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New Multiobjective Fuzzy Optimization Method and Its Application

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Abstract

This paper proposes a new multiobjective fuzzy optimization method. First, the unsatisfying function, which is more useful and effective as the expression of fuzziness for optimization problems than the membership function, is introduced. The multiobjective optimization problem is transformed into a satisficing problem by using aspiration levels, and the fuzzy satisficing problem is formulated. Then, the interactive design method to minimize the maximum unsatisfaction rating by Genetic Algorithm is proposed. The effectiveness of the proposed method is demonstrated by the design example of an active suspension system. The trade-off graph is used in order to seek a satisficing solution, which reflects the designer's preference, more interactively and graphically.

Keywords: multiobjective optimization, satisficing problem, fuzzy set theory, unsatisfying function, genetic algorithm, active suspension system

1. Introduction

In design problems, it is often required to minimize or maximize several objectives subject to several constraints, and such a problem is formulated as a multiobjective optimization problem. Since there is usually no complete optimal solution, it is important to adjust trade-off between conflicting objectives in order to obtain a Pareto optimal solution. The multiobjective optimization problem can be transformed into a satisficing problem by introducing aspiration levels for the objectives. Since the aspiration levels are usually decided according to the designer's subjective estimation or judgement based on some knowledge, information, experience, etc., they contain fuzziness. And the constraints also have impreciseness or uncertainty. Thus, it is effective to introduce the fuzzy logic [1] and formulate the problem as a fuzzy optimization (or satisficing) problem [2]. In the conventional fuzzy logic, fuzziness is defined by the membership function, and in fuzzy mathematical programming methods, the membership function usually represents the satisfaction rating (degree of satisfaction) for the constraints. However, the problem formulation and the calculation are more complicated than those of other (non-fuzzy) mathematical programming methods. Furthermore, especially when design specifications are severe, it seems preferred for the designer to formulate the design problem as the minimization of unsatisfaction rating rather than the maximization of satisfaction rating. From such viewpoints, Sunaga *et al.* [3] have proposed to express fuzziness by the unsatisfying function and formulate the fuzzy optimization/satisficing problem by using the unsatisfaction rating.

This paper treats the multiobjective optimization/satisficing problem with fuzziness. First, we introduce the unsatisfying function and the unsatisfaction rating. The fuzzy satisficing problem is formulated as the minimization of the maximum unsatisfaction rating. The effectiveness of the proposed method is demonstrated by the design example of an active suspension system, which is the simultaneous optimization of a structure and control system. Genetic Algorithms (GAs) are effective tools to solve multiobjective optimization problems [4], [5]. Since they propose some solutions by one search, the designer can select a preferred solution. Thus, we apply a GA to solve the problem, and the trade-off graph [6] is used in order to seek the satisficing solution, which reflects the designer's preference, more interactively and graphically.

2. Problem Formulation

In this section, we introduce the unsatisfying function [3] and its application to multiobjective optimization problems.

2.1 Unsatisfying Function

The membership function $\mu(x)$ used in the fuzzy logic is of a range [0, 1], and it can be transformed into the following function $\tau(x)$ with a range [0, ∞]:

$$\tau(x) = \frac{1}{\mu(x)} - 1, \tag{1}$$

$$\mu(x) = \frac{1}{1+\tau(x)}. \tag{2}$$

The relationship between these two functions are shown in **Fig.1**. The function $\tau(x)$ is named *the unsatisfying function*, and its value is called *the unsatisfaction rating.*

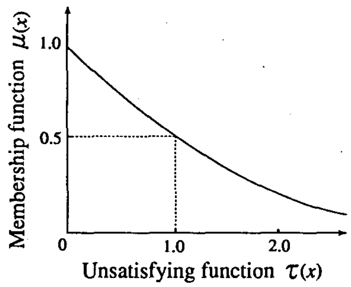

Fig.1 Relationship between membership function $\mu(x)$ and unsatisfying function $\tau(x)$.

The unsatisfying function is similar to the penalty function used in the mathematical programming (as shown in Fig.2 in sub-section 2.2). The membership function and the unsatisfying function take one-to-one correspondence, and it is possible to express fuzzy set theory by using the unsatisfying function.

In satisficing problems, as shown in the next sub-section, it is usually bad to exceed restriction values. Since it can be interpreted that

$\tau(x) = 0$ ($\mu(x) = 1$); can be sufficiently satisfied,
$\tau(x) = 1$ ($\mu(x) = 0.5$); can be accepted to this level,
$\tau(x) = \infty$ ($\mu(x) = 0$); cannot be accepted,

it is easier for a designer (decision maker) to define fuzziness by the unsatisfying function than the membership function. Furthermore, in convex programming problems where the objective function and the constraint functions are convex, the local optimal solution becomes the global optimal solution. Thus, from the viewpoint of calculation for optimization, it is convenient to treat convex functions. In general, the membership function is not a convex function, but the unsatisfying function can be easily set up to the convex function. Therefore, it can be considered that the unsatisfying function is more useful than the membership function as the expression of fuzziness in optimization problems.

2.2 Formulation of Fuzzy Satisficing Problem

In real world design problems, there are usually multi-objectives and multi-constraints, and they are formulated as the following multiobjective optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \{f_1(x), \cdots, f_m(x)\}, \\ &\text{subject to} \quad g_i(x) \le b_i \ (i = m+1, \cdots, M), \end{aligned} \tag{3}$$

where x is the vector of decision variables, $f_i(x)$ is the objective function, and $g_i(x) \le b_i$ is the constraint. Usually, there is no complete optimal solution for this problem, and we need to find a Pareto optimal solution. One of such methods is to transform this problem into a satisficing problem.

By using the aspiration level f_i^a, each objective in the problem (3) can be written as

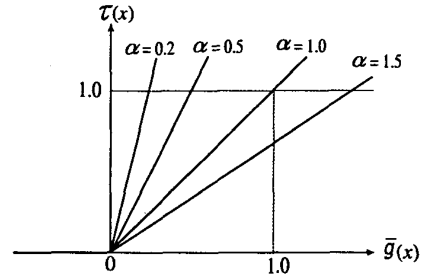

Fig.2 Unsatisfying function $\tau(x)$ and unsatisfying coefficient α.

$$f_i(x) \le f_i^a \quad (i = 1, \cdots, m). \tag{4}$$

This means that the designer wants to make $f_i(x)$ smaller than f_i^a. Then, the optimization problem (3) is considered as the satisficing problem of the constraints. That is, the multiobjective optimization problem (3) is transformed into the following satisficing problem:

$$\text{Find } x \text{ which satisfies } g_i(x) \le b_i \quad (i = 1, \cdots, M). \tag{5}$$

For calculation, we need to normalize the constraints in (5). When $b_i \neq 0$, they can be normalized as follows:

$$\bar{g}_i(x) \equiv \frac{g_i(x) - b_i}{|b_i|} \le 0, \tag{6}$$

and when $b_i = 0$, by using an appropriate value $\bar{b}_i (> 0)$, we can set

$$\bar{g}_i(x) \equiv \frac{g_i(x)}{|\bar{b}_i|} \le 0. \tag{7}$$

Consequently, the satisficing problem (5) is transformed into the following form:

$$\text{Find } x \text{ which satisfies } \bar{g}_i(x) \le 0 \quad (i = 1, \cdots, M). \tag{8}$$

It is often difficult to set up an aspiration level f_i^a appropriately for each objective $f_i(x)$. When b_i and f_i^a are severe, there may be no feasible region for (8), and the designer must re-set (relax) some of them to have feasible region. Usually, b_i and f_i^a have fuzziness, and the fuzzy set theory is very effective for such cases. Here, we introduce fuzziness to b_i and f_i^a. Suppose that the designer can accept $g_i(x)$ to a degree over $100\alpha_i\%$ for the restriction value b_i. In other words, $g_i(x)$ can be accepted to the extent of $(1 + \alpha_i)b_i$. This means that the following unsatisfying function is approximately 1:

$$\tau_i(x) = \max\left\{ \frac{1}{\alpha_i}\bar{g}_i(x),\ 0 \right\}, \tag{9}$$

where α_i is called *the unsatisfying coefficient*, and it is assumed that $\alpha_i \neq 0$ ($\alpha_i = 0$ means non-fuzzy). The relationship between $\tau_i(x)$ and $\bar{g}_i(x)$ for some α_i is shown in **Fig.2**. Of course, other types of unsatisfying function can be used, but $\tau_i(x)$ in (9) is simple and easy for formulation and calculation.

In real-world design processes, the designer usually refers to other design cases, experiences, some information, sense, etc., and aspiration levels are determined based on such ones. Since he/she considers that it is bad to violate the constraints, and wants to reduce the excess over the restriction values as much as possible, the use of unsatisfaction rating is more intuitive and useful for the designer rather than satisfaction rating.

In conventional fuzzy optimization methods using the membership function, the fuzzy decision should be made so as to unify the membership functions of the objectives and constraints. The similar decision can be applied to the method using the unsatisfying functions. In [1], p-norm of $\tau_i(\boldsymbol{x})$ is adopted. In this paper, we define $\tau_D(\boldsymbol{x})$ as

$$\tau_D(\boldsymbol{x}) = \max_{\boldsymbol{x}} \{ \tau_1(\boldsymbol{x}), \cdots, \tau_M(\boldsymbol{x}) \}, \tag{10}$$

and the decision-making is the minimization of $\tau_D(\boldsymbol{x})$. That is, the problem is given by

$$\text{Find } \boldsymbol{x} \text{ which minimizes } \tau_D(\boldsymbol{x}). \tag{11}$$

There are no distinctions between objectives and constraints in the multiobjective satisficing problem (5). And we sometimes need to solve a single-objective optimization problem whose objective is a particular constraint (e.g., for setting up the aspiration level for an objective when there is no information on it). Now, suppose that the problem is to solve a single-objective optimization with the objective $g_M(\boldsymbol{x})$. If we set up its unsatisfying coefficient α_M sufficiently larger than other $\alpha_i(i = 1, \cdots, M-1)$, it becomes equivalent to the following problem

$$\begin{aligned} &\text{Minimize } \bar{g}_M(\boldsymbol{x})_+, \\ &\text{subject to } \bar{g}_i(\boldsymbol{x}) \le 0 \ (i = 1, \cdots, M-1), \end{aligned} \tag{12}$$

where the subscript "+" denotes that $a_+ = a$ for $a \ge 0$ and $a_+ = 0$ for $a < 0$. And if we can assume that $\bar{g}_M(\boldsymbol{x}) \ge 0$, $\bar{g}_M(\boldsymbol{x})_+$ can be replaced by $\bar{g}_M(\boldsymbol{x})$ or $g_M(\boldsymbol{x})$. That is, the proposed method is available for single-objective optimization problems. (See section 4.)

3. Genetic Algorithms

3.1 Interactive Optimization Method

As the optimization method, we use Genetic Algorithms. GAs do not require the condition that the search space is differentiable and continuous. Another advantage is that GAs offer several candidate solutions to the problem. That is, most conventional optimization methods offer only one solution, but GA shows a group of solutions to the designer by one search. This property is favorable especially for multiobjective optimization problems where the unique solution does not exist.

In the proposed method, if the designer is not satisfied with any solutions obtained in the first search, he/she seeks a preferred solution by re-setting b_i and α_i. In that case, we use the trade-off graph [6] on which the previous results are plotted so as to decide the new values interactively and graphically. The same procedure will be repeated until the designer finds a satisfying solution.

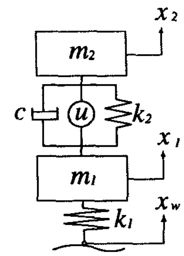

Fig.3 Active suspension model.

3.2 GA Optimization

In this paper, we consider the simple GA [7] which consists of the following operations:

1) Reproduction: We adopt the roulette-wheel selection in which the next generation is chosen at random in proportion to the fitness of each individual. Furthermore, we use the elite preserve strategy in which the individual with maximum fitness is chosen by compulsion.

2) Crossover: For a pair coupled randomly, crossover operation is performed with crossover probability P_c. The one-point crossover is used.

3) Mutation: After crossover, for each bit (binary code) in a individual, the bit value is inverted with mutation probability P_m.

4) Fitness function: The fitness function is defined as follows:

$$fitness = \frac{\beta}{1 + \tau_D(\boldsymbol{x})}, \tag{13}$$

where β is a parameter which adjusts the range of the fitness value.

4. Design of Active Suspension System

In this section, we apply the proposed method to the design of the active suspension system [8]. This is the simultaneous optimization of a structure/control system.

4.1 Active Suspension System

The active suspension system is shown in **Fig.3**, which is a simple car model. Here, each notation is as follows:

m_1: unsprung mass (tire), m_2: sprung mass (car body), k_1: tire stiffness, k_2: suspension stiffness, c: damping coefficient, x_w: road excitation, x_1: vertical displacement of m_1, x_2: vertical displacement of m_2, u: control force by actuator.

The state equation is described as follows:

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{b}u(t) + \boldsymbol{d}w(t), \tag{14}$$

where

$$\boldsymbol{x}=\begin{bmatrix} x_w-x_1 \\ x_1-x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ \frac{k_1}{m_1} & -\frac{k_2}{m_1} & -\frac{c}{m_1} & \frac{c}{m_1} \\ 0 & \frac{k_2}{m_2} & \frac{c}{m_2} & -\frac{c}{m_2} \end{bmatrix},$$

$$\boldsymbol{b}=[\,0\ \ 0\ \ \frac{1}{m_1}\ \ -\frac{1}{m_2}\,]^T,\ \boldsymbol{d}=[\,1\ \ 0\ \ 0\ \ 0\,]^T,$$

$$w(t)=\dot{x}_w(t),$$

and "T" denotes the transpose. As the control method, we consider the state feedback law as follows:

$$u(t)=-\boldsymbol{f}\boldsymbol{x}(t). \tag{15}$$

That is, the design parameter of the control system is the 1×4 gain vector $\boldsymbol{f}$.

4.2 Design Requirements

In this example, we consider the following objectives and constraints for simplicity:

1. Riding comfort J_1: In order for a person in a car to feel comfortable, it is required to reduce the vertical acceleration of the car body for a disturbance of the road surface in the specific frequency domain $4\sim 8$ Hz where a person feels most uncomfortable. Thus, we set J_1 as the area of the gain curve $(\ddot{x}_2/x_w)$ in $4\sim 8$ Hz.

2. Running stability J_2: For stable braking and cornering, it is required to reduce the fluctuation of frictional force between tire and road surface. Since a tire is modeled as an elastic spring in Fig.3, the fluctuation of the frictional force arises from one of tires. Thus, as J_2, we consider the overshoot of x_1 when a step displacement is given to x_w.

3. Posture of car body J_3: For stable steering, it is required to maintain a car body flat. Thus, we set J_3 as the overshoot of x_2 when a step displacement is given to x_w. In general, J_2 and J_3 are in competitive relations.

4. Constraints : It is known that the larger damper coefficient can reduce the vibration in resonance domain, but it will enlarge the vibration transmission ratio in high frequency domain. Thus, we give the upper bound $\bar{\gamma}$ to the damping ration γ. We also give the upper bound $\bar{U}_{\max}$ to the maximum control input $U_{\max}$. Of course, the feedback system must be stable.

In this example, the design variables are k_2, c and $\boldsymbol{f}$. Other parameters are
$m_1=50.0[kg]$, $m_2=320.0[kg]$, $k_1=1.45\times 10^5[N/m]$.
Then, the problem is formulated as follow:

$$\begin{aligned} &\text{Find} \quad k_2,\ c,\ \text{and}\ \boldsymbol{f}, \\ &\text{which minimizes} \quad J_1,\ J_2,\ \text{and}\ J_3, \\ &\text{subject to} \quad \gamma\le\bar{\gamma}, \\ &\qquad\qquad U_{\max}\le\bar{U}_{\max}, \\ &\qquad\qquad \text{stability of the feedback system.} \end{aligned} \tag{16}$$

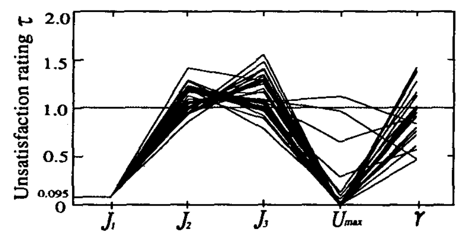

Fig.4 Trade-off graph in Step1.

Here, J_2, J_3 and $U_{\max}$ are calculated by the simulation. Furthermore, if one of the four eigenvalues of $(\boldsymbol{A}-\boldsymbol{b}\boldsymbol{f})$ is positive, we set the fitness of the individual zero in GA.

4.3 Interactive Design

The system is designed interactively using the proposed method. Let us assume that it is difficult for the designer to give the aspiration level for J_1. Thus, at first, he/she needs to solve a single-objective optimization problem on J_1 to estimate its value. Then, he/she shall solve a multiobjective optimization problem according to the design procedure.

Step 1: Estimation of aspiration level for J_1

In order to estimate J_1, a single-objective optimization problem on J_1 is formulated. The aspiration level for J_1 is set up as a small value such as $b_1=50$, and the unsatisfying coefficient is set up as a large value such as $\alpha_1=30$, comparing with others. Other parameters α_i and $b_i(i=2,\cdots,5)$ are shown in **Table 1**, and we consider the road displacement $x_w=0.02$[m].

Table 1 Parameters in Step1.

	J_1	J_2	J_3	$U_{\max}$	γ
b_i	50	0.025[m]	0.025[m]	90[N]	0.24
α_i	30	0.08	0.08	0.11	0.08
$(1+\alpha_i)b_i$	1550	0.027[m]	0.027[m]	100[N]	0.26

The result is plotted on a trade-off graph in **Fig.4**, where individuals with extremely large unsatisfaction rating are excluded. Fig.4 shows that minimum value on J_1 is about 192.5 since the unsatisfaction rating r_1 converges on about 0.095. As most of the unsatisfaction ratings for both J_2 and J_3 are in excess of 1, we find that their aspiration levels may be a little large, and they are in competitive relations. Furthermore, it seems that the unsatisfaction ratings for J_2 and J_3 can be improved if we accept deterioration of γ.

Step 2: Interactive design process

Suppose that the designer wants to improve J_3 by deteriorating J_2 and γ. He/she tries to make the satisfaction rating on J_3 smaller than 1, by setting up the parameters as $\alpha_2=0.14$ and $b_5=0.25$. Furthermore, he/she expects that J_1 also can be improved some more since the conditions on J_2 and γ are relaxed, and sets

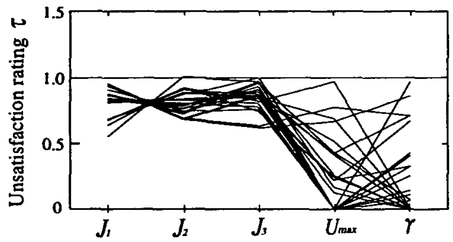

Fig.5 Trade-off graph in Step2.

up $b_1 = 180$ and $\alpha_1 = 0.07$. They are shown in **Table 2** with superscript " $*$ ".

The result is shown in **Fig.5** and **Table 3**. Fig.5 shows that the unsatisfaction ratings on J_2 and J_3 are improved. If the designer is satisfied with this result, he will choose one satisficing solution. If he/she is not satisfied, same procedure will be iterated.

Table 2 Parameters in Step2.

	J_1	J_2	J_3	$U_{\max}$	γ
b_i	180*	0.025[m]	0.025[m]	90[N]	0.25*
α_i	0.07*	0.14*	0.08	0.11	0.08
$(1+\alpha_i)b_i$	192.6	0.0285[m]	0.027[m]	100[N]	0.27

Table 3 Best fitness.

No	τ_1	τ_2	τ_3	τ_4	τ_5
1	0.8284	0.8276	0.8033	0	0.4306
2	0.8340	0.8055	0.7770	0	0.4306
3	0.8198	0.8282	0.8515	0	0.1429
4	0.8095	0.8367	0.8629	0	0.0657
5	0.8645	0.7670	0.8398	0.2221	0.3276

4.4 Simulation

The simulation result is shown for $x_w(t) = 0.02$[m] ($t \geq 0$) when the designer chooses the best fitness in Table 3. Here, the design parameters are given by

$$k_2 = 1.8209 \times 10^4 [N/m],\ c = 1.2485 \times 10^3 [Ns/m],$$

$$f = [\,-1115.46 \quad 794.521 \quad 3.914 \quad -818.004\,].$$

Fig.6 shows the responses of x_1 and x_2, and **Fig.7** shows the profile of control force by actuator. The transient responses will be improved by setting other performance indices.

5. Conclusions

In this paper, we have proposed a new interactive fuzzy optimization method using the unsatisfying function, and it has been applied to the design problem of an active suspension system. The unsatisfying function is more intuitive for a designer than the membership function. The problem is formulated as the minimization of the maximum unsatisfaction rating for each constraint. By using GA with trade-off graphs, the proposed method becomes more interactive and graphical.

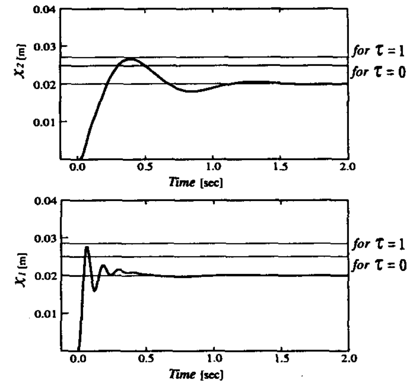

Fig.6 Step response.

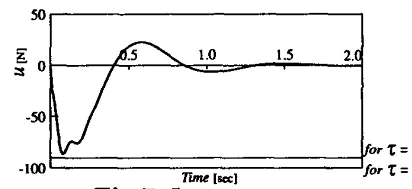

Fig.7 Input response.

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