

# Use of Genetic Algorithms for Multiobjective Optimization of Counterweight Balancing of Robot Arms\*

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## Abstract

*We present a hybrid approach to optimize the counterweight balancing of a robot arm, which uses a combination of a genetic algorithm with the min-max multiobjective optimization method to get the Pareto optimal set of solutions. This set corresponds to several possible robot designs from which the most appropriate has to be chosen by the designer. Our approach is compared to a more traditional min-max search technique in which a combination of random and sequential search was used to generate the Pareto optimal solutions. Our results show how the genetic algorithm is able to get solutions with a lower deviation from the optimal.*

**Keywords:** genetic algorithms, multiobjective optimization, robot arm optimization, design optimization, counterweight balancing

## Introduction

The use of industrial robots in different fields of technology is becoming more common every day, making it more important to be able to improve their efficiency in terms of energy consumption and working accuracy. The proper balancing of a robot manipulator is one way to improve such efficiency. There are two main methods of balancing a robot manipulator [6]: 1) by spring mechanisms, and 2) by counterweights. The second approach, which is the one selected for this work, has been frequently used in the literature for establishing better mass distributions of mechanisms and its use on robot manipulators involves the minimization of driving forces or torques as well as the support reactions at joints. Since these two criteria have to be satisfied at the same time, a multiobjective optimization approach has to be taken. The lengths and masses of balancing mechanisms of the robot arm are used as design variables, and several constraints derived from the allowable movements of the arm are imposed. The optimization model used for this work is based on the rigid-body dynamics of the PUMA-560 robot [7] [1]. We used a hybrid approach to solve this problem, in which we combined a genetic algorithm with the min-max method to get the Pareto optimal set, which corresponds to several possible robot designs from which the decision maker has to choose the most appropriate. This set was obtained by varying the importance of each of the four objective functions derived from the optimization model—two torques and two reactions—. Our approach is compared to a more traditional min-max technique in which a combination of random and sequential search is used to generate the Pareto optimal solutions. This problem has a highly non-convex search space, which implies the presence of several local minima. On the other hand, the large amount of CPU time required to evaluate the different objectives arise some interesting issues on the use of genetic algorithms in this kind of application.

## Statement of the Problem

Koski and Osyczka [6] present a multiobjective optimization model of a PUMA-560 robot arm based on its rigid-body dynamics. By using angular coordinates for the PUMA-560 robot, it is possible to calculate the generalized torques at each joint applying:

$$M_{ti} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} \quad (1)$$

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where  $\theta_i$  is the rotation at joint  $i$  and  $\dot{\theta}_i$  is the corresponding angular velocity. The term

$$L = T - V \quad (2)$$

represents the Lagrangian function of the mechanical system. Here,  $T$  is the total kinetic energy of the system and  $V$  is the total potential energy. The application of eq. (1) to a fully articulated robot arm results in the following nonlinear second-order system of differential equations

$$A\ddot{\theta} + B\dot{\theta}^2 + c - m = 0 \quad (3)$$

The manipulator is an isostatic structure, and thus it is possible to get explicit expressions for all forces and moments in the system. The friction in the joints as well as the flexibility of the arm are not included in our design model. For the application of optimization methods, a two-member robot arm, which corresponds to the two links of the PUMA-560 robot in a plane motion, is considered. This arm is assumed to move in the xy-plane only. The masses of the members are  $m_1$  and  $m_2$ . They are located as point masses at distances  $e_1$  and  $e_2$  from the joints. The external load is represented by the point mass  $m_3$ . In the model used by Koski and Osyczka, only the counterweight masses  $m_4$  and  $m_5$ , as well as their distances from the joints  $x_1$  and  $x_2$  are treated as design variables, whereas all the other quantities are fixed. Unfortunately, due to lack of space, we could not include the complete mathematical expressions for the torques and the reactions, but they may be found in Koski and Osyczka [6]. The objective of this optimization problem is to find such masses  $m_4$  and  $m_5$  for the counterweights and such joint distances  $x_1$  and  $x_2$  which will minimize the four chosen design criteria. The minimization of the torques is important because they allow the use of smaller motors, and a lower energy consumption [6]. The torques do not depend on the design variables alone, but also on the position of the robot arm  $(\theta_1, \theta_2)$ , on the angular velocities  $(\dot{\theta}_1, \dot{\theta}_2)$  and on the angular accelerations  $(\ddot{\theta}_1, \ddot{\theta}_2)$ . Usually, the working space of the robot arm is restricted, and thus constraints on the possible values of the angles are imposed. In each position of the arm, the angular velocities and accelerations may be different. In order to optimize the performance of the robot, the torques should be as small as possible at all working positions and at all existing angular velocity acceleration combinations. Thus, the first two criteria are chosen as follows:

$$f_1(\bar{x}) = \max_{\theta_1} \max_{\theta_2} \max_{\dot{\theta}_i, \ddot{\theta}_i} M_{t1}; \quad f_2(\bar{x}) = \max_{\theta_1} \max_{\theta_2} \max_{\dot{\theta}_i, \ddot{\theta}_i} M_{t2} \quad (4)$$

where notation  $\dot{\theta}_i, \ddot{\theta}_i$  is associated with the chosen angular velocity profile. The construction of joints, especially with the choice of bearings, depends largely on the reaction forces at the joints. Thus, it seems reasonable to choose the maximum values of the joint forces as two additional criteria. By using the fixed trapezoidal velocity profiles and every feasible position of the arm, these criteria can be expressed in the form

$$f_3(\bar{x}) = \max_{\theta_1} \max_{\theta_2} \max_{\dot{\theta}_i, \ddot{\theta}_i} R_1; \quad f_4(\bar{x}) = \max_{\theta_1} \max_{\theta_2} \max_{\dot{\theta}_i, \ddot{\theta}_i} R_2 \quad (5)$$

The multiobjective optimization problem [6] becomes:

$$\min (f_1(\bar{x}), f_2(\bar{x}), f_3(\bar{x}), f_4(\bar{x}))^T \quad (6)$$

subject to

$$\theta_i^l \leq \theta_i \leq \theta_i^u \quad i = 1, 2; \quad x_i^l \leq x_i \leq x_i^u \quad i = 1, 2, 3, 4 \quad (7)$$

The numerical design data for the problem under consideration is given below [6]. These values are close to those for the first two links of the PUMA-560 robot [1].

$$\begin{aligned} m_1 &= 17 \text{ kg}, & m_2 &= 6 \text{ kg}, & m_3 &= 2 \text{ kg}, & L_1 &= L_2 = 0.43 \text{ m}, & e_1 &= 0.07 \text{ m}, & e_2 &= 0.14 \text{ m}, \\ \theta_1^l &= -40^\circ, & \theta_1^u &= 220^\circ, & \theta_2^l &= -140^\circ, & \theta_2^u &= 140^\circ, & \dot{\theta}_1 \max &= 2 \frac{rad}{s}, & \dot{\theta}_2 \max &= 4 \frac{rad}{s}, \\ \ddot{\theta}_1 \max &= 10 \frac{rad}{s^2}, & \ddot{\theta}_2 \max &= 20 \frac{rad}{s^2}, & x_1^l &= x_2^l = 0, & x_1^u &= x_2^u = 0.2 \text{ m}, & x_3^l &= x_4^l = 0, \\ x_3^u &= 35 \text{ kg}, & x_4^u &= 15 \text{ kg}, & J_1 &= 0.2619 \text{ kg} \cdot \text{m}^2, & J_2 &= 0.0924 \text{ kg} \cdot \text{m}^2 \end{aligned} \quad (8)$$

### Solution Procedure

To obtain the term  $\max_{\theta_1}(\cdot)$ , we followed the procedure given by Koski and Osyczka [6]:

1. Compute the torques and joint forces at the positions  $\theta_1^l, \theta_1^l + \Delta\theta_1, \theta_1^l + 2\Delta\theta_1, \dots, \theta_1^u$ , where the increment  $\Delta\theta_1$  was chosen to be 20 degrees.

2. Select separately the maximum value for each criterion.
3. Perform the same calculations for  $\max_{\theta_2}(\cdot)$  with an increment  $\Delta\theta_2$  (we used also 20 degrees).
4. The terms  $\max_{\dot{\theta}_i, \ddot{\theta}_i}(\cdot)$  are computed using some chosen combinations of  $\dot{\theta}_i$  and  $\ddot{\theta}_i$  for given  $\theta_1$  and  $\theta_2$ .
5. After calculating  $M_{ti}$  and  $R_i$  for all the chosen values, the maximum values can be determined.

Obviously, the smaller the value of  $\Delta\theta_i$  the better the accuracy achieved, but also greater the computation time required. We experimentally found that even an apparently large increment like the one we used didn't significantly affect the final result. However, in terms of time, this value made a great difference, particularly if we consider that using either random search or the genetic algorithm, all these computations have to be performed a lot of times. To get an idea of the importance of this parameter, when an increment of one degree is used, the time required to get one set of results (i.e., the final values of the four objective functions) is of about 2 minutes and 20 seconds on a Sun Workstation with four 90 MHz HyperSparc CPUs. This time is reduced to only one second when using increments of 20 degrees, without any significant loss of precision (normally the differences were in the decimals).

### The Classical Min-Max Method

In the classical min-max method, also known as the *Global Criterion method* [8], an optimal solution is a vector of decision variables which minimizes some global criterion. A function describing this global criterion is a measurement of how close the decision maker can get to the ideal vector—i.e., the vector that contains the optimal solutions of every objective function assuming that these were treated independently—, which we'll denote by  $f^0$ . The most common form of this function is

$$f(x) = \sum_{i=1}^k \left( \frac{f_i^0 - f_i(x)}{f_i^0} \right)^p \quad (9)$$

where  $k$  is the number of objective functions.

For this formula Boychuck and Ovinnikov [2] have suggested  $p = 1$ , and Salukvadze [9] has suggested  $p = 2$ , but other values of  $p$  can also be used. Another possible measurement of "the closeness to the ideal solution" is a family of the  $L_p$ -metrics defined as follows [5]

$$L_p(f) = \left( \sum_{i=1}^k |f_i^0 - f_i(x)|^p \right)^{1/p} \quad 1 \leq p \leq \infty \quad (10)$$

Instead of deviations in the absolute sense, it is recommended to use in eq. (10) relative deviations which have a direct substantive meaning in any given context. The name min-max method is given to the global criterion method with the  $L_\infty(f)$ -metric, because for this metric the optimum  $x^*$  is defined as

$$f(x^*) = \min_x \max_i \left| \frac{f_i^0 - f_i(x)}{f_i^0} \right| \quad i = 1, \dots, k \quad (11)$$

The solution to this optimization problem yields the best compromise solution, in which all criteria are considered equally important. The use of weighting coefficients has been introduced before [5] in conjunction with this method to rank the importance of the candidate criterion, so that the min-max problem can be restated as follows

$$f(x^*) = \min_x \max_i \omega_i \left| \frac{f_i^0 - f_i(x)}{f_i^0} \right| \quad i = 1, \dots, k \quad (12)$$

where  $\omega_i$  is the weighting coefficient representing the relative importance of the  $i$ th criterion. Koski and Osyczka [6] took this approach to solve the counterweight balancing problem presented in this paper, by using the Computer Aided Multicriteria Optimization System (CAMOS) [3]. They used a method which combines random and sequential search to generate the Pareto-optima. First, they generated some points by the random search method, and the best of them were stored and used as the starting points for the sequential search procedure. Then, they minimized each objective separately, to obtain the set of optimal solutions, so that they could use the weighting min-max method described above for generating several Pareto-optimal solutions. The weights were chosen so that their sum were always equals to one. While seeking both, the ideal vector and the other Pareto-optima, they used the random search method in combination with the Nelder-Mead simplex method with a penalty function.

## Use of the Genetic Algorithm

Our approach consisted on using a genetic algorithm (GA) to obtain both, the ideal vector and the Pareto-optimal solutions. First, we ran GAs to optimize each objective separately. Then, with this vector, we introduced the following fitness function:

$$\begin{aligned}
 t_1 &= \left| \frac{f_1^0 - f_1}{f_1^0} \right| & t_2 &= \left| \frac{f_2^0 - f_2}{f_2^0} \right| & t_3 &= \left| \frac{f_3^0 - f_3}{f_3^0} \right| & t_4 &= \left| \frac{f_4^0 - f_4}{f_4^0} \right| \\
 t_5 &= \left| \frac{f_1^0 - f_1}{f_1^0} \right| & t_6 &= \left| \frac{f_2^0 - f_2}{f_2^0} \right| & t_7 &= \left| \frac{f_3^0 - f_3}{f_3^0} \right| & t_8 &= \left| \frac{f_4^0 - f_4}{f_4^0} \right| \\
 \text{if } (t_1 > t_5) \ z_1 &= t_1 \ \text{else } z_1 = t_5; & \text{if } (t_2 > t_6) \ z_2 &= t_2 \ \text{else } z_2 = t_6 \\
 \text{if } (t_3 > t_7) \ z_3 &= t_3 \ \text{else } z_3 = t_7; & \text{if } (t_4 > t_8) \ z_4 &= t_4 \ \text{else } z_4 = t_8 \\
 z &= w_1 z_1 + w_2 z_2 + w_3 z_3 + w_4 z_4 \\
 \text{fitness} &= \frac{1}{z}
 \end{aligned} \tag{13}$$

The weights  $w_i$  were also chosen such that  $w_1 + w_2 + w_3 + w_4 = 1$ .

For all the tests, we used binary tournament selection, double-point crossover, and a population size of 100 chromosomes. Instead of doing several runs with random values for the crossover and mutation probabilities, we used a nested loop in which these two values ranged from 0.1 to 0.9 at increments of 0.1, over 50 generations. This implies that 81 runs were necessary for each design. This procedure showed to be very reliable in terms of finding “good” solutions with the GA, when using a floating-point representation. Execution time becomes an issue, since each run of the GA takes about 2.5 hours on a Sun Workstation with four 90 MHz HyperSparc CPUs. However, the independence of each process made it possible to run them simultaneously on different machines, to improve the performance of the GA.

## Comparison of Results

We generated the fifteen Pareto-optimal designs presented by Koski and Osyczka [6], which include the four optimal values of the corresponding objective functions. To evaluate our results, we used as a parameter the maximum deviation from the optimum, which is defined by

$$L_p(f) = \sum_{i=1}^4 w_i \left| \frac{f_i^0 - f_i(x)}{\rho_i} \right| \tag{14}$$

where  $\rho_i = f_i^0$ , or  $f_i(x)$ , depending on which gives the maximum value for  $L_p(f)$ .

The comparison of our results with those found by Koski and Osyczka [6] are shown in Table 1. The first eight rows corresponds to the optimal solution vector, and therefore in those cases the deviation  $L_p(f)$  is computed by directly comparing the two results, taking the lower as the optimal and the difference of the other one with respect to the first as the deviation. We can clearly see how the GA provided better results in all cases. Because Koski and Osyczka didn't state the value of the increments  $\Delta\theta_i$ , we were unable to make a direct comparison to our method. Therefore, using their solution vectors, we recomputed the values of the objective functions using an increment  $\Delta\theta_i = 1$ . However, we used an increment of 20 degrees within the GA, for the sake of speed. Nevertheless, the final numerical results didn't variate too much even when we used such a large increment.

From these results we can see that the set of weights  $w_1 = 0.1$ ,  $w_2 = 0.1$ ,  $w_3 = 0.4$  and  $w_4 = 0.4$  gives the best compromise solution overall. Other interesting aspects to notice from the results to this problem is that there is a great variation in the ranges of the solutions, and that when the mass of the counterweight is close to zero, the variables  $x_1$  and  $x_2$  (joint distances) may assume any value we want, because they won't influence the solution in a significant way.

## Future Work

We are considering to use several other approaches to multiobjective optimization that have been proposed within the GA community. For example, we want to try Schaffer's VEGA (Vector Evaluated Genetic Algorithm) [10], and the weighted sum approach proposed by Hajela and Lin [4] which includes the weights of each objective in the chromosome, and promotes their diversity in the population through fitness sharing. This allows the simultaneous generation of a family of Pareto optimal designs corresponding to different weighting coefficients in a single run of the GA.

Finally, because of the intensive CPU time-consuming nature of this problem, it would be desirable to explore the use of other techniques that can reduce the number of function evaluations, such as the approximation of functions by low order polynomials over some small region. In this case a computationally expensive function

| Method | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $f_1$  | $f_2$ | $f_3$  | $f_4$  | $x_1$  | $x_2$  | $x_3$  | $x_4$ | $L_p(f)$ |
|--------|-------|-------|-------|-------|--------|-------|--------|--------|--------|--------|--------|-------|----------|
| Koski  | 1     | 0     | 0     | 0     | 112.75 | 39.06 | 750.60 | 303.09 | 0.199  | 0.199  | 34.98  | 5.77  | 20.72    |
| GA     | 1     | 0     | 0     | 0     | 92.03  | 40.21 | 687.37 | 217.41 | 0.1568 | 0.200  | 35.0   | 1.299 | 0        |
| Koski  | 0     | 1     | 0     | 0     | 216.76 | 30.21 | 713.05 | 452.31 | 0.175  | 0.114  | 10.24  | 14.86 | 0.62     |
| GA     | 0     | 1     | 0     | 0     | 168.92 | 29.59 | 891.62 | 444.14 | 0.200  | 0.0932 | 35.0   | 15.0  | 0        |
| Koski  | 0     | 0     | 1     | 0     | 133.11 | 41.94 | 374.82 | 195.23 | 0.198  | 0.14   | 0.001  | 0.002 | 0.02     |
| GA     | 0     | 0     | 1     | 0     | 133.10 | 41.94 | 374.80 | 195.21 | 0.200  | 0.200  | 0.0    | 0.001 | 0        |
| Koski  | 0     | 0     | 0     | 1     | 111.99 | 41.94 | 485.66 | 195.21 | 0.191  | 0.198  | 14.3   | 0.001 | 0.02     |
| GA     | 0     | 0     | 0     | 1     | 105.36 | 41.94 | 693.39 | 195.19 | 0.200  | 0.0932 | 35.0   | 0.001 | 0        |
| Koski  | 0.25  | 0.25  | 0.25  | 0.25  | 138.88 | 38.93 | 510.18 | 268.92 | 0.186  | 0.198  | 7.95   | 4.06  | 0.3809   |
| GA     | 0.25  | 0.25  | 0.25  | 0.25  | 133.16 | 41.87 | 375.73 | 195.92 | 0.200  | 0.200  | 0.029  | 0.045 | 0.2170   |
| Koski  | 0.3   | 0.3   | 0.2   | 0.2   | 139.91 | 37.98 | 612.36 | 298.39 | 0.171  | 0.184  | 16.9   | 5.66  | 0.4737   |
| GA     | 0.3   | 0.3   | 0.2   | 0.2   | 102.45 | 41.87 | 532.12 | 195.92 | 0.200  | 0.200  | 20.46  | 0.045 | 0.2431   |
| Koski  | 0.35  | 0.35  | 0.15  | 0.15  | 152.99 | 37.74 | 667.45 | 336.62 | 0.194  | 0.182  | 19.6   | 7.59  | 0.5540   |
| GA     | 0.35  | 0.35  | 0.15  | 0.15  | 96.99  | 40.70 | 581.09 | 209.56 | 0.200  | 0.200  | 25.055 | 0.853 | 0.2438   |
| Koski  | 0.4   | 0.4   | 0.1   | 0.1   | 152.76 | 38.85 | 800.85 | 344.61 | 0.130  | 0.193  | 32.9   | 7.84  | 0.5793   |
| GA     | 0.4   | 0.4   | 0.1   | 0.1   | 94.71  | 40.20 | 615.27 | 215.86 | 0.200  | 0.1778 | 27.689 | 1.237 | 0.2298   |
| Koski  | 0.2   | 0.2   | 0.3   | 0.3   | 136.76 | 38.91 | 505.85 | 264.17 | 0.190  | 0.197  | 8.05   | 3.82  | 0.3711   |
| GA     | 0.2   | 0.2   | 0.3   | 0.3   | 133.15 | 41.87 | 375.76 | 195.92 | 0.200  | 0.200  | 0.033  | 0.045 | 0.1742   |
| Koski  | 0.15  | 0.15  | 0.35  | 0.35  | 139.62 | 38.63 | 457.88 | 245.80 | 0.200  | 0.200  | 0.039  | 0.044 | 0.2917   |
| GA     | 0.15  | 0.15  | 0.35  | 0.35  | 133.14 | 41.87 | 375.79 | 195.91 | 0.200  | 0.200  | 0.039  | 0.044 | 0.1315   |
| Koski  | 0.1   | 0.1   | 0.4   | 0.4   | 141.63 | 39.46 | 408.89 | 228.29 | 0.103  | 0.114  | 0.138  | 2.08  | 0.1915   |
| GA     | 0.1   | 0.1   | 0.4   | 0.4   | 133.16 | 41.87 | 375.72 | 195.91 | 0.200  | 0.200  | 0.03   | 0.044 | 0.08862  |
| Koski  | 0.5   | 0.1   | 0.2   | 0.2   | 99.44  | 41.46 | 592.53 | 202.09 | 0.172  | 0.093  | 26.5   | 0.45  | 0.2036   |
| GA     | 0.5   | 0.1   | 0.2   | 0.2   | 98.91  | 41.88 | 553.41 | 195.84 | 0.200  | 0.200  | 23.244 | 0.04  | 0.1749   |
| Koski  | 0.1   | 0.5   | 0.2   | 0.2   | 153.03 | 35.75 | 645.41 | 335.46 | 0.198  | 0.157  | 17.0   | 7.84  | 0.4584   |
| GA     | 0.1   | 0.5   | 0.2   | 0.2   | 133.25 | 41.84 | 375.83 | 196.25 | 0.200  | 0.200  | 0.0    | 0.065 | 0.2533   |
| Koski  | 0.4   | 0.2   | 0.2   | 0.2   | 121.99 | 38.42 | 606.99 | 258.65 | 0.148  | 0.182  | 20.6   | 3.6   | 0.3788   |
| GA     | 0.4   | 0.2   | 0.2   | 0.2   | 98.91  | 41.87 | 553.44 | 195.91 | 0.200  | 0.200  | 23.243 | 0.044 | 0.2090   |
| Koski  | 0.2   | 0.4   | 0.2   | 0.2   | 162.68 | 39.11 | 583.60 | 319.94 | 0.152  | 0.198  | 10.3   | 6.6   | 0.5215   |
| GA     | 0.2   | 0.4   | 0.2   | 0.2   | 133.16 | 41.87 | 375.74 | 195.92 | 0.200  | 0.200  | 0.031  | 0.045 | 0.2566   |

Table 1: Pareto-optimal and minimal solutions for the robot arm considered.

is evaluated at a sufficient number of points to construct a low order polynomial approximation. Then, an iterative optimization algorithm is used for finding the minimum of the approximate function. At the point obtained the optimization model is replaced by a new approximate model, and the process continues until the improvement in the objective function can't be distinguished.

## Conclusions

A GA-based min-max approach has been proposed for a complex multiobjective optimization problem: a robot arm balancing. This problem has four objective functions to be minimized, and is highly non-convex. Furthermore, the complex calculations involved consume a lot of CPU time, and make necessary the development of heuristic techniques that need the least possible number of function evaluations. The great variation of the results obtained show that this problem would be very difficult to solve with pure random search, or with brute-force techniques. Also, to find a reasonable heuristics seems a difficult task given the factors previously mentioned, and the possible presence of local minima. The GA has showed to be very consistent in this application, finding better compromise solutions for all the instances of the problem under consideration. Also, some other GA-based approaches seem suitable for this application, especially those in which a Pareto-based selection is applied. However, time remains to be an issue to be considered in further applications of the GA to this problem, and it would be desirable to explore techniques for reducing the number of function evaluations. Nevertheless, the use of such a powerful heuristics should bring benefits to the robotics industry, and our work should be seen as a small module of a larger system whose goal is to optimize the entire design process of a robot arm.

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