

CASCADE AIRFOIL DESIGN BY MULTIOBJECTIVE GENETIC ALGORITHMS

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Abstract

Multiobjective Genetic Algorithm based on Fonseca-Fleming's Pareto-based ranking and fitness sharing techniques has been applied to aerodynamic shape optimization of cascade airfoil design. Airfoil performance is evaluated by a Navier-Stokes code. Evaluation of GA population is parallelized on Numerical Wind Tunnel, a parallel vector machine. The present multiobjective design seeks high pressure rise, high flow turning angle, and low total pressure loss at a low Mach number. Pareto solutions that perform better than existing Control Diffusion Airfoil were obtained.

Introduction

Improvements in turbomachinery is important for the present and future aerospace industry. Design of compressors plays an important role in the turbomachinery development since flow fields through compressors are more sensitive than those through turbines. Aerodynamic design of compressor blade shape is thus vital to thrust performance.

Development of aerodynamic design methods for compressor blades has always been of strong interest. Successful design methods have been reported, for example, in [1,2]. Most of them are based on an inverse approach. The inverse design method determines the shape from a prescribed pressure distribution. This is a powerful design tool for those with insights who are able to describe a favorable pressure distribution.

However, this approach leaves the problem of specifying an appropriate pressure distribution to designers at large. An arbitrarily prescribed pressure distribution may correspondingly result in an unrealistic geometry, such as a fishtail airfoil. In addition, it is difficult to include geometric constraints. Therefore, in this paper, a direct approach is employed. Direct numerical optimization methods are formed by coupling aerodynamic analysis methods with numerical optimization algorithms. They minimize (or maximize) a given aerodynamic objective function by iterating directly

on the geometry.

Goal of the compressor design is to produce the highest pressure rise at the lowest total pressure loss. The pressure rise is achieved by energy exchange from the flow velocity. Flow fields in the compressor inevitably have adverse pressure gradients and often experience boundary layer separation or shock wave. In such situations, the efficiency of energy exchange will be greatly diminished. Thus, tradeoff between high pressure rise and low total pressure loss must come to the best compromise. The design of compressors is inherently a multiobjective optimization problem.

Multiobjective optimization (MO) seeks to optimize the components of a vector-valued objective function. Unlike the single objective optimization, the solution to this problem is not a single point, but a family of points known as the Pareto-optimal set [3]. Each point in this set is optimal in the sense that no improvement can be achieved in one objective component that doesn't lead to degradation in at least one of the remaining components.

Pareto-optimal solutions might be obtained by solving appropriately formulated single-objective optimization problems on a one-by-one basis. In contrast, by maintaining a population of solutions, Genetic Algorithms (GAs) can search for many Pareto-optimal solutions in parallel [4]. Several approaches have been proposed to solve MO problems using GAs [3-9]. The keys to finding the Pareto front among these various procedures are the Pareto-based ranking and fitness sharing techniques. The Pareto-based ranking method, proposed by Fonseca and Fleming [6] as Multiple Objective Genetic Algorithm (MOGA), is adapted to the present MO problem.

To demonstrate the feasibility of the present optimization method, an existing compressor cascade model (stator airfoil) is redesigned. To evaluate the performance of each airfoil during the evolutionary process, an accurate prediction of the total pressure loss is important. Thus, a Navier-Stokes solver similar to [10] is used here. Although the present problem is two dimensional, use of the Navier-Stokes solver in MOGA requires tremendous computational time. To overcome

this, evaluations in MOGA were parallelized on Numerical Wind Tunnel (NWT, used by winners of IEEE's 1995/1996 Gordon Bell Prize), a parallel vector machine at peak performance of 279 GFLOPS with 166 processing elements, located at National Aerospace Laboratory in Japan.

MOGA

1. Coding, Crossover and Mutation

In GAs, the natural parameter set of the optimization problem is coded as a finite-length string. Traditionally, GAs use binary numbers to represent such strings: a string has a finite length and each bit of a string can be either 0 or 1. Since it is more natural to use real numbers for real function optimization, real-number coding is used here. The length of the real-number string corresponds to the number of design variables.

Airfoil geometry is split into a mean camber line and thickness distribution. These distributions are described by B-spline polygons from the leading edge to the trailing edge of the airfoil. Seven points are used for the camber line and eight points are used for the thickness distribution. Since locations of the leading edge and trailing edge are frozen, the remaining points become design variables. In addition, for the thickness distribution, one more point is constrained at the trailing edge so as to represent Control Diffusion Airfoils (CDAs) [1,2] correctly. At this point, only the y coordinate is to be determined. In total, 21 design variables are required to give locations for two B-spline polygons. Each polygon is constrained not to cross over itself. Thickness distribution is also constrained to have a positive thickness.

Crossover operator is defined by exchange of corresponding design variables. All design variables are subject to being exchanged independently at a probability of 50%. Mutation operator produces random disturbances to the design variable in the amount of ± 0.2 for the x coordinate (the chordwise direction) and ± 0.02 for the y coordinate. Probability of mutation is initially 20% and it decreases linearly to 1% over 100 generations.

2. Pareto Ranking

By maintaining a population of solutions, GAs can search for many Pareto-optimal solutions in parallel. This characteristic makes GAs very attractive for solving MO problems. The following two features are desired to solve MO problems successfully: 1) the solutions obtained are Pareto-optimal and 2) they are uniformly sampled from the Pareto-optimal set. To achieve these with GAs, the following two techniques are combined

into MOGAs [6].

To search Pareto-optimal solutions by using MOGA, the ranking selection method can be extended to identify the near-Pareto-optimal set within the population of GA. To do this, the following definitions are used: suppose $\mathbf{x}_i=(x_i,y_i)$ and $\mathbf{x}_j=(x_j,y_j)$ are in the current population and $f=(f_1,f_2)$ is the set of objective functions to be maximized,

1. \mathbf{x}_i is said to be dominated by (or inferior to) \mathbf{x}_j , if $f(\mathbf{x}_i)$ is partially less than $f(\mathbf{x}_j)$, i.e., $f_1(\mathbf{x}_i) \leq f_1(\mathbf{x}_j) \wedge f_2(\mathbf{x}_i) \leq f_2(\mathbf{x}_j)$ and $f(\mathbf{x}_i) \neq f(\mathbf{x}_j)$.
2. \mathbf{x}_i is said to be non-dominated if there doesn't exist any \mathbf{x}_j in the population that dominates \mathbf{x}_i .

Non-dominated solutions within the feasible region in the objective function space give the Pareto-optimal set.

Consider an individual \mathbf{x}_i at generation t which is dominated by p_i' individuals in the current population (Fig. 1). Its current position in the individuals' rank can be given by

$$\text{rank}(\mathbf{x}_i, t) = 1 + p_i' \quad (1)$$

All non-dominated individuals are assigned rank 1 as shown in Fig. 1.

3. Fitness Sharing

To sample Pareto-optimal solutions from the Pareto-optimal set uniformly, it is important to maintain genetic diversity. It is known that the genetic diversity of the population can be lost due to their stochastic selection process. This phenomenon is called the random genetic drift [4].

To avoid genetic drift, a practical scheme is given by taking the raw fitness and dividing through by the accumulated number of shares

$$f_s(\mathbf{x}_i) = \frac{f(\mathbf{x}_i)}{\sum_j s(d(\mathbf{x}_i, \mathbf{x}_j))} \quad (2)$$

where $s(d)$ is a sharing function that determines the neighborhood and degree of sharing. The distance $d=d(\mathbf{x}_i, \mathbf{x}_j)$ can be measured with respect to a metric in either genotypic or phenotypic space. A genotypic sharing measures the interchromosomal Hamming distance. A phenotypic sharing, on the other hand, measures the distance between the designs' objective function values. In MOGAs, a phenotypic sharing is usually preferred since we seek a global tradeoff surface in the objective function space.

A typical sharing function is given by

$$s(d) = \begin{cases} 1 - \left(\frac{d}{\sigma_{share}}\right)^\alpha & d < \sigma_{share} \\ 0 & d \geq \sigma_{share} \end{cases} \quad (3)$$

This scheme introduces new GA parameters, the niche size σ_{share} and the exponent α . In the following, the niche size σ_{share} is evaluated in the objective function space similar to [6] and the exponent α is set to 0.25 to emphasize the niche count.

Niche counts can be consistently incorporated into the fitness assignment according to rank by using them to scale individual fitness within each rank. Selection operator is defined by using the nonlinear function suggested in [11]. By implementing fitness sharing in MOGAs, one can expect to evolve a uniformly distributed representation of the global tradeoff surface.

The best- N selection [12] is incorporated further, where the best N individuals are selected for the next generation among N parents and N children. The Pareto solutions will be kept once they are formed.

Results

1. Inverse Design by Optimization

To validate the present optimization procedure, an inverse design by optimization is first performed. Aerodynamic inverse problem is to obtain airfoil geometry that produces prescribed target pressure distribution. Here, a pressure distribution computed by a Navier-Stokes code about one of CDAs [13] was used as a target. Initial designs were created randomly and the corresponding pressure distributions were computed by using the same Navier-Stokes code. Then the inverse problem was solved by minimizing differences between the target and computed pressures through a single objective GA.

For Navier-Stokes computations, the H-type grid is used, where 201 points are used in the streamwise direction and 65 points are used in the direction normal to the airfoil surface. Flow condition is set to inlet Mach number of 0.25, Reynolds number of 0.7 million based on the chord length, inlet flow angle of 40 deg, blade stagger angle of 14.4 deg, and blade pitch of 0.5988, similar to the experiment [13]. For the GA calculation, a population size of 64 is used and CFD evaluation of each member is distributed to one processing element of NWT. The CPU time necessary for one generation was roughly 20 min. In addition, since this is a single objective optimization, the basic roulette-wheel selection was used.

Figure 2 shows the results of the inverse design after 80 generations. The GA process was stopped at 80

generations since no improvement was obtained after 64 generations. Although there were minor differences in the pressure distributions near the leading edge, the designed geometry coincides with the original CDA geometry.

2. Cascade Optimization

Goal of the compressor design is to produce the highest pressure rise at the lowest total pressure loss. In addition to these two design goals, flow turning angle is maximized in the present MO optimization. Flow turning angle is an important design criteria in the classical design procedure. In general, the pressure rise increases as the flow turning angle increases. However, there is a limit in the amount of flow turning due to flow separation causing large total pressure loss.

The present MO problem can be described as

1. Maximize: Pressure rise (outlet pressure by inlet pressure), p_2/p_1
Subject to : De Haller number to be greater than 0.72
2. Maximize: Flow turning angle, $\Delta\beta$
3. Minimize: Total pressure loss, ω

All objectives are subject to

1. Maximum airfoil thickness to be the same as that of CDA, 7% of the chord length c
2. Airfoil area to be greater than that of CDA, $0.047c^2$

To maximize pressure rise, de Haller number (outlet flow velocity by inlet flow velocity) is limited more than 0.72 as suggested in [14]. For all objectives, geometric constraints based on the CDA geometry are applied as the maximum airfoil thickness to be $0.07c$ and the airfoil area to be more than $0.047c^2$.

Figure 3 shows the Pareto front of the present MOGA population after 80 generations. Each axis is linearly scaled according to the maximum and minimum fitness values. After 70 generations, the entire population consists of Rank-1 individuals. The population can be divided into two groups as marked in the figure. There are Pareto solutions satisfying high pressure rise and high flow turning angle. But they are not efficient solutions. The Pareto solutions satisfying high pressure rise and low total pressure loss generally give low flow turning angle. This is apparent from the plot of total pressure loss versus flow turning angle. It is very difficult to increase both fitness values.

Figure 4 shows variations of airfoil geometries of the Pareto solutions. To increase the pressure rise, the airfoil tends to have a sharp leading and trailing edges. To

increase flow turning angle, the airfoil has a large camber toward the trailing edge. On the other hand, to decrease the total pressure loss, the airfoil camber becomes much less near the trailing edge. This difference confirms the difficulty in satisfying both objectives.

Table 1 summarizes the performance of airfoil designs including the CDA performance. Figure 5 shows the corresponding performance diagrams. The Pareto solution having the highest pressure rise is found to give better performance than CDA in all three objectives. If reduction of total pressure loss is critical, the best Pareto solution gives 40% reduction of loss compared to CDA. Figure 6 shows one of the Pareto solutions that gives relatively high pressure rise and relatively low total pressure loss. The resulting airfoil has a relatively sharp leading edge to increase pressure rise and a blunt trailing edge to reduce total pressure loss. These results confirm that the present design procedure is capable of finding improved designs.

Conclusion

A new multiobjective optimization method for cascade airfoil design has been developed. Pareto optimal solutions are obtained from Genetic Algorithm based on Fonseca-Fleming's Pareto-based ranking and fitness sharing techniques. Airfoil performance is evaluated by a Navier-Stokes code. The computations are parallelized on Numerical Wind Tunnel. The method was validated by an inverse design of Control Diffusion Airfoil.

The present multiobjective design seeks high pressure rise, high flow turning angle, and low total pressure loss. Pareto solutions better than CDA in all three objectives are obtained. The solutions also suggests that it is difficult to design an airfoil satisfying high flow turning angle and low total pressure loss. From the Pareto solutions computed, the decision maker will be able to find a design that satisfies his design goal best.

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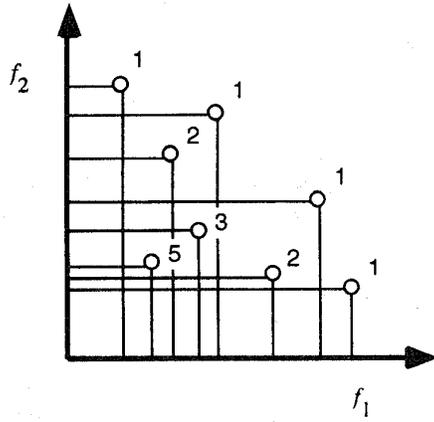


Fig. 1 Pareto ranking method

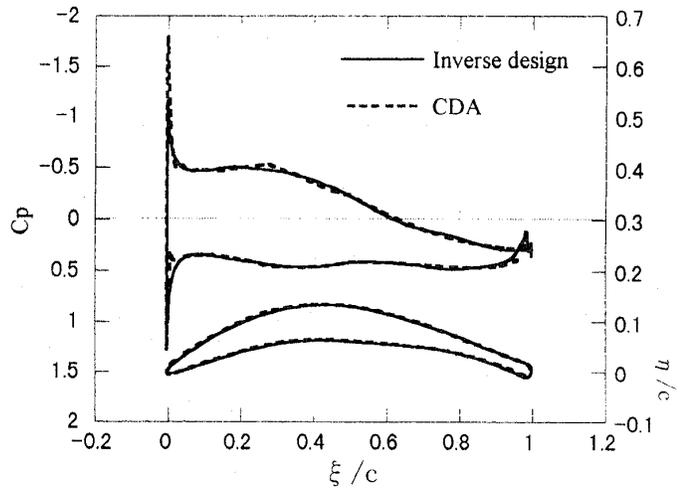


Fig. 2 Reproduction of CDA by the inverse design

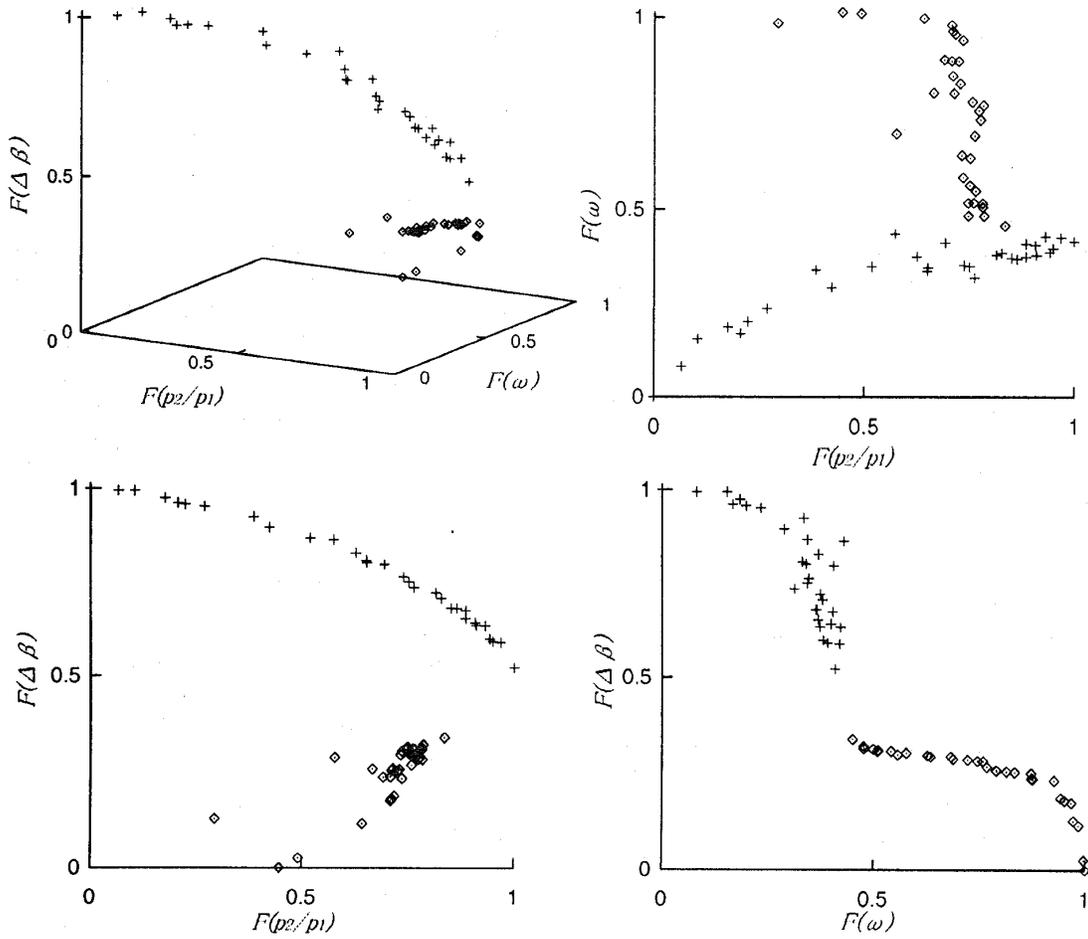


Fig.3 Pareto front in the objective space

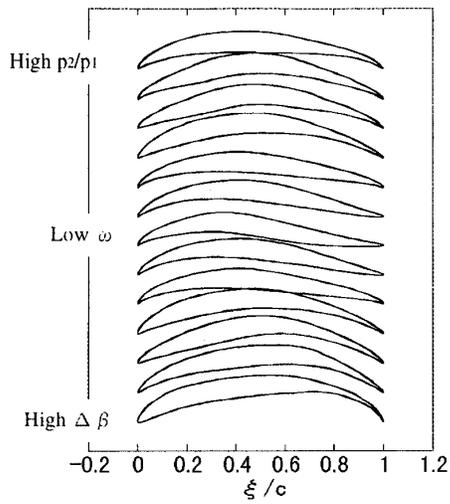


Fig. 4 Variations of Pareto solutions

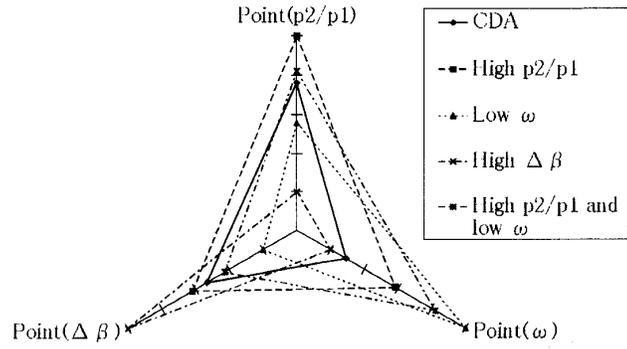


Fig. 5 Performance diagrams of Pareto solutions compared with that of CDA

Table 1 Comparison of cascade airfoil performances

CASE	p_2/p_1	ω	$\Delta \beta$ (deg.)
CDA	1.0171	0.0320	38.6
High p_2/p_1	1.0179	0.0266	41.0
Low ω	1.0164	0.0189	29.4
High $\Delta \beta$	1.0152	0.0337	51.6
High p_2/p_1 & low ω	1.0173	0.0222	35.7

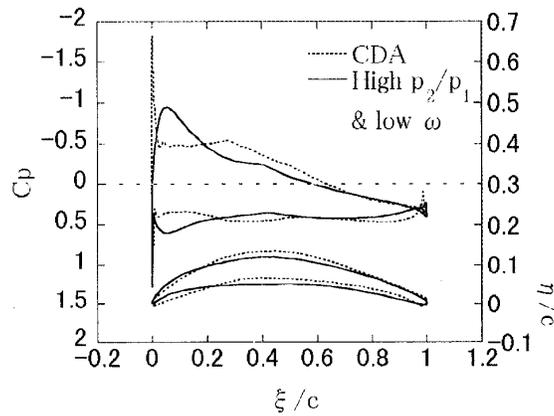


Fig. 6 Comparison of high p_2/p_1 and low ω solution with CDA