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Interactive Approaches Applied to Multiobjective Evolutionary Algorithms

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1.1 Introduction

Multiobjective Evolutionary Algorithms (MOEAs) rely on preference relations to steer the search towards high-potential regions of the search space in order to approximate the optimal solution set. In particular, a preference relation is a mean to decide if a solution is preferable over another solution in the search space.

In single-objective optimization, the determination of the optimum among a set of given solutions is clear. However, in the absence of preference information, in multiobjective optimization, there does not exist a unique preference relation to determine if a solution is better than another one. The most common preference relation adopted is known as the *Pareto dominance relation* (Pareto 1896), which leads to the best possible trade-offs among the objectives. Thus, by using this relation, it is normally not possible to obtain a single optimal solution (except when there is no conflict among the objectives), but instead, a set of good solutions (representing the best possible trade-offs among the objectives) can be produced. This set is called the *Pareto optimal set* and its image in objective space is known as the *Pareto optimal front*.

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Multiobjective optimization involves three stages: model building, search, and decision making (preference articulation). Having a good approximation of the Pareto optimal set does not completely solve a multiobjective optimization problem. The decision maker (DM) still has the task of choosing the most preferred solution out of the approximation set. This task requires preference information from the DM. Following this need, there are several methodologies available for defining how and when to incorporate preferences from the DM into the search process. These methodologies can be classified in the following categories (Coello Coello et al. 2007; Miettinen 1998):

1. Prior to the search (*a priori* approaches).
2. During the search (interactive approaches).
3. After the search (*a posteriori* approaches).

Although interactive approaches for incorporating preferences have been widely used for a long time in Operations Research (see e.g., Chankong and Haimes 1983; Miettinen 1998), it was only until very recently that the inclusion of preference information into MOEAs started to attract a considerable amount of interest among researchers (see for example, Branke 2008; Coello Coello et al. 2007). Regardless of the stage at which preferences are incorporated into a MOEA, the aim is to focus on a certain portion of the Pareto front by favoring certain objectives (or trade-offs) over others.

As noted by several researchers (Hughes 2005; Khare et al. 2003; Knowles and Corne 2007; Praditwong and Yao 2007; Purshouse and Fleming 2007; Teytaud 2007; Wagner et al. 2007), the Pareto dominance relation has a important drawback when is applied to multiobjective optimization problems with a high number of objectives (these are the so-called *many-objective problems*, e.g., Kukkonen and Lampinen 2007). That is, the deterioration of its ability to discern between good and bad solutions as the number of solutions increases. A widely accepted explanation for this problem is that the proportion of nondominated solutions (i.e., incomparable solutions according to the Pareto dominance relation) in a population increases rapidly with the number of objectives (see e.g., Bentley et al. 1978; Farina and Amato 2002). Since, incorporating preferences induces a finer order on vectors of the objective space than that achieved by the Pareto dominance relation, we believe that the use of the new preference relation is a promising approach to deal with many-objective problems. Additionally, by using an interactive optimization technique we can avoid the generation of millions or even billions of nondominated points in many-objective problems.

This chapter presents a review of recent MOEAs designed to work as interactive optimization methods. For earlier methods, the reader is referred to other specialized reviews like those presented by Branke and Deb (2005); Coello Coello (2000); Cvetković and Parmee (2002); Rachmawati and Srinivasan (2006). The contents of this chapter aims to complement these previous reviews of the field instead of aiming at being comprehensive.

1.1.1 Methods Analyzed in this Chapter

By analyzing the approaches found in other reviews and the ones covered in this chapter, we can realize that most of the approaches to incorporate preferences into MOEAs are based on methods introduced in the field of Multicriteria Decision Making. For instance, we can

find many approaches based on reference point methods. There are few methods originated in the Evolutionary Multiobjective Optimization (EMO) field or coming from other areas. We can mention, for example, a method based on the hypervolume indicator. Thus, the methods analyzed in this chapter are classified in the following categories:

- Reference point methods.
- Utility function methods.
- Miscellaneous methods.

The survey is mainly focused on interactive MOEAs. However, some a priori techniques can be easily set for selecting the location and size of the Region of Interest (RoI). This way, they can serve as a basis for interactive approaches. Therefore, we also include some of these interesting a priori techniques for incorporating preferences.

1.2 Basic Concepts and Notation

In this section, we will introduce the concepts and notation that will be used throughout the rest of the paper. Furthermore, as many interactive MOEAs are based on classical interactive methods proposed by the Operations Research community, some of these methods are first described.

1.2.1 Multiobjective Optimization Problems

Definition 1.2.1 (Multiobjective optimization problem) *A Multiobjective Optimization Problem (MOP) is defined as:*

$$\begin{aligned} &\textit{Minimize} \quad \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})]^T, \\ &\textit{subject to} \quad \mathbf{x} \in \mathcal{X}. \end{aligned} \tag{1.1}$$

The vector $\mathbf{x} \in \mathbb{R}^n$ is formed by n *decision variables* representing the quantities for which values are to be chosen in the optimization problem. The *feasible set* $\mathcal{X} \subseteq \mathbb{R}^n$ is implicitly determined by a set of equality and inequality constraints. The vector function $\mathbf{f} : \mathcal{X} \to \mathbb{R}^k$ is composed by $k \geq 2$ scalar *objective functions* $f_i : \mathcal{X} \to \mathbb{R}$ $(i = 1, \ldots, k)$. In multiobjective optimization, the sets $\mathbb{R}^n$ and $\mathbb{R}^k$ are known as *decision variable space* and *objective function space*, respectively. The image of $\mathcal{X}$ under the function $\mathbf{f}$ is a subset of the objective function space denoted by $\mathcal{Z} = \mathbf{f}(\mathcal{X})$ and referred to as the *feasible set in the objective function space*.

In order to define precisely the multiobjective optimization problem stated in definition 1.2.1 we have to establish the meaning of minimization in $\mathbb{R}^k$. That is to say, we need to define how vectors $\mathbf{z} = \mathbf{f}(\mathbf{x}) \in \mathbb{R}^k$ have to be compared for different solutions $\mathbf{x} \in \mathbb{R}^n$. In single-objective optimization the relation "less than or equal" ($\leq$) is used to compare the scalar objective values. By using this relation there may be many different optimal solutions $\mathbf{x} \in \mathcal{X}$, but only one optimal value $f^{\min} = \min\{f(\mathbf{x}) \mid x \in \mathcal{X}\}$ since the relation $\leq$ induces a total order in $\mathbb{R}$ (i.e., every pair of solutions is comparable, and thus, we can sort solutions from the best to the worst one). In contrast, in multiobjective optimization problems, there is no canonical order on $\mathbb{R}^k$, and thus, we need weaker definitions of order to compare vectors in $\mathbb{R}^k$.

In multiobjective optimization, the *Pareto dominance relation* is usually adopted. This relation was originally proposed by Fracis Ysidro Edgeworth in 1881 (Edgeworth 1881), and was generalized by the French-Italian economist Vilfredo Pareto in 1896 (Pareto 1896).

Definition 1.2.2 (Pareto dominance relation) *We say that a vector* $\mathbf{z}^1$ *dominates vector* $\mathbf{z}^2$*, denoted by* $\mathbf{z}^1 \prec \mathbf{z}^2$*, if and only if:*

$$\forall i \in \{1, \ldots, k\} : z_i^1 \leq z_i^2 \tag{1.2}$$

and

$$\exists i \in \{1, \ldots, k\} : z_i^1 < z_i^2. \tag{1.3}$$

If $\mathbf{z}^1 = \mathbf{z}^2$ or $z_i^1 > z_i^2$ for some i, then we say that $\mathbf{z}^1$ does not dominate $\mathbf{z}^2$ (denoted by $\mathbf{z^1} \not\prec_{\text{pareto}} \mathbf{z^2}$). Thus, to solve a MOP we have to find those solutions $\mathbf{x} \in \mathcal{X}$ whose images, $\mathbf{z} = \mathbf{f}(\mathbf{x})$, are not dominated by any other vector in the feasible space. It is said that two vectors, $\mathbf{z^1}$ and $\mathbf{z^2}$, are *mutually nondominated vectors* if $\mathbf{z^1} \not\prec_{\text{pareto}} \mathbf{z^2}$ and $\mathbf{z^2} \not\prec_{\text{pareto}} \mathbf{z^1}$.

Definition 1.2.3 (Pareto optimality) *A solution* $\mathbf{x}^* \in \mathcal{X}$ *is Pareto optimal if there does not exist another solution* $\mathbf{x} \in \mathcal{X}$ *such that* $\mathbf{f}(\mathbf{x}) \prec \mathbf{f}(\mathbf{x}^*)$.

Definition 1.2.4 (ρ-properly Pareto optimality) *A solution* $\mathbf{x}^* \in \mathcal{X}$ *and its corresponding vector* $\mathbf{z}^* \in \mathcal{Z}$ *are* ρ*-properly Pareto optimal (in the sense of Wierzbicki 1980b) if*

$$(\mathbf{z}^* - \mathbb{R}_\rho^k \setminus \{0\}) \cap \mathcal{Z} = \emptyset,$$

where $\mathbb{R}_\rho^k = \{\mathbf{z} \in \mathbb{R}^k | \max_{i=1,\ldots,k} z_i + \rho \sum_{i=1}^k z_i \geq 0\}$*, and* ρ *is some scalar. The trade-offs among the objectives are bounded by* ρ *and* $1/\rho$*.*

Definition 1.2.5 (Pareto optimal set) *The Pareto optimal set,* P_{opt}*, is defined as:*

$$P_{\text{opt}} = \{\mathbf{x} \in \mathcal{X} \mid \nexists \mathbf{y} \in \mathcal{X} \,:\, \mathbf{f}(\mathbf{y}) \prec \mathbf{f}(\mathbf{x})\}. \tag{1.4}$$

Definition 1.2.6 (Pareto front) *For a Pareto optimal set,* P_{opt}*, the Pareto front,* PF_{opt}*, is defined as:*

$$PF_{\text{opt}} = \{\mathbf{z} = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})) \mid \mathbf{x} \in P_{\text{opt}}\}. \tag{1.5}$$

In decision variable space, these vectors are referred to as decision vectors of the Pareto optimal set, while in objective space, they are called objective vectors of the Pareto optimal set. In practice, the goal of a posteriori approaches is finding the "best" approximation set of the Pareto optimal front. An approximation set is a finite subset of $\mathcal{Z}$ composed of mutually nondominated vectors and is denoted by PF_{approx}. Currently, it is well accepted that the best approximation set is determined by the closeness to the Pareto optimal front, and the spread over the entire Pareto optimal front (Coello Coello et al. 2007; Deb et al. 2002b; Zitzler et al. 2003).

In interactive optimization methods it is useful to know the lower and upper bounds of the Pareto front. The *ideal point*, $\mathbf{z}^\star$, represents the lower bounds and is defined by $z_i^\star = \min_{z \in \mathcal{Z}}\{z_i\}\ \forall i = 1, \ldots, k$. In turn, the upper bounds are defined by the *nadir point*, $\mathbf{z}^{\text{nad}}$, which is given by $z_i^{\text{nad}} = \max_{z \in PF_{\text{opt}}}\{z_i\}\ \forall i = 1, \ldots, k$. In order to avoid some problems when the ideal and nadir points are equal or very close, a point strictly better than the ideal point is usually defined. This point is called the *utopian point*, $\mathbf{z}^{\star\star}$, and is defined by $z_i^{\star\star} = z_i^\star - \epsilon, \forall i = 1, \ldots, k$, where $\epsilon > 0$ is a small scalar.

1.2.2 Classical Interactive Methods

Reference Point Methods

These kinds of methods are based on the achievement scalarizing function approach proposed by Wierzbicki (1980a,b). An achievement scalarizing function uses a reference point to capture the desired values of the objective functions.

Definition 1.2.7 (Achievement scalarizing function) *An achievement scalarizing function (or achievement function for short) is a parameterized function $s_{\mathbf{z}^{\mathrm{ref}}}(\mathbf{z}) : \mathbb{R}^k \to \mathbb{R}$, where $\mathbf{z}^{\mathrm{ref}} \in \mathbb{R}^k$ is a reference point representing the decision maker's aspiration levels. Thus, the multiobjective problem is transformed into the following scalar problem:*

$$\begin{aligned} \textit{Minimize} \quad & s_{\mathbf{z}^{\mathrm{ref}}}(\mathbf{z}), \\ \textit{subject to} \quad & \mathbf{z} \in \mathcal{Z}. \end{aligned} \tag{1.6}$$

A common achievement function is based on the Chebyshev distance (L_∞ metric) (see e.g., Ehrgott 2005; Miettinen 1998).

Definition 1.2.8 (Chebyshev distance) *For two vectors $\mathbf{z}^1, \mathbf{z}^2 \in \mathbb{R}^k$ the Chebyshev distance is defined by*

$$d_\infty(\mathbf{z}^1, \mathbf{z}^2) = ||\mathbf{z}^1 - \mathbf{z}^2||_\infty = \max_{i=1,\ldots,k} |z_i^1 - z_i^2|. \tag{1.7}$$

Definition 1.2.9 (Weighted achievement function) *The weighted achievement function (or achievement function for short) is defined by*

$$s_\infty(\mathbf{z}, \mathbf{z}^{\mathrm{ref}}) = \max_{i=1,\ldots,k} \{\lambda_i(z_i - z_i^{\mathrm{ref}})\} + \rho \sum_{i=1}^{k} \lambda_i(z_i - z_i^{\mathrm{ref}}), \tag{1.8}$$

where $\mathbf{z}^{\mathrm{ref}}$ is a reference point, $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_k]$ is a vector of weights such that $\forall i\ \lambda_i \geq 0$ and, for at least one i, $\lambda_i > 0$, and $\rho > 0$ is an augmentation coefficient sufficiently small (usually $\rho = 10^{-6}$). The main role of ρ is to avoid the generation of weakly Pareto optimal solutions.

We should note that, unlike the Chebyshev distance, the achievement function does not use the absolute value in the first term. This small difference allows the achievement function to correctly assess solutions that improve the reference point.

The achievement function has some convenient properties over other scalarizing functions. As proved, for instance by Steuer (1986), Miettinen (1998) and Ehrgott (2005), the minimum of eq. (1.8) is a Pareto optimal solution and we can find any ρ-properly Pareto optimal solution (see def. 1.2.4).

Light Beam Search Method

The Light Beam Search (LBS) method proposed by Jaszkiewicz and Slowinski (1999), is an iterative method which combines the reference point idea and tools of Multi-attribute Decision Analysis (MADA). At each iteration, a finite sample of nondominated points is generated. The sample is composed of a current point called *middle point* (which is obtained

at a previous iteration), and J nondominated points from its neighborhood. A local preference model in the form of an *outranking relation* S is used to define the neighborhood of the middle point. It is said that a vector $\mathbf{z}^1$ outranks vector $\mathbf{z}^2$ ($\mathbf{z}^1 S \mathbf{z}^2$) if $\mathbf{z}^1$ is considered to be at least as good as $\mathbf{z}^2$. The outranking relations are defined by the DM, which specify three preference thresholds for each objective, namely: *indifference threshold, preference threshold* and *veto threshold*. The DM has the possibility to scan the inner area of the neighborhood along the objective function trajectories between any two characteristic neighbors or between a characteristic neighbor and the middle point. In Algorithm 1 the general scheme of the LBS procedure is shown.

Algorithm 1 General Scheme of the Light Beam Search Procedure.

Step 1: Ask the DM to specify the starting aspiration and reservation points.
Step 2: Compute the starting middle point.
Step 3: Ask the DM to specify a local preferential information used to build an outranking relation.
Step 4: Present the middle point to de DM.
Step 5: Calculate the characteristic neighbors of the middle point and present them to the DM.
Step 6: **If** DM is satisfied **then**
STOP.
else
6.1: Ask DM to choose one of the neighboring points to be the new middle point, or
6.2: Update the preferential information, or
6.3: Define new aspiration point and/or reservation point.
6.4: Go to **Step 3**.

1.3 MOEAs based on Reference Point Methods

1.3.1 A Weighted Distance Metric

Deb and Sundar (2006) incorporated a reference point approach into the Nondominated Sorting Genetic Algorithm II (NSGA-II) (Deb et al. 2002a). They introduced a modification in the crowding distance operator in order to select from the last nondominated front the solutions that would take part of the new population. They used the following achievement function based on the Euclidean distance

$$d(\mathbf{z}, \mathbf{z}^{ref}) = \sqrt{\sum_{i=1}^{k} w_i \left(\frac{z_i - z_i^{ref}}{z_i^{\max} - z_i^{\min}} \right)^2}, \tag{1.9}$$

where $\mathbf{z} \in \mathbb{R}^k$ is a solution, $\mathbf{z}^{\text{ref}}$ is a reference point, $z_i^{\max} = \max_{z \in P}\{z_i\}$ and $z_i^{\min} = \min_{z \in P}\{z_i\}$ $\forall i = 1, \ldots, k$, calculated with respect to the current population, P, and the weight vector should satisfy $w_i \in [0, 1]$ and $\sum_{i=1}^{M} w_i = 1$.

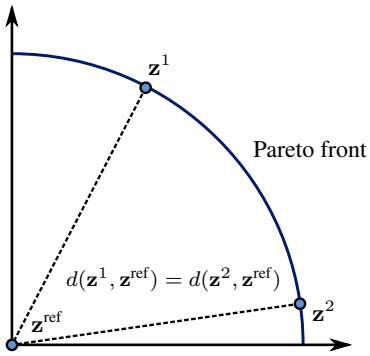

Figure 1.1 The location of the reference point and the shape of the Pareto front might avoid reducing the region of interest.

The value of this function was used to sort and rank the population accordingly (the solution closest to the reference point receives the best rank). In order to control the spread of the RoI, solutions whose achievement value differs by an amount of ϵ or less receive the same rank. This way, a set of solutions clustered around the best ranked solution forms the RoI. This method was designed to take into account a set of reference points, i.e., several independent RoIs can be generated. A drawback of this scheme is that it might generate some non-Pareto optimal solutions, particularly in MOPs with disconnected Pareto fronts. Furthermore, we want to point out that the location of the reference point and shape of the Pareto front also determine the size of the RoI and, in general, the order in which solutions are ranked. Let's take for example the Pareto front of the 2-objective DTLZ2 problem (see Fig. 1.1). If we choose the ideal point as reference point, then for any value of ϵ, all the solution in the Pareto front will be equally ranked since they are equidistant to the origin (for example solutions $\mathbf{z}^1$ and $\mathbf{z}^2$ in Fig. 1.1).

An interesting case is observed when the reference point is farther away from the origin. In that case, solutions in the extreme of the Pareto front are closer to the origin than the solution in the center of the Pareto front. This means that the DM should have some knowledge about the shape and lower bounds of the Pareto front in order to avoid these situations.

1.3.2 Light Beam Search combined with NSGA-II

A similar approach was also proposed by Deb and Kumar (2007), in which the LBS procedure (Jaszkiewicz and Slowinski 1999) was incorporated into NSGA-II. Similar to the previous approach, they modified the crowding operator to incorporate DM's preferences. They used a weighted achievement function to assign a crowding distance to each solution in each front. Thus, the solution with the least distance will have the best crowding rank. Like in the previous approach, this algorithm finds a subset of solutions around the optimum of the achievement function adopting the outranking relation proposed by Jaszkiewicz and Slowinski (1999). In Jaszkiewicz and Slowinski (1999) three kinds of thresholds are defined to determine if one solution outranks another one. However, in Deb and Kumar (2007) the

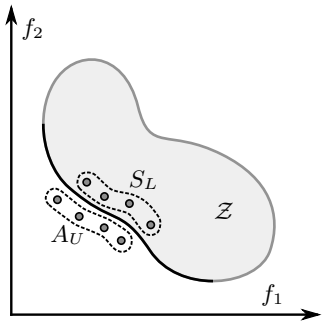

Figure 1.2 Controlling accuracy of the Pareto front approximation.

veto threshold is the only one used. This relation depends on the crowding comparison operator.

Although these two previous techniques were presented and implemented as an *a priori* technique, it is possible to formulate an interactive method with the same basic idea.

1.3.3 Controlling the accuracy of the Pareto front approximation

Kaliszewski et al. (2012) presented an interesting interactive approach coupled with a MOEA. The main contribution of this work is a mechanism to control the accuracy of the subset of nondominated solutions obtained. The authors adopted an achievement function to generate a set of weakly Pareto optimal solutions. In order to control the accuracy of the Pareto front approximation, two sets are generated and updated during the search: a set of feasible nondominated solutions, S_L, and a set of infeasible nondominated solutions, A_U, such that $S_L \not\prec A_U$ (see Fig. 1.2). The basic idea is to enclose the Pareto front with two approximation sets, one approaching from below and another one from above. The accuracy of the Pareto front approximation is determined by a distance measure between S_L and A_U.

The decision maker's preferences are expressed by means of the so-called *vector of concessions* which represents proportions in which the DM agrees to sacrifice unattainable values of the objectives represented by the ideal point in the hope of getting Pareto optimal solutions.

1.3.4 Light Beam Search combined with PSO

Wickramasinghe and Li (2009) proposed the combination of the LBS method (Jaszkiewicz and Slowinski 1999) and a Particle Swarm Optimization (PSO) technique to guide the swarm towards a RoI according to the aspiration and reservation points provided by the DM. The main idea is ranking the population using the outranking relation instead of the usual Pareto dominance. By changing the threshold parameters of the outranking relations (indifference threshold, preference threshold, veto threshold) the size of the RoI near the point that minimizes eq. (1.8) can be regulated. The authors used the Multi-objective

Differential Evolution and Particle Swarm Optimization (MDEPSO) algorithm as a framework to incorporate LBS. The key points in which LBS is inserted are the following:

- Each time a new particle is created its achievement function value is calculated.
- The leaders to guide the population are sorted according to its achievement function value using the outranking relation. This is the step in which the spread of the RoI is controlled.
- The personal best of each particle will take the value of the updated position only if the achievement function value is improved.
- In order to obtain the new population, the original and updated particles are mixed and sorted according to the outranking relation. Finally, the best half of the mixed population will form the new population.

1.3.5 A preference relation based on a weighted distance metric

Said et al. (2010) use the achievement function given by eq. (1.9) to create a new preference relation called r-dominance. This relation combines the usual Pareto dominance and the achievement function in the following way.

Definition 1.3.1 *Given a set of solutions P and a reference point $\mathbf{z}^{\mathrm{ref}}$, a solution $\mathbf{z}^1$ is said to r-dominate a solution $\mathbf{z}^2$ if:*

1. $\mathbf{z}^1 \prec \mathbf{z}^2$*, or*
2. $\mathbf{z}^1$ *and* $\mathbf{z}^2$ *are mutually non-dominated solutions, and* $D(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^{\mathrm{ref}}) < -\delta$*, where* $\delta \in [0, 1]$ *and*

$$D(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^{\mathrm{ref}}) = \frac{d(\mathbf{z}^1, \mathbf{z}^{\mathrm{ref}}) - d(\mathbf{z}^2, \mathbf{z}^{\mathrm{ref}})}{Dist_{\max} - Dist_{\min}},$$

$$Dist_{\max} = \max_{\mathbf{z} \in P} d(\mathbf{z}, \mathbf{z}^{\mathrm{ref}}),$$

$$Dist_{\min} = \min_{\mathbf{z} \in P} d(\mathbf{z}, \mathbf{z}^{\mathrm{ref}}).$$

The authors prove that the r-dominance relation, in the same way as the Pareto dominance, defines a strict partial order on a set of solutions since the relation is irreflexive, asymmetric and transitive. They also prove that r-dominance is complete with the Pareto dominance and compatible with the non Pareto dominance, i.e., if $\mathbf{z}^1$ r-dominates $\mathbf{z}^2$, then $\mathbf{z}^1 \prec \mathbf{z}^2$, and if $\mathbf{z}^1$ r-dominates $\mathbf{z}^2$, then $\mathbf{z}^2 \not\prec_{\mathrm{par}} \mathbf{z}^1$. In spite of these desirable properties, the r-dominance relation has some drawback due to the Euclidean distance function adopted. The size of the RoI is mainly determined by the threshold δ. If $\delta = 1$, r-dominance is equivalent to the Pareto dominance relation. In turn, if $\delta = 0$, in most cases, the relation becomes more stringent since only those solutions minimizing eq. (1.9) will be the most preferred by the relation.

The r-dominance relation was inserted in the interactive method presented in Algorithm 2 using a variant of NSGA-II as the search engine.

1.3.6 The Chebyshev Preference Relation

This preference relation, proposed by López-Jaimes et al. (2011), is based on the Chebyshev achievement function (see eq. (1.8)), and provides a simple way to integrate preferences into different types of MOEAs. The basic idea of the Chebyshev preference relation is to combine the Pareto dominance relation and an achievement function to compare solutions in objective function space.

First, the achievement function value, $s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$, is computed for each solution $\mathbf{z}$. Then, the objective space is divided into two regions. One region defines the RoI and contains those solutions with an achievement value less or equal to $s^{\min} + \delta$, where $s^{\min} = \min_{\mathbf{z} \in \mathcal{Z}} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$, and δ is a threshold that determines the size of the RoI. Fig. 1.3 shows the RoI defined by means of the achievement function. Solutions in this region are compared using the usual Pareto dominance relation, while solutions outside of the RoI are compared using their achievement function value.

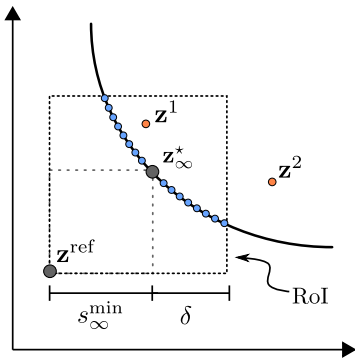

Figure 1.3 Nondominated solutions with respect to the Chebyshev relation.

Formally, the Chebyshev preference relation is defined as follows.

Definition 1.3.2 *A solution* $\mathbf{z}^1$ *is preferred to solution* $\mathbf{z}^2$ *with respect to the Chebyshev relation (*$\mathbf{z}^1 \prec_{\text{cheby}} \mathbf{z}^2$*), if and only if:*

1. $s_\infty(\mathbf{z}^1, \mathbf{z}^{\text{ref}}) < s_\infty(\mathbf{z}^2, \mathbf{z}^{\text{ref}}) \wedge \{\mathbf{z}^1 \notin R(\mathbf{z}^{\text{ref}}, \delta) \vee \mathbf{z}^2 \notin R(\mathbf{z}^{\text{ref}}, \delta)\}$, *or,*

Algorithm 2 Interactive Optimization using the r-dominance relation.

Step 1:	Ask the DM for the following parameter values: population size, number of generations, reference solution, weight vector and threshold δ.
Step 2:	Apply r-NSGA-II the number of generations required.
Step 3:	Present to the DM the set of preferred solutions.
Step 4:	**If** the DM is satisfied with the provided set of solutions, **then** Stop the process. **Otherwise** Ask the DM for new values for the parameters and return to **Step 2**.

2. $\mathbf{z}^1 \prec \mathbf{z}^2 \wedge \{\mathbf{z}^1, \mathbf{z}^2 \in R(\mathbf{z}^{\text{ref}}, \delta)\}$,

where $R(\mathbf{z}^{\text{ref}}, \delta) = \{\mathbf{z} \mid s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}}) \leq s^{\min} + \delta\}$ *is the Region of Interest with respect to the vector of aspiration levels* $\mathbf{z}^{\text{ref}}$.

The threshold δ is set in terms of the user parameter $\tau \in [0, 1]$ according to $\delta = \tau \cdot (s^{\max} - s^{\min})$, where $s^{\max} = \max_{\mathbf{z} \in P} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$ and $s^{\min} = \min_{\mathbf{z} \in P} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$. In this way, if $\tau = 1$, all the solutions in the population P are compared adopting the usual Pareto dominance relation. On the other hand, if $\tau = 0$, then all the solutions are compared using the achievement function value.

Unlike some distance metrics, the achievement function (eq. (1.8)) allows a MOEA to find points in problems with nonconvex Pareto fronts.

López-Jaimes et al. (2011) also proposed a variant of the Chebyshev relation that uses an approximation of the ideal point as reference point in definition 1.3.2. This variant is called the *central-guided Chebyshev relation* since it focuses the search towards the ideal point.

Algorithm 3 shows the interactive process using the Chebyshev relation.

Algorithm 3 Interactive technique using the Chebyshev preference relation.

Step 1: Ask the DM to specify the threshold τ.
If the DM has some knowledge about the problem, he/she can provide a reference point. Otherwise, the central-guided preference relation can be used to converge towards the ideal point.

Step 2: **If** a reference point was provided, **then**
Execute the MOEA using the Chebyshev relation with the reference point provided by the decision maker.
else
Execute the MOEA using the central-guided Chebyshev relation.

Step 3: Ask the DM to define how many solutions of the current approximation should be shown.
Additionally, from the use of the central-guided relation the DM can be informed of the current ideal point in order to decide new aspiration levels.

Step 4: **If** the DM is satisfied with some solution of the current set, **then**
STOP.
else
Go to **Step 1**.

1.4 MOEAs based on Value Function Methods

1.4.1 Progressive approximation of a value function

Deb et al. (2010) proposed a method in which the DM's value function is progressively approximated through pairwise comparisons of a small set of solutions.

After applying a MOEA, a selection of well-distributed solutions of the achieved Pareto front approximation is presented to the DM (in the experiments reported by the authors, five solutions are shown). Then, for every pair of solutions the DM should establish which one is preferred over the other, or if they are incomparable. Based on this preference

information a polynomial value function, $V(\mathbf{z})$, is built. For 2-objective problems, this function is the product of two linear functions whose parameters must be determined using the preference information as constraints of an optimization problem. The created value function is employed to define a preference relation in order to select the parents and, later, to decide which individuals will survive to the next generation. This preference relation uses the value function value (denoted by V_2) of the second-best solution found in the population.

Definition 1.4.1 *A solution $\mathbf{z}^1$ is preferred to solution $\mathbf{z}^2$ with respect to an value function value V_2 if and only if:*

1. $\mathbf{z}^1 \prec \mathbf{z}^2$, *or*
2. $V(\mathbf{z}^1) > V_2$ *and* $V(\mathbf{z}^2) < V_2$.

The value function is also used to formulate a termination criterion of the interactive process. The idea is to perform a linear search along the gradient of the value function taking the best solution in the population as initial point. If this solution is improved by a given threshold value, the interactive process continues and the MOEA is applied again. Otherwise, the process stops and the solution found in the linear search is considered the most preferred solution.

1.4.2 Value function by ordinal regression

For a given class of value function it is possible that many of its instances generated by changing the parameters can be compatible with the provided preference information. In many approaches, like the one presented in Section 1.4.1, only one specific instance is used to evaluate the set of solutions. However, as Branke et al. (2010) pointed out, since this selection is rather arbitrary, a more robust approach should take into consideration all the set of value functions compatible with the preference information (see Fig. 1.4). This motivation was the origin of the Robust Ordinal Regression (ROR) proposed by Branke et al. (2010). When all the compatible value function are considered, two different preference relation can be defined:

1. Necessary preference relation: a solution $\mathbf{z}^1$ is ranked at least as good as $\mathbf{z}^2$ if $\mathbf{z}^1$ is preferred over $\mathbf{z}^2$ in all compatible instances of the value function.
2. Possible preference relation: a solution $\mathbf{z}^1$ is ranked at least as good as $\mathbf{z}^2$ if $\mathbf{z}^1$ is preferred over $\mathbf{z}^2$ in at least one compatible instance of the value function.

The necessary preference relation is robust in the sense that any pair of solutions is compared the same whatever the compatible instance of the value function.

In order to define a necessary preference ranking, the preference information is obtained by asking the DM to make a pairwise comparison of a small set of alternative solutions. Then, the necessary preference relation is computed by solving a linear programming problem. In some decision-making situations is useful to know the most representative value function among all the compatible ones. The authors considered the most representative value function as the one which maximizes the difference of scores between alternatives related by preference in the necessary ranking.

The concept of ROR was integrated into NSGA-II making two important changes:

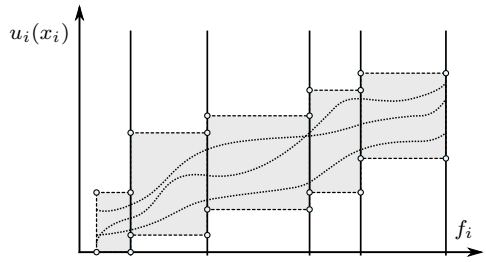

Figure 1.4 Range of compatible value function for objective f_i.

1. The Pareto dominance is replaced by the necessary preference relation in such a way that the selection for reproduction and the selection for survival are carried out according to the necessary preference rank.
2. The crowding distance is computed taking into account the multidimensional scaling given by the most representative value function.

During the main loop of NSGA-II, after k generations, the DM is asked for new preference information.

1.5 Miscellaneous Methods

1.5.1 Desirability functions

Wagner and Trautmann (2010) proposed transforming each objectives of the problem by a Desirability Function (DF) that maps the original objective to a domain $[0, 1]$ This new function has a bias towards the preferred solutions according to the desired values provided by the DM.

Let's consider that the image of objective f_i is $\mathcal{Z}_i \subseteq \mathbb{R}$, then a DF is defined as any function $d_i : \mathcal{Z}_i \to [0, 1]$ that specifies the desirability of different regions of the domain $\mathcal{Z}_i$ for objective f_i. The authors adopted two types of DF which were introduced by Harrington (1965): one designed for maximization or minimization of the objectives (one-sided function), and another one for target value problems (two-sided function). Thus, the original MOP is transformed into

$$\begin{aligned} \text{Minimize} \quad & -d(\mathbf{z}) = -d[\mathbf{f}(\mathbf{x})] = -(d_1[f_1(\mathbf{x})], \ldots, d_k[f_k(\mathbf{x})])^T \\ \text{where} \quad & \mathbf{x} \in \mathcal{X}, \\ & \mathbf{z} \in \mathcal{Z}, \\ & d(\mathbf{z}) : \mathcal{Z} \to [0, 1],\ i = 1, \ldots, k. \end{aligned} \tag{1.10}$$

Since the transformed problem is also a MOP, the multi-objective optimization algorithm adopted to solve the original problem can be used without modification to solve the new problem. However, the Pareto optimal solutions of the modified problem will present a biased

distribution towards the RoI of the original MOP. For the one-sided DF, the DM states his/her preferences by setting two points of each objective's DF: the first point, $(z_i^{(1)}, d_i^{(1)})$ represents the most desirable value, $d_i^{(1)} = 1$, in the domain $\mathcal{Z}_i$, while the second point, $(z_i^{(2)}, d_i^{(2)})$, denotes the least desirable value, $d_i^{(2)} = 0$, in the domain. The values for $z_i^{(1)}$ and $z_i^{(2)}$ are taken from the range $[z_i^\star, z_i^{\mathrm{nad}}]$, i.e., the range of the Pareto optimal front. For example, in order to focus the search on the center of a Pareto front, $z_i^{(1)} = z_i^\star$ and $z_i^{(2)} = z_i^{\mathrm{nad}}/2$ should be used.

1.6 Conclusions and Future Work

This chapter has presented a short review of recent efforts to design interactive MOEAs. It is clear that most of the proposals are based on classical well-known techniques originated in the OR field. From the proposed interactive methods, the most popular approach is the reference point method. This opens important paths of future research. There are many interactive techniques proposed by the OR community and, therefore, one of the obvious future research paths is the development of new interactive MOEAs using techniques based on a classification of the objectives, trade-off methods, or marginal rates of substitution. Another interesting possibility would be the development of interactive MOEAs using concepts from other sources. For example, the incorporation of preferences through the bias of the hypervolume (or other indicators used for assessing performance of MOEAs (Zitzler et al. 2003)).

The use of interactive MOEAs to deal with problems with a high number of objectives has become a popular research trend within evolutionary multi-objective optimization. We believe that there are two main reasons for this. On the one hand, the incorporation of preferences avoids the problem of visualizing a huge number of solutions in high dimensionality. On the other hand, emphasizing a region of interest introduces a stringent criterion that allows comparing nondominated solutions.

Summarizing, we believe that the incorporation of preferences into MOEAs is a very important research topic, not only because this is a fundamental part of the decision making process involved in the solution of a multi-objective optimization problem, but also because it can help to deal with problems having a large number of objectives. This topic, however, is still scarcely researched in the current literature, mainly because of its strong links with Operations Research, which makes it necessary to have a good background in such a discipline as well as in evolutionary multi-objective optimization. However, as more efforts are being made to bring together these two communities (OR and EMO) (Branke et al. 2008) we expect to see much more research in this area in the next few years.

Acknowledgements

The authors acknowledge support from CONACyT project no. 103570.

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