortés. Solving Multiobjective Optimization Problems using an Artificial Immune System. *Genetic Programming and Evolvable Machines*, 6(2):163–190, June 2005.

[18] C. A. Coello Coello and G. B. Lamont, editors. *Applications of Multi-Objective Evolutionary Algorithms*. World Scientific, Singapore, 2004. ISBN 981-256-106-4.

[19] C. A. Coello Coello, G. B. Lamont, and D. A. Van Veldhuizen. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer, New York, second edition, September 2007. ISBN 978-0-387-33254-3.

[20] C. A. Coello Coello and G. Toscano Pulido. Multiobjective Optimization using a Micro-Genetic Algorithm. In L. Spector et al., editors, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'2001)*, pages 274–282, San Francisco, California, 2001. Morgan Kaufmann Publishers.

[21] C. A. Coello Coello, G. Toscano Pulido, and M. Salazar Lechuga. Handling Multiple Objectives With Particle Swarm Optimization. *IEEE Transactions on Evolutionary Computation*, 8(3):256–279, June 2004.

[22] C. A. Coello Coello. *An Empirical Study of Evolutionary Techniques for Multiobjective Optimization in Engineering Design*. PhD thesis, Department of Computer Science, Tulane University, New Orleans, LA, April 1996.

[23] D. Corne, M. Dorigo, and F. Glover, editors. *New Ideas in Optimization*. McGraw-Hill, London, 1999.

[24] J. Crichigno and B. Barán. Multiobjective Multicast Routing Algorithm. In J. Neuman de Souza, P. Dini, and P. Lorenz, editors, *Telecommunications and Networking. 11th International Conference on Telecommunications (ICT'2004)*, pages 1029–1034. Springer, Lecture Notes in Computer Science, Vol. 3124, Fortaleza, Brazil, August 1-6 2004. ISBN 978-3-540-22571-3.

[25] D. Cvetković and I. C. Parmee. Preferences and their Application in Evolutionary Multiobjective Optimisation. *IEEE Transactions on Evolutionary Computation*, 6(1):42–57, February 2002.

[26] D. Dasgupta, editor. *Artificial Immune Systems and Their Applications*. Springer-Verlag, Berlin, 1999.

[27] K. Deb. Multi-Objective Genetic Algorithms: Problem Difficulties and Construction of Test Problems. *Evolutionary Computation*, 7(3):205–230, Fall 1999.

[28] K. Deb. *Multi-Objective Optimization using Evolutionary Algorithms*. John Wiley & Sons, Chichester, UK, 2001. ISBN 0-471-87339-X.

[29] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. In M. Schoenauer et al., editors, *Proceedings of the Parallel Problem Solving from Nature VI Conference*, pages 849–858, Paris, France, 2000. Springer. Lecture Notes in Computer Science No. 1917.

[30] K. Deb, M. Mohan, and S. Mishra. Evaluating the ϵ-Domination Based Multi-Objective Evolutionary Algorithm for a Quick Computation of Pareto-Optimal Solutions. *Evolutionary Computation*, 13(4):501–525, Winter 2005.

[31] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA–II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, April 2002.

[32] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable Test Problems for Evolutionary Multiobjective Optimization. In A. Abraham, L. Jain, and R. Goldberg, editors, *Evolutionary Multiobjective Optimization. Theoretical Advances and Applications*, pages 105–145. Springer, USA, 2005.

[33] V. Devireddy and P. Reed. Efficient and Reliable Evolutionary Multiobjective Optimization Using ε-Dominance Archiving and Adaptive Population Sizing. In K. Deb et al., editor, *Genetic and Evolutionary Computation–GECCO 2004. Proceedings of the Genetic and Evolutionary Computation Conference. Part II*, pages 390–391, Seattle, Washington, USA, June 2004. Springer-Verlag, Lecture Notes in Computer Science Vol. 3103.

[34] A. Dietz, A. Aguilar-Lasserre, C. Azzaro-Pantel, L. Pibouleau, and S. Domenech. A fuzzy multiobjective algorithm for multiproduct batch plant: Application to protein production. *Computers & Chemical Engineering*, 32(1-2):292–306, January-February 2008.

[35] K.F. Doerner, W.J. Gutjahr, R.F. Hartl, C. Strauss, and C. Stummer. Pareto ant colony optimization with ILP preprocessing in multiobjective portfolio selection. *European Journal of Operational Research*, 171(3):830–841, June 2006.

[36] M. Dorigo and G. Di Caro. The Ant Colony Optimization Meta-Heuristic. In David Corne, Marco Dorigo, and Fred Glover, editors, *New Ideas in Optimization*, pages 11–32, London, 1999. McGraw-Hill.

[37] M. Dorigo and T. Stützle. *Ant Colony Optimization*. The MIT Press, 2004. ISBN 0-262-04219-3.

[38] L. Dridi, M. Parizeau, A. Mailhot, and J.-P. Villeneuve. Using evolutionary optimization techniques for scheduling water pipe renewal considering a shor planning horizon. *Computer-Aided Civil and Infrastructure*, 23(8):625–635, November 2008.

[39] J. J. Durillo, J. García-Nieto, A. J. Nebro, C. A. Coello Coello, F. Luna, and E. Alba. Multi-Objective Particle Swarm Optimizers: An Experimental Comparison. In M. Ehrgott, C. M. Fonseca, X. Gandibleux, J.-K. Hao, and M. Sevaux, editors, *Evolutionary Multi-Criterion Optimization. 5th International Conference, EMO 2009*, pages 495–509. Springer. Lecture Notes in Computer Science Vol. 5467, Nantes, France, April 2009.

[40] M. Ehrgott and X. Gandibleux. Multiobjective Combinatorial Optimization—Theory, Methodology, and Applications. In M. Ehrgott and X. Gandibleux, editors, *Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys*, pages 369–444. Kluwer Academic Publishers, Boston, 2002.

[41] A.E. Eiben and J.E. Smith. *Introduction to Evolutionary Computing*. Springer, Berlin, 2003. ISBN 3-540-40184-9.

[42] M. Emmerich, N. Beume, and B. Naujoks. An EMO Algorithm Using the Hypervolume Measure as Selection Criterion. In C. A. Coello Coello, A. Hernández Aguirre, and E. Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 62–76, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.

[43] A. P. Engelbrecht. *Computational Intelligence: An Introduction*. John Wiley & Sons, 2003. ISBN 0-47084-870-7.

[44] A. P. Engelbrecht. *Fundamentals of Computational Swarm Intelligence*. John Wiley & Sons, Ltd, 2005. ISBN 0-470-09191-6.

[45] M. Erickson, A. Mayer, and J. Horn. The Niched Pareto Genetic Algorithm 2 Applied to the Design of Groundwater Remediation Systems. In E. Zitzler, K. Deb, L. Thiele, C. A. Coello Coello, and D. Corne, editors, *First International Conference on Evolutionary Multi-Criterion Optimization*, pages 681–695. Springer-Verlag. Lecture Notes in Computer Science No. 1993, 2001.

[46] S.-K. Fan and J.-M. Chang. A parallel particle swarm optimization algorithm for multi-objective optimization problems. *Engineering Optimization*, 41(7):673–697, July 2009.

[47] A. Farhang-Mehr and S. Azarm. Diversity Assessment of Pareto Optimal Solution Sets: An Entropy Approach. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 723–728, Piscataway, New Jersey, May 2002. IEEE Service Center.

[48] M. Fleischer. The Measure of Pareto Optima. Applications to Multi-objective Metaheuristics. In C. M. Fonseca, P. J. Fleming, E. Zitzler, K. Deb, and L. Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 519–533, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.

[49] L. J. Fogel. *Artificial Intelligence through Simulated Evolution*. John Wiley, New York, 1966.

[50] C. M. Fonseca and P. J. Fleming. Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization. In S. Forrest, editor, *Proceedings of the Fifth International Conference on Genetic Algorithms*, pages 416–423, San Mateo, California, 1993. University of Illinois at Urbana-Champaign, Morgan Kauffman Publishers.

[51] M. P. Fourman. Compaction of Symbolic Layout using Genetic Algorithms. In *Genetic Algorithms and their Applications: Proceedings of the First International Conference on Genetic Algorithms*, pages 141–153. Lawrence Erlbaum, 1985.

[52] F. Freschi, C. A. Coello Coello, and M. Repetto. Multiobjective Optimization and Artificial Immune Systems: A Review. In H. Mo, editor, *Handbook of Research on Artificial Immune Systems and Natural Computing: Applying Complex Adaptive Technologies*, pages 1–21. Medical Information Science Reference, Hershey, New York, 2009. ISBN 978-1-60566-310-4.

[53] F. Freschi and M. Repetto. VIS: an artificial immune network for multi-objective optimization. *Engineering Optimization*, 38(8):975–996, December 2006.

[54] X. Gandibleux and M. Ehrgott. 1984-2004 – 20 Years of Multiobjective Metaheuristics. But What About the Solution of Combinatorial Problems with Multiple Objectives? In C. A. Coello Coello, A. Hernández Aguirre, and E. Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 33–46, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.

[55] C. García-Martínez, O. Cordón, and F. Herrera. A taxonomy and an empirical analysis of multiple objective ant colony optimization algorithms for the bi-criteria TSP. *European Journal of Operational Research*, 180(1):116–148, July 2007.

[56] F. Glover. Heuristics for integer programming using surrogate constraints. *Decision Sciences*, 8:156–166, 1977.

[57] F. Glover. Tabu search for nonlinear and parametric optimization (with links to genetic algorithms). *Discrete Applied Mathematics*, 49:231–255, 1994.

[58] C.-K. Goh, Y.-S. Ong, and K. C. Tan, editors. *Multi-Objective Memetic Algorithms*. Springer, Berlin, Germany, 2009. ISBN 978-3-540-88050-9.

[59] D. E. Goldberg. *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley Publishing Company, Reading, Massachusetts, 1989.

[60] D. E. Goldberg and J. Richardson. Genetic algorithm with sharing for multimodal function optimization. In J. J. Grefenstette, editor, *Genetic Algorithms and Their Applications: Proceedings of the Second International Conference on Genetic Algorithms*, pages 41–49, Hillsdale, New Jersey, 1987. Lawrence Erlbaum.

[61] M. M. Golias, M. Boile, and S. Theofanis. Berth scheduling by customer service differentiation: A multi-objective approach. *Transportation Research Part E - Logistics and Transportation Review*, 45(6):878–892, November 2009.

[62] L. Grandinetti, F. Guerriero, G. Lepera, and M. Mancini. A niched genetic algorithm to solve a pollutant emission reduction problem in the manufacturing industry: A case study. *Computers & Operations Research*, 34(7):2191–2214, July 2007.

[63] J. Grobler and A. P. Engelbrecht. Hybridizing PSO and DE for improved vector evaluated multi-objective optimization. In *2009 IEEE Congress on Evolutionary Computation (CEC'2009)*, pages 1255–1262, Trondheim, Norway, May 2009. IEEE Press.

[64] J. Grobler, A. P. Engelbrecht, and V. S. S. Yadavalli. Multi-Objective DE and PSO Strategies for Production Scheduling. In *2008 Congress on Evolutionary Computation (CEC'2008)*, pages 1154–1161, Hong Kong, June 2008. IEEE Service Center.

[65] C. Grosan. Improving the Performance of Evolutionary Algorithms for the Multiobjective 0/1 Knapsack Problem using ε-dominance. In *2004 Congress on Evolutionary Computation (CEC'2004)*, volume 2, pages 1958–1963, Portland, Oregon, USA, June 2004. IEEE Service Center.

[66] M. Guntsch and M. Middendorf. Solving Multi-criteria Optimization Problems with Population-Based ACO. In C. M. Fonseca, P. J. Fleming, E. Zitzler, K. Deb, and L. Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 464–478, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.

[67] K. Harada, K. Ikeda, and S. Kobayashi. Hybridizing of Genetic Algorithm and Local Search in Multiobjective Function Optimization: Recommendation of GA then LS. In M. Keijzer et al., editor, *2006 Genetic and Evolutionary Computation Conference (GECCO'2006)*, volume 1, pages 667–674, Seattle, Washington, USA, July 2006. ACM Press. ISBN 1-59593-186-4.

[68] J. H. Holland. Concerning efficient adaptive systems. In M. C. Yovits, G. T. Jacobi, and G. D. Goldstein, editors, *Self-Organizing Systems—1962*, pages 215–230. Spartan Books, Washington, D.C., 1962.

[69] J. Horn, N. Nafpliotis, and D. E. Goldberg. A Niched Pareto Genetic Algorithm for Multiobjective Optimization. In *Proceedings of the First IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Intelligence*, volume 1, pages 82–87, Piscataway, New Jersey, June 1994. IEEE Service Center.

[70] S. Huband, P. Hingston, L. Barone, and L. While. A Review of Multiobjective Test Problems and a Scalable Test Problem Toolkit. *IEEE Transactions on Evolutionary Computation*, 10(5):477–506, October 2006.

[71] S. Huband, P. Hingston, L. White, and L. Barone. An Evolution Strategy with Probabilistic Mutation for Multi-Objective Optimisation. In *Proceedings of the 2003 Congress on Evolutionary Computation (CEC'2003)*, volume 3, pages 2284–2291, Canberra, Australia, December 2003. IEEE Press.

[72] A. K. Hutzschenreuter, P. A. N. Bosman, and H. La Poutré. Evolutionary Multiobjective Optimization for Dynamic Hospital Resource Management. In M. Ehrgott, C. M. Fonseca, X. Gandibleux, J.-K. Hao, and M. Sevaux, editors, *Evolutionary Multi-Criterion Optimization. 5th International Conference, EMO 2009*, pages 320–334. Springer. Lecture Notes in Computer Science Vol. 5467, Nantes, France, April 2009.

[73] H. Ishibuchi, Y. Hitotsuyanagi, and Y. Nojima. Scalability of Multiobjective Genetic Local Search to Many-Objective Problems: Knapsack Problem Case Studies. In *2008 Congress on Evolutionary Computation (CEC'2008)*, pages 3587–3594, Hong Kong, June 2008. IEEE Service Center.

[74] H. Ishibuchi, N. Tsukamoto, and Y. Nojima. Evolutionary many-objective optimization: A short review. In *2008 Congress on Evolutionary Computation (CEC'2008)*, pages 2424–2431, Hong Kong, June 2008. IEEE Service Center.

[75] H. Ishibuchi, N. Tsukamoto, Y. Sakane, and Y. Nojima. Hypervolume Approximation Using Achievement Scalarizing Functions for Evolutionary Many-Objective Optimization. In *2009 IEEE Congress on Evolutionary Computation (CEC'2009)*, pages 530–537, Trondheim, Norway, May 2009. IEEE Press.

[76] S. Jeong, S. Hasegawa, K. Shimoyama, and S. Obayashi. Development and Investigation of Efficient GA/PSO-Hybrid Algorithm Applicable to Real-World Design Optimization. *IEEE Computational Intelligence Magazine*, 4(3):36–44, August 2009.

[77] Y. Jin and B. Sendhoff. Incorporation of Fuzzy Preferences into Evolutionary Multiobjective Optimization. In W.B. Langdon et al., editors, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'2002)*, page 683, San Francisco, California, July 2002. Morgan Kaufmann Publishers.

[78] J. Kennedy and R. C. Eberhart. Particle Swarm Optimization. In *Proceedings of the 1995 IEEE International Conference on Neural Networks*, pages 1942–1948, Piscataway, New Jersey, 1995. IEEE Service Center.

[79] J. Kennedy and R. C. Eberhart. *Swarm Intelligence*. Morgan Kaufmann Publishers, San Francisco, California, 2001.

[80] J. Knowles and D. Corne. Properties of an Adaptive Archiving Algorithm for Storing Nondominated Vectors. *IEEE Transactions on Evolutionary Computation*, 7(2):100–116, April 2003.

[81] J. D. Knowles and D. W. Corne. Approximating the Nondominated Front Using the Pareto Archived Evolution Strategy. *Evolutionary Computation*, 8(2):149–172, 2000.

[82] J. D. Knowles, D. W. Corne, and M. Fleischer. Bounded Archiving using the Lebesgue Measure. In *Proceedings of the 2003 Congress on Evolutionary Computation (CEC'2003)*, volume 4, pages 2490–2497, Canberra, Australia, December 2003. IEEE Press.

[83] M. Laguna and R. Martí. *Scatter Search : Methodology and Implementations in C*. Kluwer Academic Publishers, 2003. ISBN 1-402-07376-3.

[84] A. Lara, G. Sanchez, C. A. Coello Coello, and O. Schütze. HCS: A New Local Search Strategy for Memetic Multi-Objective Evolutionary Algorithms. *IEEE Transactions on Evolutionary Computation*, 14(1):112–132, February 2010.

[85] M. Laumanns, L. Thiele, K. Deb, and E. Zitzler. Combining Convergence and Diversity in Evolutionary Multi-objective Optimization. *Evolutionary Computation*, 10(3):263–282, Fall 2002.

[86] R. Martí. Scatter Search–Wellsprings and Challenges. *European Journal of Operational Research*, 169:351–358, 2006.

[87] K. M. Miettinen. *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, Boston, Massachusetts, 1999.

[88] J.N. Morse. Reducing the size of the nondominated set: Pruning by clustering. *Computers and Operations Research*, 7(1–2):55–66, 1980.

[89] L. N. de Castro and J. Timmis. *An Introduction to Artificial Immune Systems: A New Computational Intelligence Paradigm*. Springer, London, 2002. ISBN 1-85233-594-7.

[90] A. J. Nebro, F. Luna, E. Alba, B. Dorronsoro, J. J. Durillo, and A. Beham. AbYSS: Adapting Scatter Search to Multiobjective Optimization. *IEEE Transactions on Evolutionary Computation*, 12(4):439–457, August 2008.

[91] C. K. Oei, D. E. Goldberg, and S.-J. Chang. Tournament Selection, Niching, and the Preservation of Diversity. Technical Report 91011, Illinois Genetic Algorithms Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois, December 1991.

[92] V. Pareto. *Cours D'Economie Politique*, volume I and II. F. Rouge, Lausanne, 1896.

[93] S. Raiagopal and R. Ganguli. Conceptual design of UAV using Kriging based multi-objective genetic algorithm. *Aeronautical Journal*, 112(1137):653–662, November 2008.

[94] M. Reyes Sierra and C. A. Coello Coello. Improving PSO-Based Multi-objective Optimization Using Crowding, Mutation and ϵ-Dominance. In C. A. Coello Coello, A. Hernández Aguirre, and E. Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 505–519, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.

[95] M. Reyes-Sierra and C. A. Coello Coello. Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art. *International Journal of Computational Intelligence Research*, 2(3):287–308, 2006.

[96] V. Romero-Garcia, J. V. Sanchez-Perez, L. M. Garcia-Raffi, J. M. Herrero, S. Garcia-Nieto, and X. Blasco. High Optimization processes for increasing the attenuation properties of acoustic metamaterials by means of creation of defects. *Applied Physics Letters*, 93(22), December 2008.

[97] G. Rudolph and A. Agapie. Convergence Properties of Some Multi-Objective Evolutionary Algorithms. In *Proceedings of the 2000 Conference on Evolutionary Computation*, volume 2, pages 1010–1016, Piscataway, New Jersey, July 2000. IEEE Press.

[98] L. V. Santana-Quintero, N. Ramírez, and C. Coello Coello. A Multi-objective Particle Swarm Optimizer Hybridized with Scatter Search. In A. Gelbukh and C. A. Reyes-Garcia, editors, *MICAI 2006: Advances in Artificial Intelligence, 5th Mexican International Conference on Artificial Intelligence*, pages 294–304. Springer, Lecture Notes in Artificial Intelligence Vol. 4293, Apizaco, Mexico, November 2006.

[99] J. D. Schaffer. Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. In *Genetic Algorithms and their Applications: Proceedings of the First International Conference on Genetic Algorithms*, pages 93–100. Lawrence Erlbaum, 1985.

[100] H.-P. Schwefel. Kybernetische evolution als strategie der experimentellen forschung in der strömungstechnik. Dipl.-Ing. thesis, 1965. (in German).

[101] M. Sefrioui and J. Periaux. Nash Genetic Algorithms: examples and applications. In *2000 Congress on Evolutionary Computation*, volume 1, pages 509–516, San Diego, California, July 2000. IEEE Service Center.

[102] A. J. Soto, R. L. Cecchini, G. E. Vazquez, and I. Ponzoni. Multi-Objective Feature Selection in QSAR Using a Machine Learning Approach. *QSAR & Combinatorial Science*, 28(11-12):1509–1523, December 2009.

[103] N. Srinivas and K. Deb. Multiobjective Optimization Using Nondominated Sorting in Genetic Algorithms. *Evolutionary Computation*, 2(3):221–248, Fall 1994.

[104] G. Syswerda and J. Palmucci. The Application of Genetic Algorithms to Resource Scheduling. In R. K. Belew and L. B. Booker, editors, *Proceedings of the Fourth International Conference on Genetic Algorithms*, pages 502–508, San Mateo, California, 1991. Morgan Kaufmann.

[105] K.C. Tan, E.F. Khor, and T.H. Lee. *Multiobjective Evolutionary Algorithms and Applications*. Springer-Verlag, London, 2005. ISBN 1-85233-836-9.

[106] J. Teo, L. D. Neri, M. H. Nguyen, and H. A. Abbass. Walking with EMO: Multi-Objective Robotics for Evolving Two, Four, and Six–Legged Locomotion. In L. T. Bui and S. Alam, editors, *Multi-Objective Optimization in Computational Intelligence: Theory and Practice*, pages 300–332. Information Science Reference, Hershey, PA, USA, 2008. ISBN 978-1-59904-498-9.

[107] G. Toscano-Pulido, C. A. Coello Coello, and L. V. Santana-Quintero. EMOPSO: A Multi-Objective Particle Swarm Optimizer with Emphasis on Efficiency. In S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 272–285, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.

[108] D. A. Van Veldhuizen. *Multiobjective Evolutionary Algorithms: Classifications, Analyses, and New Innovations*. PhD thesis, Department of Electrical and Computer Engineering. Graduate School of Engineering. Air Force Institute of Technology, Wright-Patterson AFB, Ohio, May 1999.

[109] D. A. Van Veldhuizen and Gary B. Lamont. On Measuring Multiobjective Evolutionary Algorithm Performance. In *2000 Congress on Evolutionary Computation*, volume 1, pages 204–211, Piscataway, New Jersey, July 2000. IEEE Service Center.

[110] J. Wang and J. P. Terpenny. Interactive Preference Incorporation in Evolutionary Engineering Design. In Yaochu Jin, editor, *Knowledge Incorporation in Evolutionary Computation*, pages 525–543. Springer, Berlin Heidelberg, 2005. ISBN 3-540-22902-7.

[111] L. Wang and C. Singh. Stochastic combined heat and power dispatch based on multi-objective particle swarm optimization. *International Journal of Electrical Power & Energy Systems*, 30(3):226–234, March 2008.

[112] L. While, P. Hingston, L. Barone, and S. Huband. A Faster Algorithm for Calculating Hypervolume. *IEEE Transactions on Evolutionary Computation*, 10(1):29–38, February 2006.

[113] P. B. Wienke, C. Lucasius, and G. Kateman. Multicriteria target optimization of analytical procedures using a genetic algorithm. *Analytica Chimica Acta*, 265(2):211–225, 1992.

[114] L. Xu, B. Zhu, and E. D. Goodman. An Improved MOCC with Feedback Control Structure Based on Preference. In *2009 ACM SIGEVO Summit on Genetic and Evolutionary Computation (GEC'2009)*, pages 651–656, Shanghai, China, June 12-14 2009. ACM Press. ISBN 978-1-60558-326-6.

[115] S. Y. Zeng, L. S. Kang, and L. X. Ding. An Orthogonal Multi-objective Evolutionary Algorithm for Multi-objective Optimization Problems with Constraints. *Evolutionary Computation*, 12(1):77–98, Spring 2004.

[116] Q. Zhang and H. Li. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6):712–731, December 2007.

[117] X. Zhang, B. Lu, S. Gou, and L. Jiao. Immune Multiobjective Optimization Algorithm Using Unsupervised Feature Selection. In F. Rothlauf et al., editor, *Applications of Evolutionary Computing. EvoWorkshops 2006: EvoBIO, EvoCOMNET, EvoHOT, EvoIASP, EvoINTERACTION, EvoMUSART, and EvoSTOC*, pages 484–494, Budapest, Hungary, April 2006. Springer, Lecture Notes in Computer Science Vol. 3907.

[118] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms on Test Functions of Different Difficulty. In A. S. Wu, editor, *Proceedings of the 1999 Genetic and Evolutionary Computation Conference. Workshop Program*, pages 121–122, Orlando, Florida, July 1999.

[119] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation*, 8(2):173–195, Summer 2000.

[120] E. Zitzler and S. Künzli. Indicator-based Selection in Multiobjective Search. In X. Yao et al., editor, *Parallel Problem Solving from Nature - PPSN VIII*, pages 832–842, Birmingham, UK, September 2004. Springer-Verlag. Lecture Notes in Computer Science Vol. 3242.

[121] E. Zitzler, M. Laumanns, and L. Thiele. SPEA2: Improving the Strength Pareto Evolutionary Algorithm. In K. Giannakoglou, D. Tsahalis, J. Periaux, P. Papailou, and T. Fogarty, editors, *EUROGEN 2001. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems*, pages 95–100, Athens, Greece, 2002.

[122] E. Zitzler and L. Thiele. Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach. *IEEE Transactions on Evolutionary Computation*, 3(4):257–271, November 1999.

[123] E. Zitzler, L. Thiele, and J. Bader. SPAM: Set Preference Algorithm for Multiobjective Optimization. In Günter Rudolph, Thomas Jansen, Simon Lucas, Carlo Poloni, and Nicola Beume, editors, *Parallel Problem Solving from Nature–PPSN X*, pages 847–858. Springer. Lecture Notes in Computer Science Vol. 5199, Dortmund, Germany, September 2008.

[124] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. Grunert da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. *IEEE Transactions on Evolutionary Computation*, 7(2):117–132, April 2003.

[125] V. Zoete, A. Grosdidier, and O. Michelin. Docking, Virtual High Troughput Screening and in Silico Fragment-Based Drug Design. *Journal of Cellular and Molecular Medicine*, 13(2):238–248, February 2009.

Carlos A. Coello Coello
CINVESTAV-IPN
Departamento de Computación
Av. IPN No. 2508
Col. San Pedro Zacatenco
México, D.F. 07360, México
ccoello@cs.cinvestav.mx