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Pareto simulated annealing for fuzzy multi-objective combinatorial optimization

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Abstract

The paper presents a metaheuristic method for solving fuzzy multi-objective combinatorial optimization problems. It extends the Pareto simulated annealing (PSA) method proposed originally for the crisp multi-objective combinatorial (MOCO) problems and is called fuzzy Pareto simulated annealing (FPSA). The method does not transform the original fuzzy MOCO problem to an auxiliary deterministic problem but works in the original fuzzy objective space. Its goal is to find a set of approximately efficient solutions being a good approximation of the whole set of efficient solutions defined in the fuzzy objective space. The extension of PSA to FPSA requires the definition of the dominance in the fuzzy objective space, modification of rules for calculating probability of accepting a new solution and application of a defuzzification operator for updating the average position of a solution in the objective space. The use of the FPSA method is illustrated by its application to an agricultural multi-objective project scheduling problem.

Key words

Fuzzy multi-objective combinatorial optimization, Metaheuristics in fuzzy objective space, Simulated annealing, Fuzzy multi-objective project scheduling.

1. Introduction

In a recent manifesto on Fuzzy Information Engineering, Dubois, Prade and Yager (1997) are pointing out three basic semantics associated with the use of fuzzy sets. A first semantics (and historically the oldest one) is the expression of closeness, proximity, similarity and the like; this is the usual understanding of fuzzy sets in clustering, recognition and classification tasks. A second semantics is related to representation of incomplete or vague states of information under the form of possibility distributions; this view of fuzzy sets enables representation of imprecise or uncertain information in mathematical models of decision problems considered in operations research. A third semantics for a fuzzy set expresses preferences concerning satisfaction of flexible constraints and/or attainment of goals; this is especially important for exploiting information in decision making. The gradedness introduced by fuzzy sets refines the simple binary distinction made by ordinary constraints. It also refines the crisp specification of goals and “all-or-nothing” decisions.

In this paper, we are focusing our attention on the second semantics. Our interest in it follows from a long experience in modeling and solving project scheduling problems under uncertainty concerning time parameters of activities (Hapke et al., 1994). This uncertainty being of possibilistic character, the best way of handling it is through the use of fuzzy sets. Precisely, the uncertain time parameters are modeled as fuzzy numbers, i.e. normalized, convex fuzzy subsets of the real line. To be yet more realistic, we were considering multi-objective fuzzy project scheduling problems (Hapke et al., 1997). These problems are clearly fuzzy multi-objective combinatorial optimization (MOCO) problems whose deterministic equivalents are NP-hard. Due to this complexity, it was advisable to use a metaheuristic procedure for solving these problems (approximately).

Before developing a metaheuristic procedure for the fuzzy MOCO problems, we formulated the following postulates it should fulfil:

- it should not aggregate a priori the fuzzy objective functions into a single objective but generate a representative approximation of the non-dominated set for further exploration using an interactive procedure,
- it should consider non-dominated points in the original fuzzy objective space and not non-dominated points in the objective space of an auxiliary deterministic problem (no defuzzification).

In this paper, we are taking these postulates into account within a Pareto Simulated Annealing (PSA) procedure previously developed by Czyzak and Jaszkiewicz (1996, 1997) for deterministic MOCO problems. The main problems which have to be solved when extending PSA to the fuzzy objective space are:

- definition of the dominance in the fuzzy objective space,
- adaptation of the simulated annealing scheme to fuzzy values of objective functions.

In order to adapt the PSA scheme to fuzzy values of objective functions, one has to test if a newly generated neighbourhood solution dominates, is non-dominated or is dominated by the current solution. Furthermore, if the new solution is not dominated by the current one it should be used to update the set of approximately efficient solutions in the fuzzy objective space. In order to define the dominance, one has to propose first a way of comparing fuzzy scores on objective functions. The comparison rule of fuzzy scores should also be used in the step where a probability of accepting a newly generated solution is calculated. The PSA procedure extended to handle fuzzy MOCO problems will be called fuzzy Pareto simulated annealing (FPSA). The comparison rule used in FPSA should be connected with the semantics of fuzzy numbers.

Existing metaheuristic approaches to fuzzy MOCO problems are quite different in the following points:

- they use the third semantics of the fuzzy sets characterized above and aggregate the objective functions a priori, i.e. they use fuzzy goals for objective functions and tend to maximize the smallest satisfaction degree (Sakawa et al., 1994, 1997),
- they consider defuzzified objectives which are aggregated a priori (Fortemps, 1997),

- they aggregate both fuzzy objectives and fuzzy constraints with the meaning of the third semantics into a single function (Slany, 1996).

The aim of the present paper is to develop a multi-objective metaheuristic procedure working truly in the fuzzy objective space. The paper is organized as follows. In the next section, we introduce basic concepts and definitions connected with the fuzzy set modeling of uncertainty, with the comparison of two fuzzy numbers and with the dominance and fuzzy MOCO problems. Section 3 presents the metaheuristic FPSA procedure for the fuzzy MOCO problems. In section 4, the FPSA is applied to an agricultural multi-objective project scheduling problem. The final section groups conclusions.

2. Basic concepts and definitions

2.1. Uncertainty modeling

Set A in a base set X can be described by a membership function $\mu_A : X \rightarrow \{0,1\}$ with $\mu_A(x)=1$ if $x \in A$ and $\mu_A(x)=0$ if $x \notin A$. If it is uncertain, whether or not element x belongs to set A, the above model can be extended such that the membership function maps into interval [0,1]. A high value of this membership function implies high possibility while a low value a poor possibility. This leads to the definition of a fuzzy set (Dubois, Prade 1980).

Let X be a base set and μ_A a function from X into the unit interval [0,1].

Then the set

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\} \quad (1)$$

is called a fuzzy set in X and $\mu_{\tilde{A}}$ is called the membership function of $\tilde{A}$.

Another useful definition is that of a level set or a level cut. Let $\tilde{A}$ be a fuzzy set in X and $\alpha \in (0,1]$. The α-level set or α-level cut of $\tilde{A}$ is the set

$$\tilde{A}_\alpha = \{ x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \} \quad (2)$$

Lower and upper bounds of α-cut $\tilde{A}_\alpha$ are equal to $\inf_{x \in X} \tilde{A}_\alpha$ and $\sup_{x \in X} \tilde{A}_\alpha$, respectively.

Flat fuzzy number is a special kind of a fuzzy set. Precisely, a flat fuzzy number is defined as a normalized convex fuzzy subset of real line R, i.e. it is defined according to (1), where $X=\Re$.

The precise form of a fuzzy number can be described by an expert only rarely. A practical way of getting suitable membership functions of fuzzy data has been proposed by Romelfanger (1990). He proposes that the expert expresses his/her optimistic and pessimistic information about parameter uncertainty on some prominent membership levels α, e.g.:

$\alpha=1$:	$\mu(x) = 1$	-means that value x certainly belongs to the set of possible values,
$\alpha=\lambda$:	$\mu(x) > \lambda$	-means that the expert estimates that value x with $\mu(x) \geq \lambda$ has a good chance of belonging to the set of possible values,
$\alpha=\varepsilon$:	$\mu(x) < \varepsilon$	-means that value x with $\mu(x) < \varepsilon$ has only a very little chance of belonging to the set of possible values, i.e. the expert is willing to neglect the corresponding values of x with $\mu(x) < \varepsilon$.

Thus a flat fuzzy number in six-point convention is represented by six real numbers. Formally, a flat fuzzy number $\tilde{M}$ is represented by the following list of symbols:

$$\tilde{M} = (\underline{m}^{\varepsilon}, \underline{m}^{\lambda}, \underline{m}, \overline{m}, \overline{m}^{\lambda}, \overline{m}^{\varepsilon}) \tag{3}$$

We assume that λ = 0.6, ε = 0.1. An exemplary flat fuzzy number in the six-point convention is presented in Figure 1.

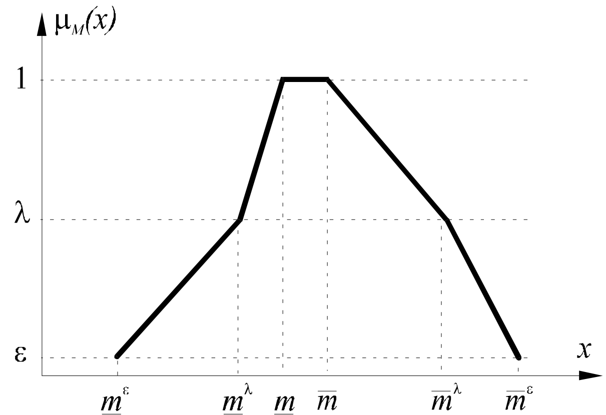


Figure 1. *Flat fuzzy number* $\tilde{M}$ *in six-point representation*

It is worth pointing out that well-known trapezoidal and triangular fuzzy numbers can be modeled in this representation as well. The presented approach of acquisition of uncertain information is well-suited to the second semantics of fuzzy numbers. Membership functions of fuzzy numbers in the third semantics can be acquired in a different way (see Slany, 1996).

2.2. Fuzzy arithmetics

Let $\tilde{A}$ and $\tilde{B}$ be fuzzy numbers of the universe X and Y. Let * denote any basic arithmetic operations (+, –, ×, /). Then any operation $\tilde{A} * \tilde{B}$ can be described by Zadeh's extension principle (Zadeh, 1975):

$$\mu_{\tilde{A}*\tilde{B}}(z) = \max_{x*y=z} \{\min [(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)]\} \tag{4}$$

In the case of arithmetic operations on fuzzy numbers in piece-wise representation the equation (4) corresponds to

$$\tilde{C}_{\alpha} = (\tilde{A} * \tilde{B})_{\alpha} = \tilde{A}_{\alpha} * \tilde{B}_{\alpha}, \text{ for any } \alpha \in (0, 1]. \tag{5}$$

Above equation shows that the α-cut on a general arithmetic operation on two fuzzy numbers is equivalent to the arithmetic operation on the respective α-cuts of the two fuzzy numbers. Both $(\tilde{A} * \tilde{B})_{\alpha}$ and $\tilde{A}_{\alpha} * \tilde{B}_{\alpha}$ are interval quantities the operations on which can make use of classical interval analysis (Ross, 1995). Thus one can define the following arithmetic operations for fuzzy numbers in six-point representation:

$$\tilde{A} \oplus \tilde{B} = \left(\underline{a}^{\varepsilon} + \underline{b}^{\varepsilon},\ \underline{a}^{\lambda} + \underline{b}^{\lambda},\ \underline{a} + \underline{b},\ \overline{a} + \overline{b},\ \overline{a}^{\lambda} + \overline{b}^{\lambda},\ \overline{a}^{\varepsilon} + \overline{b}^{\varepsilon}\right) \tag{6}$$

$$\tilde{A} \ominus \tilde{B} = \left(\underline{a}^{\varepsilon} - \overline{b}^{\varepsilon},\ \underline{a}^{\lambda} - \overline{b}^{\lambda},\ \underline{a} - \overline{b},\ \overline{a} - \underline{b},\ \overline{a}^{\lambda} - \underline{b}^{\lambda},\ \overline{a}^{\varepsilon} - \underline{b}^{\varepsilon}\right) \tag{7}$$

$$\tilde{A} \times \tilde{B} = \left(\underline{a}^{\varepsilon} \times \underline{b}^{\varepsilon},\ \underline{a}^{\lambda} \times \underline{b}^{\lambda},\ \underline{a} \times \underline{b},\ \overline{a} \times \overline{b},\ \overline{a}^{\lambda} \times \overline{b}^{\lambda},\ \overline{a}^{\varepsilon} \times \overline{b}^{\varepsilon}\right) \tag{8}$$

$$\tilde{A} / \tilde{B} = \left(\underline{a}^{\varepsilon} / \overline{b}^{\varepsilon},\ \underline{a}^{\lambda} / \overline{b}^{\lambda},\ \underline{a} / \overline{b},\ \overline{a} / \underline{b},\ \overline{a}^{\lambda} / \underline{b}^{\lambda},\ \overline{a}^{\varepsilon} / \underline{b}^{\varepsilon}\right) \tag{9}$$

and moreover:

$$\widetilde{\max}(\tilde{A}, \tilde{B}) = (\max(\underline{a}^{\varepsilon},\underline{b}^{\varepsilon}), \max(\underline{a}^{\lambda},\underline{b}^{\lambda}), \max(\underline{b},\underline{a}), \max(\overline{b},\overline{a}), \max(\overline{b}^{\lambda},\overline{a}^{\lambda}), \max(\overline{b}^{\varepsilon},\overline{a}^{\varepsilon})) \quad (10)$$

$$\widetilde{\min}(\tilde{A}, \tilde{B}) = (\min(\underline{a}^{\varepsilon},\underline{b}^{\varepsilon}), \min(\underline{a}^{\lambda},\underline{b}^{\lambda}), \min(\underline{b},\underline{a}), \min(\overline{b},\overline{a}), \min(\overline{b}^{\lambda},\overline{a}^{\lambda}), \min(\overline{b}^{\varepsilon},\overline{a}^{\varepsilon})) \quad (11)$$

2.3. Comparison of two fuzzy numbers

The definition of dominance and the combinatorial search in a fuzzy objective space require comparing fuzzy scores on objective functions. As it was stressed in the introduction, the way of comparing fuzzy scores is connected with the semantics of fuzzy numbers. Indeed, the fuzzy numbers can be seen as imprecise probability distributions (see Dempster, 1967; Shafer, 1976). In this perspective, the comparison of two fuzzy numbers can be substituted by the comparison of their mean values defined consistently with the well-known definition of expectation in probability theory. The idea exploited by Dubois and Prade (1987) rely on the mathematical fact that, with respect to a fuzzy number, the possibility measure corresponds to an upper probability distribution, while the necessity measure, to a lower probability distribution of the corresponding random variable in the sense of Dempster (1967). Then it is reasonable to define the mean value of a fuzzy number as a closed interval whose bounds are expectations of upper and lower probability distributions. The comparison of two fuzzy numbers boils then down to the comparison of arithmetic means of these bounds, which is computationally equivalent to another intuitive comparison principle based on the area compensation determined by the membership functions of two fuzzy numbers being compared (Kołodziejczyk, 1986; Chanas, 1987; Roubens, 1990; Fortemps and Roubens, 1996).

The main advantage of the area compensation method is that it answers not only which one of two fuzzy numbers is greater but also what is the degree to which one fuzzy number is greater than another one. Working on fuzzy scheduling problems (Hapke et al., 1997) the authors have used this comparison method under the name weak comparison rule (WCR) in order to distinguish it from a more restrictive comparison method used to observe precedence constraint and called strict comparison rule (SCR).

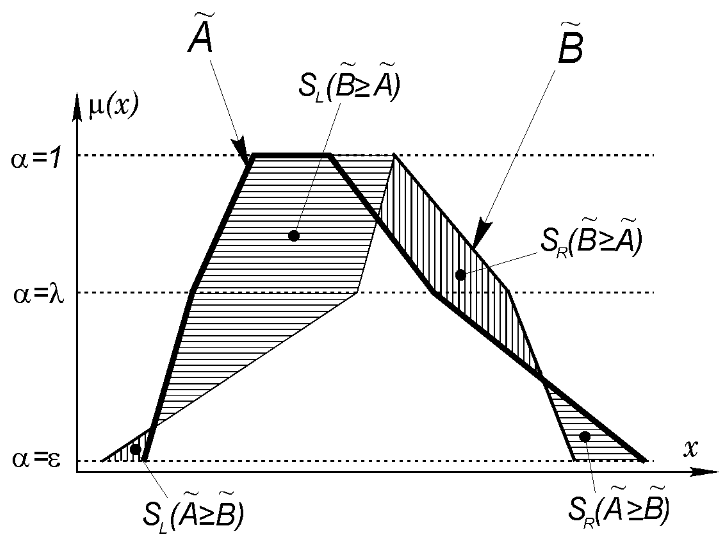


Figure 2. *The comparison of two fuzzy numbers based on area compensation*

Let $\tilde{A}, \tilde{B}$ be two fuzzy numbers and $S_L(\tilde{A} \geq \tilde{B})$, $S_R(\tilde{A} \geq \tilde{B})$, $S_L(\tilde{A} \leq \tilde{B})$, $S_R(\tilde{A} \leq \tilde{B})$ the areas determined by their membership functions (Figure 2):

$$S_L(\tilde{A} \geq \tilde{B}) = \int_{U(A,B)} (\inf_{x\in\mathbf{R}} A_\alpha - \inf_{x\in\mathbf{R}} B_\alpha) d\alpha \tag{12}$$

$$S_R(\tilde{A} \geq \tilde{B}) = \int_{V(A,B)} (\sup_{x\in\mathbf{R}} A_\alpha - \sup_{x\in\mathbf{R}} B_\alpha) d\alpha \tag{13}$$

where

$$U(\tilde{A},\ \tilde{B}) = \{\alpha \mid \inf_{x\in\mathbf{R}} A_\alpha \geq \inf_{x\in\mathbf{R}} B_\alpha, \varepsilon \leq \alpha \leq 1\} \tag{14}$$

$$V(\tilde{A},\ \tilde{B}) = \{\alpha \mid \sup_{x\in\mathbf{R}} A_\alpha \geq \sup_{x\in\mathbf{R}} B_\alpha, \varepsilon \leq \alpha \leq 1\} \tag{15}$$

$S_L(\tilde{A} \leq \tilde{B})$ and $S_R(\tilde{A} \leq \tilde{B})$ are defined analogously.

$S_L(\tilde{A} \geq \tilde{B})$ and $S_R(\tilde{A} \geq \tilde{B})$ are the areas that are related to the possibility of $A^* \geq B^*$, where A^* and B^* are potential realizations of $\tilde{A}$ and $\tilde{B}$. The degree $C(\tilde{A} \geq \tilde{B})$ to which $\tilde{A} \geq \tilde{B}$ is calculated as a sum of areas in which $A^* \geq B^*$ minus a sum of areas in which $A^* \leq B^*$.

Definition 1. (Weak Comparison Rule - WCR)

According to the WCR, the degree C to which fuzzy number $\tilde{A}$ is greater than or equal to $\tilde{B}$ is defined as:

$$C(\tilde{A} \geq \tilde{B}) = \frac{1}{2}\{S_L(\tilde{A} \geq \tilde{B}) + S_R(\tilde{A} \geq \tilde{B}) - S_L(\tilde{B} \geq \tilde{A}) - S_R(\tilde{B} \geq \tilde{A})\} \tag{16}$$

Using WCR, one can define three relations between $\tilde{A}$ and $\tilde{B}$, corresponding to weak inequality, strict inequality and equivalence between fuzzy numbers, respectively:

$$\tilde{A} \geq \tilde{B} \text{ iff } C(\tilde{A} \geq \tilde{B}) \geq 0 \tag{17}$$

$$\tilde{A} > \tilde{B} \text{ iff } C(\tilde{A} \geq \tilde{B}) > 0 \tag{18}$$

$$\tilde{A} \sim \tilde{B} \text{ iff } C(\tilde{A} \geq \tilde{B}) = C(\tilde{B} \geq \tilde{A}) = 0 \tag{19}$$

It is easy to note that $C(\tilde{A} \geq \tilde{B}) = -C(\tilde{B} \geq \tilde{A})$.

The proposed WCR is related to the comparison of "mean values of fuzzy numbers" (Fortemps, 1997), as defined in (Dubois and Prade, 1987) in the framework of the Dempster-Shafer theory. The mean value of a fuzzy number $\tilde{A}$ is the interval $[E(\tilde{A})_*, E(\tilde{A})^*]$, where $E(\tilde{A})_*$ and $E(\tilde{A})^*$ are the mean values related to the cumulative possibility distribution (left spread), and to the cumulative necessity distribution (right spread), respectively. The natural way of defuzzifying such an interval is to calculate the arithmetic mean of its limits. The defuzzification function $\Im(\tilde{A})$ is defined as follows (Chanas, 1987; Fortemps, 1997):

$$\Im(\tilde{A}) = \frac{E_*(\tilde{A}) + E^*(\tilde{A})}{2} \tag{20}$$

The following properties of the function $\Im(\tilde{A})$ and of the WCR will be useful.

Property 1. (Hapke, 1997)

For fuzzy number in the six-point representation, the function $\Im(\tilde{A})$ can be calculated as:

$$\Im(\tilde{A}) = \frac{1}{4(1-\varepsilon)}\left\{(\lambda-\varepsilon)(\underline{a}^{\varepsilon} + \underline{a}^{\lambda} + \overline{a}^{\lambda} + \overline{a}^{\varepsilon}) + (1-\lambda)(\underline{a}^{\lambda} + \underline{a} + \overline{a} + \overline{a}^{\lambda})\right\} \tag{21}$$

Property 2. (Fortemps, 1997)

The degree $C(\tilde{A} \geq \tilde{B})$ to which $\tilde{A}$ is greater than or equal to $\tilde{B}$, calculated according to the WCR, can be calculated using $\Im(\tilde{A})$ and $\Im(\tilde{B})$ as:

$$C(\tilde{A} \geq \tilde{B}) = \Im(\tilde{A}) - \Im(\tilde{B}) = \frac{E_*(\tilde{A}) + E^*(\tilde{A})}{2} - \frac{E_*(\tilde{B}) + E^*(\tilde{B})}{2} \tag{22}$$

From properties 1 and 2 immediately follows the formula for $C(\tilde{A} \geq \tilde{B})$ where $\tilde{A}$ and $\tilde{B}$ are six-point fuzzy numbers.

2.4. Fuzzy multi-objective combinatorial optimization and fuzzy dominance

Definition 2. (Fuzzy MOCO problem)

The general fuzzy multi-objective combinatorial optimization (fuzzy MOCO) problem is formulated as:

$$\max\{\tilde{f}_1(\mathbf{x}),\ldots,\tilde{f}_J(\mathbf{x})\} \text{ s.t. } \mathbf{x} \in D, \tag{23}$$

where: *solution* $\mathbf{x} = [x_1,\ldots,x_I]$ is a vector of discrete and crisp *decision variables*, D is a finite set of feasible solutions, $\tilde{f}_1,\ldots,\tilde{f}_J$ are fuzzy objective functions (criteria) which, for given $\mathbf{x}$ take values called *fuzzy scores*.

Thus, the image of solution $\mathbf{x}$ in the objective space is a vector $\tilde{\mathbf{f}}^{\mathbf{x}} = [\tilde{f}_1(\mathbf{x}),\ldots,\tilde{f}_J(\mathbf{x})]$ composed of J fuzzy numbers. We will say that $\tilde{\mathbf{f}}^{\mathbf{x}}$ is a *fuzzy point* in the objective space. In order to define a dominance relation in the objective space we will use the WCR for the fuzzy scores. The dominance relation defined in this way will be called *WCR-dominance.*

Definition 3. (WCR dominance)

Fuzzy point $\tilde{\mathbf{f}}^{\mathbf{x}}$ *WCR-dominates* fuzzy point $\tilde{\mathbf{f}}^{\mathbf{y}}$ iff $\tilde{f}_j(\mathbf{x}) \geq \tilde{f}_j(\mathbf{y})\ \forall j$ and $\tilde{f}_j(\mathbf{x}) > \tilde{f}_j(\mathbf{y})$ for at least one j, i.e. iff $C(\tilde{f}_j(\mathbf{y}) \geq \tilde{f}_j(\mathbf{x})) \geq 0\ \ \forall j$ and $C(\tilde{f}_j(\mathbf{y}) \geq \tilde{f}_j(\mathbf{x})) > 0$ for at least one j.

Definition 4. (WCR-non-dominated points and WCR-efficient solutions))

A fuzzy point $\tilde{\mathbf{f}}^{\mathbf{x}}$ is *WCR-non-dominated* (*WCR-Pareto-optimal*) if there is no other fuzzy point $\tilde{\mathbf{f}}^{\mathbf{y}}$ ($\mathbf{y} \in D$) that WCR-dominates $\tilde{\mathbf{f}}$. A solution $\mathbf{x}$ is *WCR-efficient* if its image is WCR-non-dominated. The set of all WCR-efficient solutions will be denoted by N.

Definition 5. (Fuzzy ideal point)

The fuzzy *ideal point* is a fuzzy point $\tilde{\mathbf{f}}^*$ in the objective space composed of the best attainable fuzzy values of objectives, i.e.:

$$\forall j = 1,\ldots,J\ \forall \mathbf{x} \in D\ \ \tilde{\mathbf{f}}_j^* \geq \tilde{\mathbf{f}}_j^{\mathbf{x}}\ . \tag{24}$$

Definition 6. (Fuzzy nadir point)

The fuzzy *nadir point* is a fuzzy point $\tilde{\mathbf{f}}_*$ in the objective space composed of the worst attainable fuzzy values of objectives in the set of WCR-non-dominated points, i.e.

$$\forall j = 1,...,J \ \forall \mathbf{x} \in N \ \ \tilde{\mathbf{f}}_j^* \leq \tilde{\mathbf{f}}_j^{\mathbf{x}} . \tag{25}$$

In Definitions 5 and 6, the fuzzy scores are compared with WCR.

3. Fuzzy Pareto simulated annealing - FPSA

In order to solve fuzzy MOCO problems we propose to use adaptation of Pareto simulated annealing (PSA) procedure proposed by Czyzak and Jaszkiewicz (1996, 1997) and extended in (Jaszkiewicz, 1996) for crisp MOCO problems. The adaptation of PSA to the fuzzy case will yield a procedure called fuzzy PSA (FPSA).

In the crisp case, PSA generates a set of approximately efficient solutions, considered as an approximate representation of the whole set of efficient solutions. In the fuzzy case, FPSA will generate a set of approximately WCR-efficient solutions *N'*. The goal of FPSA is to generate a set *N'* that would be a good approximation of the set of all WCR-efficient solutions *N*. The term “good approximation” means that the decision maker (DM) should be able to find in *N'* a solution close to the solution that he/she would select if set *N* was known.

Adaptation of PSA to fuzzy case requires:

- the use of WCR-dominance in updating the set of approximately WCR-efficient solutions,
- calculating the probability of accepting a new solution basing on fuzzy scores,
- updating the average position of a fuzzy point in the objective space.

In each iteration of the procedure, a sample (population) of solutions (fuzzy points in the objective space), called generating sample, is used. The main idea of FPSA is to assure a tendency for approaching the set of WCR-efficient solutions as well as for dispersing the solutions constituting the generating sample over the whole WCR-efficient set *N*. The first tendency is taken into account when calculating the probability of acceptance. The inclination for dispersing the solutions from the generating sample over the whole set *N* is obtained by controlling the weights of particular objectives used in an aggregation formula.

In the case of single objective SA a new solution is accepted with probability equal to one if it is not worse than the current solution. Otherwise, it is accepted with probability less than one. In the (fuzzy) multi-objective case, one of the following three situations may appear while comparing a new solution **y** with the current solution **x**:

- **y** may (WCR-)dominate **x**,
- **y** may be (WCR-)dominated by **x**,
- **y** and **x** may be mutually (WCR-)non-dominated.

In the first situation the new solution may be considered as not worse than the current one and accepted with probability equal to one. In the second situation the new solution may be considered as worse than the current one and accepted with probability less than one. Serafini (1994) and Ulungu (1993) have proposed several multi-objective rules for acceptance probability which in different way treat the third situation. In some previous experiments (see e.g. Czyzak and Jaszkiewicz, 1997) we observed that the following rule gives the best results in PSA:

$$P(\mathbf{x},\mathbf{y},T,\Lambda^{\mathbf{x}}) = \min\left\{1, \exp\left(\sum_{j=1}^{J} \lambda_j^{\mathbf{x}} \left(f_j(\mathbf{y}) - f_j(\mathbf{x})\right)/T\right)\right\}, \tag{26}$$

where **x** is one of the generating solutions, **y** is a solutions obtained from **x** be performing a randomly selected basic move, T is a parameter called temperature, $\Lambda^{\mathbf{x}} = \left[\lambda_1^{\mathbf{x}},...,\lambda_J^{\mathbf{x}}\right]$ is the vector of weights associated with generating solution **x**. In other words, the differences on particular objectives are locally aggregated with a simple weighted sum (see Figure 3). Hansen (to appear) has also observed that this kind of local aggregation gives better results than local aggregation with a weighted Tchebycheff function.

In FPSA the probability of acceptance is calculated as follows:

$$P(\mathbf{x},\mathbf{y},T,\Lambda^{\mathbf{x}}) = \min\left\{1, \exp\left(\sum_{j=1}^{J} \lambda_j^{\mathbf{x}} \left(C\left(\tilde{f}_j(\mathbf{y}) \geq \tilde{f}_j(\mathbf{x})\right)\right)/T\right)\right\}. \tag{27}$$

In other words the distance between two fuzzy scores $\tilde{f}_j(\mathbf{x})$ and $\tilde{f}_j(\mathbf{y})$ is calculated according to the WCR. Note that this rule assures that **y** is accepted with probability equal to 1 if it WCR-dominates **x**.

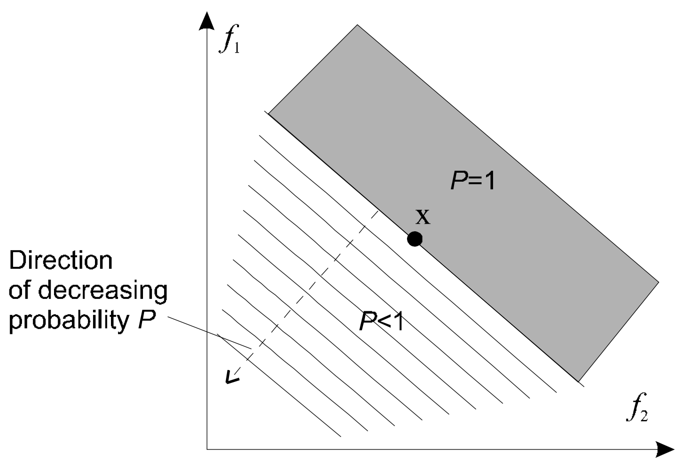


Figure 3. *Acceptance probability P in bi-objective case, maximized objectives*

Please note, that the higher the weight associated with a given objective, the lower the probability of accepting moves that decrease the value on this objective and the lower the probability of worsening the value of this objective. Consider for example the situation presented in Figure 4a. In this case $\lambda_1^{\mathbf{x}} > \lambda_2^{\mathbf{x}}$. Observe that probability of acceptance decreases rapidly when the value of the first objective decreases. So, by controlling the weights one can control the probability of worsening/improving values of the particular objectives.

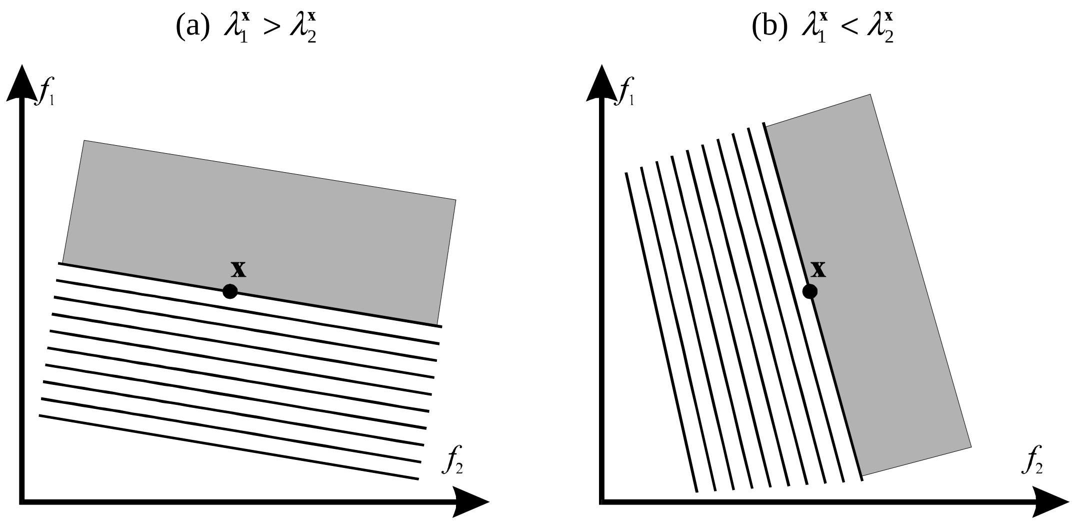


Figure 4. *The role of weights in the multi-objective rules for acceptance probability*

The outcome of the procedure is the set of approximately WCR-efficient solutions N'. At the beginning of the procedure, set N' is empty. In order to update the set, the following operations are performed, whenever a new solution **y** dominating **x** is generated:

- remove from set N' all solutions WCR-dominated by **y**,
- add **y** to N' if it is not WCR-dominated by any solution from this set.

The general scheme of the PSA procedure may be written as follows:

Select a starting sample of generating solutions $S \subset D$
for each $\mathbf{x} \in S$ **do**
    Update set N' of approximately WCR-efficient solutions with **x**
$T := T_o$
repeat
    Update the number of the generating solutions
    Update weights of all generating solutions
        for each $\mathbf{x} \in S$ **do**
            Construct $\mathbf{y} \in V(\mathbf{x})$
        if $\tilde{\mathbf{f}}^{\mathbf{y}}$ is not WCR-dominated by $\tilde{\mathbf{f}}^{\mathbf{x}}$ **then**
            Update set N' of approximately WCR-efficient solutions with **y**
        x := **y** (accept **y**) with probability $P(\mathbf{x}, \mathbf{y}, T, \Lambda^{\mathbf{x}})$
    if the conditions of changing the temperature are fulfilled **then**
        decrease T
until the stop conditions are fulfilled

where $V(\mathbf{x}) \subset D$ is the set of feasible solutions that can be obtained from **x** by performing a single move.

The goal of updating the weights is to assure a dispersion of generating solutions in the objective space. As, however, position of each generating solution may change significantly in subsequent iterations, it is more important to assure dispersion of average positions of generating solutions. The average position of solution **x** is a point in the objective space, denoted by $\mathbf{v}^{\mathbf{x}}$. In the standard PSA the average positions are calculated by exponential smoothing and updated whenever the generating solution is changed (a move is accepted). In the FPSA the fuzzy scores are defuzzified before applying the exponential smoothing:

$$v_j^{\mathbf{x}} := \beta v_j^{\mathbf{x}} + (1-\beta)\Im(\tilde{f}_j(\mathbf{x})),\ \mathrm{j} = 1,...,\mathrm{J}, \tag{28}$$

where $\beta < 1$ is smoothing factor close to one (e.g. β=0.95).

The algorithm for updating the weights is as follows:

<u>for each</u> $\mathbf{x} \in S$ **<u>do</u>**

Select solution $\mathbf{x}' \in S$ such that $\mathbf{v}^{\mathbf{x}'}$ is closest (e.g. according to Euclidean distance) to $\mathbf{v}^{\mathbf{x}}$ and such that $\mathbf{x}'$ and $\mathbf{x}$ are mutualy WCR-non-dominated

<u>if</u> there is no such solution $\mathbf{x}'$ or it is the first iteration with $\mathbf{x}$ **<u>then</u>**

Set random weights such that:

$$\forall_j \lambda_j^{\mathbf{x}} \geq 0 \text{ and } \sum_{\mathrm{j}} \lambda_j^{\mathbf{x}} = 1$$

<u>else</u>

<u>for each</u> objective f_j

$$\lambda_j^{\mathbf{x}} = \begin{cases} \alpha\lambda_j^{\mathbf{x}} & ,\text{if } \mathrm{v}_{\mathrm{j}}^{\mathbf{x}'} \geq \mathrm{v}_{\mathrm{j}}^{\mathbf{x}} \\ \lambda_j^{\mathbf{x}}/\alpha & ,\text{if } \mathrm{v}_{\mathrm{j}}^{\mathbf{x}'} < \mathrm{v}_{\mathrm{j}}^{\mathbf{x}} \end{cases}$$

normalize the weights such that $\sum_{\mathrm{j}} \lambda_j^{\mathbf{x}} = 1$

where $\alpha > 1$ is a constant close to one (e.g. $\alpha = 1.01$).

The size of the sample of the generating solutions is not constant but can be updated in each iteration. Some non-interesting solutions may be removed from the sample and some solutions may be duplicated. A solution is removed if in a specified number of iterations it is dominated by some other generating solutions. A solution is duplicated if in specified number of iterations it is neither dominated nor dominates any other generating solution. The starting size of the generating sample is equal to J+1 and typically increases when temperature decreases.

Please note, that except of weights updating, each of the generating solutions operates independently, so, the algorithm is naturally parallel.

As FPSA is a metaheuristic procedure, it defines only a general scheme of the calculations. This general scheme has to be customized for a given MOCO problem. The customization consists in defining the way a new solution is generated from the neighborhood of the current solution.

PSA and FPSA have been implemented in standard C++ as two libraries of classes. Both the implementations share the same set of most classes. FPSA includes classes that are responsible for comparing, storing, reading and performing mathematical operations on fuzzy numbers. The main aim of the object-oriented implementation was to assure easy adaptation to a given problem.

In order to use FPSA library to solve a given problem one is only required to define two new classes. One of the classes is responsible for reading and storing information about an instance of a given problem. Another class corresponds to a solution of a given problem. It should contain fields to store information about a single solution and operations responsible for finding the first solution, performing basic moves and storing the solutions. More information about the implementation can be found at http://www.cs.put.poznan.pl/~and_j/psa.html.

4. Application to an agricultural multi-objective project scheduling problem

We will illustrate the use of FPSA on a real example of project scheduling taken from an agricultural concern (see Słowiński et al., 1991). The project consists of 40 farm operations subject to precedence and resource constraints. The list of activities of the agricultural project is given in Table 1. A graphical representation of precedence constraints in the set of activities is given in Figure 5. The activities require two types of renewable resources: manpower (R^r_1) and tractors (R^r_2) available in 200 and 150 units, respectively. The only nonrenewable resource, money (R^n_2), is available in 20000 units at the beginning of the project. For majority of project activities there are specified three performing modes differing by resource requirements and duration.

Three following criteria are to be taken into account: the project completion time - T, the manpower resource smoothness - R (presented as an average deviation from the average resource usage) and the total project cost - C (the usage of nonrenewable resources). As durations of most the activities are given as fuzzy numbers, the project completion time is also a fuzzy number. We assume that a cost of a given activity depends partially on its actual duration, so it can be expressed as a fuzzy number. Thus, the total project cost is also a fuzzy number. Resource smoothness is a crisp objective. A more detailed description of the problem is available from the authors upon request.

1. harrowing wheat	21. tedding grassland
2. harrowing rape	22. raking grassland
3. sowing rape	23. cropping grassland
4. cropping harv. lupin	24. desication potato
5. drying lupine	25. cropping sugar beet
6. cropping of strorer	26. fertilisation lupin
7. cutting down lucerne	27. cropping fodder beet
8. cutting down grass	28. fertilization wheat
9. tedding grassland	29. ploughing
10. raking grassland	30. skimming lupin
11. cropping grassland	31. harrowing lupin
12. desication potato	32. harrowing wheat
13. harrowing wheat	33. sowing wheat
14. harrowing rape	34. cropping sugar beet
15. sowing rape	35. cropping potato
16. cropping harv. lupin	36. harrowing potato
17. drying lupine	37. ploughing
18. cropping of strorer	38. sowing corn
19. cutting down lucerne	39. fertilisation fodder
20. cutting down grass	40. ploughing corn

Table 1. *List of activities of the agricultural project*

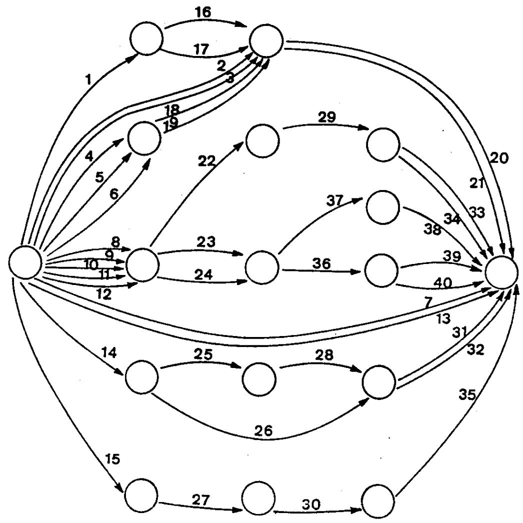
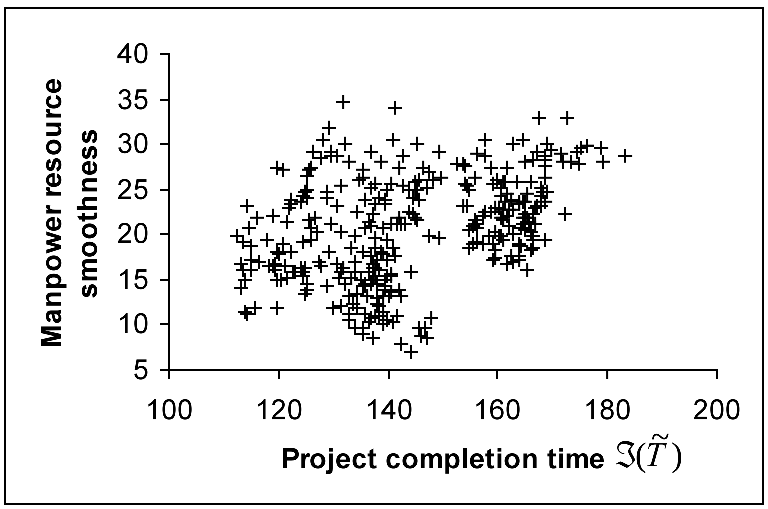
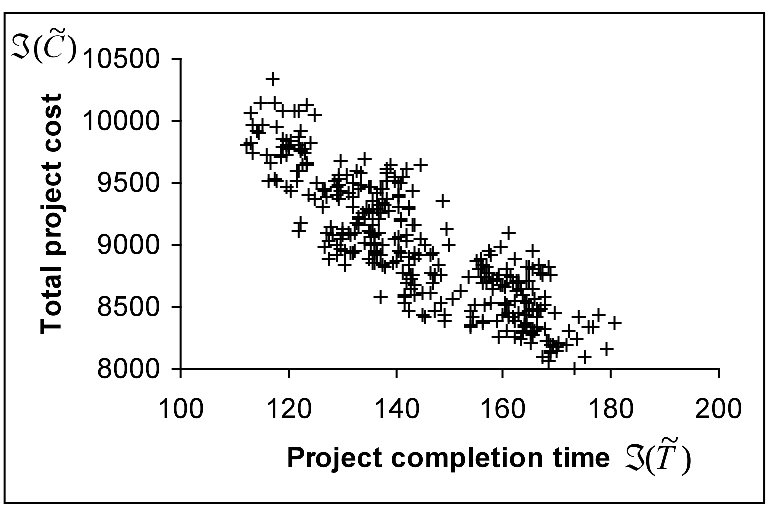


Figure 5. *Precedence constraints in the set of agricultural activities*

After performing 88000 steps of FPSA, the set N' of 364 approximately WCR-efficient solutions was found.





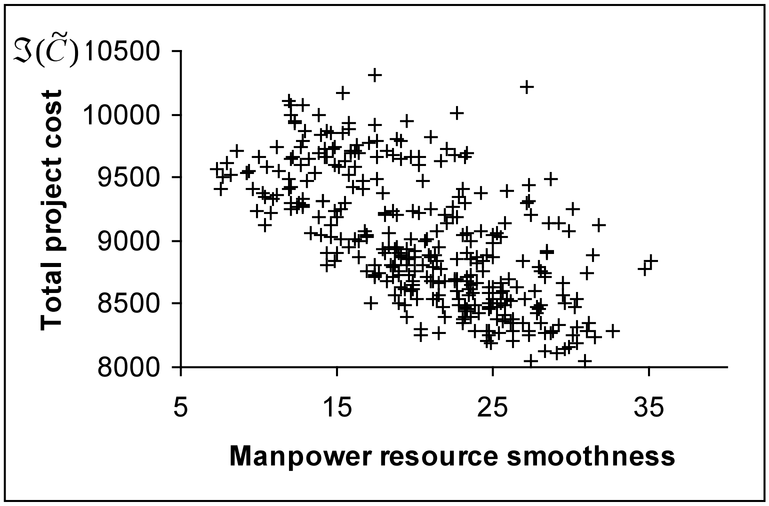


Figure 6. *Two-dimensional projections of the set N'*

The approximate fuzzy ideal point calculated form set *N'* is as follows:

$\tilde{T}$ = (116, 116, 117, 117, 120, 121), $\Im(\tilde{T})$ =117.8,

R = 9.29,

$\tilde{C}$ =(7937, 8028, 8210, 8210, 8302, 8393), $\Im(\tilde{C})$ =8117.5

while the approximate fuzzy nadir point is as follows:

$\tilde{T}$ = (174, 176, 180, 180, 182, 184), $\Im(\tilde{T})$ =179.3,

R = 33.56,

$\tilde{C}$ =(10000, 10086, 10172, 10172, 10431, 10517), $\Im(\tilde{C})$ =10234.8.

In Figure 6 three two-dimensional projections of the set *N'* are presented. Note, that the fuzzy scores were defuzziefied using formula (21).

Generation of 364 approximately WCR-efficient solutions does not complete the solution process. The DM should now select from set *N'* compromise solution that best fits his/her preferences. As the number of solutions is relatively large, the DM could be supported by an interactive procedure in the search for the best compromise solution over a set *N'*. Such a procedure has been described in (Hapke et al. 1997).

Before starting the interactive analysis, the DM may be interested in learning some general properties of the problem. To this aim, correlations of defuzzified values of the objectives in the set *N'* were calculated. The results are presented in Figure 7. One can observe significant negative correlation between project completion time and project cost. This is caused by the fact that activity performing modes with short duration are in general more expensive. Another interesting observation is significant positive correlation between project completion time and resource smoothness. Such relationship is not trivial and the DM may not expect it. The information presented in Figure 7 suggests that the DM should not expect a significant improvement of the project completion time at the expense of resource smoothness and vice versa. The two objectives, however, can be significantly improved by increasing the project cost.

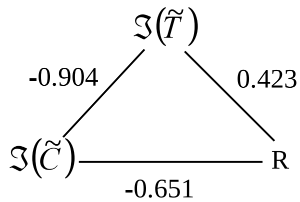


Figure 7. *Correlations of objectives in set N'*

In the case of fuzzy objectives the DM may not only be interested in their minimization but also in finding solutions with low uncertainty. We have tested a hypothesis that low (good) values of fuzzy objectives will also correspond to low uncertainty. In order to test it, we analyzed correlations between defuzzified values of fuzzy objectives and their degree of fuzziness. The degree of fuzziness of a fuzzy number $\tilde{M}$ was expressed as $Fz(\tilde{M}) = \overline{m}^{\varepsilon} - \underline{m}^{\varepsilon}$. The correlation between $\Im(\tilde{T})$ and $Fz(\tilde{T})$ was equal to 0.392, while correlation between $\Im(\tilde{C})$ and $Fz(\tilde{C})$ was equal to 0.432, so, the correlations although significantly above zero were relatively low. This suggests that the degree of fuzziness of some objectives may be used as a separate objective to be minimized.

Conclusions

We have proposed an extension of the Pareto simulated annealing multi-objective metaheuristic procedure to the case of fuzzy MOCO problems. The method does not work in the objective space of a defuzzified auxiliary problem but in the fuzzy objective space of the original MOCO problem. The defuzzification operator is used only when fuzzy solutions are compared or their average positions are updated.

The general concepts used to adapt PSA to the case of fuzzy MOCO problems may also be used to adapt some other multi-objective metaheuristic procedures to fuzzy MOCO problems.

The use of FPSA have been illustrated by its application to an agricultural project scheduling problem. It was shown that a simple statistical analysis of the set of approximately WCR-efficient solution delivers interesting information about correlations of particular objectives and correlations between mean values of fuzzy objectives and their degree of fuzziness.

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