

Comparison Study of SPEA2+, SPEA2, and NSGA-II in Diesel Engine Emissions and Fuel Economy Problem

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Abstract- Recently, the technology that can control NOx and Soot values of diesel engines by changing the electronically controllable parameters has been developed. However, there is a trade-off relationship between fuel economy and NOx values. Therefore, the diesel engines that can change their characteristics with along to the driving environment should be emerged in the future. For designing these kinds of engines, the Pareto solutions that can express the trade-off between fuel economy and NOx values are needed. In that case, the derived non dominated solutions should have the diversity not only in the objective space but also in the design variable space. SPEA2+ is one of multi objective genetic algorithms and is developed based on SPEA2. The derived non dominated solutions by SPEA2+ have the diversity in both objective space and design variable space. In this study, the diesel engines that have high fuel economy and small amounts of NOx and Soot are designed by SPEA2+. The results are compared with those of SPEA2 and NSGA-II. From the discussions, it is found that the solutions of SPEA2+ have the diversity not only in the objective space but also in the design variable space. These characteristics are very suitable for designing diesel engines whose parameters are changing against the driving environment.

1 Introduction

The diesel engine has excellent fuel economy, and is widely used especially in commercial vehicles, machine tools, and in large-scale ocean vessels. In addition, it is widely used in Europe because the levels of carbon dioxide in the exhaust are low. On the other hand, diesel engines are not widely used by the general public in Japan because the implementation of catalyzers in Japan lags considerably behind Europe. In addition, there are ever increasing concerns with regard to environmental problems, drawing attention to air pollution caused by the NOx (nitric oxide) and Soot exhaust from diesel engines. Restrictions on automobile engines are becoming increasingly severe every year. To satisfy these restrictions, it is necessary to develop a design method that is both quick and efficient. Therefore, the traditional trial and error design process is now shifting towards a process using computer simulations. Thus, construction of a system by computer simulation is very useful to design diesel engines that decrease amounts of NOx and Soot exhaust without adversely affecting the output or fuel economy.

When defining design parameters by computer simulation, optimization methods can be used. However, in real-world problems, such as the design of diesel engines, it is usually necessary to consider multiple items. In many cases, there are trade-off relationships between these items, and thus it is difficult to minimize or maximize these items simultaneously. Such processes cannot be handled by methods that optimize only one objective function. Thus, a multi-objective optimization method that can handle multiple objects is required.

Although many multi-objective optimization methods exist, evolutionary multi-objective optimization (EMO), which applies evolutionary computation to multi-objective optimization, has attracted a great deal of attention recently[1, 2, 3, 4]. There are many varieties of multi-objective genetic algorithms (MOGA), but the best studied is the application of genetic algorithms (GA) to the multi-objective optimization problem. In multi-objective optimization, a common goal is to obtain a Pareto-optimal set that indicates a trade-off relationship. Such studies are common because one characteristic of GAs is the multipoint search, and multiple Pareto-optimal sets can be obtained by a single search. Many MOGA have been proposed. However, SPEA2[3] proposed by Zitzler and NSGA-II[4] proposed by Deb provide excellent results as compared with other multi-objective genetic algorithms proposed. Here, we have proposed SPEA2+[5], an enhancement of SPEA2. In addition to the mechanism used by SPEA2 to efficiently search for a solution, mechanisms to obtain more accurate solutions, such as neighborhood crossover, and archives to obtain diverse solutions even in the design variable space, have been added to SPEA2+.

In this study, SPEA2+ was applied to the fuel emission scheduling problem in the diesel engine, and the results were compared with those obtained by SPEA2 and NSGA-II. The target objective functions were amounts of Specific fuel consumption (SFC), NOx and Soot. The design variables were parameters that can be controlled electronically, such as injection shape of the crank angle, and the start angle. The results indicated that SPEA2+ is suitable for fuel emission scheduling optimization of diesel engines, and multi-objective optimization is also a very effective tool suitable for designing diesel engines.

2 Multi-objective Genetic Algorithm

2.1 Multi-objective optimization problem

The multi-objective optimization problem is a problem of minimization or maximization of multiple evaluation criteria that conflict with each other. It is difficult to say that the solution that is an optimum for one criterion is the optimal solution for multi-objective optimization, because the multiple criteria have trade-off relationships with each other. Therefore, in multi-objective optimization, the concept of a Pareto-optimal solution is used in the search. In a Pareto-optimal solution, there are multiple, or sometimes an infinite number of solutions. In multi-objective optimization, as it is mentioned, getting a Pareto-optimal solution is one of the goals and an approach to obtain a wide range of Pareto-optimal solutions at equal intervals is required.

2.2 SPEA2+

Currently, there is a great deal of active research regarding algorithms and their applications in a variety of multi-objective problems. Among the multi-objective Genetic Algorithms reported to date, NSGA-II proposed by Deb and SPEA2 proposed by Zitzler show excellent results. These algorithms include important search mechanisms, such as preservation of good solutions discovered in the search and appropriate reduction of the possible Pareto-optimal solutions.

However, the crossover method, which is one of the GA operators, has not been discussed in these algorithms. In addition, mechanisms to maintain diversity in the design variable space have not been considered.

SPEA2+ is a new model of a multi-objective genetic algorithm that improves the search performance of SPEA2 by considering these problems. SPEA2+ is SPEA2 with the addition of the following mechanisms:

1. Neighborhood crossover to allow crossing over individuals located close to each other in objective space
2. Mating selection that reflects all good solutions preserved in the archive
3. Application of two archives to maintain diverse solutions in the objective space and the design variable space

Neighborhood Crossover

Neighborhood crossover crosses over individuals located close to each other in objective space. In the multi-objective genetic algorithm that performs a global search, because it is possible that parental individuals that are not close to each other in the objective space may cross over, there are problems in the search efficiency. Therefore, by crossing over individuals for which the search direction is the same, offspring individuals are generated close to the parental individuals.

Watanabe showed that the neighborhood crossover is very effective in the multi-objective genetic algorithm[6]. The mechanism of neighborhood crossover is shown below.

Step 1 Create an empty population (Q_t) . $i=0, k=0$.

Step 2 From search population (P_t) , search for individual x_0 which has either a maximum or a minimum value of the objective function, and add the individual x_0 to the population (Q_t) . Remove individual x_0 from the search population (P_t) .

Step 3 Search for individual x_{i+1} close to individual x_i in the objective space from the search population (P_t) , and add individual x_{i+1} to the population (Q_t) . Remove individual x_{i+1} from search population (P_t) .

Step 4 $i=i+1$. Repeat Step 3 until the search population (P_t) is empty and resume to Step 5 once empty.

Step 5 In the neighborhood crossover, two neighborhood individuals are paired. However, against population (Q_t) , perform a neighborhood shuffle with intervals of less than 10% of the total number of individuals in the population.

Two individuals are selected randomly within the arbitrary range. Then, they are replaced for crossover pair.

Step 6 Against the shuffled population (Q_t) , select individuals individual x_k and x_{k+1} .

Step 7 Perform a crossover between the two selected individuals.

Step 8 $k=k+2$, and once all population (Q_t) has been renewed, resume to Step 9; if not, return to Step 6.

Step 9 Copy population (Q_t) to empty search population (P_t) and end search.

In this study, the interval of the neighborhood shuffle was 4.5%.

Mating selection

Selection of the next generation's search population from the archive population is called mating selection. In SPEA2, two individuals are selected using binary tournament selection[7], and the individual with higher fitness is added to the next generation's search population.

However, in SPEA2, as the good solutions discovered during the search are preserved in the archive, in the latter half of the search, all the individuals preserved in the archive often form a non-dominated solution. Therefore, overlapping individuals may be selected as a search population when binary tournament selection is used, and the search becomes inefficient.

All of the good solutions preserved in the archive are copied in SPEA2+ to reflect the excellent solution preserved in the archive, and used as the next generation's search population.

Archives for good solutions in objective and design variable space

Mechanisms to maintain diversity in the design variable space are not considered in many multi-objective genetic algorithms. However, when the decision maker selects a solution from the obtained solution set, both the objective

function value and the design variable value are important. If comparable objective function values can be generated using a different design variable value, diversity of non-dominated solutions in the design variable space becomes very useful for the decision maker.

In SPEA2+, a new archive (design variable archive) is used to preserve good solutions in the design variable space in addition to the archive (objective archive) that preserves various good solutions in the objective space. Environmental selection[3] of SPEA2 is used to renew the design variable archive. However, when the number of non-dominated solutions exceeds the archive size of the design variable archive, the proximity of the individuals is obtained using the Euclidean distance based on the value of the design variable. Then, the archive truncation method[3] based on proximity is executed in SPEA2, and the number of individuals is reduced.

The algorithm of SEPA2+ is shown below:

- Step 1** Generate initial individual (P_0). Generation is $t=0$. Evaluate each individual, and assign fitness using SPEA2's fitness assignment method[3]. Copy this initial population to the objective archive population (OA_0), and design variable archive population (VA_0).
- Step 2** $t=t+1$. If the archive truncation method was used in Step 5, copy design variable archive population (VA_{t-1}) to the search population (P_t). If not, copy objective archive population (OA_{t-1}) to the search population (P_t).
- Step 3** If the objective archive population (OA_{t-1}) was copied to the search population (P_t), perform neighborhood crossover, mutation, and evaluation on the search population (P_t). If the design variable archive population (VA_{t-1}) was copied to the search population (P_t), perform neighborhood crossover based on the value of the design variable, mutation, and evaluation on the search population (P_t). Neighborhood crossover based on the value of the design variable perform on the base of the design variable space instead of the objective space. As a result, the search population (P_t) is renewed.
- Step 4** Assign fitness to the search population (P_t) using the fitness assignment method of SPEA2.
- Step 5** The search population (P_t), objective archive population (OA_{t-1}), and design variable archive population (VA_t) are joined, and the objective archive population (OA_t) and design variable archive population (VA_t) are renewed. At this time, an environmental selection of SPEA2 is used as a method of renewing the objective archive.
- Step 6** Judge whether the end condition is met. If the end condition is fulfilled, end search; if not, return to Step 2.

A design variable archive exists in SPEA2+, but is not directly related to genetic operation. However, in this study,

SPEA2+ is the design variable archive was added to genetic manipulation. When the number of non-dominated solutions exceeds the size of the archive, the design variable archive is copied to the next generation's search population. Conversely, when the number of non-dominated solutions is smaller than the archive size, the objective archive is copied to the next generation's search population. By reflecting the design variable archive in the search, more diverse solutions have possibilities of obtaining in the design variable space.

3 Diesel engine fuel emission scheduling problem

3.1 Outline of the diesel engine fuel emission scheduling problem

In this study, a diesel engine was designed to minimize the amounts of SFC, NOx and Soot. SFC is an index that when minimized the fuel economy is maximized. There are many design parameters for the diesel engine. In this study, we didn't target shape parameters, such as bore diameter and stroke length, we targeted parameters that can be controlled electronically, such as EGR, swirl rate, and fuel injection ratio. Target shape parameter was related to physical size are pre-determined by the specification, and the degree of design freedom is low. On the other hand, the parameters that can be controlled electronically are controllable or are new technologies that are becoming controllable, and will be used for engines in the near future. By targeting parameters that can be controlled electronically, the designed engine will not have one fixed solution but will have a dynamic design that can be adapted according to requirements. This is a so-called, flexible system, and will also be one of the forms of future engine design.

3.2 HIDECS

The simulation of diesel engine is very complicated. Therefore many researchers have proposed many models of diesel engine combustions. These models are classified into two categories: phenomenological model and detailed multi-dimensional model. In the past 30 years, the most sophisticated phenomenological spray-combustion model, HIDECS, has shown great potential as a predictive tool for both performance and emissions in a wide range of direct injection diesel engines. It was developed originally at the University of Hiroshima and was named eHIDECS only recently. A detailed discussion of this model and examples of its successful application are given in the references[8, 9, 10, 11, 12, 13, 14]. In this study, HIDECS was used as an analyzer to determine the target function values in optimization.

In HIDECS, the required calculation load is very light. KIVA code of a detailed multi-dimensional model is a well-known diesel engine combustion analyzer; however, this model requires a very large calculation load for analysis for one trial. The genetic algorithm used in the present study exhibits high optimum solution search ability. The downside is that the calculations must be repeated many

times. However, HIDECS allows use of the genetic algorithm within a practicable time frame.

3.3 Performance Metrics of Pareto solution

Evaluation of the solution set obtained is essential to evaluate the performance of the applied optimization method. Especially, in multi-objective optimization, the solution cannot be evaluated uniquely. In general, from the obtained solution set, we expect proximity to the Pareto-optimal front (accuracy), breadth against the entire Pareto-optimal front, and the equal distribution within the Pareto-optimal front.

In this study, the ratio of non-dominated individuals (RNI) was used to evaluate the accuracy of the obtained solution set. In addition, the cover rate was used to evaluate the breadth and to what extent the obtained solution has an even distribution.

To compare the accuracies of the two methods, their accuracies were evaluated based on the ratio of non-dominated individuals by obtaining the number of solutions that were inferior to the other. Let union of the solution sets S_1 and S_2 obtained by the two methods be S_U . From S_U , select non-dominated solutions, and let the selected solution set be S_P . Calculate ratio of each solution set against S_P . The closer this ratio is to 100%, the more superior it is compared to other methods. That is, it will obtain a solution close to the Pareto-optimal front.

The cover rate is the ratio of the number of divided areas, where each area is the area between the maximum and minimum values divided into arbitrary sizes, that has the obtained solution set, to the total number of divided areas. From the cover rate, we can evaluate the breadth of the area covered by the solution and how evenly distributed the solutions is against the Pareto-optimal front. The cover ratio I_{cover_k} against an objective function f_k can be obtained using equation 1. N is the number of divisions, and N_k is the number of divided areas covered by the solution set.

$$I_{cover_k} = \frac{N_k}{N} \quad (1)$$

Cover ratio I_{cover} can be obtained by calculating the average of the cover ratio (equation 2). Let M be the number of objective functions.

$$I_{cover} = \frac{1}{M} \sum_{k=1}^M I_{cover_k} \quad (2)$$

The maximum cover rate is 1 and the minimum value is 0. The closer the value is to the maximum value of 1, the solution set covers a greater area and is more evenly distributed. As the Pareto-optimal front was unknown in this study, the maximum and minimum values in each objective function were used values from experience. In addition, the number of divisions was defined as the number of individuals (Population Size)/2, and as the objective function is expressed by the logarithm, the area was divided by the logarithm (\log_{10}).

In this study, the diversity of the solutions should be discussed. Generally, the distribution of the solutions can be evaluated with generalized co-variance value. However, co-variance is not suitable for the solutions that have multiple peaks. In such a case, though the co-variance value is high, the diversity of the solutions is very low. This is the reason why we use cover rate for evaluating the diversity instead of co-variance. The cover rate of each design variable is evaluated in this study. When the cover rate is derived, the maximum and minimum values of design variable are used for the range and the division number is determined using the bit number of design variable for GA.

4 Numerical experiment

4.1 Target of Diesel engine

The Specification of targeting diesel engine is shown in Table 1.

Table 1: Specification of the target diesel engine

Bore (m)	0.1329
Stroke (m)	0.0825
Connected Rod (m)	0.26
Compress Ratio	14.7
Nozzle Diameter (m)	0.00029
Nozzle Number	8
Engine Speed (rpm)	2200

4.2 Injection shape of fuel

In this study, the amounts of SFC, NOx, and Soot were used as the objective function, and we tried to minimize them simultaneously. As shown in Fig. 1, the injection shape of the fuel is two-step injection where the fuel is injected in two pulses. Moreover, the fuel injection duration angles of the first and second pulses are the same, and the amount of total fuel is constant.

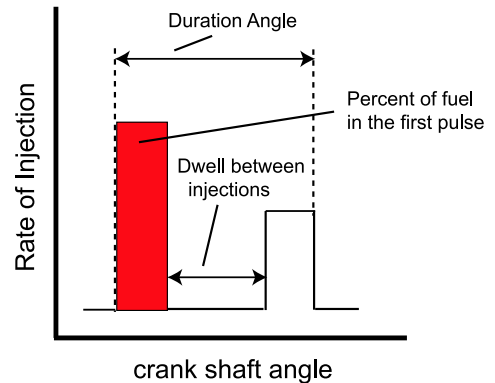


Figure 1: Description of two-pulse injection shape

To achieve two-step injection in this study, the percent-age of fuel in the first pulse, dwell between injections, and

duration angle are necessary. Therefore, two-step injection was achieved by handling these as design variables in this study. Moreover, start angle, exhaust gas recirculation (EGR), boost pressure, and swirl ratio are also handled as design variables in addition to those described above. These parameters can be controlled electronically. The constraint conditions of each design variable are shown in Table 2.

Table 2: Range of design variables

Item	Min	Max	bit for GA
Percentage of first pulse	50	84	7
Dwell between injections (CA)	3.0	15.0	7
Start Angle (ATDC)	-10.0	10.0	8
Duration Angle (CA)	25.0	40.0	5
Boost Pressure (kg/cm ²)	3.45	3.65	5
EGR rate	0.0	0.30	5
Swirl Ratio	0.0	6.0	5

The GA parameters used in this experiment are shown in Table 3.

Table 3: GA Parameter

Population Size	200
Terminal Generation	100
Crossover Rate	1.0
Mutation Rate	1/42
Runs	5

5 Results

5.1 Comparison between SPEA2+, SPEA2 and NSGA-II

Figures 2 ~ 5 show the solution sets obtained by SPEA2+, SPEA2, and NSGA-II. Fig. 6 shows the results of evaluation of the obtained solution set using the ratio of non-dominated individuals (RNI). Fig. 7 shows the results of evaluation of the obtained solution set using the cover rate.

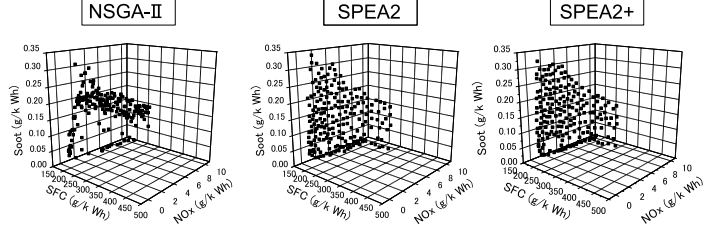


Figure 2: Pareto-optimal Solutions (SFC, NOx, Soot)

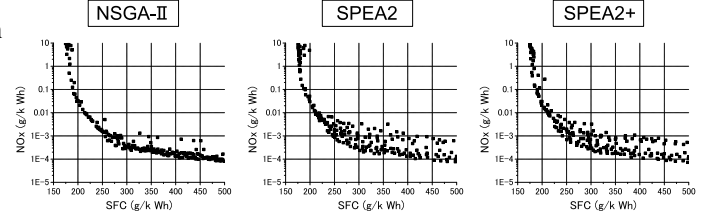


Figure 3: Pareto-optimal Solutions (SFC, NOx)

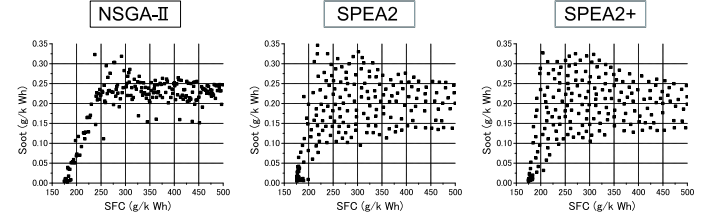


Figure 4: Pareto-optimal Solutions (SFC, Soot)

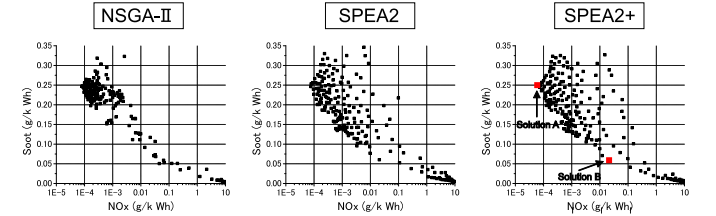


Figure 5: Pareto-optimal Solutions (NOx, Soot)

As shown Fig. 6, the accuracy of the solution set obtained by SPEA2+ was slightly higher than those of SPEA2 and NSGA-II. In addition, it was that the accuracies of the solution sets obtained by SPEA2 and NSGA-II were almost equivalent. From Fig. 7, it is evident that range covered and closeness of the interval of the solution set obtained

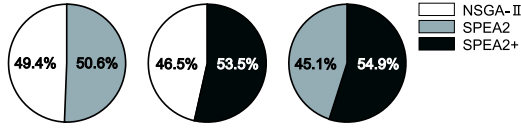


Figure 6: Comparison of SPEA2+, SPEA2, and NSGA-II by RNI

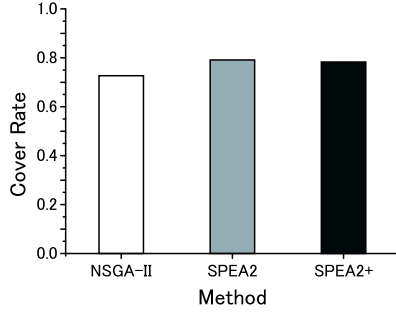


Figure 7: Comparison of SPEA2+, SPEA2, and NSGA-II by cover rate

with NSGA-II was slightly inferior to those of SPEA2 and SPEA2+. In addition, SPEA2+ and SPEA2 showed almost equivalent proximity and breadth of the obtained solution sets.

Figures 8 and 9 show the relations between NOx and start and duration angles from the solution sets obtained by SPEA2+, SPEA2, and NSGA-II.

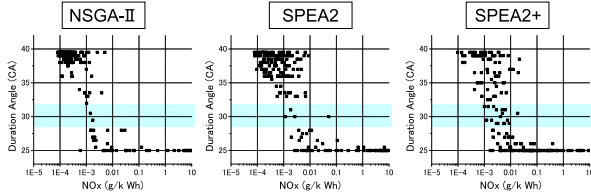


Figure 8: Relation of NOx and Duration Angle

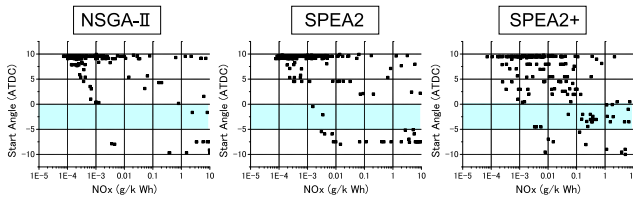


Figure 9: Relation of NOx and Start Angle

As shown Fig. 8 and Fig. 9, the design variable archive of SPEA2+ was the most diverse design variable values.

In Fig. 10, the cover rates of each design variable are shown for evaluating the diversity of the derived solutions in design variable space. The results of Fig. 10 are shown with along to the GA generation. These results of SPEA2+ are in the design variable archive.

From Fig. 10, it is confirmed that the solutions of

SPEA2+, besides of the results of the swirl ratio, have the higher diversity compared to the results of the other methods. Therefore, it is concluded that SPEA2+ can derive the solutions that have the diversity in the design variable space. On the other hand, the results of NSGA-II do not have the diversity compared to the other methods. In the results of start angle, the cover rates of the all results are low. 8 bit is used for this design variable. Therefore, there are 256 divisions for this design variable instead of the total population is 200.

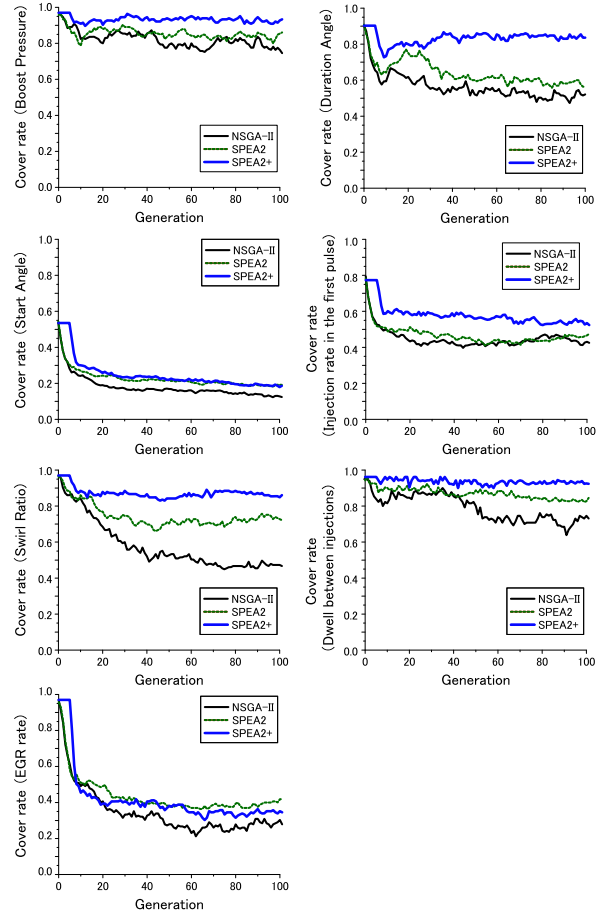


Figure 10: Cover rate of SPEA2+, SPEA2, and NSGA-II

The results of the experiment indicated that SPEA2+ showed slightly superior solution search ability as compared to SPEA2 and NSGA-II. In addition, by using two archives, SPEA2+ was able to obtain a more diverse solution set in the design variable space as compared to SPEA2 and NSGA-II. Therefore, we concluded that SPEA2+ is a very effective method for addressing the fuel emission scheduling problem of diesel engines.

The injection shape of solution A in Fig. 5 is shown in Fig. 11. While the minimum NOx value was obtained at this point, a very high SFC value of 0.245 (g/k Wh) was seen, and therefore this is not a realistic solution. On the other hand, when an alternative solution of about 0.06 (g/k Wh) is desired, the decision maker may obtain solution B, by referring to the shape of the Pareto front of Fig. 5. The injection shape of solution B is shown in Fig. 12. As dis-

cussed above, because many alternative solutions can be obtained as the Pareto solution set in multi-objective GA, it is very effective for diesel engine design.

Another advantage of using SPEA2+ is the diversity of the solution in the design variable space. In Fig. 8, when considering the Pareto solution obtained by SPEA2, a Pareto solution of duration angle around 30 cannot be selected. The same thing may be happened when NCGA-II is used. On the other hand, as the diversity of the design variable is considered in SPEA2+, a duration angle of around 30 can also be selected. This increases the possible choices, and becomes a great advantage for the decision maker.

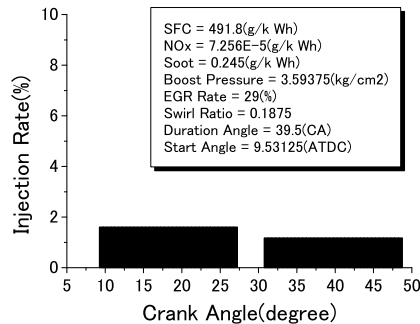


Figure 11: Injection Shape in Solution A

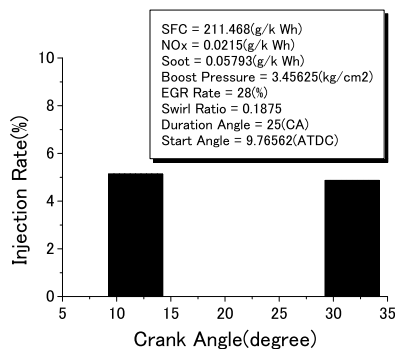


Figure 12: Injection Shape in Solution B

6 Conclusions

In this study, the effectiveness of SPEA2+ in the diesel engine fuel emission scheduling problem was compared with those of SPEA2, and NSGA-II. The results of the experiment indicated that SPEA2+ showed slightly superior solution search ability as compared to SPEA2 and NSGA-II. In addition, by using two archives, SPEA2+ was able to obtain a more diverse solution set in the design variable space as compared to SPEA2 and NSGA-II. Therefore, we concluded that SPEA2+ is a very effective method for addressing the fuel emission scheduling problem in diesel engine design.

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