New multi-objective stochastic search technique for economic load dispatch

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Abstract: A new multi-objective stochastic search technique (MOSST) for the multi-objective economic dispatch problem in power systems is presented. It is a highly constrained problem with both equality and inequality constraints. The MOSST heuristic has been designed as a combination of real coded genetic algorithms (GA) and simulated annealing (SA). It incorporates a genetic crossover operator BLX – α and a problem specific mutation operator with a local search heuristic to provide a better search capability. Extensive simulations are carried out on standard test systems, considering various aspects, and the results are compared with other methods. These results indicate that the new MOSST heuristic converges rapidly to improved solutions. MOSST is a truly multi-objective technique, as it provides the values of various parameters for optimising different objectives, as well as the best compromise between them, all in a single run. Perturbation analysis shows that the solutions obtained by MOSST are truly pareto-optimal, i.e. no objective can be further improved without degrading the others.

1 Introduction

The economic load dispatch (ELD) problem involves allocation of generations to different thermal units to minimise the cost of generation, while satisfying the equality and inequality constraints of the power system and keeping pollution within limits [1]. The two objectives may be conflicting in nature and a compromise has to be reached to obtain an acceptable power dispatch strategy within the various system constraints.

Mathematically, the problem is represented as minimising F_T, where

$$F_T = \sum_{i=1}^{N_g} F_i(P_{gi}) \tag{1}$$

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and

$$F_i(P_{gi}) = a_i P_{gi}^2 + b_i P_{gi} + d_i + |e_i \sin\{f_i(P_{gi\,min} - P_{gi})\}| \tag{2}$$

subject to the power balance constraint

$$\sum_{i=1}^{N_g} P_{gi} - P_D - P_{loss} = 0 \tag{3}$$

and the capacity constraint

$$P_{gi\,min} \le P_{gi} \le P_{gi\,max} \tag{4}$$

where F_T is the total cost of generation, F_i is the operating cost of the ith generator (\$/h), P_{gi} is the loading of the ith generator, a_i, b_i, d_i, e_i, f_i are coefficients of the cost curve of the ith generator, P_{gimin}, P_{gimax} are the minimum and maximum limits on the loadings of the ith generator, P_D is the total demand, and P_{loss} is the real power loss in transmission lines.

The emission of sulfur dioxide, nitrogen oxides, carbon monoxide gases etc., which cause atmospheric hazards, can be mathematically modelled as

$$emission = 10^{-2}(\alpha_i + \beta_i P_{gi} + \gamma_i P_{gi}^2) + \xi_i \exp(\varepsilon_i P_{gi}) \tag{5}$$

where α, β, γ, ξ, ε are coefficients of generator emission characteristics [2].

Some papers model the problem at hand as a multi-objective problem using a constraint method [3, 4] or weighting method [5]. In the former, the multi-objective problem is reduced to a single objective problem by treating the emissions as a constraint. The latter approach linearly combines the objectives as a weighted sum. The objective function so formed may lose significance due to the incorporation of multiple non-commensurable factors into a single function. Alternatively, the objectives are considered one at a time [2]. This approach does not provide any idea regarding the trade-offs involved.

In this paper, a new multi-objective optimisation approach, MOSST, is presented. It is an adaptation of the method proposed by Yip *et al.* [6] for single criterion function optimisation. The solution provided by MOSST is a family of points known as the pareto-optimal set. Each point in this surface is optimal, in the sense that no improvement can be achieved in an objective that does not lead to a degradation in at least one of the remaining objectives. A comparison of simulation results on standard examples from the literature indicates that the present method discovers better solutions by examining an extremely small fraction of the feasible solution space.

2 Multi-objective stochastic search technique (MOSST) for ELD

MOSST is a hybrid approach which incorporates SA in the selection process of GA. Thus, MOSST provides the advantages of both GA and SA. In this Section, the basic steps and the problem-specific implementation details are discussed. The MOSST algorithm can be expressed concisely in the form of a pseudo-code as given below:

Pseudo-code 1

1. Initialise
   (i) Set initial temperatures T_a and T_b
   (ii) Randomly select N parent strings
   (iii) Number of children to be generated by each parent
   (iv) Initialise pareto-optimal set (PS)
2. For each parent i, generate $m(i)$ children using crossover
3. Perform mutation with a probability pm
4. Find the best child for each parent
5. Select the best child as the parent for the next generation as per pseudo-code 2 given in the explanation of Step 5
6. Repeat Step 7 to Step 10 for each family
7. $count = 0$
8. Repeat Step 9 for each child; goto Step 10
9. Increase count as per pseudo-code 3 given in the explanation of Steps 6-9
10. Acceptance number of the family is equal to *count* (A)
11. Sum the acceptance numbers of all the families (S)
12. For each family i, calculate the number of children to be generated in the next generation according to the following formula:

$$m(i) = (T * A)/S$$

where T = total number of children generated by all the families

13. Update PS
14. Decrease the temperature
15. Repeat Step 2-14 until a certain number of iterations has been reached

A more detailed explanation of each step is as follows:

Step 1: (i) As in SA, the selection of temperatures is such that initially the probability of acceptance of a bad move, i.e. when the best child is worse than the parent is high ($\simeq 1$), but as the temperatures are lowered this probability is successively decreased to zero [7]. As there are two objective values with widely different ranges, separate temperatures T_a and T_b are maintained. The initial and final temperatures are calculated as follows:

Initially, the probability of accepting a bad move is approximately 1, i.e.:

$$\exp(-\Delta X_{average}/T_1) = 0.99 \tag{6}$$

and finally

$$\exp(-\Delta X_{average}/T_{MAXIT}) = 0.0001 \tag{7}$$

therefore

$$T_1 = -\Delta X_{average}/\log(0.99)$$
$$T_{MAXIT} = -\Delta X_{average}/\log(0.0001) \tag{8}$$

where T_1 is the initial temperature, T_{MAXIT} is the final temperature, and $\Delta X_{average}$ is the difference in the objective X for any two neighbouring points in the search space.

(ii) Generation of the initial population of $N = 50$ parents is random. Care is taken to ensure that only feasible strings are generated. Power generations from the generators are coded as strings of floating point numbers. The generation capacity constraint (eqn. 4) is embedded in the coding itself using the following encoding:

$$X_i = (P_{gi} - P_{gi\,min})/(P_{gi\,max} - P_{gi\,min}) \tag{9}$$

where X_i is the ith element of chromosome X, i.e.:

$$P_{gi} = X_i(P_{gi\,max} - P_{gi\,min}) + P_{gi\,min} \tag{10}$$

The equality constraint of real power balance (eqn. 3) has also been embedded in the coding using the approach of [8]. The method randomly selects N_g - 1 generator loadings and treats the N_gth generator as the dependent generator. The loading of this generator is calculated using the B-coefficients in such a way as to satisfy the power balance equation. MOSST focuses the search effort on feasible solutions only, thereby reducing the search time significantly.

(iii) Initially, all the parents generate an equal number of children given by $m(i) = M$. In this implementation, $M = 10$. The total number of children in a generation is fixed and is given by

$$T = \sum_{i=1}^{N} m(i) \tag{11}$$

(iv) A set containing non-inferior solutions, PS, is maintained and is updated in every generation. Initially PS is empty.

Step 2: For each parent i, $m(i)$ children are generated. A blend crossover operator ($BLX - \alpha$) based on the theory of interval schemata [9] has been employed in this study with $\alpha = 0.5$. $BLX - \alpha$ operates by randomly picking a point in the range ($p1 - \alpha(p2 - p1)$, $p2 + \alpha(p2 - p1)$), where $p1$ and $p2$ are the two parent points and $p1 < p2$. This crossover may produce a child which does not satisfy the power balance constraint even when the parents satisfy it. Therefore, one of the generators is selected at random and its power generation is adjusted to restore the real power balance as in Step 1 (ii).

Step 3: A local optimisation step is used for mutation in the proposed method. Cost mutation and emission mutation are performed over the population with small probability (0.1 in each case). Mutation is carried out in two steps. First, the generators with highest and lowest incremental costs (emissions) are taken as the candidates for mutation in the selected chromosome. Second, the loading of the generator with highest incremental cost (emission) is reduced by an amount ΔP_g, and an equal and opposite change is made on the generator with minimum incremental cost (emission). ΔP_g is given by the following expression:

$$\Delta P_g = K * \min\{(P_{g_1} - P_{g_{1\,min}}), (P_{g_{2\,max}} - P_{g_2})\} \tag{12}$$

where Pg_1, Pg_2 are the loadings of the generators having maximum and minimum incremental costs, respectively, Pg_{1min} is the minimum limit of generator 1, Pg_{2max} is the maximum limit of generator 2, and K is constant (0.1 in this case).

The change is made repeatedly until either the overall cost (emission) of the chromosome starts increasing, or the generator loading reaches its limit. Inclusion of the mutation operator speeds up the convergence considerably and also leads to better solutions.

Step 4: The children in the same family (i.e. generated from the same parent) compete with each other and only the best child survives. Child 1 is better than child 2 if

$$(x_1 - x_2)/x_1 + (y_1 - y_2)/y_1 \leq 0 \qquad (13)$$

where $x_1(x_2)$ and $y_1(y_2)$ are the objective functions values of child 1 (child 2) for objectives x and y, respectively. The best child is better than other children in the family.

Step 5: Child C is preferred over its parent P for the next generation for the objectives X and Y according to the following pseudo-code:

Pseudo-code 2

if $C_x \leq P_x$ and $C_y \leq P_y$ (Child is superior to parent)
  then the child is selected as the parent for the next generation;
else if $C_x < P_x$ and $C_y > P_y$ (Child is non-inferior to parent)
  if $\exp\{P_y - C_y)/T_b\} \geq \rho$
    then the child is selected;
  endif;
else if $C_x > P_x$ and $C_y < P_y$
  if $\exp\{(P_x - C_x)/T_a\} \geq \rho$
    then the child is selected;
  endif;
else the child is rejected; (Child inferior to Parent)
endif;

where C_x, C_y are the objective function values of the child for objectives x and y, respectively, P_x, P_y are the objective function values of the parent for objectives x and y, respectively, ρ is a random number in the range (0, 1), and T_a, T_b are the temperatures for the two objectives.

Steps 6-12: The number of children allocated to each family for the next generation is proportional to a parameter called the acceptance number. If a region is found to contain a large number of good candidate solutions (as measured by the acceptance number), more search is allocated in that region. The acceptance number is calculated by counting the good solutions as under:

Pseudo-code 3

if $C_x \leq P_x$ and $C_y \leq P_y$
  then increment the count
else if $C_x < P_x$ and $C_y > P_y$
  if $\exp\{(LOWEST_Y - C_Y)/T_b\} \geq \rho$
    then increment the count;
  endif;
else if $C_x > P_y$ and $C_y < P_y$
  if $\exp\{(LOWEST_X - C_X)/T_a\} \geq \rho$
    then increment the count;
  endif;
endif

where $LOWEST_X$ and $LOWEST_Y$ are the lowest values of objectives X and Y obtained so far. The acceptance number of the family is equal to the count.

Step 13: A set of the non-inferior solutions is maintained and updated in every generation.

Step 14: A cooling schedule given in [7] has been employed in this method and is given below:

$$T_{a,i+1} = T_{a,i}/(1 + \beta_a T_{a,i})$$
$$T_{b,i+1} = T_{b,i}/(1 + \beta_b T_{b,i}) \; i = 1, 2, \ldots, MAXIT-1 \qquad (14)$$

where β_a and β_b are constants whose values are specified as

$$\beta_a = (T_{a,1} - T_{a,MAXIT}) \,/\, (T_{a,1}T_{a,MAXIT}(MAXIT - 1))$$
$$\beta_b = (T_{b,1} - T_{b,MAXIT}) \,/\, (T_{b,1}T_{b,MAXIT}(MAXIT - 1)) \qquad (15)$$

This cooling schedule is logarithmic in nature, i.e. the reduction in the temperature is large in each iteration at high temperatures but become progressively smaller at lower temperatures. This implies that more search is devoted to lower temperatures.

Step 15: The algorithm stops when the maximum number of iterations (*MAXIT*) is reached.

3 Results of computational experience

The proposed algorithm has been implemented in MATLAB on a Pentium 133MHz and has been tested on various test systems available in the literature considering non-monotonic and discontinuous cost curves. The versatility and enhanced modelling power of MOSST is demonstrated by presenting a variety of examples which include consideration of different aspects of the ELD problem. In a single run, the proposed heuristic gives:

(i) best solutions with respect to individual objectives;

(ii) the best compromise solution obtained by considering a specific choice of weights for the two objectives at hand (equal weightage in this case);

(iii) The complete pareto-optimal set of solutions providing the trade-off curve (in contrast to other methods which require multiple runs).

3.1 Examples

In this Section, some of the results of examples taken from the literature are reported. Transmission losses are neglected in the examples considered for the sake of comparison with existing methods, although the MOSST permits their consideration. The results have been obtained by setting population size to 50 and the total number of evaluations to 15,000. In the tables given below, best cost and best emission indicate the minimum cost and minimum emission taken individually. The best compromise indicates the minimum cost when both objectives are combined with equal weightage.

Table 1: Test results of cost minimisation subproblem for 6-generator, 30-bus system

Generator	a	b	d	P_{gmin}	P_{gmax}	LP	MOSST
1	100	200	10	0.05	1.5	0.15	0.1125
2	120	150	10	0.05	1.5	0.30	0.3020
3	40	180	20	0.05	1.5	0.55	0.5311
4	60	100	10	0.05	1.5	1.05	1.0208
5	40	180	20	0.05	1.5	0.46	0.5311
6	100	150	10	0.03	1.5	0.35	0.3625
Operating cost $/h						606.31	605.89

Table 2: Test results of emission minimisation subproblem for 6-generator, 30-bus system

Generator	α	β	γ	ξ	ε	LP	MOSST
1	4.091	–5.554	6.490	2.0E–4	2.857	0.40	0.4095
2	2.543	–6.047	5.638	5.0E–4	3.333	0.45	0.4626
3	4.258	–5.094	4.586	1.0E–6	8.000	0.55	0.5426
4	5.326	–3.550	3.380	2.0E–3	2.000	0.40	0.3884
5	4.258	–5.094	4.586	1.0E–6	8.000	0.55	0.5427
6	6.131	–5.555	5.151	1.0E–5	6.667	0.50	0.5142
Emission index						0.19424	0.19418

Table 3: Different solutions obtained in a single run of MOSST

Type of solution	MOSST		LP	
	Cost $/hr	Emission index	Cost $/hr	Emission index
Best cost	605.8890	0.2222	606.314	0.2233
Best emission	644.1118	0.19418	639.6	0.19423
Best compromise	621.7582	0.1968	—	—

Table 4: Assumed emission data for 13-generator, 57-bus system

Generator	α	β	γ	ξ	ε
z1	5.326	–3.55	3.38	2.0E–3	2.0
z2, z3	4.258	–5.094	4.586	1.0E–6	8.0
a1–a4, b1, b2	4.091	–5.554	6.490	2.0E–4	2.857
b3, c1–c3	2.543	–6.047	5.638	5.0E–4	3.333

Table 5: Generator loadings (MW) and operating costs for different algorithms

Generator	BGA	IGA	GAA	GAA2	SABED	MOSST
z1	638.60	628.32	627.05	628.32	668.40	628.13
z2	357.29	356.80	359.40	356.49	359.78	297.34
z3	357.15	359.45	358.95	359.43	358.20	299.09
a1	110.88	159.73	158.93	159.73	104.28	159.02
a2	152.51	109.86	159.73	109.86	60.36	159.59
a3	160.06	159.73	159.68	159.73	110.64	159.45
a4	161.45	159.73	159.53	159.63	162.12	159.68
b1	161.21	159.73	158.89	159.73	163.03	159.59
b2	116.09	159.73	110.15	159.73	161.52	159.61
b3	76.63	76.92	77.27	77.31	117.09	114.45
c1	75.00	75.00	75.00	75.00	75.00	75.00
c2	60.00	60.00	60.00	60.00	60.00	60.00
c3	93.13	55.00	55.41	55.00	119.58	89.07
Operating cost $/h	24703.32	24398.63	24418.99	24398.23	24970.91	24261
Emission index	0.2948	0.2950	0.2950	0.2950	0.2958	0.2897

Example 1: A 6-generator, 30-bus standard test system given in [2] is adopted in this example. The MOSST algorithm is implemented to minimise the operating costs as well as the emissions in a single run, unlike the LP method proposed in [2] which takes two separate runs for the two objectives. The results of the MOSST are compared with those obtained from the linear programming approach presented in [2] for both cases in Tables 1 and 2. Table 3 gives all three types of solution obtained from MOSST.

Example 2: A practical power system with 13 generators given in [10] is adopted here. The generator data as given in [10] is utilised. Emission curves are assumed and are given in Table 4. The best cost of the MOSST algorithm is compared in Table 5 with those reported in [8, 10] for different variations in genetic algorithms, viz. BGA, IGA, GAA, GAA2 and simulated annealing. Table 6 gives all three types of solution obtained from MOSST.

Table 6: Different solutions obtained in a single run of MOSST

Type of solution	Cost \$/hr	Emission index
Best cost	24261	0.2897
Best emission	25334	0.2865
Best compromise	24264	0.2896

3.2 Performance evaluation

The most important characteristic of any iterative optimisation algorithm is its ability to converge rapidly to the optimal solution. The convergence graphs of a single run of MOSST on Example 1 are presented in Figs. 1 and 2 for best cost, best emission and best compromise solutions, respectively. These graphs clearly indicate that MOSST converges very rapidly to the optimal solution, while evaluating a small fraction of the search space.

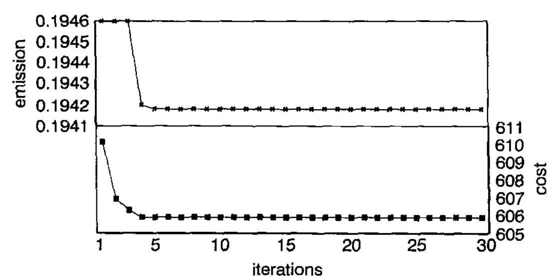

Fig. 1 *Convergence of best emission and best cost solutions*

The final solutions remaining in the pareto-optimal set *PS* are plotted in Fig. 3. In order to validate *PS*, several runs of MOSST are performed with varying weightages of the two objectives. The best compromise solutions obtained in each case are superimposed on Fig. 3 and plotted in Fig. 4, which shows that the two are almost identical. This amply validates that a single run of MOSST is sufficient to obtain the pareto-optimal set and repeated runs with modified weightages are not required.

3.3 Perturbation analysis

A further perturbation analysis is carried out to prove the authenticity of the pareto-optimal set (the points shown in Fig. 3) obtained by the proposed technique.

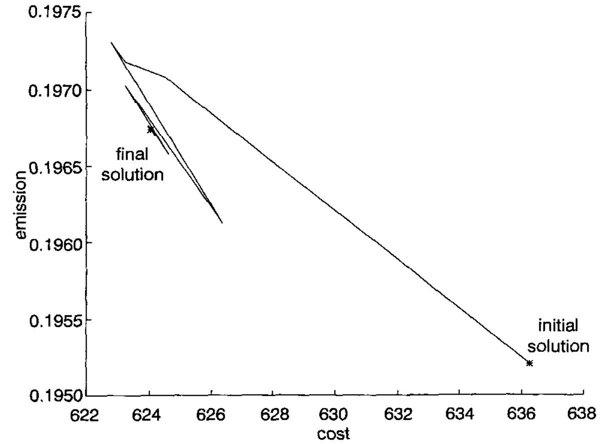

Fig. 2 *Convergence of best compromise solution*

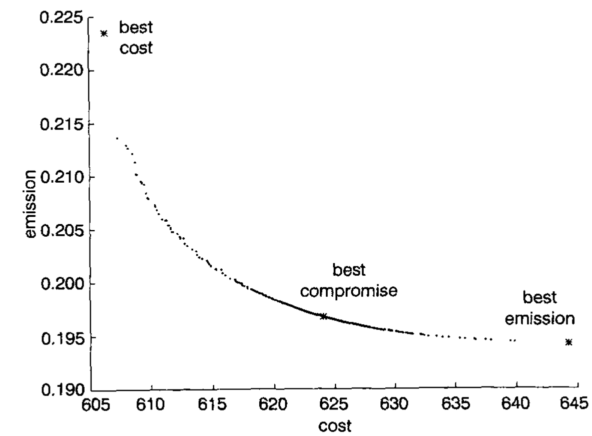

Fig. 3 *Pareto-optimal set obtained by a single run of MOSST*

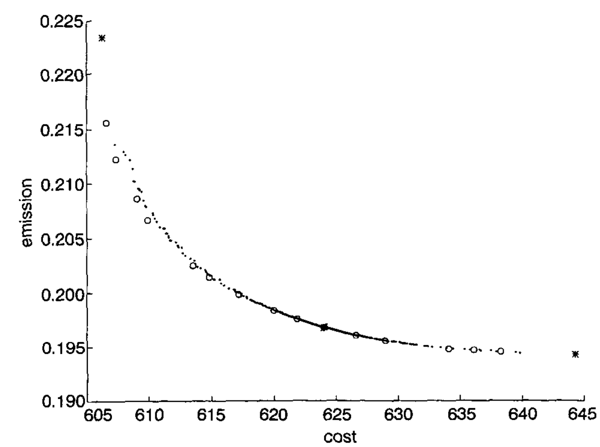

Fig. 4 *Superimposition of compromise solutions, shown by circles, obtained by varying the weights of the two objectives in multiple runs, on Fig. 3*

Each solution in the pareto-optimal set is subjected to a systematic perturbation to obtain a reduction in the cost. This is done by reducing the generation of a generator in small steps, followed by an equal increase in the generator with minimum incremental cost, as long as the total cost of generation reduces. The process is similar to the cost mutation operator discussed earlier. However, all the generators, except the one with maximum incremental cost, are considered in the increasing order of incremental costs in this analysis. Each step change is 10% of the maximum change possible until one of the two generators reaches its limit. Thus, the analysis takes a maximum of 10 steps for each pair of generators. The emission index is also computed for the modified generations thus obtained. For all the elements of the pareto-optimal set, a reduction in the cost is always accompanied by an increase in the emission index and vice versa. This proves that the final set of solutions obtained are truly pareto-optimal.

4 Conclusions

A new heuristic MOSST has been developed, with the incorporation of genetic operators into the generation process and simulated annealing in the selection process, for multi objective optimisation. The versatility and enhanced modelling power of MOSST has been demonstrated by implementing it for the economic-emission-dispatch problem. A suitable implementation restricts the search process to the feasible solution search space and ensures that the heuristic is very effective in determining near-optimal solutions, after examining a small fraction of the total solution space. Thus, MOSST is extremely fast and is time applicable even to large practical power systems. The heuristic is general and can be suitably implemented for a variety of problems. Unlike the other methods available in the literature, this method gives optimal values of different objectives, the best compromise between them, as well as the pareto-optimal set depicting the trade-offs involved in a single run. In the examples presented it outperforms the other methods designed expressly for the particular problem. This, coupled with the speed of the algorithm, makes the approach highly suitable for the multi-objective problems.

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