OR Spektrum (2000) 22: 425–460

OR Spektrum
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A survey and annotated bibliography of multiobjective combinatorial optimization

Multikriterielle Kombinatorische Optimierung: Übersicht und Kommentierte Bibliographie

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Received: February 7, 2000 / Accepted: April 14, 2000

Abstract. This paper provides a survey of the research in and an annotated bibliography of multiple objective combinatorial optimization, MOCO. We present a general formulation of MOCO problems, describe the main characteristics of MOCO problems, and review the main properties and theoretical results for these problems. The main parts of the paper are a section on the review of the available solution methodology, both exact and heuristic, and a section on the annotation of the existing literature in the field organized problem by problem. We conclude the paper by stating open questions and areas of future research.

Zusammenfassung. Der Artikel bietet einen Überblick und eine kommentierte Bibliographie über die Forschung in multikriterieller kombinatorischer Optimierung (MOCO, multiple objective combinatorial optimization). Wir stellen eine allgemeine Formulierung von MOCO Problemen vor, beschreiben die wichtigsten Charakteristika und Eigenschaften solcher Probleme und fassen die wesentlichen theoretischen Ergebnisse in diesem Forschungsgebiet zusammen. Die Hauptteile des Artikels sind die Abschnitte 4 über exakte und heuristsiche Lösungsverfahren und 6, der – problemweise untergliedert – die vorhandene Literatur kommentiert. Am Ende des Artikels steht ein Abschnitt zu offenen Fragen und Richtungen für zukünftige Forschung.

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Key words: Multicriteria optimization – Combinatorial optimization – Bibliography – Exact methods – Heuristics

Schlüsselwörter: Multikriterielle Optimierung – Kombinatorische Optimierung – Bibliographie – Exakte Verfahren – Heuristiken

1 Introduction

Combinatorial optimization is a field extensively studied by many researchers. Due to its potential for application in real world problems it has prospered over the last few decades. A good survey of the state of the art is provided by [42]. But as far as real world decision making is concerned, it is also well known, that decision makers have to deal with several – usually conflicting – objectives. The growth in the interest of theory and methodology of multicriteria decision making (MCDM) over the last thirty years is witness of this fact, see [195] for a survey of the activities in the field, and [228] for a list of MCDM applications.

Thus it is somewhat surprising that a combination of both, i.e. multicriteria or multiobjective combinatorial optimization (MOCO) has not been studied widely. A few papers in the area have been published in the seventies, then the classical problems have been investigated in the eighties. Only in recent years – approximately since 1990 – a profound interest in the topic is evident. Since then several PhD theses have been written, specific methodologies have been developed, and the number of research papers in the field has grown considerably.

In this paper we intend to give an overview over the literature in the field of multiobjective combinatorial optimization. In the following sections, we first present a brief introduction to the field, including a general problem formulation, description of several types of MOCO problems, and the most important theoretical properties of these problems (Sects. 2 and 3). Then we review existing methods to solve MOCO problems in Section 4. In Section 5 we explain the classification of literature that we used. This consists first of a classification of the problem treated and secondly of the methodology applied to solve it. Section 6 paper is devoted to the annotation of the literature. The paper is concluded by a brief discussion of open questions and areas of future research (Sect. 7).

Let us now describe the focus of this paper. We compiled the literature on multiobjective combinatorial optimization accessible to us, especially concentrating on new developments since 1994, when the last survey [215] was published, including some older references missing there or fundamental in the field. Besides the general survey on MOCO, [215], there are two specifically devoted to multiobjective network design, [32,33], which have additional references. In terms of new research since 1994, our bibliography for instance includes the new direction of using metaheuristics for MOCO problems, literature on the so-called two phases method (see Sect. 4), and new theoretical results.

The aim of this survey is twofold. First we want to provide a starting point for researchers and students interested in the field, giving a brief introduction and commenting on, thus guiding through, existing literature. For the experienced researcher the list is intended as structured overview of the field.

2 Multiple objective combinatorial optimization problems

The feasible set of a (multiobjective) combinatorial problem is defined as a subset $X \subseteq 2^A$ of the power set of a finite set $A = \{a_1, \ldots, a_n\}$. Typically, in combinatorial optimization two types of objective functions are considered, namely the sum and the bottleneck objective:

$$z(S) = \sum_{a \in S} w(a), \text{ or}$$

$$z(S) = \max_{a \in S} w(a),$$

where $S \in X$ and $w : A \to \mathbb{Z}$ is some weight function.

In a multicriteria combinatorial problem several weight functions $w_j : A \to \mathbb{Z}$ are given, yielding several objective functions $z^j, \ j = 1, \ldots, Q$ (usually of the sum or bottleneck type). The problem is then to solve

$$\text{“}\min_{S \in X}\text{”}(z^1(S), \ldots, z^Q(S)) \qquad \text{(MOCO)}$$

where the meaning of “min” has still to be defined.

Most often the minimization in (MOCO) is understood in the sense of efficiency (or Pareto optimality). A subset $S \in X$ is called efficient if there does not exist another feasible solution $S' \in X$ such that $z^j(S') \leq z^j(S)$ for all $j = 1, \ldots, Q$ with strict inequality for at least one of the objectives. The corresponding vector $z(S) = (z^1(S), \ldots, z^Q(S))$ is called nondominated. The set of Pareto optimal (efficient) solutions of (MOCO) will be denoted by E, the set of nondominated vectors by ND throughout the paper. Sometimes we shall use the term nondominated frontier for the set of all nondominated vectors, especially in the bicriteria context.

However, besides efficiency, there are other definitions of the “min” term in the formulation of (MOCO). For example, one could consider lexicographic minimization, when objective vectors are compared lexicographically: $z(S_1) <_{lex} z(S_2)$ if $z^j(S_1) < z^j(S_2)$, where j is the smallest index such that $z^j(S_1) \neq z^j(S_2)$. This could be done with respect to one, or all permutations of the objective functions z^j.

Another possibility is to minimize the worst objective function, i.e.

$$\min_{S \in X} \max_{j=1,\ldots,Q} z^j(S).$$

We call this the max-ordering problem (following [56]) in order to distinguish it from the single objective bottleneck problem (note that both are often called min-max problems, which may create confusion).

A combination of the latter two is the lexicographic max-ordering problem, where the vector of objective values $z(S)$ is first resorted in a nonincreasing order of its components, and the resulting vectors are compared lexicographically, see [46,48] for details.

In a real world decision context, finally a compromise has to be made among the many efficient solutions that (MOCO) may have. This is the reason why often the existence of a utility function is implicitly or explicitly assumed. A utility function assigns each criterion vector $z(S)$ a scalar overall utility. Then methods are developed to find a solution of maximum utility. This is a typical approach in interactive methods described later.

Closely related to combinatorial problems are multiobjective integer programming problems. These can be formulated as follows.

$$
\begin{array}{rrcl}
 & \text{“}\min\text{”}\, Cx & & \\
\text{subject to} & Ax & = & b \\
 & x_i & \geq 0 & i = 1, \dots, n \\
 & x_i & \text{integer} & i = 1, \dots, n
\end{array}
\tag{MOIP}
$$

Here C is a $Q \times n$ objective matrix, A is an $m \times n$ constraint matrix, and $x \in \mathbb{R}^n$. There is a considerable amount of literature on these problems. We refer to some surveys that exist but will not include the literature in the bibliography. In this respect, [23, 199, 234] provide surveys of techniques to find efficient solutions for (MOIP), [198] gives an overview of interactive methods for (MOIP), and [166] surveys (MOIP) with binary variables.

In general, combinatorial optimization problems can be considered as special cases of integer (in particular binary) programming. A MOCO problem is distinguished by a specific set of constraints, that provides a structure to the problem. We focussed on these problems for this bibliography.

We have to comment on scheduling here. Scheduling problems can be considered as combinatorial optimization problems. However, they are problems with their own specific theory and methodology, which is quite different from other problems of combinatorial optimization, so that we decided not to include scheduling problems in this survey. The interested reader is referred to recent surveys on multiobjective scheduling problems, [203, 204].

To conclude this section, let us mention one particular case, namely, when the set of feasible solutions is an explicitly given finite set, e.g. $X = A$. In this case, all problems discussed above are efficiently solvable. Algorithms can be found in [49, 50] and [123]. For this reason, these problems are mathematically not particularly interesting and we omit them from further discussion.

To summarize, (MOCO) is a discrete optimization problem, with n variables x_i, $i = 1, \dots, n$, Q objectives z^j, $j = 1, \dots, Q$ and a specific constraint structure defining the feasible set X. This definition includes multiobjective versions of the shortest path, minimum spanning tree, assignment, knapsack, travelling salesperson, or set covering problems, to mention only a few.

3 Properties of multiobjective combinatorial optimization problems

In this section we discuss some of the properties of MOCO problems. It is in order to mention here that there is a considerable number of erroneous statements, even in papers published in international standard refereed journals. We will point out the most important of these throughout the paper, in the appropriate places.

By its nature, multiobjective combinatorial optimization deals with discrete, non continuous problems, although the objectives are usually linear functions. An essential consequence of this fact when trying to determine the set of all efficient solutions (or nondominated vectors in objective space) is, that it is not sufficient to aggregate the objectives through weighted sums.

It is long known that for multiobjective linear programming problems

$$\min\{Cx : Ax = b, x \geq 0\}$$

the set of efficient solutions is exactly the set of solutions that can be obtained by solving LP's

$$\min\left\{\sum_{j=1,\dots,Q} \lambda_j c^j x : Ax = b, x \geq 0\right\},$$

where $\sum_{j=1}^{Q} \lambda_j = 1, \lambda_j > 0, j = 1, \dots, n,$ see e.g. [107]. But the discrete structure of the MOCO problem makes this result invalid. Thus there usually exist efficient solutions, which are not optimal for any weighted sum of the objectives. This is true even in cases where the constraint matrix is totally unimodular, contrary to a proposition in [121] (see [216] for an example). These solutions are called nonsupported efficient solutions NE, whereas the remaining are called supported efficient solutions, SE. In early papers referring to MOCO, NE was usually not considered. Most authors focussed on scalarizing the objectives by means of weighting factors λ_j.

Nevertheless, the set NE is important. Usually there are many more nonsupported than supported efficient solutions, see e.g. [222] for numerical results. Moreover, the nonsupported solutions contribute essentially to the difficulty of MOCO problems. Below, we shall briefly discuss the concepts of computational complexity of (MOCO). For introductions to the theory of $\mathbb{NP}$-completeness and $\#\mathbb{P}$-completeness we refer to [74] and [221] and references therein, respectively. These notions deal with the difficulty of finding a, respectively counting the number of solutions of a (MOCO). The corresponding appropriate definitions of decision and counting problems for MOCO problems can be found in [52] and [186].

It turns out that the respective versions of (MOCO) in the sense of finding or counting efficient solutions are in general $\mathbb{NP}$- and $\#\mathbb{P}$-complete, respectively. This is true even for problems which have efficient algorithms in the single objective case. We refer to [52,58] and [186] for results in this respect. Therefore the development of heuristics with guaranteed worst case performance (bounded error) is interesting. However, not much is known in this regard: [52] gives some general results on approximating the efficient set by a single solution, [157] uses a Tchebycheff metric to measure the error, and [174,175] consider the existence of such algorithms. Some specific results about flow problems, shortest path problems and the TSP are discussed in Section 6.

Another aspect related to the difficulty of MOCO is the number of efficient solutions. It turns out that it may be exponential in the problem size, thus prohibiting any efficient method to determine all efficient solutions. Such results are known for the spanning tree, matroid base, shortest path, assignment, and travelling salesperson problem (see [49, 59, 85, 93, 188] for details). Consequently such problems are called intractable. Even the size of the set SE may be exponential, see [172]. However, numerical results available on the knapsack problem [222] show the number of supported solutions grows linearly with the problem size, but the number of nonsupported solutions grows following an exponential function.

As far as the other definitions of optimality in (MOCO) are concerned, we note that the max-ordering problem with sum objectives is $\mathbb{NP}$-hard in general (see [22]), but can be reduced to a single objective problem in the case of bottleneck objectives [49]. Bounds and heuristic methods for the former problem have been investigated in [160]. At least one solution of the max-ordering problem is always efficient, but possibly nonsupported. Similarly, a lexicographic max-ordering solution, although always efficient and optimal for the max-ordering problem may be nonsupported [49].

For lexicographic optimization it is known that a lexicographically optimal solution is always efficient, and even a supported efficient solution, see [85]. Lexicographic optimization can also be viewed as a special case of algebraic optimization, see [233].

In view of the new trend to apply metaheuristics and local search in MOCO problems (see Sect. 4 below), it is interesting to consider the issue of neighbourhoods of feasible solutions, and their relations to efficient solutions. Using a neighbourhood corresponding to Simplex basis pivots for the shortest path problem and exchanges of one edge for the spanning tree problem it was shown in [54, 55] that the set of efficient solutions can be an unconnected subset of X with respect to the neighbourhood. So depending on the definition of neighbourhood it is possible that local search methods are not able to find all efficient solutions.

4 Solution methods for MOCO problems

In the context of multiobjective programming (MOP), it is usual to distinguish the methods following the role of the decision maker in the resolution process. Information provided by the decision maker often concerns his preferences. In "a priori mode", all the preferences are known at the beginning of the decision making process. The techniques used seek for a solution on the basis of these parameters. The best example is given by goal-programming methods. In "a posteriori mode" the set of all efficient solutions is generated for the considered problem. At the end, this set is analyzed according to the decision maker's preferences. Many approximation (heuristic) methods are conceived following this resolution mode. In the "interactive mode", the preferences are introduced by the decision maker during the resolution process. The methods involve a series of computing steps alternated with dialogue steps and can be viewed as the interactive determination of a satisfying compromise for the decision maker. Thus they require a high participation level on the part of the decision maker. Practical problems are often solved according to the interactive mode.

The appropriate resolution mode is chosen considering the situation of the decision process. The method involved in the process could be exact or approximation methods.

4.1 Exact methods

Here we discuss some of the methods used to solve MOCO problems. Many of these essentially combine the multiple objectives into one single objective. The most popular, and the one used first, is weighted sum scalarization. The problem solved is

$$\min\left\{\sum_{j=1}^{Q}\lambda_j z^j(x) : x \in X\right\}, \tag{P_λ}$$

where $0 \leq \lambda_j \leq 1$ and $\sum_{j=1}^{Q}\lambda_j = 1$. Varying the weights, it is known that all supported efficient solutions can be found, using results from [77] and linear programming [107]. The advantage of the method (especially for problems where the single objective version is solvable in polynomial time) is that for each $\lambda \in \mathbb{R}^Q$ the problem (with sum objectives) is only as difficult as the single objective counterpart of (MOCO). Parametric programming can be used to solve the problem for all λ.

The approach has been applied to many MOCO problems: see e.g. [98, 227] for shortest path, [5, 43, 108, 194] for the transportation problem [39] for assignment, [127, 137] for network flow, [85, 183] for spanning tree, [45, 171] for knapsack and [135] for location problems. In many of these papers, the existence of nonsupported efficient solutions was either not known, or ignored. When a sum and a bottleneck objective are present, the minimization of the sum of the objectives has been discussed in [143] and [159] for general combinatorial optimization problems.

A second well known approach in multicriteria optimization is the compromise solution method [230], where one tries to minimize the distance to an ideal point z^I or to a utopian point $z^U = z^I - \epsilon e$, where $e = (1, \ldots, 1) \in \mathbb{R}^Q$ is the vector of all ones, and $\epsilon > 0$. The ideal point is defined according to the individual minima of each objective

$$z_j^I := \min_{x \in X} z^j(x).$$

Usually, the Tchebycheff norm is used as distance measure:

$$\min\left\{\max_{j=1}^{Q}\{\lambda_j |z^j(x) - z_j^I|\} : x \in X\right\}.$$

Unfortunately, when we consider sum objectives, this type of problem is usually $\mathbb{NP}$-complete, see e.g. [147] for references on the shortest path problem. This explains why it is rarely used, even though, theoretically the whole of the efficient set can be found, see e.g. [178]. Using another norm, e.g. an l_p norm, $p \notin \{1, \infty\}$ leads to nonlinear objectives, and we found no reference using this approach for MOCO.

Note that for $p = 1$, the compromise solution method with l_1 norm coincides with the weighted sums approach.

A special approach to multiobjective optimization is goal programming, see e.g. [106,129] for details. Here, for each of the objectives a target value (goal) is specified by the decision maker. The overall aim is to minimize the deviation from the specified goals. This approach is very popular and although it is sometimes considered a different field from multiobjective optimization we list some references in the bibliography.

One approach that is successful for bicriteria problems is the use of ranking methods. Define

$$z_j^N := \min_{x \in X} \left\{ z^j(x) : z^i(x) = z_i^I \right\}, \quad j = 1, 2; \;\; i \neq j. \tag{1}$$

The ideal point $z^I = (z_1^I, z_2^I)$ and Nadir point $z^N = (z_1^N, z_2^N)$ define lower and upper bounds on the objective values of efficient solutions. Then starting from a solution with $z^1(x) = z_1^I$, and finding second best, third best, ... , K-best solutions with respect to the first objective until z_1^N is reached, the efficient set can be determined. The approach has been used for the shortest path problem [24] and the transportation problem [43]. Note that computation of the Nadir point z^N in the bicriteria case essentially means the solution of two lexicographic optimization problems.

A generalization of this approach to more than three objectives (stated without proof in [141]) is not possible without knowledge of the Nadir point, which is difficult to obtain when $Q > 2$, see [119]. Note that a generalization of (1) does not necessarily provide an upper bound on objective values of efficient solutions. Not even considering lexicographic optimization with respect to all permutations of objectives is guaranteed to produce upper bounds on objective values of efficient solutions, see [57].

Moreover, the ranking approach can be effectively used to solve max-ordering problems with any number of criteria. First a weighting vector is chosen, then K-best solutions x^K are created according to the combined objective $\sum \lambda_j z^j$. When for the first time

$$\min_{k=1,\ldots,K-1} \max_{j=1,\ldots,Q} z^j(x^k) \leq \sum_{j=1}^{Q} \lambda_j z^j(x^K)$$

an optimal solution is among $\{x^1, \ldots, x^K\}$. We refer to [47] and [83] and [85] for applications to the uniform matroid, network flow problem, and spanning tree problem, respectively.

Let us now look at methods adapted from single objective combinatorial optimization. Among the very well established procedures is dynamic programming [11]. The method applies to sequential decision problems, which admit a recursion formula such as

$$\min \left(g_N(x_n) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \right),$$

where g is a cost function depending on the state variable x_k and control variable u_k at stage k. Theoretically, this recursion can easily be adapted to the multiobjective case. Therefore dynamic programming algorithms appear most often for problems, where they have been established for the single objective versions earlier. These are the shortest path problem [19,97,98,120,165], the knapsack problem [21,45, 115,114,116], the TSP [63,205] and the transportation problem [68,169].

An implicit enumeration algorithm, which is widely used to solve hard combinatorial optimization problems is branch and bound. Its philosophy is to partition the problem into mutually disjoint and jointly exhaustive subproblems. Bounds are computed for subproblems and the process continues until an optimal solution is found. Much to our surprise, we could only find a few papers applying branch and bound for MOCO – to the knapsack problem [214,217,222] and the max-ordering shortest path problem [165]. The adaptation of branch and bound poses one difficult problem. Since we deal with nondominated vectors, bounds play the role of Nadir points for subproblems. Thus they may be difficult to compute, or bad, i.e. not discarding enough feasible, nonefficient solutions.

Many authors used available single objective methods for a particular problem and adapted them to the multiobjective case. The more natural such a generalization is, the bigger the number of papers pursuing such an approach. We note the following, representing (recent) examples.

- Shortest Path: [93,139] for label setting and [14,31,145,192,206,207] for label correcting methods
- Spanning Tree: [29,85] for adaptations of Prim's algorithm and [183,186] for the greedy algorithm
- Assignment: [162,213,216] for the Hungarian method
- Network Flow: [51,126–128] for the out-of-kilter algorithm and [17,158] for the network simplex method
- TSP: [52] for Christofides' algorithm

Finally, we explain a general framework for the exact solution of the problem of determining the efficient set for bicriteria (MOCO), the two phases method. The name goes back to [211] and [216] and is telling: In the first phase SE is found using the scalarization technique, and solving single objective problems. The necessary weights are easy to compute using information generated in the process. The second phase consists of finding the nonsupported efficient solutions by problem specific methods, using bounds, reduced costs, etc. In fact, most of the algorithms known to the authors (with exception of the shortest path problem) that are capable of determining the whole of E are some modification of the two phases method, e.g. [51,128] (Network Flow), [214,222], (Knapsack), [216] (Assignment) and [164](Spanning Tree).

4.2 Approximation methods

The last two decades have been highlighted by the development and the improvement of approximative solution methods, usually called "heuristics and metaheuristics". A heuristic is defined by [167] as a technique which seeks good (i.e. near-optimal) solutions at a reasonable computational cost without being able to guarantee either feasibility or optimality. Often heuristics are problem-specific, so that a method which works for one problem cannot be used to solve a different one.

In contrast, metaheuristics are powerful techniques generally applicable to a large number of problems. A metaheuristic refers to an iterative master strategy that guides and modifies the operations of subordinate heuristics by combining intelligently different concepts for exploring and exploiting the search space [78, 155]. A metaheuristic may manipulate a single solution or a collection of solutions at each iteration. Metaheuristics include, but are not limited to, constraint logic programming, genetic algorithms, evolutionary methods, neural networks, simulated annealing, tabu search, non-monotonic search strategies, greedy randomized adaptive search, ant colony systems, variable neighbourhood search, scatter search, and their hybrids. A comprehensive list of 1380 references on the theory and application of metaheuristics is presented in [155]. The success of these methods is due to the capacity of such techniques "to solve in practice" some hard combinatorial problems.

As in the single objective case, a reasonable alternative to exact methods for solving large-scale instances of MOCO problems is to derive an approximation method. Such methods yield a good tradeoff between the quality of an approximation of the efficient solutions set, denoted by $\widehat{E}$, and the time and memory requirements. The adaptation of metaheuristic techniques for the resolution of MOP problems, denoted by multiobjective metaheuristics, MOMH, has mushroomed. Generally, the first adaptations use the components known in the single-objective methods to deal with the efficient solution concept, too. Chronologically, adaptations have concerned genetic algorithms (GA, 1985), neural networks (NN, 1990), simulated annealing (SA, 1992), tabu search (TS, 1994), and more recently, the greedy randomized adaptive search procedure (GRASP, 1998).

Two main approaches appear in these methods. The first is based on the principle of search directions. The second approach takes advantage of information carried by the population of solutions, using the notion of domination.

- *Methods of local search in objective space.* Starting from an initial solution, the procedure approximates a part of the nondominated frontier corresponding to a given search direction λ. A local aggregation mechanism of the objectives, often based on a weighted sum, produces the effect to focus the search on a part of the nondominated frontier. The principle is repeated for several search directions to approximate completely the nondominated frontier. Following the methods, the directions can be defined a priori [69,211], guided [70,89] or aleatory [35,146]. At any time the search mechanism uses only one solution and an iteration tries to attract the solution generated towards E along direction λ. The efficiency of theses adaptations is strongly depending on the definition of λ.

– *Population based methods.* Contrary to the first approach, where only one individual is attracted toward the nondominated frontier, here all the population contributes to the evolution process toward the nondominated frontier. By maintaining a population of solutions, such a method can search for many efficient solutions in parallel via self adaptation and cooperation. This characteristic makes population-based methods very attractive for solving multiobjective problems. Most operational procedures are based on genetic algorithms, ([25] counts more than 320 papers). However, only a few concern MOCO problems. Evolutionary algorithms (EA) would also fall in this category.

We now suggest some guidelines according to which existing methods can be differentiated.

– A first distinction concerns the case of a general method versus a dedicated method. With some minor adaptation (definition of a solution, neighbourhood structure, etc.) in their implementation, general methods can be applied to a wide variety of problems (for example [35,70,89,146,211]). Specific methods are designed for particular MOCO problems as e.g. [118] or result from a strong customization of a general method as e.g. [69].
– A second distinction is the interaction mode. We distinguish the a priori mode, the interactive mode [3,94,181,210], and the a posteriori mode [35,70,89,146, 211].
– The kind of method is a third distinctive feature. We can separate the local search based procedures (SA, TS, GRASP), population based procedures (GA, EA), specific procedures (e.g. stochastic methods) and hybridization.
– The last distinction refers to technical components integrated in the procedure, such as, e.g., the identification of the kind of initial solutions used by the method.

4.2.1 Simulated annealing The use of simulated annealing as a technique for MOP problems was discussed first in [187]. When solution x^1 is compared with solution x^2 according Q objectives $z^j(x)$, $j = 1 \dots Q$, and where Δz^j is the difference between solution x^1 and x^2 in the objective j, three situations are possibles:

Case 1: $\forall j \quad \Delta z^j \leq 0$
Case 2: $\exists j, j' \quad \Delta z^j < 0$ and $\Delta z^{j'} > 0$
Case 3: $\forall j \quad \Delta z^j \geq 0$

The main idea of using SA for solving MOP problems consists in using a weighted norm component in the acceptance of a solution of lower quality (cases 2 and 3).

In [218,211], an independent SA process is defined using a direction λ. A scalarizing function $s(x, \lambda) = \sum_{j=1 \dots Q} \lambda_j z^j(x)$ is used to compute the difference $\Delta s = s(x^2, \lambda) - s(x^1, \lambda)$ between two solutions. Then let us consider a current solution x^t and $y \in \mathcal{N}(x^t)$, a solution randomly selected in the neighbourhood $\mathcal{N}(x^t)$ of x^t. In computing Δs for y and x^t, a strategy consists in the following decisions:

a) If $\Delta s < 0$ then $x^{t+1} \leftarrow y$.

b) If $\Delta s \geq 0$ then $x^{t+1} \leftarrow y$ with probability p and $x^{t+1} \leftarrow x^t$ with probability $1-p$.

Other alternative rules for the probability of accepting a new solution have been suggested and discussed in [187]. The set of potential efficient solutions in direction λ is updated except if $\Delta z^j \geq 0\ \forall j$. A feasible initial solution x^0 is built at random [211] or using a greedy algorithm according the search direction [208]. Several lists of potentially efficient solutions $\widehat{E_1}$, $\widehat{E_2}$, $\widehat{E_3}, \dots$ are generated according to different weighting vectors $\lambda^1, \lambda^2, \lambda^3, \dots$ and merged to provide $\widehat{E}$.

In the method of [35], the main differences with the previous SA adaptation concern the management of weights and the consideration of a set of current solutions. Here, each solution in this set is "optimized" iteratively following the same mechanisms explained above (neighbouring solutions that may be accepted according a probabilistic strategy). But the weights are tuned dynamically in such a way that a solution will tend to move away from the other efficient solution. This will hopefully lead to an approximation uniformly spread along the nondominated frontier. Details about general procedures and algorithmic aspects are discussed in: [211,218], an SA adaptation called MOSA; [210], an interactive version of MOSA; [35,95], an SA adaptation called PSA; [94], PSA in an interactive way.

4.2.2 Tabu search The first papers describing the use of TS as technique for solving MOP problems dealt with a single objective strategy. In [36] a family of (P_λ) problems are solved to generate a set $\widehat{SE}$ approximating SE. In [100] the method consists in solving a sequence of single objective problems considering in turn each objective z^j associated with a penalty term. More recently, other tabu search approaches capable of generating both supported and nonsupported efficient solutions have been discussed.

In [70], principles of the TS method have been extended to determine a good approximation of E. This TS adaptation uses the utopian point z^U as point of reference with a scalarizing function $s(x, \lambda)$ to browse the nondominated frontier. Considering an iteration t and x^t, a current solution and its (sub)neighbourhood $\mathcal{N}(x^t)$ obtained according to a move defined in relation to the feasible set of the considered problem. At each iteration, z^U is updated according to the values $z(x)$ for all $x \in \mathcal{N}(x^t)$. The new current solution x^{t+1} is the best non tabu solution according to the current search direction following $s(x, \lambda)$. A tabu memory connected with the objectives and based on an improvement measure of each objective is suggested. This structure memorizes the improvement measured for each objective (indifference, weak improvement, strong improvement). It is used to update the search direction in order to browse, in an equilibrium way, all the efficient frontier. Intensification, diversification and tabu daemon (usually aspiration criteria) are discussed in the MOP context. A new direction is then defined by giving more importance of the improvement obtained for each objectives (indifference, weak improvement, strong improvement).

In [2], two weight vectors λ^a, λ^b belonging to the canonical basis of $\mathbb{R}^Q$ are selected at each iteration. They correspond to the two worst objectives a and b

according to decreasing values of the ratios $z^j(x^t)/z_j^I;\ j = 1, \ldots, Q$, where x^t is a given current solution. Then new weights are randomly generated for (λ^a, λ^b).

In [89] a set of "generation solutions", each with its own tabu list is considered. These solutions are dispersed along the objective space in order to allow a search in areas of the nondominated frontier. Weights are defined for each solution to force the search into a certain direction of the nondominated frontier and away from other current solutions that are efficient with respect to it. Diversification is ensured by the set of generation solutions and a drift criterion. Details about general procedures and algorithmic aspects are discussed in: [70, 71], a TS adaptation called MOTS; [89, 90] another TS adaptation also called MOTS; [2], a hybrid resolution process based on TS and GA; [3], a hybrid and interactive resolution process based on SA and TS.

4.2.3 Genetic algorithms (population-based methods) Since VEGA (vector evaluated genetic algorithm) in 1985 [181], many procedures based on genetic algorithm principles have been developed to deal with multiple objectives (multiple objective genetic algorithm [64], nondominated sorting GA [193], niched Pareto GA [104], MOGA [146], GA based on a min-max strategy [26, 28]). Significant progress in the literature concerns corrections of shortcomings observed in previous algorithms and propositions of new algorithmic primitives to generate a better approximation of E. For example, [80] suggests the use of non-domination ranking and selection to move a population toward the nondominated frontier. This concept is used to avoid the phenomenon of producing solutions only on the extremity of the nondominated frontier, where one performance is optimal. The author also suggested a kind of niche method to keep the GA from converging to a single point on the frontier. This concept is used to avoid a premature convergence of the algorithm and maintain individuals all along the nondominated frontier. These ideas have been implemented later in [64], and [104]. [146] presented a procedure not based on the Pareto ranking principle but on a weighted sum of objective functions to combine them into a scalar fitness function. The weight values are generated randomly for each iteration ensuring a good distribution of solutions along the nondominated frontier. Other papers concerning GA and EA (evolutionary algorithms) based procedures are [10, 25, 27, 86, 87, 112, 110].

4.2.4 Other approaches and new developments Other adaptations of heuristic procedures are found like dedicated heuristics [118], a stochastic search method [197], neural network based methods [134, 196] or the GRASP method [73]. We mention also a paper concerning a comparison of neighbourhood search techniques for MOP [138].

After a large interest in the extension of usual metaheuristics (SA, TS, GA, etc.) to the multiobjective context, actual research takes various orientations. Some hybrid methods, marrying for example TS and GA [2], or SA and TS [3] are designed. The idea here is to take advantage of the power of hybrid concepts in order to obtain a more efficient whole. Other research adds new components to MOMH in order to grasp the specifics of MOCO problems, for example in using a "generation set" in tabu search [89]. Also a greedy procedure is now often used, for example for the generation of initial solutions [69, 72, 110, 208]. As a greedy initial solution is closer to the nondominated frontier than a randomly chosen feasible solution, the solution procedure saves time during the approximation process. Using the first phase of the GRASP method, greedy randomized initial solutions are also used [73].

Recently some research exploits available information about the problem to be solved in order to reduce the search domain. Such knowledge is exploited to focus the search process on promising areas in terms of efficient solutions. For example domination situations are used to prune part of the domain proved to be void of efficient solutions [69].

5 Classification of the literature

In this section, we describe the classification scheme we used below to annotate the references. We classify a paper according to four categories, namely combinatorial structure, objective function type, problem type, and method applied. The first three pertain to the description of the problem discussed in a given paper.

As indicated in Section 2, to classify a certain paper, we first have to identify the problem discussed. This consists of the combinatorial structure (i.e. shortest path, knapsack, etc.), the number and type of objectives (i.e. sum, bottleneck, or eventually something else), and the type of problem (e.g. finding the efficient set, max-ordering, lexicographic).

In addition to the identification of the problem, we give the methodology used in the paper. We can distinguish between exact and approximation (or heuristic) methods, where exact means that the optimal solutions mentioned in the problem description are found, whereas approximation means that only some solutions representing this set, not necessarily optimal, are found.

So, we introduce a classification using positions

$$Pos1/Pos2/Pos3/Pos4.$$

Below, we provide tables where the different entries for each position are listed. Table 1 refers to the combinatorial structure of the problem.

Entries for $Pos2$ do not need a table, they simply define the number and type of objective functions considered. We could restrict ourselves to the sum and bottleneck objectives, with occasional exceptions explained where appropriate. Most of the papers that deal with other types of objectives, are listed separately, because almost each of them would have required its own entry here as well as for $Pos1$. Note that Q stands for an arbitrary number of objectives. As an example for a typical entry, 1-$\sum Q$-$\max$ denotes a problem with 1 sum and Q (i.e. any finite number of) bottleneck objectives.

Table 2 lists the various types of problem, which we introduced in Section 3.

$Pos4$ is used to describe the solution method applied and refers to the discussion in Section 4. We use the entries given in Table 3.

We note that sometimes two entries appear in one position. This means that one paper falls under two categories or that the approach applied in the paper is a

Table 1. Entries for $Pos1$ – Combinatorial structure

Entry	Explanation
SPP	Shortest Path Problem
AP	Assignment Problem
TP/TS	Transportation/Transshipment Problem
NF	Network Flow Problem
ST	Spanning Tree Problem
MB/MI	Matroid Base/Matroid Intersection Problem
TSP	Travelling Salesperson Problem
KP	Knapsack Problem
DL/NL	Discrete/Network Location Problem
SCP	Set Covering Problem

Table 2. Entries for $Pos3$ – Type of problem

Entry	Explanation
E	Finding the efficient set
e	Finding a subset of the efficient set
SE	Finding supported efficient solutions
$\widehat{\bullet}$	Finding an approximation of $\bullet$
lex	Solving the lexicographic problem
MO	Solving the max-ordering problem
lexMO	Solving the lexicographic max-ordering problem
U	Optimizing a utility function
C/S	Finding a compromise/satisfying solution

combination of two methods. It may also happen that a single paper appears under several classifications if more than one problem was considered, or several methods proposed.

6 Annotation of the literature problem by problem

In this section we will give an annotated overview over the literature. We found it most convenient to organize the section according to the combinatorial structure of MOCO problems. Thus, we introduce ten subsections, dealing with the most important combinatorial problems, in terms of the number of papers available. In a last subsection we briefly mention other MOCO problems that have appeared in papers, but to a definitely smaller extent.

As an exception to this order, we briefly mention PhD theses in the subject, since they are also witness of the growing research efforts in the field. An increasing number of dissertations have been written on MOCO in recent years. Those that we found were not all dedicated to MOCO specifically, but use some MOCO

Table 3. Entries for $Pos4$ – Solution method applied

Entry	Explanation
SP	Exact algorithm specifically designed for the problem
LS/LC	Label setting/label correcting method
DP	Algorithm based on dynamic programming
BB	Algorithm based on branch and bound
IA	Interactive method
H	Heuristic specifically designed for the problem
SA	Simulated annealing algorithm
TS	Tabu search algorithm
GA	Genetic or evolutionary algorithm
GRASP	Greedy randomized adaptative search procedure
GP	Goal programming
2P	Two phases method
A	Approximation algorithm with worst case performance bound
LP	Method based on linear programming

problems in another context: [31] deals with the multiobjective shortest path problem for routing of hazardous material, [131] contains information about bicriteria spanning trees, [26] is about evolutionary techniques in multiobjective optimization, and [49] presents some results for certain general MOCO problems. Among those which are specifically dedicated to MOCO problems we mention [60] and [126] on the flow problem and [103] and [202] in scheduling. [90] explores the use of metaheuristics for MOCO, and [211] introduces the two-phases method and develops it for the assignment and knapsack problem. Finally fast approximation algorithms for MOCO problems are discussed in [174].

6.1 Shortest path problems

The multiobjective shortest path problem consists in finding in a network with vector weights on the edges "optimal" paths. The papers we found usually consider the problem with specified starting and ending node, or from a given starting node to all other nodes. The shortest path problem belongs to the most widely studied MOCO problems. There exists a survey on the topic [212] and a bibliography on the Internet, containing an abstract collection [140].

Most problems in this category are $\mathbb{NP}$-complete: See [186] for the efficient paths problem with two sum objectives, [93] for intractability of the same problem. In [93] ten bicriteria shortest path problems are introduced and analyzed. In [54] an example shows that a result from [139] about the connectedness of efficient solutions is wrong. $\mathbb{NP}$-completeness of the max-ordering problem is mentioned in [147]. However, the multicriteria shortest path problem is an exceptional kind of problem, because a fully polynomial time approximation scheme is known, as presented in [224].

SPP/2-$\sum$/E/LC:	[14], [192], [206]	SPP/2-$\sum$/E/LS:	[93]
SPP/2-$\sum$/E/2P,LC:	[145]	SPP/2-$\sum$/E/SP:	[141],[24],[105]
SPP/2-$\sum$/E/DP:	[98]	SPP/2-$\sum$/$\widehat{E}$/A:	[93]
SPP/1-$\sum$ 1-$\max$/E/SP:	[93], [156]	SPP/2-$\sum$/C/IA:	[61]
SPP/2-$\sum$/U/SP:	[98]	SPP/2-$\sum$/U/IA:	[148]
SPP/2-$\sum$/ne/IA:	[30]	SPP/3-$\sum$/E/LC:	[67]
SPP/Q-$\sum$/SE/SP:	[98], [227]	SPP/3-$\sum$/C/IA:	[67]
SPP/Q-$\sum$/E/LC:	[31], [207]	SPP/Q-$\sum$/E/LS:	[139]
SPP/Q-$\sum$/E/DP:	[97], [120]	SPP/Q-$\sum$/$\widehat{E}, \widehat{MO}$/A:	[224]
SPP/Q-$\sum$/C/IA:	[99]	SPP/Q-$\sum$/U/DP:	[19]
SPP/Q-$\sum$/U/SP:	[144]	SPP/Q-$\sum$/MO/DP,BB:	[165]
SPP/Q-$\sum$/MO/LC:	[147]	Other:	[142]

A variety of algorithms based on dynamic programming (e.g. [98,120]), label setting [93,139] and label correcting methods (e.g. [14,145,192]) are available, with computational experiments [14,105,192] comparing different methods. In the biobjective case an algorithm based on ranking paths has also been proposed, [141,24]. The general idea is also applicable to other MOCO problems with two objectives, as explained in Section 4.

Besides, several papers present formulations of specific problems in terms of multicriteria shortest paths, or consider other variations of the classical problem, see also [32,33].

6.2 The assignment problem

Total unimodularity of the constraint matrix of the assignment problem guarantees that an optimal integer solution is found by linear programming methods. This does no longer hold in the multiobjective case.

The literature on the multiobjective assignment problem is again focussed on the determination of (supported) efficient solutions. In fact, it belongs to the first MOCO problems studied. However, the first papers only deal with SE, using convex combinations of objectives [39], or goal programming [20]. However, nonsupported efficient solutions exist [216], and the problem is $\mathbb{NP}$-complete [186] and $\#\mathbb{P}$-complete [152] and an exponential number of efficient solutions may exist.

Exact algorithms to determine the whole set E [162,216] have been developed. They make use of single objective methods and duality properties of the assignment problem. Recently we can also observe the application of metaheuristic techniques for the problem [208]. Some papers deal with a special version of the problem: [9,226]. Other papers deal with variations of the problem or applications. These cannot really be classified according to the problem and methodology applied or discussed in detail. We list them separately.

AP/2-$\sum$/SE/SP:	[39]	AP/2-$\sum$/E/2P,SP:	[162], [213], [216]
AP/2-$\sum$/$\widehat{E}$/SA:	[208]	AP/Q-$\sum$/E/SP:	[184]
AP/Q-$\sum$/$\widehat{E}$/SA:	[200]	AP/Q-$\sum$/S/GP:	[20]
Other:	[6],[7], [9], [226]		

TP/2-$\sum$/SE/LP:	[5]	TP/1-$\sum$ 1-$\max$/SE/LP:	[5], [163], [194]
TP/Q-$\sum$/se, S/IA:	[169]	TP/Q-$\sum$/SE/LP:	[43], [108], [190]
TP/Q-$\sum$/SE/DP:	[68]	TP/Q-$\sum$/S/SP:	[37]
TP/Q-$\sum$/$\widehat{E}$/GA:	[76], [75]	TP/Q-$\sum$/C/SP:	[130]
Other:	[209]		

6.3 Transportation and transshipment problems

Both are generalizations of the assignment problem, where the right hand side of the constraint may take positive integer values, and the variables any nonnegative integer. The transshipment problem has transshipment nodes in addition to demand and supply nodes.

Again, in the single objective case total unimodularity and integer right hand sides imply that an optimal solution of the linear relaxation is also an optimal solution of the problem itself. Making use of this fact, most of the papers use a scalarization by means of weighted sums or goal programming approaches. Thus, NE continues to be neglected in this area.

6.4 Network flow problems

The network flow problem is a problem that actually is on the borderline between combinatorial and linear optimization. It is well known that with a single objective there always exist integer optimal solutions of the LP, due to the unimodularity of A, which is the reason for considering it a combinatorial problem.

In the multiobjective flow problem we have to distinguish between the linear and the integer case. In the linear case, we know that $SE = E$. We deal with the papers in their relevance for the integer case. [172] demonstrated that an exponential number (in the number of node of the network) of extreme points among SE may occur. Most of the algorithms in the literature generalize methods for the single objective flow problem, e.g. the out-of-kilter method [127,137] or elements from network simplex [17,158]. The algorithms for MO and lexMO problems [51,83] are based on ranking approaches. For linear bicriteria network flow problems algorithms approximating the efficient set to any given precision ϵ are presented in [66,15, 173] and generalized to bicriteria quadratic network flow problems in [229].

NF/2-$\sum$/SE/SP:	[127], [158], [137]	NF/Q-$\sum$/E/SP:	[51]
NF/2(3)-$\sum$/E/SP:	[126], [128], [150], [151], [185]	NF/Q-$\sum$/SE/SP:	[117]
NF/2-$\sum$/$\widehat{SE}$/A:	[15], [176], [173], [66]	NF/Q-$\sum$/lex/SP:	[16], [17]
NF/Q-$\sum$/MO/SP:	[83]	NF/Q-$\sum$/lexMO/SP:	[51]
NF/Q-$\sum$/C/IA:	[60], [62]	Other:	[149], [229]

6.5 The spanning tree problem

The spanning tree problem is to find among all spanning trees of a given graph one that is "minimal" with respect to the edge weights. This problem appears in network design. It is known that the problem to find efficient solutions is $\mathbb{NP}$-complete [18] and intractable [85]. $\mathbb{NP}$-completeness also holds for the max-ordering problem [85]. The complexity status of a variety of multiobjective spanning tree problems, involving other than the typical sum and bottleneck objectives is studied in [18, 40, 41]. The algorithms that have been proposed to find efficient trees range from minimizing weighted sums [161, 182, 183] over generalizations of Prim's [29] and Kruskal's [183] method to approximation [85] and genetic algorithms [231]. A counterexample to a sufficient condition for a spanning tree to be efficient proposed in [29] has been given in [85]. As far as local search methods are concerned, it is important to note that, defining trees to be adjacent, if they have $n-2$ edges in common can imply that the set of efficient spanning trees is not connected [54].

ST/2-$\sum$/SE/SP:	[85]	ST/1-$\sum$ 1-$\max$/SE/SP:	[161]
ST/2-$\sum$/E/2P,SP:	[164]	ST/2-$\sum$/$\widehat{E}$/H:	[4], [85], [111]
ST/Q-$\sum$/SE/SP:	[182], [183]	ST/Q-$\sum$/E/SP:	[29]
ST/Q-$\sum$/$\widehat{E}$/GA:	[231]	ST/Q-$\sum$/MO/SP:	[85]
Other:	[40], [41]		

6.6 Matroids and matroid intersections

The matroid base problem is a generalization of the spanning tree problem. With a single objective it can be solved by the greedy algorithm. A generalization of this result for finding efficient bases is given in [186]: For each efficient basis B, there exists a topological sorting of the elements (e.g. edges of a graph), such that the greedy algorithm finds B. A topological sorting is a total or linear order that respects the partial order given by the vector weights. The problem is $\mathbb{NP}$-complete, as was shown e.g. in [47, 186]. A matroid intersection problem is to find a set of minimal weight which is independent with respect to two matroids.

Few papers deal with these problems in the multiobjective case. We identified the following, mostly presenting exact algorithms, theoretical properties [81, 223], and complexity issues [47, 186]

MB/2-$\sum$/SE, E/SP:	[47], [186]	MI/Q-$\sum$,1-$\max$ 1-$\sum$/Lex/SP:	[232]
MB/Q-$\sum$/MO/SP:	[47], [81]	MB/Q-$\sum$/$\widehat{\text{MO}}$/H:	[223]

6.7 The travelling salesperson problem

In combinatorial optimization, the TSP is widely studied. To find a shortest tour among n cities is $\mathbb{NP}$-complete even with one objective, for both the sum and bottleneck case. Moreover, the number of efficient solutions is expected to be exponential, see [59]. For approximation results, we refer to [52], where limits on the possibility of approximating efficient solution by one heuristic solution are derived and generalizations of the tree and Christofides heuristic are analyzed.

These might be reasons why investigation of the multiobjective version is not so common, and why research concentrates on exact algorithms based on dynamic programming as well as heuristics. Some papers discuss special versions or generalizations of the TSP, such as various formulations of vehicle routing problems.

TSP/1-$\sum$ 1-Π^1 /E/DP:	[63]	TSP/2,3-$\sum$/$\widehat{E}$/GA:	[110]
TSP/3-$\sum$/E/SP:	[13]	TSP/Q-$\sum$/E/DP:	[205]
TSP/Q-$\sum$/$\widehat{E}$/A:	[52]	TSP/Q-$\sum$/$\widehat{E}$/TS:	[91]
Other:	[79], [102]		

[1] Π denotes an objective defined by the products of weights

6.8 Knapsack problems

The knapsack problem is one of the fundamental $\mathbb{NP}$-complete combinatorial optimization problems. All papers that we found deal with the problem to identify or approximate SE or E. Finding E or SE are obviously $\mathbb{NP}$-complete, too. Thus it is not surprising that the algorithms proposed are either based on implicit enumeration methods such as dynamic programming [45,114–116], branch and bound [214,217] or apply heuristic procedures, especially metaheuristics to approximate E [69,88,176,177]. Some papers also deal with an extension to time-dependent knapsack problems [115,116]. An interactive decision support system for the capital budgeting problem is proposed in [201].

6.9 Location problems

Location planning is a very active area of research. The objective in a location problem is to find one (or more) locations, such that some objective, usually related to

KP/2-$\sum$/SE/SP:	[171]	KP/2-$\sum$/SE/DP:	[45]
KP/2-$\sum$/$\widehat{SE}$/H:	[171]	KP/2-$\sum$/E/2P,BB:	[214], [217], [222]
KP/2-$\sum$/$\widehat{E}$/TS:	[69]	KP/2-$\sum$/$\widehat{E}$/H:	[177]
KP/2-$\sum$/$\widehat{E}$/H:	[176]	KP/2-$\sum$/$\widehat{E}$/GA+TS:	[2]
KP/2-$\sum$/$\widehat{e}$/SA+TS:	[3]	KP/2,3-$\sum$/$\widehat{E}$/GA:	[72]
KP/Q-$\sum$/E/DP:	[114], [115], [116]	KP/Q-$\sum$/$\widehat{E}$/TS:	[88], [89]
KP/Q-$\sum$/$\widehat{E}$/SA:	[35], [200],[219], [218]	KP/Q-$\sum$/U/DP:	[21]
KP/Q-$\sum$/S/GP:	[113]		

the distance to a set of existing facilities is minimized or maximized. These objectives usually are the weighted sum or maximum of individual distances. Moreover, location problems can be divided into three categories, namely planar, network and discrete problems. In planar location, the feasible set is (a subset of) the Euclidean plane. Network location problems deal with a network of nodes and arcs, new facilities can be built either on the nodes only, or also on arcs. Finally, for discrete location problems a set of potential sites is specified. Problems of the latter category are usually formulated as mixed integer programs. From the point of few of MOCO, we will consider only network and discrete location problems. For details about planar problems and single objective location problems, we refer to the specialized literature, e.g. [124, 125] for surveys. We refer also to two reviews on the topic in MOCO context, [34] and [168]. Most of the applications use a goal programming approach.

NL/Q-$\sum$/lex,E/SP:	[84]	NL/Q-$\sum$/MO,lexMO/SP:	[56]
DL/Q-$\sum$, Q-max/E/SP:	[153]	DL/Q-$\sum$/SE/SP:	[135]
DL/Q-$\sum$/lexMO/SP:	[154]	DL/Q-$\sum$/U,S/IA,GP:	[136]
DL/Q-$\sum$/S/GP:	[8]	Other:	[101]

6.10 The set covering problem

The set covering problem is an $\mathbb{NP}$-complete problem with applications in the location of emergency facilities. Suppose there are m sites of potential emergency and n potential locations for emergency facilities, incurring cost c_i to build this site. Then the aim is to select – at minimal cost – enough sites to cover all risks.

The multiobjective set covering problem has not gained much attention in the literature, and the main results in one of the references [179] are wrong. [96] deals with a particular problem. Note also that some of the problems discussed in the shortest path Section 6.1 above and in the other MOCO problems Section 6.11 below deal with aspects of "covering".

SCP/Q-$\sum$/E/SP:	[179]	SCP/Q-$\sum$/SE/SP:	[38]
SCP/2-$\sum$/$\widehat{E}$/GRASP:	[73]		

6.11 Other MOCO problems

In the previous sections we have discussed the most important multiobjective combinatorial optimization problems. Besides these there is some literature on other problems: Some classical problems have been discussed only in a few papers, others deal with problems that are so specific that they would require their own category. All of these are discussed summarily here.

In [82] a lexicographic flow problem is used to determine minimal cuts with a minimal number of arcs in a network. [191] deals with the one dimensional cutting stock problem with two objectives in a lexicographic context (priorities on the objectives). Both an exact and a heuristic algorithm are given. In [1] an interactive approach is proposed to solve the multiobjective cutting stock problem.

We also found few references [109,133] on the quadratic assignment problem in a multicriteria context. This is closely related to the facility layout problem which is discussed in a number of papers. They actually use approaches based on the quadratic assignment problem: [44,65,170,132,220]. Other references on the facility layout problem are [109,122,189,225]

Many of the papers listed in the surveys [33] and [32] about multiobjective transportation network design are also among these specific problems. A variety of multiobjective routing problems is discussed in [12].

7 Open questions and conclusions

Our survey of the state of the art in multiobjective combinatorial optimization clearly identifies potential areas of research and weak points in the existing literature. We briefly outline these below.

7.1 General remarks

Very few theoretical results are available about the properties of MOCO problems, like characterization of efficient solutions, the number of efficient solutions (supported and nonsupported) both in the worst case or on average, the topology of the nondominated frontier, the elicitation of lower and upper bounds, etc. Taking into account the fact that MOCO problems are almost always very hard in terms of computational complexity the need for a thorough theoretical understanding of MOCO problems is all the more evident. It is also clear that a better theoretical comprehension of these problems will contribute to the development of efficient solution methods.

Many of the current extensions of methods useful for single objective optimization to the multiobjective situation have exhibited some difficulties for finding E.

One such example is the the VEGA method. MOCO problems have specific properties and need specific techniques to cope in an efficient way with these. Some adaptations such as MOSA, PSA, etc. could produce good results on a particular problem like the knapsack problem. The question is, whether such method show good performances when applied to other problems. From the evolution of these methods over the last years, one can have some doubts. No comparative studies on the performance of solution strategies like branch and bound or dynamic programming on a variety of problems are available.

Few papers refer to practical application of MOCO problems. Moreover, when the MOCO problem is extracted from a practical context, the resolution is often reduced to a single objective problem. For example, this is the case to the channel minimization problem of [36], but also for a lot of scheduling problems (see [202]). Thus there is a need to attract the attention of decision makers to the area of MOCO and solve the problems arising in practice in a real multicriteria context.

7.2 Remarks on exact methods

For exact methods, there is a huge gap between the bicriteria and the general case. Many procedures have been developed especially for bicriteria problems and cannot be modified to deal with the general case, a remark that is especially true for the two phases method. This gap is probably caused by the lack of theoretical understanding of MOCO problems with three or more objectives, as pointed out above.

The two phases approach proved to be a key development for bicriteria MOCO problems. However, as far as we know, there are no general procedures to compute supported efficient solutions in the multiobjective case. This would be of course the first step to an application of the two phases method in three or more criteria MOCO.

For the effective adaptation of some bicriteria methods to the general case, knowledge of good lower and upper bounds on the efficient set is needed. The computation of the Nadir point (which is pretty easy in bicriteria problems) is an unsolved problem in general. Another research area would be to consider the computation of sets of solutions that constitute a set of lower and upper bounds on E. The lack of such results makes it impossible to adapt certain procedures to general MOCO at this time.

There is a wide variety of combinatorial problems that have never been investigated in a multicriteria context, as is evident from the problems list in Section 6.

An important concept in multiobjective programming (MOP) is that of level sets. It can be seen as a general framework for MOP, which allows a characterization of efficient solutions [53], as well as interactive procedures. Applications to MOCO could be promising but are not existing now.

7.3 Remarks on heuristic methods

Closely related to the remark about adaptation of single objective methods is the question of multiobjective metaheuristics to solve MOCO problems. We are not convinced of the efficiency of a real metaheuristic in the sense of a meta-method able to solve efficiently any MOCO. Each problem has its own specifics and a general MOMH cannot cope with all of these. One research direction is the identification of techniques for which the computational results obtained are promising. For example, greedy algorithms are more and more used in procedures for the generation of initial solutions.

If a heuristic method defined according to the “a posteriori mode” is available, it is easy and alway possible to transform it to the “interactive mode”. The main challenge for heuristic methods is then how to obtain very quickly a good approximation of the whole nondominated frontier. With such an approximation, the procedure could then be to continue either in increasing the approximation quality for the nondominated frontier or in focusing the approximation on a part of the nondominated frontier following the preference of a decision maker in the context of an interactive procedure.

An important question in the context of approximation methods is: How to measure and compare approximations, and how to evaluate the quality of an approximation for problems with multiple objectives? Ideas have been put forward in [208, 92, 180]. Some attributes like coverage, uniformity and cardinality to judge the approximation to be satisfactory or not by a decision maker have been defined. Such attributes are also useful when defining stopping rules in approximation methods, and again when the tuning of heuristic algorithms is examined.

Bounds and domination conditions should be used to reduce the search space. All available information to bracket and reduce the decision space is welcome. Such information could be used for scanning the “core” of the problem, identifying and discarding irrelevant aspects of the problem investigated. Information could be derived from the decision space as well as from the objective space.

For some MOCO problems, the resolution could be decomposed in several steps. For example, in a first step the procedure could try to identify the supported efficient solution using an exact method. Information could be extracted from the first results to reduce the search space and in a second step try to identify the nonsupported solutions by a heuristic method. Such a “semiexact” method is especially attractive for problems that can be efficiently solved as single objective combinatorial problems. Note that usually the cardinality of the sets SE is much smaller than the number of nonsupported efficient solutions.

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