Novel Fibre Bragg Grating design using Multiobjective Evolutionary Algorithms

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Abstract- A multi-objective evolutionary optimisation algorithm is applied to a Fibre Bragg Grating (optical filter) design problem. The design specified a dual wavelength filter with four required spectral characteristics - total bandwidth, peak separation, peak width and minimum transmission. Five parameters which described the apodised grating profile were used to define the search space and the transfer matrix method was used to numerically evaluate the transmission spectrum of candidate solutions. Various constraints on the search space were included in the design algorithm. Two separate selection schemes were tested, a distance based approach as used in the Nondominated Sorting Genetic Algorithm (NSGA-II) and a conglomerative clustering approach as used in the Strength Pareto Evolutionary Algorithm (SPEA). Non-dominated solutions are found and it is evident that particular objectives can be achieved more easily than others. Preliminary results are discussed and future work is introduced.

1 Introduction

As more demands are made on telecommunications and other applications of photonics such as sensing, demand for the complexity and functionality of the interconnecting devices increases. Fibre Bragg Gratings (FBG) for optical filters and switching are an inherent part of such systems. The design of these devices is a complicated task, and forms an excellent set of problems of which to use sophisticated design algorithms such as Evolutionary Algorithms. Our work in this area of photonic design has so far focused on using an Evolutionary Strategy for the design of a complex optical fibre [1].

The design of FBG's using Genetic Algorithm's (GA) has been previously explored using single objective techniques [2] [3]. Typically, these have been simple designs (for example, bandpass filters), and GA's have proven to be very successful in this domain. In these cases the objective function was defined as the minimisation of the difference between the target spectrum and design spectrum. Weights were used to increase the relative importance of regions of interest.

In this paper we consider the *generalised* case of multiple objectives without reducing the problem to a single objective. The four objectives used here relate to four spectral features of interest of a wavelength filter, which were explicitly defined and calculated for each solution. The advantage of multiobjective techniques over single-objective techniques is that we can find groups of solutions which inherently represent all possible compromises between the various target design objectives, leaving the final decision of *optimality* to a human decision maker.

2 Fibre Bragg Gratings

Permanent gratings in optical fibre were first demonstrated experimentally in the late 1970's. Since then, the theoretical and experimental aspects of FBG's have flourished, resulting in a multitude of applications. FBG's have be used as stand alone devices, for example, sensing applications for strain, temperature and voltage measurement. They have also been incorporated into fibre communications systems where they are used to combine, divide and filter digital light signals. The manufacture of FBG's has reached a level now where designs can be quickly and easily fabricated, making FBG design ideal for sophisticated design algorithms. An overview of FBG history and technological developments is available in [4].

The theoretical aspects of FBG's are well developed, and many techniques have been published which allow us to obtain a grating structure given a desired spectrum. A recent example is presented in [5]. A common disadvantage of these methods is that the required transmission spectrum and other properties such as the group delay spectrum have to be specified over a large wavelength range. Further to that, the inverse algorithms do not allow the inclusion of other constraints, where the ability to specify important spectral features that should be present would be extremely useful. Here we have used a multiobjective evolutionary approach since it allows us to quantify the particular features of interest, without the need to over-specify the transmission spectrum for an inversion algorithm.

Experimentally, FBG's are produced by exposing a short length of optical fibre to an intense optical interference pattern, which results in a lengthwise modulation of the refractive index of the silica (glass) of the fibre. This high

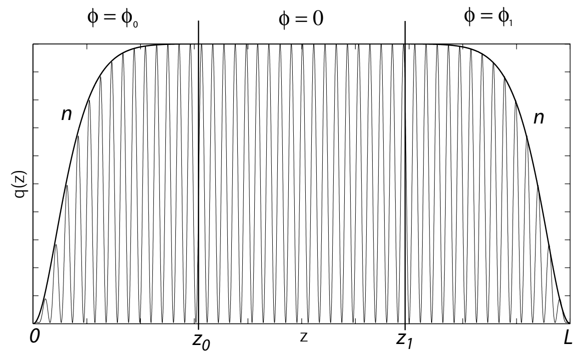

Figure 1: Graphical representation of the seven design variables $q_0, z_0, z_1, n, L, \phi_0, \phi_1$. The intrinsic grating structure is shown as the high-frequency curve. The resulting apodised profi le (envelope) is also shown.

frequency modulation is typically in the order of a μm, and forms an overall envelope profile which is cm's in length (Figure 1). The main idea of FBG design is to find a profile which leads to the FBG exhibiting particular spectral properties of interest, where different wavelengths can be selectively reflected (or transmitted) in varying degrees.

Generally, the strongest interaction of the FBG with light occurs at the Bragg wavelength λ_{B}, defined by

$$\lambda_{\mathrm{B}} = 2n_{\mathrm{eff}}\Lambda \tag{1}$$

where n_{eff} is the model effective index of the propagative wave and Λ is the grating period. For the purposes of this study we set $\lambda_{\mathrm{B}} = 1.55\mu m$, but the factor Λ has itself previously been used as a design variable [2]. Since we can tune this with relative ease during the manufacturing process, it is not included as a design variable.

One of the simplest FBG's consists of a uniform grating, which leads to a sinc like function transmission spectrum. To smooth out these side ripples in the spectrum, an *apodisation* can be introduced at each end of the grating. This process of apodisation refers to gradually changing the strength of the grating, rather than an abrupt change as in the uniform FBG. In addition to that, phase changes can be introduced in the grating to induce transmission peaks in the spectrum. Various functions are used to describe this type of FBG, where we have a rolloff on the ends and a constant central region. The raised cosine $\cos^2(z)$ is one of these commonly used functions.

2.1 Design parameters and manufacturing constraints

The design parameters used in this problem are associated with details of the raised cosine function which describes the apodised profile. The most general form of the raised cosine grating structure is defined as

$$q(z) = q_0 \cos^2\left(\frac{\pi}{2}\left|\frac{2z}{L}\right|^n\right) e^{i\phi(z)} \tag{2}$$

where

$$\begin{aligned} \phi(z) &= \phi_0, \qquad z < z_0 \\ &= \phi_1, \qquad z > z_1 \\ &= 0, \qquad \text{otherwise} \end{aligned}$$

containing seven design parameters: q_0 is the peak strength of the grating, L is the total length of the grating, n controls the curvature of the end drops and z_0, z_1 relate to the locations of the ϕ_0, ϕ_1 phase changes respectively. Since the spectrum of interest in this study is symmetrical about $\lambda = \lambda_{\mathrm{B}}$, we impose $\phi_0 = \phi_1$ and $z_0 = -z1$, resulting in a search space of five dimensions. The contribution of the various parameters to the FBG design is shown graphically in Figure 1.

Constraints were included in the design process since various manufacturing limits exist relating to factors such as the maximum strength of the grating and maximum manufacturable length. These were taken as $L_{\max} = 15cm$ and $q_{0\max} = 500m^{-1}$. Some constraints also exist due to the definition of the design, such as $0 \le \phi_0, \phi_1 < 2\pi$, $n > 1.0$ and $0 \le z_0 < z_1 \le L$.

2.2 Numerical Modelling

Derived from Maxwell's equations, the coupled mode equations describing a simple FBG can be written as

$$+iu'(z,\delta) + \delta u(z,\delta) + q(z)e^{+i\phi(z)}v(z,\delta) = 0 \tag{3}$$

$$-iv'(z,\delta) + \delta v(z,\delta) + q(z)e^{-i\phi(z)}u(z,\delta) = 0 \tag{4}$$

where $q(z)$ represents the apodisation (overall strength envelope) of the FBG, $\phi(z)$ encodes the chirp (phase change) of the grating, and δ is defined as the de-tuning value

$$\delta = 2\pi\bar{n}\left(\frac{1}{\lambda} - \frac{1}{\lambda_{\mathrm{B}}}\right) \tag{5}$$

where $\bar{n}$ is the average refractive index of the FBG. The forward and backward propagating waves are represented by u and v respectively.

We solve the coupled mode equations using a transfer matrix approach. The fibre grating is divided into discrete layers of width h along z between 0 and L. The transfer matrices propagate the fields from z to $z + h$, layer by layer through the structure. Both matrices T_δ and T_κ depend on the choice of h, but T_δ only depends on the detuning value δ, whereas T_κ depends on the grating properties defined by $q(z)$.

$$T_\kappa = \begin{bmatrix} \cosh(qh) & \sinh(qh)e^{+i\phi} \\ \sinh(qh)e^{-i\phi} & \cosh(qh) \end{bmatrix} \tag{6}$$

$$T_\delta = \begin{bmatrix} e^{+i\delta h} & 0 \\ 0 & e^{-i\delta h} \end{bmatrix} \quad (7)$$

At $z = L$ we specify the boundary conditions $u(L) = 1$ and $v(L) = 0$. Layer by layer the matrices are applied to $[u, v]$

$$\begin{bmatrix} u(0) \\ v(0) \end{bmatrix} = T_\kappa T_\delta \cdots T_\kappa T_\delta \begin{bmatrix} u(L) \\ v(L) \end{bmatrix} \quad (8)$$

and the final reflectance at λ is given by $r(\lambda) = \frac{v(\lambda)}{u(\lambda)}$.

This is repeated for a range of wavelengths to obtain the spectrum over the range $\lambda_{\mathrm{B}} \pm 1\mathrm{nm}$. The resulting transmission spectrum in decibel units is evaluated using

$$T_{\mathrm{dB}}(\lambda) = 10 \log_{10}(1.0 - |r(\lambda)|^2) \quad (9)$$

3 Multi-objective Evolutionary Optimisation

Real-world engineering problems generally involve multiple objectives, and to simultaneously meet these most optimisation techniques will combine these multiple objectives into a single one. In these cases, different objectives are assigned different levels of importance in the form of weights, which directs the design algorithm to emphasize a particular type of solution in its search.

One of the advantages of Pareto based design schemes is that they automatically find solutions with all possible weights, that is, a range of structures where different features are expressed to varying degrees. This translates to the fact that we don't need to make a previous decision about the importance of objectives relative to others, since this may not even be known. Ultimately it places the decision process in the hands of a human decision maker, which is desirable in many real engineering problems.

Evolution-based design algorithms naturally lend themselves to such techniques since they are population based. The non-dominated solutions can exist simultaneously and through the processes of mutation, recombination, breeding and selection an accurate representation of the Pareto set of that problem can be formed. The most important issue in the design of multiobjective optimisation algorithms is the convergence to and quality of the non-dominated set, which should ultimately be very close to the true Pareto set associated with that particular design problem.

All current Evolutionary Algorithms, both single and multi-objective mainly differ in the choices of mutation and selection operators. The main focus of current research on improving multiobjective algorithms resides in the selection operator. Some recent papers have examined different mutation schemes, but it is still in very early stages.

Our initial work on photonic design focused on the design of an optical fibre using a single objective Evolutionary Strategy algorithm [1]. The expansion to a multiobjective algorithm was straightforward, where only the selection operators had to be included in the pre-exisiting Object Oriented framework. The adaptive mutation operators, as discussed in [6] were not altered.

3.1 Mutation operator

The adaptive mutation operator functions on the premise that the optimal mutation strategy is not known for each particular optimisation problem. Apart from the usual gene vector, two other vectors are introduced - strategy variables and correlation variables.

The strategy variables refer to the standard deviation σ of the normally distributed random mutations applied to each individual gene, and control the distribution of the search along the design parameter axes. The correlation variables α generalise these search directions. It is unclear at this stage how this adaptive mutation operator affects convergence and the quality of the multiobjective algorithm, and is expected to be the basis of further research.

For a more detailed overview of adaptive mutation, we refer the reader to [6].

3.2 Selection Operator

During the selection process we want to reduce a population of size $N_{\mathrm{parents}} + N_{\mathrm{children}} \rightarrow N_{\mathrm{parents}}$. In this paper two types of selection operators were used.

The first used was the *crowding distance approach* as used in the NSGA-II (Non Dominated Sorted Genetic Algorithm-II) [7]. Firstly, the non-dominated set (size N_{dom}) is extracted from the whole population. If $N_{\mathrm{dom}} \leq N_{\mathrm{parents}}$, only the non-dominated set is used to produce children. If $N_{\mathrm{dom}} > N_{\mathrm{parents}}$, some individuals need to be culled. Relative to each objective, all individuals are sorted in order in turn. We then step through this list and each individual is assigned a crowding distance which is equal to the average distance between its two closest neighbours. These individuals are then sorted from greatest distance (most sparse) to least distant (most densely packed in objective space), and the top N_{parents} are chosen. Individuals are thus sorted on the basis of *closeness* to each other. By default, the individuals which lie in the extreme corners of the objective space are chosen. These corner individuals will generally have excellent performance in one objective, but poor performance in all other objectives.

The other approach used to reduce the population is a clustering technique. This has been previously used in the SPEA (Strength Pareto Evolutionary Algorithm) [8]. Here the non-dominated set is chosen, but we also use the remaining non-dominated levels to fill up the parent population if required. If $N_{\mathrm{dom}} > N_{\mathrm{parents}}$, the population is reduced using clustering. The clustering algorithm used here is known as *hierarchical conglomeration*. Initially, every single individual forms a cluster. Through multiple iterations, clusters are joined together by comparing their Euclidian distances

Objective	Target trait value	Objective
Peak separation (A)	10Ghz (0.08nm)	$min\,\lvert A-0.08\rvert$
Average depth (B)	<-30dB	$min(B)$
Peak width (C)	0.8pm	$min\,\lvert C-0.8\rvert$
Total bandwidth (D)	1nm	$min\,\lvert D-1.0\rvert$

Table 1: Outline of the four objectives used (see Figure 2). The target trait values for the initial design specification are shown, along with the conversion to the target objective values which are to be minimised.

in the objective space. In the cases where there is more than one individual in a cluster, a *centroid* (average cluster position) is used to measure the objective distance.

Often in practice, when dealing with $n_{\rm obj} > 2$, there will be more than $N_{\rm parents}$ on the first non-dominated front. The clustering (or whichever reduction technique we choose) is then only applied to this set of individuals, which all have an equal breeding probability. Constant population sizes of 100 are often used in multiobjective algorithms. We expect that as the dimensionality of objectives increase, more individuals are required to give a good representation of this non-dominated front, in the order of 10^n where n is the number of objectives.

It is important to note here than when distances are measured in objective space, these measures must be normalised. This is often not necessary when test functions are used since each objective value exists in the same range. This is not so for real problems such as the FBG one dealt with here, where the objective values are of different units and different orders of magnitude, and are not known. To find the distance between two solutions a, b we use

$$d_{\rm norm} = \sqrt{\sum_{m=1}^{N_{\rm obj}} \left(\frac{a_m - b_m}{o_{\rm max}(m) - o_{\rm min}(m)} \right)^2} \qquad (10)$$

where a_m refers to the *m*th objective of individual a and $o_{\rm min}(m), o_{\rm max}(m)$ refer to the minimum and maximum values of the *m*th objective in the population.

3.3 Constraint handling

Constraints were included in the design process since various manufacturing limits exist relating to factors such as the maximum strength of the grating and maximum manufacturable length. The constraints were enforced in a fairly simple manner at the breeding stage. If individuals did not conform upon breeding, they were removed. Breeding continued until enough individuals were created to fill the population.

In future more complicated methods of constraints will be introduced, such as that outlined in [7]. Here individuals which do not conform are kept in the population and

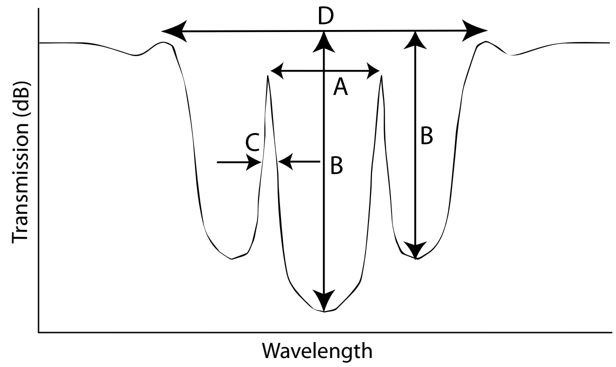

Figure 2: Diagram of the type of transmission spectrum we are optimising for, showing the 4 quantitative values used as extracted from the spectrum. Objective B is shown twice to indicate that it was taken as the average of the depth of the two transmission troughs.

included in the non-dominated selection scheme. The selection then takes place based on (1) their objective values, and (2) the closeness of conformation to constraints. Such methods are less *artificial* than the current method used, and may be ideal for finding non-dominated solutions which exist near the constraint boundaries.

3.4 Computational Implementation

One of the many advantages of Evolutionary based algorithms over other more traditional search schemes is the easy adoption of a parallel scheme for the calculation of traits. Once the genetic material of a population of individuals have been determined, the trait calculations can proceed independently of one another. Our code was parallelised using the Message Passing Interface (MPI). Here we used the freely available MPICH library [9].

The runs of this software are typically very efficient over 11 processors, 10 serving as slave processors with a single master distributing the workload. The speedup of the algorithm is close to the expected $(n_{\rm cpu} - 1)^{-1}$ relation, due to the short MPI communication times relative to the trait calculation times. The use of more than 11 processors does not give much of an advantage due to incomplete parallelisation (Ahmdahl's law). Further details are available at [1].

The C++ Standard Template Library data structures were used heavily, as they greatly simplify the process of data storage and data manipulation in algorithm such as the clustering technique described previously.

4 Fibre Bragg Grating design

Four spectral features were chosen for quantitative optimisation.

- Trait A defines the distance between the the two central wavelength transmission peaks,
- Trait B refers to the average depth of the two troughs,
- Trait C is the thickness of the peaks at the halfway transmission point between the two troughs,
- Trait D refers to the bandwidth of the FBG.

For an outline of the resulting objectives refer to Table 1 and Figure 2.

Once the transmission spectrum $T(\lambda)$ was calculated as a function of the five gene values, a peak finding routine based on $\frac{dT}{d\lambda}$ was used to extract the four trait features.

Runs of the multiobjective algorithm were conducted using both the SPEA and NSGA-II selection algorithms. Population sizes of 50 parents and 50 children were generally used, but runs with 100 to 500 parents and children were also conducted to ascertain if better coverge of the non-dominated set could be achieved, given the 4D objective space. The runs were conducted over 500 generations.

4.1 Results

A behaviour that stood out in all the MOEA runs was that two main types of solutions would often dominate the population:

- Individuals which were very dominant in objective B (very deep troughs) but poor in all other objectives (this correlated strongly with large shallow side peaks),
- Individuals with no distinguishable side peaks (corresponding to objective $C = 0$).

During the early generations of runs, where the mutation rates are high, the parent population tended to converge to these areas of the objective space. This perhaps indicates that the gene space responsible for features such as the expression of dual peaks along with deep troughs is quite limited in scope.

The objective functions were re-evaluated in an attempt to divert the search space away from these features. Firstly, individuals with no discernible central peaks were given a bad objective value (objective $C = 1$). Secondly, individuals with an average transmission trough less than -60dB were given an equal objective value, such that

$$\begin{aligned} B_{\mathrm{obj}} &= B_{\mathrm{trait}}, \qquad B_{\mathrm{trait}} > -60\mathrm{dB} \\ &= -60\mathrm{dB}, \qquad \mathrm{otherwise} \end{aligned}$$

This placed the individuals with large negative B on an equal footing during the selection process. The combination of these two objective function adjustments successfully improved the diversity of the non-dominated set found, and typically resulted in the examples shown in Figure 3.

The problem of a large proportion of individuals occurring in the population with large negative troughs still per-

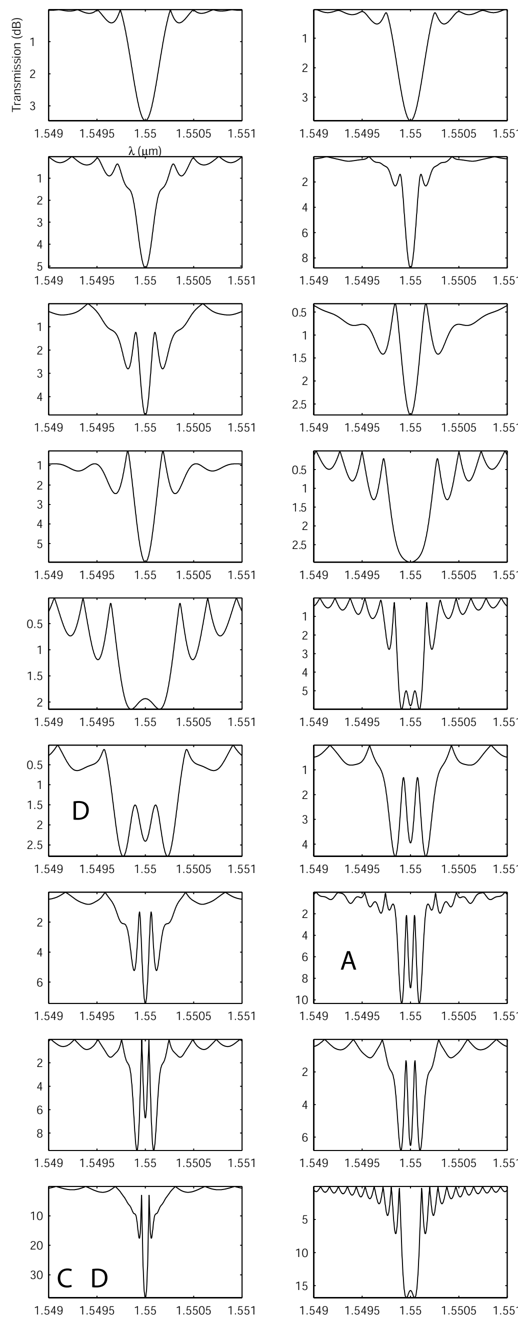

Figure 3: Transmission spectra typical of those found on the final non-dominated set of a MOEA run. Generally 20% of individuals were of this diverse kind. The rest consisted of a single deep transmission trough (strong reflection peak at λ_B). Some extremal designs are labelled with the corresponding objective(s) they excel at. The corresponding traits are $A = 0.084\mathrm{nm}, B = -21.5\mathrm{dB}, C = 0.08\mathrm{nm}, D = 0.85\mathrm{nm}$.

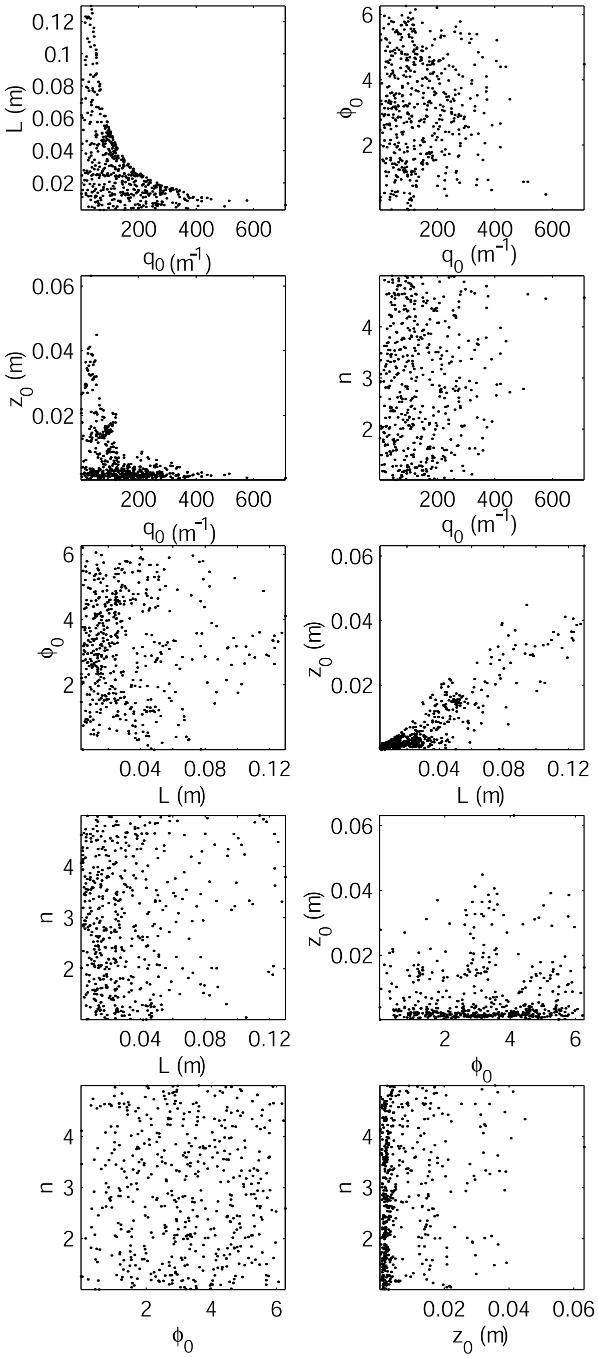

Figure 4: Pairwise plots along the direction of each design variable of the non-dominated set after 500 generations using $N_{\text{parents}} = 500, N_{\text{children}} = 500$.

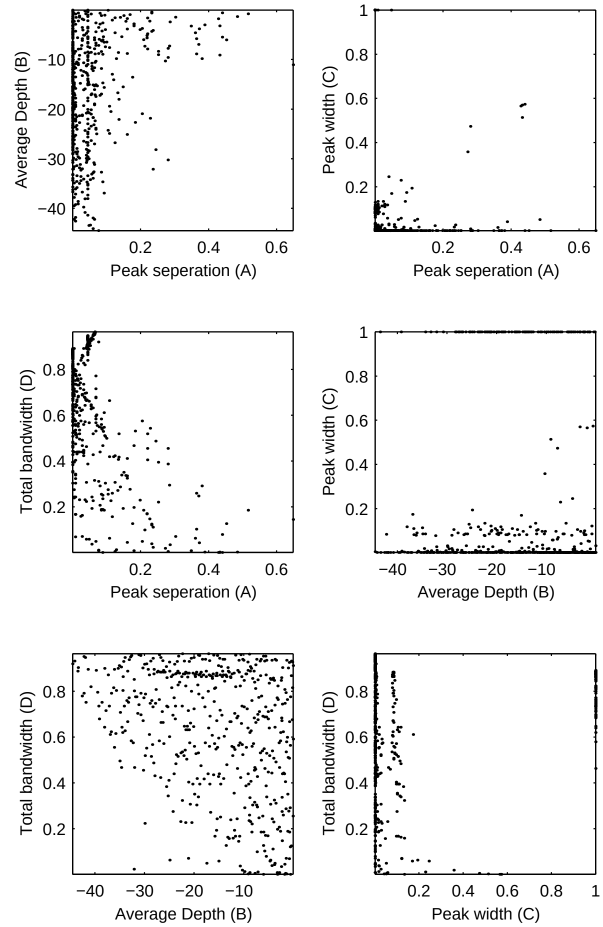

Figure 5: Pairwise plots along the direction of each objective (as outlined in Table 1) of the non-dominated set after 500 generations using $N_{\text{parents}} = 500, N_{\text{children}} = 500$. Most of the population is concentrated in the region where Peak width (C) ≈ 0.

sisted. Examination of the gene (design) parameters suggested that the population remained quite diverse in the design space as compared to the objective space (Figures 5 and 4). This suggests that the genetic space which is responsible for those particular features must be very small since they are rarely expressed. In an attempt to alleviate this and increase coverage of the non-dominated front, the total size of the population was increased to 400 and 1000. Although this did provide more viable solutions, a large proportion of population members still existed with large negative transmissions. An example of the resulting non-dominated set for a run with $N_{\mathrm{parents}}, N_{\mathrm{children}} = 500$ is shown in Figure 4 (design space) and Figure 5 (objective space).

The crowding and clustering size reduction schemes aim at maintaining diversity, but it is difficult when the number of diverse solutions is much less than the parent population size. So at this point in time, no discernible difference was found between the use of the crowding based selection operator and the clustering selection operators. It is difficult to make judgement on this till the non-dominated diversity is improved.

Two methods which will be implemented in an attempt to reduce this problem are

- Further reduce the size of the parent population so that at each generational loop there is a larger proportion of individuals which have better diversity in all objectives,
- Favour such diverse individuals during the breeding process.

Other opportunities exist in defining the objectives more explicitly, such as the flatness of the troughs (which would be advantageous for wavelength filters). Such changes in the objective definitions would be ideal, but we must simultaneously keep the dimensionality of the objective space small.

5 Conclusions

A Multi-Objective Evolutionary Algorithm was applied to a Fibre Bragg Grating design problem where four objectives were simultaneously optimised for a twin wavelength filter device. Small sub-groups of diverse non-dominated solutions were successfully found, but the population tended to converge to particular solution types, limiting diversification over time. Future work will involve limiting the number of selected individuals to more accurately represent non-dominated members.

The research work in its current state is in early days, with a large scope present for future work involving the improvement of the design algorithm to diversify the non-dominated sets found, and visualisation of these non-dominated sets to further our understanding of FBG design.

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