Genetic Algorithm Approach to Production Ordering Problems with Inconsistent Constraints

Hitoshi Iima
Dept. of Electronics & Information Science
Kyoto Institute of Technology
Matsugasaki, Sakyo-ku, Kyoto 606-8585
Japan
iima@si.dj.kit.ac.jp

Nobuo Sannomiya
Dept. of Electronics & Information Science
Kyoto Institute of Technology
Matsugasaki, Sakyo-ku, Kyoto 606-8585
Japan
sanmiya@si.dj.kit.ac.jp

Makoto Wakasugi
Uniden Corporation
Hatchobori, Chuo-ku, Tokyo 104
Japan
kgg02030@niftyserve.or.jp

Abstract – **A method for solving a scheduling problem with many complicated constraints is proposed. For a case study of such problems, we deal with an optimal production ordering problem in acid rinsing process of a steelmaking plant. Strong requirements assigned to the constraints usually deteriorate the objective value, and in many cases it is impossible to satisfy all the constraints completely. Therefore, fundamental constraints of the problem are treated as hard constraints, the other constraints as soft or relaxable constraints. The genetic algorithm is applied to this problem. A two-phase method is proposed in such a way that in Phase 1, relaxable constraints conflicting strongly with the objective function are found, and in Phase 2, the Pareto optimal solutions for representing the tradeoff between the objective value and the relaxation of the constraints are obtained. This method is effective because computation load for tuning penalty parameters is decreased. Furthermore, the method gives a better result than the real operation data.**

I. INTRODUCTION

In order to develop a flexible manufacturing system, many theoretical and technical problems must be solved. A typical problem is to decide an optimal arrangement of products for production systems. It includes difficulties such as complicated constraints and many local optimum solutions. Especially strong requirements assigned to the constraints deteriorate the objective value, and in many cases make it impossible to satisfy all the constraints completely. Therefore the problem must be solved by finding out an appropriate point of compromise between attainment of the objective and requirement on several constraints. Sannomiya and Nishikawa [1] proposed a method for treating such a compromise for a linear programming problem. The present paper extends the method to a production ordering problem which can not be expressed as a linear programming problem.

The genetic algorithm (GA) [2] is applied to this problem. A two-phase method is proposed in such a way that in Phase 1, relaxable constraints conflicting with the objective function are found, and in Phase 2, the Pareto optimal solutions for representing the tradeoff between the objective value and the relaxation of the constraints are obtained.

As an application to a real problem, we execute an experimental trial of an optimal production ordering problem in an acid rinsing process of a steelmaking plant. Numerical results are compared between the present method and the penalty parameter method which was proposed previously by the authors [3].

II. PROBLEM STATEMENT

We consider the following problem:

(P1) min $Z(x)$ subject to

$$x \in X_1, \tag{1}$$

$$x \in X_2(y) \triangleq \{\, x \mid f_q(x) \le y_q \quad q = 1, 2 \cdots Q\}, \tag{2}$$

where x is the decision variable and $y \in R^Q$ is the parameter representing the set value of the constraint. In production ordering problems x is defined as an array of several elements. The constraint (1) is a hard constraint, that is, the constraint that can not be changed by planner's intention. We assume that X_1 has a feasible region.

The constraint (2) is called a soft constraint, that is, the constraint that could be relaxed, to some extent, by planner's consideration. The value of y is ideally wanted to set equal to y^*. But $X_1 \cap X_2(y^*)$ becomes infeasible for x. Then the problem is how to choose y for making a satisfactory compromise between the objective and the constraint.

III. OUTLINE OF ALGORITHM

A two-phase method is proposed in the following manner. In Phase 1 we solve the bicriterion problem given by

$$\text{(P2)} \quad \min \begin{bmatrix} Z(x) \\ F(x) \end{bmatrix} \text{ subject to (1),}$$

where $F(x)$ represents the degree to which the solution x does not satisfy (2) at $y = y^*$.

By applying GA to (P2) we obtain the Pareto optimal solutions. They give us information regarding the components of the soft constraint whose set values must be relaxed for getting feasibility of the original problem and/or for improving the objective value.

Since (P2) is a bicriterion problem, its Pareto optimal solutions are obtained as restricted ones. Therefore it is not reasonable to determine a preferred value of y on a basis of this solution set. Instead we select several components of the soft constraint as $q_1, q_2, \cdots, q_m \in \{1, 2, \cdots, Q\}$ $(m < Q)$. These m constraints are considered to have a strong conflict with the objective. The set values y_q of the remaining $(Q - m)$ soft constraints are fixed appropriately, and these constraints are appended to (1).

Thus in Phase 2 we solve the following multiobjective optimization problem:

$$\text{(P3)} \quad \min \begin{bmatrix} Z(x) \\ f_{q_1}(x) \\ \vdots \\ f_{q_m}(x) \end{bmatrix} \text{ subject to (1).}$$

Applying GA to (P3) leads to its Pareto optimal solutions from which we determine a suboptimal solution of (P1) as well as reasonable values of y_q for $q = 1, 2, \cdots, Q$.

IV. APPLICATION TO AN PRODUCTION ORDERING PROBLEM IN ACID RINSING PROCESS

A. Problem Statement

As shown in Fig. 1. we consider a process of rinsing hot strips with acid in a steelmaking plant. Hot strip coils are sent from a hot strip mill, and are made flat by an uncoiler. The strips are welded together by a welding machine in order to go through continuously in a rinsing tank. The function of the acid rinsing process is to remove an oxide film from the surface of the strip.

We decide the welding order of M strip coils, i.e. $\{C_1, C_2, \cdots, C_M\}$ so as to satisfy several constraints. The decision variable x is the welding order, i.e.

$$x = C_{k_1} C_{k_2} \cdots C_{k_M}, \quad k_i \in \{1, 2, \cdots, M\}. \tag{3}$$

The data of hot strip coils to be welded consists of the following nine items.

1) The thickness of strip [mm]
2) The width of strip [mm]
3) The inside diameter of coil (two classes : Class α and Class β)
4) The kind of oil (eight classes : Class 0, 1, 2 to Class 7)
5) The kind of edge (two classes : Class R and Class Y)
6) The kind of steel (two classes : Class N and Class S)
7) The quality of special steel (eight classes : Class A, B, C to Class I)
8) The grease painting (two classes : Class U and Class W)
9) The state of color of strip (two classes : Class γ and Class σ)

The objectives and the constraints for this problem are as follows:

(a) On the basis of coil characteristics, the coils are classified into L groups such as $\{G_1, G_2, \cdots, G_L\}$. In the acid rinsing process, the coils belonging to a same group must be arranged successively.

(b) A set-up cost Z_D is defined for the welding order of L groups. A smaller value of Z_D is better.

(c) The smaller difference in thickness is better in the case where the strips whose edge is of Class Y are welded together. The smaller difference in width is better in the case where the strips whose edge is of Class R are welded together. These differences are called a set-up cost Z_T.

(d) As for an arrangement of coils, an extraordinary difference of thickness or width of the strip between successive coils is not allowed. For such an arrangement, an additional strip should be inserted between them in order to decrease the difference. But few additional strips are better.

(e) There are three constraints P_{G_j} $(j = 1, 2, 3)$ for the welding order of L groups. If possible, we want to obtain a solution satisfying the constraints.

(f) There are ten constraints P_{C_j} $(j = 1, 2, \cdots, 10)$ for the welding order of M coils. If possible, we want to obtain a solution satisfying the constraints.

Item (a) is a hard constraint. Item (b), Item (c) and Item (d) are objectives. Item (e) and Item (f), which are explained in detail in our earlier paper [3], are soft constraints.

Thus, we have the following optimization problems:

$$\text{(P4)} \quad \min Z = Z_D + Z_A + Z_T,$$

$$Z_D = \sum_{\ell=1}^{L-1} d(G_{k_\ell}, G_{k_\ell+1}), \tag{4}$$

$$Z_A = v_1 A, \tag{5}$$

$$Z_T = v_2 \sum_{m=1}^{M-1} h(C_{k_m}, C_{k_{m+1}}) + v_3 \sum_{m=1}^{M-1} w(C_{k_m}, C_{k_{m+1}}), \tag{6}$$

subject to

$$Z_I = 0 \quad \text{(a hard constraint)}, \tag{7}$$

$$Z_G = 0 \quad \text{(a soft constraint)}, \tag{8}$$

$$Z_C = 0 \quad \text{(a soft constraint)}, \tag{9}$$

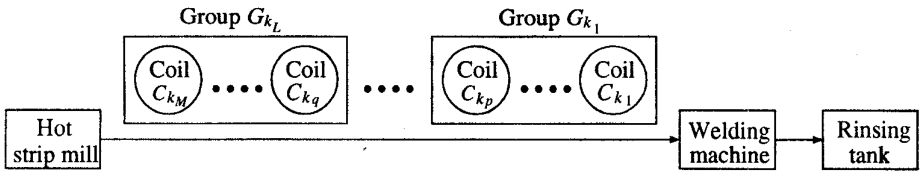


Fig. 1. Production process

$$Z_G = \sum_{j=1}^{3} Z_G^j, \tag{10}$$

$$Z_C = \sum_{j=1}^{10} Z_C^j, \tag{11}$$

where

C_{k_m} : The coil arranged in the m-th order for the coil order given by (3)

G_{k_ℓ} : The group arranged in the ℓ-th order for the group order shown in Fig. 1

$d(G_{k_\ell}, G_{k_{\ell+1}})$: The set-up cost for the group sub-order $\{G_{k_\ell} G_{k_{\ell+1}}\}$

A : The number of additional strips

$h(C_{k_m}, C_{k_{m+1}})$: $\begin{cases} \text{The difference in thickness between } C_{k_m} \text{ and } C_{k_{m+1}} \\ \text{(if the edges of both } C_{k_m} \text{ and } C_{k_{m+1}} \text{ are of Class } Y) \\ 0 \quad \text{(otherwise)} \end{cases}$

$w(C_{k_m} C_{k_{m+1}})]$: $\begin{cases} \text{The difference in width between } C_{k_m} \text{ and } C_{k_{m+1}} \\ \text{(if the edges of both } C_{k_m} \text{ and } C_{k_{m+1}} \text{ are of Class } R) \\ 0 \quad \text{(otherwise)} \end{cases}$

$v_i (i = 1, 2, 3)$: The weight coefficients

Z_1 : The number of times at which the constraint (a) is violated in the coil order

Z_G^j : The number of times at which the constraint P_{G_j} is violated in the group order

Z_C^j : The number of times at which the constraint P_{C_j} is violated in the coil order.

B. Formulation in Phase 1

In Phase 1 we solve the problem given by

$$\text{(P5)} \quad \min \begin{bmatrix} Z \\ F = Z_G + Z_C \end{bmatrix} \text{subject to (7).}$$

C. Application of GA in Phase 1

C.1. Outline of the Algorithm

We consider a set of individuals $p_n(t)$ $(n = 1, 2, \cdots, N)$, belonging to the population $P(t)$ at generation t. The population size N is assumed to be constant, irrespective of generation. The genetic operators generate the population $P(t+1)$ at the next generation $t+1$.

The algorithm is summarized as follows:

Step 1. Select at random the initial population $P(1)$. Set $t \leftarrow 1$.

Step 2. (Reproduction). Calculate the fitness functions for individuals, and reproduce the individuals of $P(t)$ according to the distribution of their fitness values.

Step 3. Go to Step 4 with probability P_c, or go to Step 5 otherwise.

Step 4. (Crossover). Mate the members of $P(t)$ at random, and carry out the crossover operation for each pair of individuals. Then we have a new population.

Step 5. Go to Step 6 with probability P_m, or go to Step 7 otherwise.

Step 6. (Mutation). Carry out the mutation operation for $p_n(t)$ selected at random, and a new individual is obtained. By replacing $p_n(t)$ with the new one, we have a new population.

Step 7. The population thus obtained is defined as $P(t+1)$.

Step 8. If $t = t^*$, the individual with the highest fitness value is adopted as a suboptimal solution of the problem. If $t < t^*$, set $t \leftarrow t+1$ and return to Step 2.

C.2. Individual Description

In (P4) we have to decide both the group order and the order of coils in each group as a solution. The solution has to satisfy the hard constraint (a). For this purpose each individual is represented in terms of $L+1$ sequences $X, Y_1, Y_2, \cdots, Y_L$. X is the welding order of the groups, i.e. $X = G_{k_2} G_{k_3} \cdots G_{k_{L-1}}$. Y_ℓ is the welding order of the coils which belong to G_ℓ.

Since the first coil (C_{k_1}) and the last coil (C_{k_M}) are fixed in advance, they do not exist in the sequences. Similarly, since the first group (G_{k_1}) and the last group (G_{k_L}) are also fixed in advance, they do not exist in the sequence.

From the individual description mentioned above, a solution is determined by the following decoding rule.

Step 1. Set $x \leftarrow C_{k_1}$.

Step 2. Set $x \leftarrow xY_{k_1}$.
Step 3. Set $s \leftarrow 2$.
Step 4. Take out G_{k_s} from X, then set $x \leftarrow xY_{k_s}$.
Step 5. If $s = L-1$, go to Step 6. If $s < L-1$, set $s \leftarrow s+1$ and return to Step 4.
Step 6. Set $x \leftarrow xY_{k_L}$.
Step 7. Set $x \leftarrow C_{k_M}$.

In the case where L=4, M=10, $G_1 = \{C_1, C_2, C_3\}$, $G_2 = \{C_4, C_5\}$, $G_3 = \{C_6, C_7\}$ and $G_4 = \{C_8, C_9, C_{10}\}$, an example of decoding is as follows:

$$\begin{array}{l} X : G_3 G_2 \\ Y_1 : C_3 C_2 \\ Y_2 : C_5 C_4 \\ Y_3 : C_6 C_7 \\ Y_4 : C_8 C_9 \end{array} \quad \rightarrow \quad x : C_1 C_3 C_2 C_6 C_7 C_5 C_4 C_8 C_9 C_{10}.$$

C.3. Genetic Operators

As the reproduction operator, we use Pareto reservation strategy proposed by Tamaki et al. [4]. In the method, non-dominated individuals are all reserved in the next generation. If the number of non-dominated individuals are less than population size, the rest of the population is filled by adopting the parallel selection method. In the parallel selection method, which is proposed by Schaffer [5], sub-populations are reproduced separately from the current population according to each objective. The strategy proposed by Sannomiya et al. [6] is used for each sub-population.

The cycle crossover [2] is used as the crossover operator. The mutation operator is constructed by exchanging two genes selected randomly. The genetic parameters are given as follows:

Population size	= 500,
Maximum generation	= 5000,
Crossover rate	= 0.7,
Mutation rate	= 0.1.

D. Computational Results in Phase 1

As an example we treat a case where L=7 and M= 200. The data of this case study is obtained from a real process. The details are omitted here. The weight coefficients are set to be v_1=2000 and v_2=v_3=1.

Fig. 2. shows the Pareto optimal solutions obtained by solving (P5) in Phase 1. We have 21 Pareto optimal solutions. For each Pareto optimal solution, Table I shows the objective function value and the number of times at which each constraint is violated.

Let us find out the soft constraints having a strong conflict with the objective. From Table I we confirm that Z_G^1, Z_C^1, Z_C^2, Z_C^4, Z_C^5 and Z_C^8 have the strong conflict with Z. Moreover Z_C^7 has another strong conflict with them.

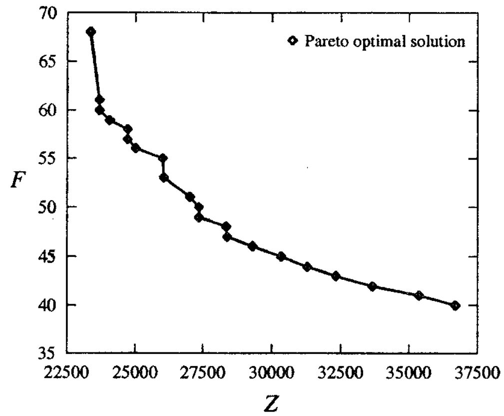


Fig. 2. Distribution of Pareto optimal solutions for (P5)

E. Formulation in Phase 2

From the results in Phase 1, we formulate (P3) with m=2 in Phase 2. The values of $\{Z_G^1, Z_C^1, Z_C^2, Z_C^4, Z_C^5, Z_C^8\}$ and Z_C^7 are defined as two kinds of objective function. In addtion to (7), the remaining soft constraints are treated as the constraints in such a way that Z_G^2=0, $Z_G^3 \leq 2$, $Z_C^3 \leq 7$, $Z_C^6 \leq 2$ and $Z_C^9 \leq 4$.

Thus the following problem is solved as (P3):

$$\text{(P6)} \quad \min \begin{bmatrix} Z \\ f_1 = Z_G^1 + Z_C^1 + Z_C^2 + Z_C^4 + Z_C^5 + Z_C^8 \\ f_2 = Z_C^7 \end{bmatrix},$$

subject to (7) and

$$Z_G^2 = 0, \tag{12}$$
$$Z_G^3 \leq 2, \tag{13}$$
$$Z_C^3 \leq 7, \tag{14}$$
$$Z_C^6 \leq 2, \tag{15}$$
$$Z_C^9 \leq 4. \tag{16}$$

F. Application of GA in Phase 2

The design of GA in Phase 2 is the same as that in Phase 1, except that population size =750. If an individual violates even one of the constraints (7), (12), (13), (14), (15) and (16), the penalty value is added to the objective function values Z, f_1 and f_2 respectively.

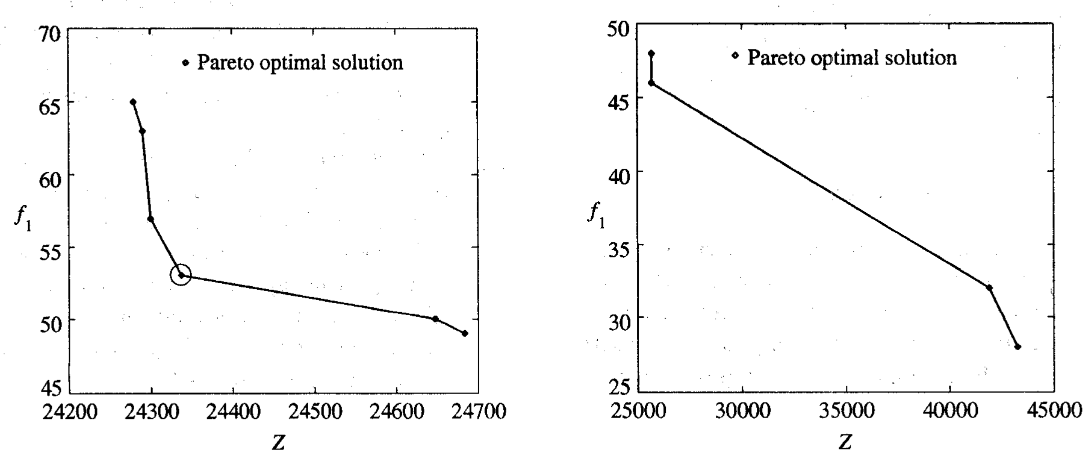
G. Computational Results in Phase 2

Fig. 3. shows the Pareto optimal solutions for (P6), where ten solutions are obtained. From this figure we can determine a preferred solution, an example of which is shown by a circle. The result of Phase 2 is shown in Table II, where a comparison is made among an operation data, Phase 2 of the proposed method and the penalty parameter method. In this table Z' is

TABLE I
ALL PARETO OPTIMAL SOLUTIONS FOR (P5)

Z	F	Z_D	A	Z_T	Z_G^1	Z_G^2	Z_G^3	Z_C^1	Z_C^2	Z_C^3	Z_C^4	Z_C^5	Z_C^6	Z_C^7	Z_C^8	Z_C^9
23359.97	68	2400	19	1959.97	2	0	2	6	4	6	26	4	1	0	15	2
23695.67	61	2400	19	2295.67	2	0	2	2	4	6	27	4	1	0	11	2
23698.07	60	2400	19	2298.07	2	0	2	2	4	6	26	4	1	0	11	2
24037.42	59	2400	19	2637.42	2	0	2	2	4	6	26	4	1	0	9	3
24718.87	58	2400	20	2318.87	2	0	2	2	4	4	26	4	1	0	11	2
24728.85	57	2400	20	2328.85	1	0	2	2	4	6	24	3	1	1	11	2
25028.61	56	2400	20	2628.61	2	0	2	2	2	7	25	3	1	0	9	3
26007.25	55	2400	21	2607.25	2	0	2	2	2	7	24	3	1	0	9	3
26049.39	53	2400	21	2649.39	1	0	2	2	2	7	23	2	1	1	9	3
27018.56	51	2400	22	2618.56	1	0	2	2	1	7	23	2	2	1	8	2
27340.66	50	2400	22	2940.66	1	0	2	2	1	7	24	2	1	1	5	4
27343.06	49	2400	22	2943.06	1	0	2	2	1	7	23	2	1	1	5	4
28321.70	48	2400	23	2921.70	1	0	2	2	1	7	22	2	1	1	5	4
28363.86	47	2400	23	2963.86	1	0	2	2	1	5	23	2	1	1	5	4
29293.90	46	2400	24	2893.90	1	0	2	2	1	5	22	2	1	1	5	4
30345.70	45	2400	25	2945.70	1	0	2	2	1	5	21	2	1	1	5	4
31311.10	44	2400	26	2911.10	1	0	2	2	1	5	21	2	1	1	5	3
32312.30	43	2400	27	2912.30	1	0	2	2	1	5	20	2	1	1	5	3
33641.30	42	2400	28	3241.30	1	0	2	2	1	5	20	2	1	1	5	2
35365.44	41	2400	30	2965.44	1	0	2	2	1	6	18	1	1	1	5	3
36694.44	40	2400	31	3294.44	1	0	2	2	1	6	18	1	1	1	5	2

Note : $Z_A = v_1 A = 1000A$



(a) f_2=0 (b) f_2=1

Fig. 3. Distribution of Pareto optimal solutions for (P6)

TABLE II

COMPARISON OF THE RESULT AMONG OPERATION DATA, THE PROPOSED METHOD AND THE PENALTY PARAMETER METHOD

	Operation Data	Proposed Method	Penalty Parameter Method
Z_D	2400	2400	2400
A	24	20	19
Z_T	3077.68	1937.06	2216.42
Z_G^1	2	2	2
Z_G^2	0	0	0
Z_G^3	2	2	2
Z_C^1	2	4	2
Z_C^2	4	4	4
Z_C^3	6	6	4
Z_C^4	41	27	24
Z_C^5	4	4	4
Z_C^6	2	2	1
Z_C^7	0	0	0
Z_C^8	6	12	11
Z_C^8	3	2	2
Z_C^{10}	0	0	0
Z_{total}	72	65	56
Z'	36477.68	31997.06	29636.42

the value of the objective function defined in the penalty parameter method, and Z_{total} is the total number of times at which any soft constraint is violated. The penalty parameter method was proposed previously by Sannomiya et al. [3]. In that method skillful adjustment of many penalty parameters is necessary, and consequently the best result is obtained from the viewpoints of A, Z_{total} and Z'. On the other hand, the proposed method does not need any parameter adjustment, and the result obtained is better than the operation data and is similar to the penalty parameter method.

V. CONCLUSION

A two-phase method with multicriteria has been proposed for solving scheduling problems with inconsistent constraints. In the two-phase method, some of the constraints are relaxed. In each phase the genetic algorithm has been applied. The proposed method gives us a reasonable solution for a coupled business of mathematical modeling and decision making in a scheduling problem, that is, simultaneous determination of both the set values in the soft constraints and the preferred solution.

As an application to a real problem, we have executed an experimental trial of an optimal production ordering problem in an acid rinsing process of a steelmaking plant. It is concluded from the numerical results that the proposed method is effective because computation load for tuning penalty parameters is decreased. Furthermore, the method gives a better result than the real operation data and results in a similar accuracy as compared to the penalty parameter method.

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