MULTIPLICITY IN GENETIC ALGORITHMS TO FACE MULTICRITERIA OPTIMIZATION*

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Abstract- When establishing the Pareto-optimal front, effective multicriteria optimization involves simultaneous parallel search for multiple members of a genetic algorithm population.

In one of these approaches due to Eiben and Lis [2] rather than conducting multiple independent single objective searches, all the individuals in the population, "speciated" by criterion, explore the problem space expecting that the increased parallel processing of schemata improves effectiveness to find more solutions in the Pareto-optimal range.

The present paper investigates the problem of using a genetic algorithm on a set of test functions which allows a multisexual population, multiple parents and multiple crossovers per mating, attempting to build a Pareto optimal set of larger size. Also, while creating a new population a selection process for replacement favours those new created solutions that are inclined to appertain to the Pareto front.

As a result, the performance of the method produce an evenly distributed and larger set of efficient points.

Keywords: Genetic algorithms, multiobjective optimization, Pareto optimal solutions.

1 Introduction

In the case of multiobjective optimization it is not always clear how to determine the "goodness" of a solution, because every solution has a number of fitness values, one per each objective. A particular solution can be good for some objectives and bad for other objectives.

Common existing methods for handling multiple criteria problems are classified as aggregative approaches and usually combine the various objectives into a single utility function. In that way, a genetic algorithm performs as usual finding the fittest individuals for that single aggregated function.

The simplest approach, of the aggregative family, is a linear scalar combination which assigns weights (constant coefficients) to each criterion (attribute) according to the judgement of a human decision maker (DM). Bhanu and Lee [1] and Vemuri and Cedeño [21], worked on it.

In order to avoid linear relationships among the criteria, other researchers introduced nonlinear relationships by exponentiating critical attributes, by multiplying highly related attributes or by setting thresholds (upper and lower values) for each criterion. This latter approach involves handling of constraints and penalty or repair functions. Simpson et al [18], Savic and Walters [15] Krause and Nissen [11] and Stanley and Mudge [20] gave insights in this sense.

Nevertheless, aggregative approaches are criticised of being rather simplistic. Furthermore, a question remains; how to combine separate values of fitness into a reasonable single fitness value? Sometimes it is neither possible nor rational to combine different, most of the times conflicting, criteria into a single one.

A better alternative to escape of this dilemma is to use the concept of Pareto optimality.

Vilfredo Pareto [14] in 1896 established that there exists a partial ordering in the searching space of a multiobjective problem. The Pareto criterion simply states that a solution is better than another one if it is so good in all attributes, and better in at least one of these attributes.

More formally, for an *n*-objective optimization problem the search space can be seen as an *n*-dimensional space and therefore each solution is an *n*-vector of attribute components.

In a maximization problem given two solutions $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$, the Pareto criterion says that, x dominates y iff $x_i \geq y_i \; \forall i$ and $\exists j$ such that $x_j > y_j$.

This is the well-known covering relationship in a vectorial space and defines the partial ordering stated by Pareto. In the problem space some pairs of solutions will be non-commesurate, in the sense that neither dominates the other. Some solutions will be dominated by other solutions and some other solutions will not be dominated by any of the solutions in the problem space. The latter, nondominated set, corresponds to the *Pareto front*, also

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known as the *acceptable set*, the *efficient points* and the *Pareto optimal set*.

Knowledge of the Pareto front is of utmost importance when search is applied before decision making. This information provides the DM with the trade-offs to establish interactions between different criteria, hence simplifying the decision process to choose an acceptable range of solutions for a multicriteria problem.

Evolutionary algorithms are suitably fitted to face the seeking of the Pareto front due to their implicit parallel search.

According to Horn [10] taxonomy, *cooperative population searches (CPS) with criterion selection* is one of the first methods implemented by Schaffer [16], [17], Fourman [8] and then by Kursawe, [12], [13] and others. Goldberg [9] suggested another subclass of CPS called *Pareto ranking*. Here a *non-dominated sorting* to rank the population according to the Pareto optimality criterion was introduced. Also niching and speciation on the Pareto front were proposed. The Goldberg's original proposal of nondominated sorting was effectively implemented by Srinivas and Deb [19].

The central idea in CPS, is to make a parallel single criterion search, where all members of the population of an evolutionary algorithm are involved in a cooperative search to build the Pareto front. Many of these approaches use niching to obtain an evenly distributed set of the *efficient points*.

Eiben and Lis [2] recently introduced a variant addressing the problem, called multisexual genetic algorithm (MSGA), where basically,

- Each individual is provided with a sex (speciation) intimately related with an optimizing criterion.
- Individuals are evaluated according to their performance within their (sexual) subpopulations.
- A multiparent crossover requiring one parent per sex is used for recombination.

According to their report, this approach showed to be effective to build a set of nondominated solutions of reasonable size by merging those *locally*[1] nondominated solutions obtained through the evolutive process.

We continued working on these basic ideas modifying some features of MSGA and adding some other characteristics. In our proposal, the whole process attempting to build the Pareto front is using a *multiplicity approach* expressed in a multisexual population, where multiple parents per sex are chosen for mating and once selected undergo multiple crossovers. Moreover, from the various children generated after crossover those that are *globally* nondominated are favoured for insertion in the next generation. This new approach will be called *multi-sexual-parents-crossovers genetic algorithm* (MSPC-GA).

2 Background

In this section a brief overview on MCPC and MSGA is given.

Multiple crossover per couple (MCPC) is a newly introduced [4] crossover method which in contrast with the conventional single crossover per couple approach (SCPC), permits more than one crossover operation for each mating pair. MCPC was applied to optimize classic testing functions and some harder (non-linear, non-separable) functions. The goodness of this approach prevailed under all tests and revealed that, when MCPC is applied with 2, 3 and 4 crossovers per couple, results as good as under SCPC can be expected with an additional benefit in processing time. This performance was obtained through the ability showed by MCPC of exploiting the recombination of good, formerly found solutions. But on the other hand, those experiments also showed that, in some cases, the method increased the risk of premature convergence due to a loss of genetic diversity. To overcome this problem further successful approaches were undertaken by combining MCPC with an alternative selection method; *fitness proportional couple selection* [7], by using self-adaptation of MCPC parameters [5], or binding MCPC to alternative selection mechanisms, [6].

As above explained MSGA assigns a sex, or gender, to each individual. There are as many sexes as optimization criteria exist. In this way each individual is specialized and tries to fulfil certain optimization criterion. An individual is represented as the corresponding genetic code plus a sex marker, which initially can be randomly set. Consequently the population is divided into subpopulations, each one in accordance with some optimization criterion and individuals are evaluated correspondingly through the appropriate fitness function.

Once individuals are evaluated they are sorted according to their fitness and the rank obtained is the basis for future selection. Ranks are determined independently for each sex.

Recombination is performed as follows: one individual is chosen, as a parent, from each sex and then they undergo *uniform scanning crossover*, which generates a single offspring. Each gene in the child is provided from any of the corresponding genes in the parents with equal probability. The sex of the child is inherited from that parent, which supplied the largest number of genes. If more than one parent supply to the offspring the same maximal number of genes then the sex of the offspring is randomly chosen from these parent's sexes. Mutation takes place only in the genetic code of the chromosome.

The set of *locally* nondominated solutions is determined in the current generation and then merged with the previously found nondominated solutions. During merging those nondominated solutions that become dominated are discarded. After execution this updated set will be the output of the MSGA.

[1] According to the literature we use the following terms: *locally nondominated* referring to those solutions that are nondominated in the current population and *globally nondominated* referring to those solutions that were determined as nondominated, so far, at the current stage of the evolutionary process.

3 Brief description of MSPC-GA

Basically the new approach:

1) uses proportional selection.
2) selects multiple parents per sex.
3) uses an extension of *multiple crossovers per couple* (MCPC), called *multiple crossovers per mating* (MCPMA).
4) for insertion in the next population, gives preference to those offspring, which are classified so far, as *globally* nondominated.

Some discussion is necessary to clarify the difference with MSGA approach. First, rather than ranking selection, proportional selection is used to favour best performer individuals under each criterion. Second, rather than one parent per sex an equal number of multiple parents per sex are selected to provide genetic diversity. Third, rather than one crossover operation on the parent's set multiple crossovers are applied in an attempt to increase exploitation of good solutions previously found.

As above indicated, in the experiments reported in this paper MCPMA is used. Here multiple crossovers are allowed now between multiple parents. Finally, rather than inserting the single child generated by uniform scanning crossover another selection process is applied to select those offspring which resulted globally nondominated.

Using the *multiparent approach* of Eiben [3] mitigates the possible loss of diversity generated by MCPC and no extra adjustments seems to be necessary. Consequently exploitation and exploration of the problem space are suitably balanced.

The last point above mentioned 4), implies to maintain the updated set of solutions found so far as belonging to the Pareto front. Let us call it $P_{current}$. This set is updated at the end of each generation cycle.

To build the new population, each time the new offspring are created by application of MCPMA, we apply the following procedure:

While the new population is created
do
  Select n_1 parents from each sex,
  Apply MCPMA with uniform scanning crossover to obtain n_2 offspring and mutate,
  By consulting $P_{current}$ determine the subset O_{nond} of these new offspring that are globally nondominated,
  If $O_{nond} \neq \Phi$ then insert O_{nond} into the new population
    else insert $n_2/2$ offspring randomly chosen into the new population
od

The number n_1 of parents and the number n_2 of crossovers are parameters of the GA.

Essentially the proposed MSPC-GA,

- Augments implicit parallel search by encouraging crossbreeding among "species".
- Increases exploration and attempts to avoid premature convergence through multiparent recombination.
- Increases exploitation of good solutions previosly found through multiple crossovers per mating.
- Favours for insertion in the next generation those solutions which are, at the present stage, globally nondominated. If no one is found then genetic diversity is favoured by a random selection.

Consequently, it is expected a contribution of the method to speed the search and to find a larger set size when seeking the Pareto optimal set.

4 Experiments

To evaluate the performance of the MSPC-GA, some problems reported in the works of Eiben-Lis and Schaffer were used for experiments. These problems and the corresponding MSPC-GA parameters are listed bellow.
In Problems 1 and 2, where Eiben and Lis reported details on parameters, our experiments replicated the same parameters setting to contrast results.

Problem I: Eiben-Lis Test1 (Srinivas and Deb [18])

Minimize $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ where

$$f_1(x_1,x_2) = (x_1^2 + x_2^2)^{\frac{1}{8}}$$

$$f_2(x_1,x_2) = ((x_1 - 0.5)^2 + (x_2\, 0.5)^2)^{\frac{1}{4}}$$

with $-5 \le x_1, x_2 \le 10$

Population size:	30
Crossover rate :	0.8
Mutation rate :	0.01
Chromosome length:	32

Problem 2: Eiben-Lis Test 2 (Schaffer F3 [16])

Minimize $f_1(x)$ and $f_2(x)$ where

$$f_1(x) = \begin{cases} -x & \text{if } x \le 1 \\ -2 + x & \text{if } 1 < x \le 3 \\ 4 - x & \text{if } 3 < x \le 4 \\ -4 + x & \text{if } 4 < x \end{cases}$$

$$f_2(x) = (x-5)^2$$

with $-5 \le x \le 10$

Population size:	100
Crossover rate :	0.3
Mutation rate :	0.001
Chromosome length:	16

Problem 3: Schaffer function F2 [16].

Minimize $f_{21}(x)$ and $f_{22}(x)$ where

$$f_{21}(x) = x^2$$

$$f_{22}(x) = (x-2)^2$$

with $-6 \le x \le 6$

Population size : 100
Crossover rate : 0.85
Mutation rate : 0.01
Chromosome length: 14

5 Results

Experiments were undertaken for diverse values of n_1 and n_2 on each function. In these preliminary tests when i crossovers were allowed, i parents per sex were also selected ($n_1 = n_2$). In general, good results were achieved for values of n_1 and n_2 between 2 and 4. In this section we report results obtained assigning values of 2, 3 and 4 for both, n_1 and n_2 .

In *Problem 1* after 100 generations 441 non-dominated solutions, were found almost evenly distributed on the Pareto front, with $n_1 = n_2 = 3$. They are shown in Figure 1. When $n_1 = n_2 = 4$, 263 nondominated solutions were obtained.

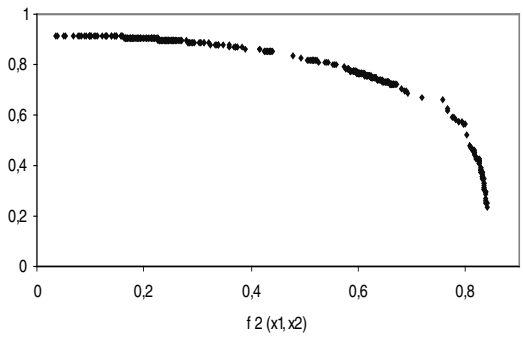

Fig. 1 – The Pareto front for Problem 1, with 3 parents per sex and 3 crossovers.

In *Problem 2* after 600 generations 597 and 585 non-dominated solutions, were found almost evenly distributed on the Pareto front for n_1 and n_2 set both to 2 and 3, respectively. They are shown in Figures 2 and 3 respectively.

Figures 4 and 5 show the solutions found as output of MSPC-GA in the variable and criteria domain for Problem 2.

In *Problem 3* after 200 generations 583 and 580 non-dominated solutions, were found evenly distributed on the Pareto front for n_1 and n_2 set both to 3 and 4, respectively. They are shown in Figures 6 and 7 respectively.

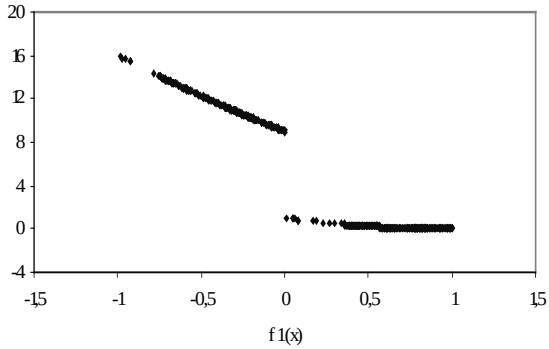

Fig. 2 – The Pareto front for Problem 2, with 2 parents per sex and 2 crossovers.

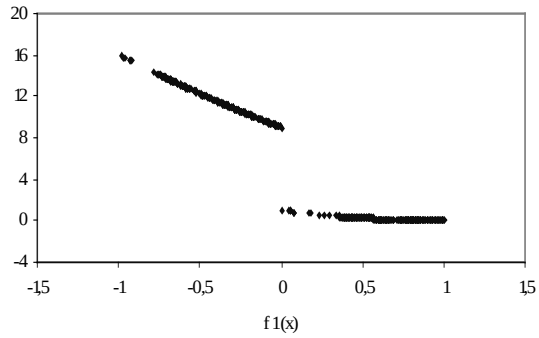

Fig. 3 – The Pareto front for Problem 2, with 3 parents per sex and 3 crossovers.

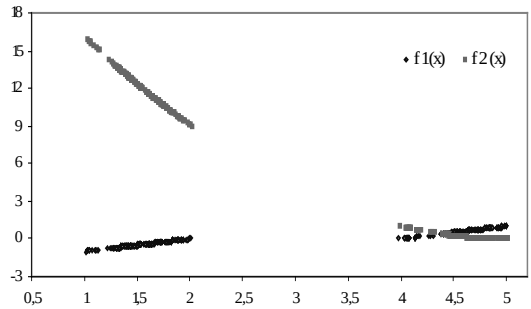

Fig. 4 – Variable and criteria domain for Problem 2, with 2 parents per sex and 2 crossovers

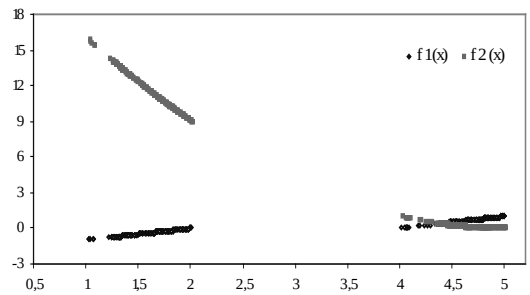

Fig. 5– Variable and criteria domain for Problem 2, with 3 parents per sex and 3 crossovers

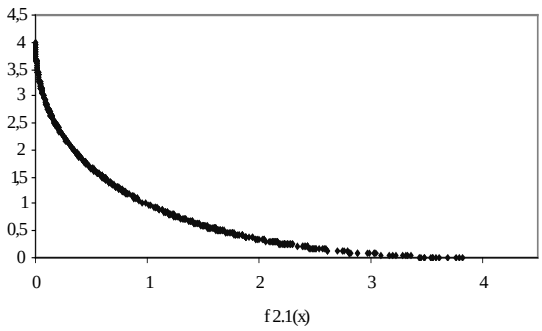

Fig. 6 – The Pareto front for Problem 3, with 3 parents per sex and 3 crossovers.

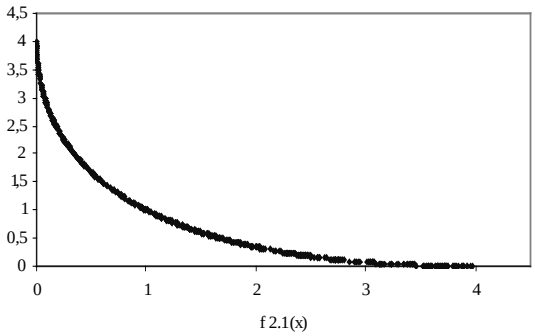

Fig. 7 – The Pareto front for Problem 3, with 4 parents per sex and 4 crossovers.

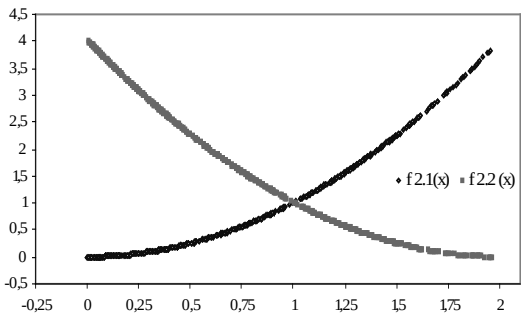

Fig. 8 – Variable and criteria domain for Problem 3, with 3 parents per sex and 3 crossovers.

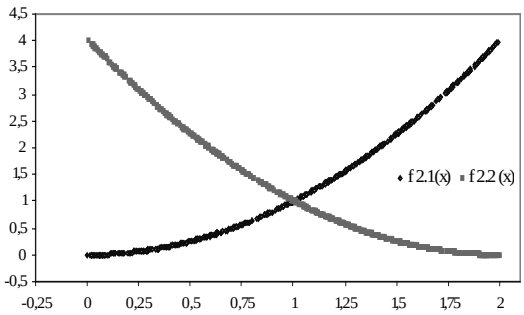

Fig. 9 – Variable and criteria domain for Problem 3, with 4 parents per sex and 4 crossovers.

Figures 8 and 9 show the solutions found as output of MSPC-GA in the variable and criteria domain for Problem 3.

As can be observed in the above figures, a larger size of the Pareto optimal set can be obtained with an almost even distribution of the nondominated points.

Table 1 sumarizes results when contrasting MSCP-GA and MSGA.

Problem	Problem 1			Problem 2		
Method	MSGA	MSPC-GA		MSGA	MSPC-GA	
		(3,3)	(4,4)		(2,2)	(3,3)
#nond	129	441	263	494	597	585
#gen.	197	100	100	631	600	600

Table 1 – Results under MSGA and MSPC-GA

For the same problems and using the same parameters setting MSPC-GA, provides in a fewer number of generations, a Pareto front larger than the previously reported for MSGA. This results holds for all the values chosen for n_1 and n_2.

It also can be observed that simultaneous increments of n_1 and n_2 do not benefit the performance. Optimal n_1 and n_2 combination is yet an open question.

6 Conclusions

The present paper describes a new approach to face multicriteria problems where emphasis is put in applying multiplicity to some of the techniques used in an evolutionary algorithm.

The idea of using multiple sexes to subdivide the population in subpopulations of cooperative individuals according to the multiobjectives of the problem, enforces crossbreeding by restricting mating to individuals of different sex. This approach showed its benefits in the Eiben and Lis cited work.

The present proposal extends this idea by allowing now multiple parents per sex and multiple crossovers per mating in order to balance the explorative and exploitative efforts, which are present in any evolutionary algorithm. Parents selection rewards for reproduction those best performer individuals. Moreover, offspring selection for replacement favours those new created individuals that are prone to reside in the Pareto front. If no one fulfilling that condition exits then a random selection is done to maintain genetic diversity

A set of preliminary experiments was carried out by simply assigning as many parents per sex as crossovers are allowed during mating.

When a comparison with other techniques was possible, current results outperform previous findings with similar parameters setting on the same problem set. The number of efficient points is larger and their distribution is at least satisfactory.

It worth to remark that no fitness sharing for niching was necessary to apply in order to obtain a well-delineated Pareto front. Distance comparisons between all members (or a sample) of the population at each fitness sharing calculation do not need to be computed. Consequently a lesser computational effort is expected because comparisons are restricted to determine dominance.

These results are promising and encourage us to deep forward investigation by varying adaptively MSPC-GA parameters, essentially the trade-offs between the number of multiple parents and crossovers to establish the abilities and possible limitations of this approach.

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