Multicriteria Optimization of Air-Cored Solenoids with Multiple Windings

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Abstract The multiobjective optimization of an eight-coil actively-shielded solenoid is performed, suggesting the problem as a benchmark and focusing the attention on the optimization strategy. A goal-attainment multicriteria optimization based on a first-order evolution strategy algorithm has been carried out; a set of Pareto-optimal solutions has been calculated, showing that the final solutions are spread in a region both in the design and in the variable space.

1. Introduction

In previous works [1-4], the optimization problem of Loney's solenoid has been widely studied [7], suggesting it as an inverse magnetostatic benchmark problem and focusing on different possible multicriteria optimization methodologies. Many other multiple-coil configurations [6] are possible and are widely used in designing highly homogeneous NMR solenoids. Moreover, when it is important to have a very low stray field outside the magnet, the most efficient solution is to use actively-shielded magnets in which the external coils carry opposite currents.

2. Geometry of the Device and Optimization Strategy

The device we deal with is inspired by some NMR solenoids and consists of eight cylindrical coils, four of which are bigger and carry a positive current, while the other four carry an opposite current and are smaller. In Fig. 1, the design variables are shown.

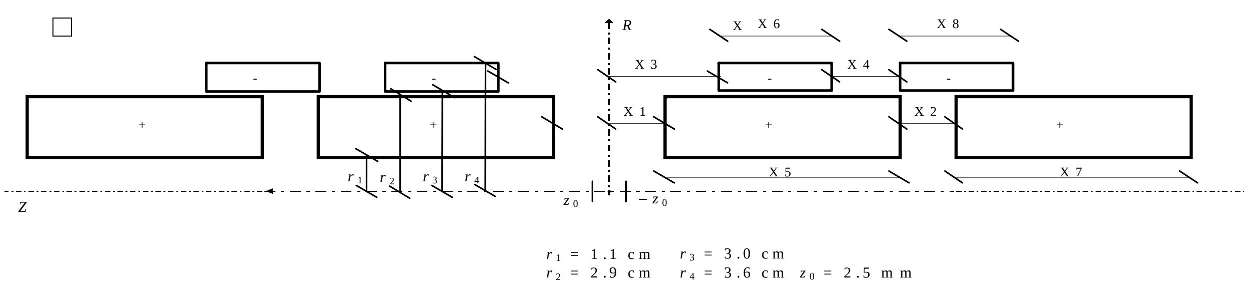

Fig.1 - Geometry of the eight-coil device.

For the analysis, a discretization of the coils in terms of current sheets has been performed, in order to compute magnetic field via Biot-Savart's law and elliptic integrals. On the other hand, the following two objective functions have been considered for the synthesis:

$$F_1 = \frac{(B_{max} - B_{min})}{B_0} \qquad\qquad F_2 = B_{stray} \qquad (1)$$

where B_{max} and B_{min} are the maximum and the minimum values of the magnetic induction in the interval ($-z_0$, z_0) on the axis and B_0 =1 [T] is the prescribed value at the center of the solenoid. The value of current density is modified at each iteration to ensure the prescribed value of B_0. B_{stray} is the maximum value of the field evaluated in two different sampling points, located at (r , z) = (24 , 0) and (r , z) = (0 , 10.8) respectively (values in cm).

A goal-attainment multicriteria strategy [5], [8] has been implemented: at each iteration, the normalized discrepancy between the objective function value and the associated goal stated by the designer is evaluated for each objective function; then, the bigger of the two discrepancies is minimized. In an equivalent way the goal-attainment multicriteria strategy can be stated as

$$\min_{X} \max_{i=1,2} \frac{\left| F_i(X) - G_i \right|}{G_i} \, . \qquad (2)$$

where X is the vector of design variables, shown in in Fig. 1, G_1=0.1 ppm is the desired goal for relative field inhomogeneity and G_2=1.E-3 [T] is the desired goal for absolute stray field. A minimizer based on an evolution strategy of lowest order has been used as optimization engine [3]. After performing several optimization runs with different starting points randomly generated, in order to select the best optimal solutions, a sorting algorithm based on the definition of Pareto-optimality has been implemented.

3. Study of 2D Objective Functions for Simplified Configurations

In order to study 2D objective functions, simplified configurations have been considered, for which 6 of 8 variables are fixed to a prescribed value. The first case we deal with is defined

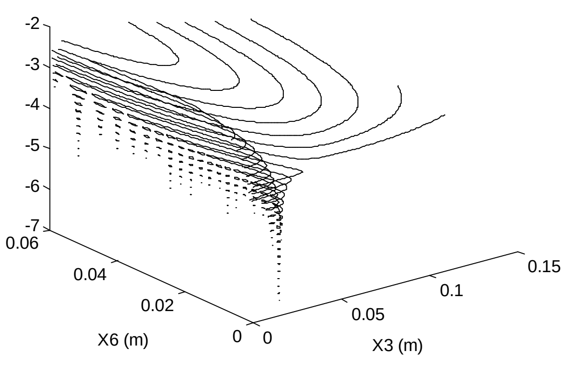

Fig.2 - Case a): log of F_1

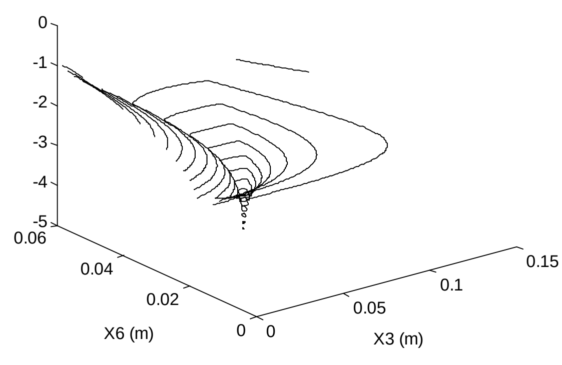

Fig.3 - Case a): log of F_2

In this first case the inhomogeneity objective function F_1 has a non-analytic and ill-conditioned behaviour shown in Fig.2. A deep “valley” can be noted along which several much deeper minima are disseminated. On the other hand, the stray-field objective function has a quite regular behaviour and exhibits a deep unique minimum, as can be seen in Fig.3.

As shown in Fig.4, a similar behaviour of the inhomogeneity can be obtained considering the following case b) $X_2= 0$ $X_4= 0$ $X_7=0$ $X_8=0$ $X_5= 6$ cm $X_6=3$ cm.
The behaviour of the two functions shown below is typical of the family of problems we are dealing with and characterizes even the full eight-variable problem.

Before tackling the final and most complex eight-variable problem, a more simple four-variable optimization has been performed on a device defined by the following
case c) $X_2= 0$, $X_4= 0$, $X_7=0$, $X_8=0$.

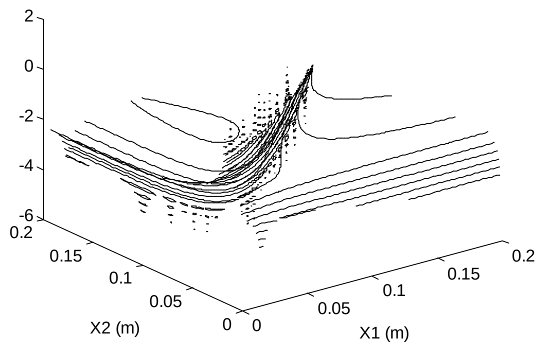

Fig. 4 - Case b) log of F_1.

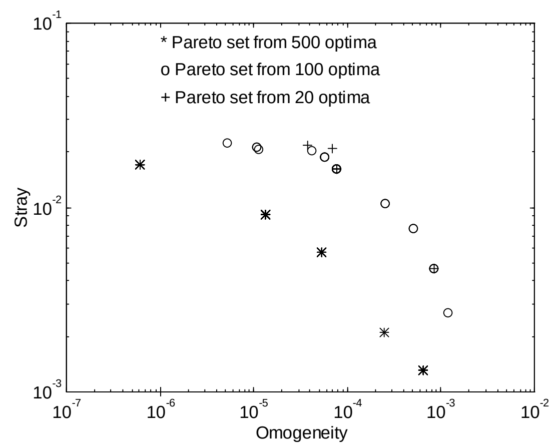

Fig. 5 - Case c): Pareto optimal sets.

The study of the four-coil four-variable configuration is interesting in order to see how the Pareto-optimal set changes when the number of optimization runs is increased. From Fig.5 it can be seen that the Pareto set moves towards the goal point when the total number of solutions increases.

4. Final Global Optimization

In order to tackle the problem of the sensitivity between initial configuration and final results we have repeated the optimization runs, generating a grid of starting points in a random way. After 10 hours of cpu-time on a 32 bit Digital Workstation, 40 optimizations have been performed. Applying the selection criterion mentioned in section 2 we obtain the Pareto set plotted in Fig. 6, where the corresponding initial values are also shown. Remarkable improvements in both the objective functions are evident.

In Fig.7 all the 7 final Pareto optimal configurations can be seen; it is evident that a big shape variability is possible. As a consequence, the uniqueness of the solution cannot be claimed.

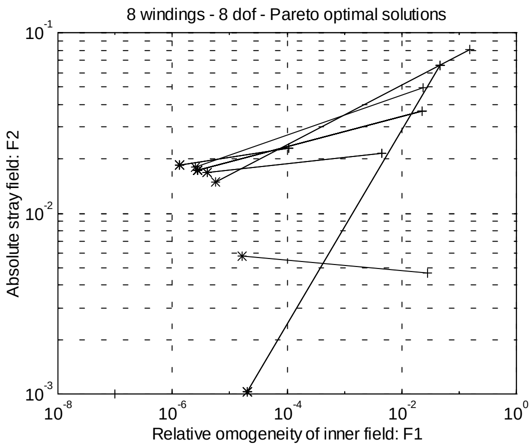

Fig. 6 - Pareto optimal set in the objective function space.

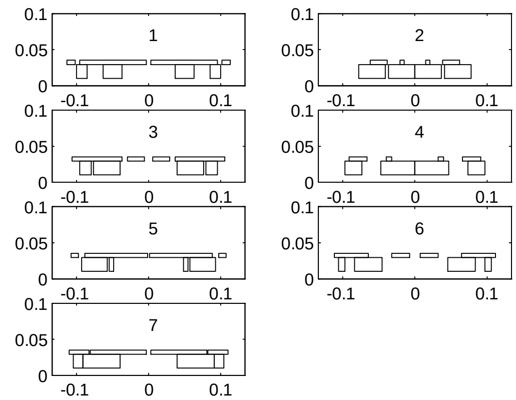

Fig. 7 – Pareto-optimal shapes.

5. Conclusion

1) The optimization of multiple-coil shielded solenoids seems to be a very suitable problem for testing the multicriteria non-deterministic optimization strategies, since it is necessary to cope with objectives in conflict, ill conditioning of the objective functions, and a high number of design variables and constraints.
2) Optimization has been performed for various configurations up to a eight-coil eight-variable solenoid geometry which presents a quite high complexity; several solutions have been obtained, characterized by 1-10 ppm of field inhomogeneity and 1.E-2 [T] of stray field.
3) Numerical experiments show that, though using a non-deterministic minimizer, a high sensitivity exists between a starting and a final point of an optimization trajectory. Consequently, it is of fundamental importance to perform many optimization runs with different starting points and to extract a Pareto optimal set in order to have few, yet not unique, final optima.

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