

# An Elitist Genetic Algorithm for Multiobjective Optimization

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## 1 Introduction

Solving multiobjective engineering problems is a very difficult task due to, in general, in these class of problems, the objectives conflict across a high-dimensional problem space. In these problems, there is no single optimal solution, the interaction of multiple objectives gives rise to a set of efficient solutions, known as the Pareto-optimal solutions. During the past decade, Genetic Algorithms (GAs)[1] were extended in order to track this class of problems, as the Non-dominated Sorting Genetic Algorithm (NSGA) suggested by Srinivas and Deb [2]. These multiobjective approaches explore some features of GAs to tackle these kind of problems, in particular:

- as GAs work with populations of candidate solutions, they can, in principle, find multiple Pareto-optimal solutions in a single run;
- GAs using some diversity-preserving mechanisms can find widely different Pareto-optimal solutions.

In spite of the success of the application of these approaches to several multiobjective problems, some authors [5] suggested that elitism can speed up the performance of GAs and also prevent the loss of good solutions found during the search process. In this paper, a GA with a new elitist approach is proposed to solve multiobjective optimization problems.

In section 2, the elitist GA is described. Next, the results of the application to several problems are presented. Finally, some conclusions and future work are addressed.

## 2 An Elitist GA

One of the first evolutionary algorithms proposed for multiobjective optimization was the NSGA [2]. This approach differs from conventional GAs with respect to the selection operator emphasizing the non-domination of solutions. Non-domination is tested at each generation in the selection phase, thus defining an approximation to the Pareto optimal set. Crossover and mutation operators remain as usual. On the other hand, a sharing method is used to distribute the solutions in population over the

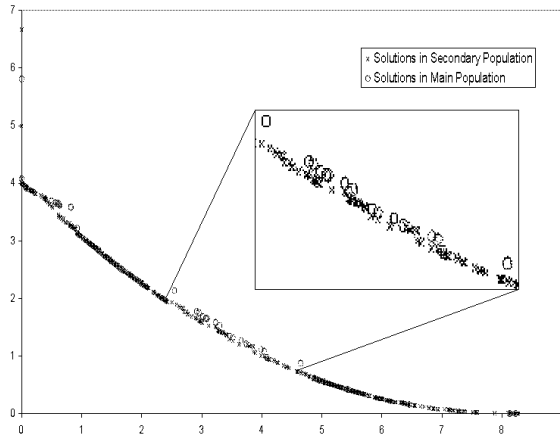


Figure 1: Non-dominated Solutions in Secondary and Main Populations after 200 generations

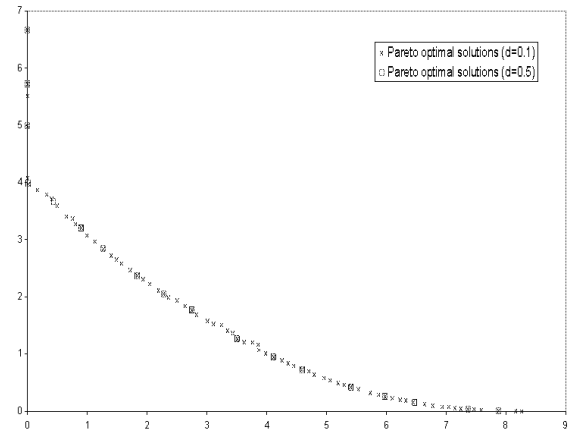


Figure 2: Non-dominated Solutions in Secondary Population for distances of 0.1 and 0.5 after 200 generations

Pareto-optimal region. The multiobjective algorithm here considered is, in terms of fitness sharing, similar to NSGA, but using an elitist technique. This technique is based on a separate population, the secondary population (SP) composed of all (or a part of) potential Pareto optimal solutions found so far during the search process. In this sense, the SP is completely independent of the main population, not participating in the search process.

In its simplest form, in all generations the new potential Pareto optimal solutions found are stored in SP. The SP update implies the determination of Pareto optimality of all solution stored so far, in order to eliminate those that became dominated. As the size of SP grows, the time to complete this operation may become significant. So, in order to prevent this, in general, a maximum SP size is imposed. Thus, the algorithm consists on, for all generations, to store, in SP, each Pareto optimal solution  $x_{nd}$  found in the main population if:

1. all solutions in SP are different of  $x_{nd}$ ;
2. none of the solutions in SP dominates  $x_{nd}$ .

Next, all solutions in SP that became dominated are eliminated. In order to illustrate this algorithm, let us consider the following multiobjective problem:

$$\begin{aligned} \min f_1(x_1, x_2) &= (x_1 + x_2)^2 \\ \min f_2(x_1, x_2) &= (x_1 x_2 - 2)^2 \end{aligned}$$

The two variables were represented by 32 bits strings. In the experiment, a population size of 100, two point crossover with probability 0.7, an uniform mutation with probability 0.001 and a sigma share of 0.05 were considered. Figure 1 shows the non-dominated (potential Pareto optimal) solutions, after 200 generations, in main and secondary populations for this problem. In this figure, it is possible to observe that a significant number of the potential Pareto optimal solutions in main population is dominated by solutions in secondary population. This highlights the importance of the use of a secondary population preventing the loss of good solutions. Furthermore, the distribution of the solutions of main population in the objective space is not very uniform.

As mentioned, as the size of SP increases, the execution time and memory requirements also increase. So, it is convenient to keep relatively small sizes of SP. In this sense, the previous algorithm can be

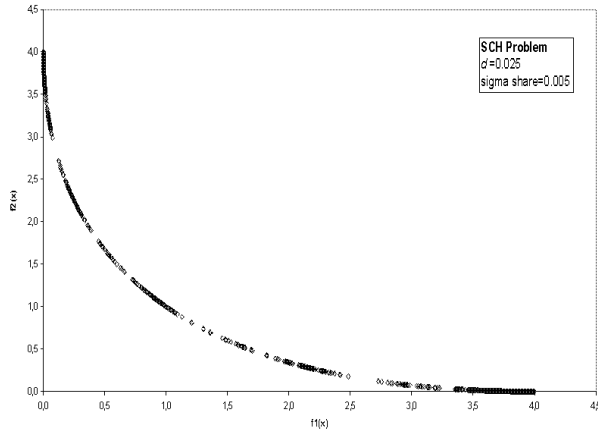


Figure 3: Non-dominated Solutions for SCH Problem

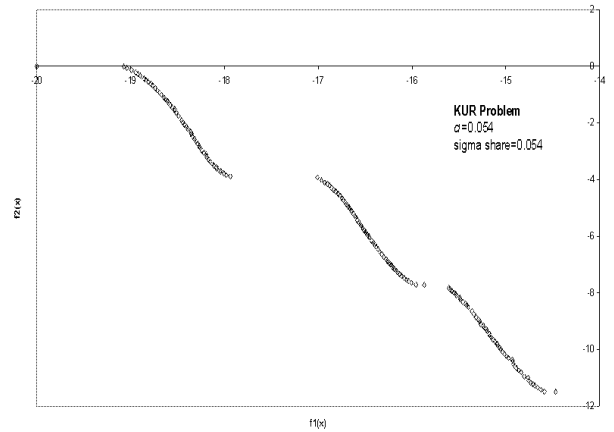


Figure 4: Non-dominated Solutions for KUR Problem

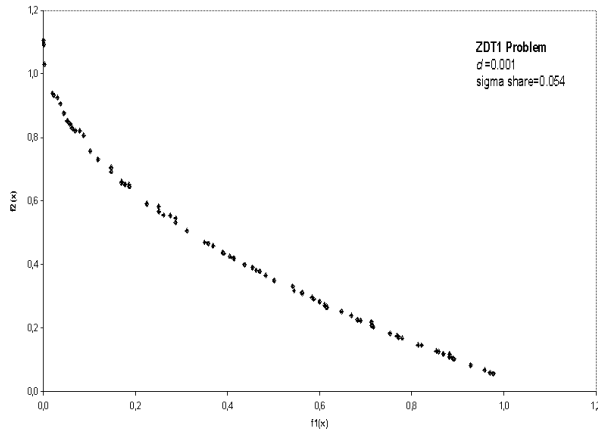


Figure 5: Non-dominated Solutions for ZDT1 Problem

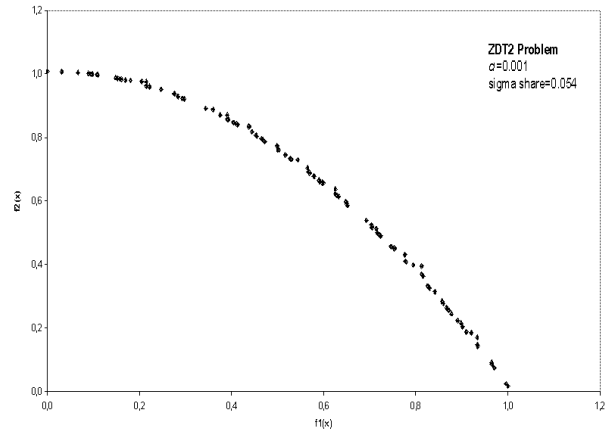


Figure 6: Non-dominated Solutions for ZDT2 Problem

modified accordingly. A new parameter  $d$  is introduced, stating the minimum desirable distance in objective space between potential Pareto optimal solutions in SP. So, step 2. of the algorithm is modified as:

2. none of the solutions in SP that are not dominated by  $x_{nd}$  are closer than  $d$  (euclidean distance measured on objective space).

Figure 2 presents the non-dominated (potential Pareto optimal) solutions in SP after 200 generations for  $d = 0.1$  and  $d = 0.5$ . The solutions, in this figure, are widely distributed in objective space according to the value of  $d$ .

Bearing in mind the previous examples and figures, the elitist strategy presented, shows two main advantages, when compared with the results from the main population:

- the set of all non-dominated solutions in SP constitutes a far better approach to the optimal

Problem	Variable bounds	Objective functions	Optimal solutions	Comments
SCH $n = 1$	$[-10^3, 10^3]$	$f_1(x) = x^2$ $f_2(x) = (x - 2)^2$	$x \in [0, 2]$	convex
FON $n = 3$	$[-4, 4]$	$f_1(x) = 1 - \exp(-\sum_{i=1}^3 (x_i - \frac{1}{\sqrt{3}})^2)$ $f_2(x) = 1 - \exp(-\sum_{i=1}^3 (x_i + \frac{1}{\sqrt{3}})^2)$	$x_1 = x_2 = x_3$ $\in [-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$	nonconvex
POL $n = 2$	$[-\pi, \pi]$	$f_1(x) = [1 + (A_1 - B_1)^2 + (A_2 - B_2)^2]$ $f_2(x) = [(x_1 + 3)^2 + (x_2 + 1)^2]$ $A_1 = 0.5 \sin 1 - 2 \cos 1 + \sin 2 - 1.5 \cos 2$ $A_2 = 1.5 \sin 1 - \cos 1 + 2 \sin 2 - 0.5 \cos 2$ $B_1 = 0.5 \sin x_1 - 2 \cos x_1 + \sin x_2 - 1.5 \cos x_2$ $B_2 = 1.5 \sin x_1 - \cos x_1 + 2 \sin x_2 - 0.5 \cos x_2$		nonconvex, disconnected
KUR $n = 3$	$[-5, 5]$	$f_1(x) = \sum_{i=1}^{n-1} (-10 \exp(-0.2 \sqrt{x_i^2 + x_{i+1}^2}))$ $f_2(x) = \sum_{i=1}^n ( x_i ^{0.8} + 5 \sin x_i^3)$		nonconvex
ZDT1 $n = 30$	$[0, 1]$	$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)}]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	convex
ZDT2 $n = 30$	$[0, 1]$	$f_1(x) = x_1$ $f_2(x) = g(x)[1 - (x_1/g(x))^2]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	nonconvex
ZDT3 $n = 30$	$[0, 1]$	$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)} - \frac{x_i}{g(x)} \sin(10\pi x_1)]$ $g(x) = 1 + 9(\sum_{i=2}^n x_i)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	convex, disconnected
ZDT4 $n = 10$	$x_1 \in [0, 1]$ $x_i \in [-5, 5]$ $i = 2, \dots, n$	$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)}]$ $g(x) = 1 + 10(n-1) + \sum_{i=2}^n [x_i^2 - 10 \cos(4\pi x_i)]$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	nonconvex
ZDT6 $n = 10$	$[0, 1]$	$f_1(x) = 1 - \exp(-4x_1) \sin^6(4\pi x_1)$ $f_2(x) = g(x)[1 - (f_1(x)/g(x))^2]$ $g(x) = 1 + 9[(\sum_{i=2}^n x_i)/(n-1)]^{0.25}$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, n$	nonconvex, nonuniformly spaced

Table 1: Multiobjective Problems

Pareto set;

- the solutions in SP clearly present a balanced distribution along the Pareto front;
- parameter  $d$  allows the definition of the concentration of points all along the Pareto front;
- the size of the SP is small when compared to other approaches;
- the additional computational time required is negligible considering the quality of the results when considering just one main population and, moreover, is much lower than the time required to maintain all non-dominated points in SP.

### 3 Results

The multiobjective problems were chosen from a number of significant past studies in that area [6][7][8][9][5]. All problems have two objective functions and no constraints. Table 1 describes these problems, showing the number of variables, their bounds, the Pareto-optimal solutions known, and the nature of Pareto-optimal front for each problem.

The Elitist GA was applied to each problem with a reasonable set of values for the parameters (no effort was made in finding the best parameter setting for each problem). The stopping criterion was terminate the execution after 250 generations. All variables were coded using 30 bits strings. A two point crossover and an uniform mutation were used. The crossover probability was, for all problems, 0.7. The mutation probability was given by  $1/b$  where  $b$  is the binary string length ( $b = 30n$  bits). The value of sigma share and  $d$  were varied according to the problem considered. Figures 3 to 6 present the non-dominated (potential Pareto optimal) solutions in SP after 250 generations for SCH, KUR, ZDT1 and ZDT2 problems.

## 4 Conclusions and Future Work

In real multiobjective problems the Pareto front is not known. Thus, any algorithm designed to search for this front must assure an uniformly distribution of the solutions all along the Pareto front, but also, to prevent premature convergence.

In this work an Elitist Genetic Algorithm for Multiobjective Optimization is presented. This new approach, tested on several problems, assumes a well balanced distribution of the solutions on the Pareto front. Furthermore, the concentration of points can be controlled by a parameter and, for reasonable values, the increase on computational time is negligible.

Future work will concentrate on the use of the secondary population in guiding the search for the Pareto optimal set.

## References

- [1] D. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, 1989.
- [2] N. Srinivas and K. Deb. *Multi-Objective function optimization using non-dominated sorting genetic algorithms*. Evolutionary Computation, Vol. 2, pp. 221-248, 1995.
- [3] H-P. Schwefel. *Evolution and Optimal Seeking*. John Wiley and Sons, 1995.
- [4] C. M. Fonseca and P. J. Fleming. *Genetic Algorithms for multi-objective optimization: Formulation, discussion and generalization*. Proceedings of the Fifth International Conference on Genetic Algorithms, S. Forrest, Ed. San Mateo, CA: Morgan Kauffman, pp. 416-423, 1993.
- [5] E. Zitzler, K. Deb and L. Thiele. *Comparison of multiobjective evolutionary algorithms: Empirical results*. Evolutionary Computation, Vol. 8, pp. 173-195, 2000.
- [6] J. D. Schaffer. *Multiple objective optimization with vector evaluated genetic algorithms*. Proceedings of the First International Conference on Genetic Algorithms, J.J Grefenstette, Ed. Hillsdale, pp.93-100, 1987.
- [7] C. M. Fonseca and P. J. Fleming. *Multiobjective optimization and multiple constraint handling with evolutionary algorithms-Part II: Application example*. IEEE Transactions on Systems, Man, and Cybernetics: Part A: Systems and Humans, vol. 28 no. 1, pp. 38-47, 1998.
- [8] C. Poloni. *Hybrid GA for multiobjective aerodynamic shape optimization*. Genetic Algorithms in engineering and computer science, G. Winter, J. Periaux, M. Galan, and P. Puesta, Ed. Hillsdale, pp. 397-414, 1997.
- [9] F. Kursawe. *A variant of evolution strategies for vector optimization*. Parallel Problem Solving from Nature, H.-P. Schwefel and R. Manner, Eds Berlin: Germany: Springer, pp. 193-197, 1990.