

MULTIOBJECTIVE OPTIMIZATION OF COMPONENT PLACEMENT ON PRINTED WIRING BOARDS

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Abstract

This paper presents a solution methodology for multiobjective optimization problems in the context of models for the placement of components on printed wiring boards. The methodology combines the use of a flow and heat transfer solver, a genetic algorithm for the adaptive search of optimal or near-optimal solutions, and a multiobjective optimization strategy (Pareto optimization or Multiattribute utility analysis). Using as the optimization criterion the minimization of an estimate of the failure rate of the system of components due to thermal overheating (via an Arrhenius relation), the effectiveness of the present solution methodology is demonstrated by reference to a case with *known* optimal solutions. The results obtained using the same solution methodology for a multiobjective optimization problem (a variation of the case study) involving the minimization of the aforementioned total failure rate of the system as well as the minimization of the total wiring length (given some interconnectivity requirements) are presented and discussed for both Pareto optimization and Multiattribute utility analysis.

1 Introduction

The optimal placement of components on printed wiring boards requires satisfying multiple, possibly conflicting, design objectives. As pointed out by Moresco [1], these design objectives may be very different in nature - geometrical, electrical, thermal, mechanical, and cost (manufacturing and maintenance) - which makes finding the "best" design a complicated task.

Two major design objectives are related to the reliability (thermal/mechanical/cost) and the routing (electrical/cost) requirements of the component placement design. Specifically, the minimization of estimates of the *failure rate* of the system and *total wiring length*

are design objectives of prominent interest. The former optimization criterion imposes major heat transfer requirements on the design because of the combined effects of: i) rapidly increasing packaging density and power dissipation demands; and, ii) potentially high costs associated with the failure of electronic components, as pointed out by Weiss et al. [2] and Wessely et al. [3] among others. The latter is critical because of electrical performance, speed and transmission line requirements and its impact on the manufacturing costs.

Most optimization studies regarding component placement have considered a single design objective, such as reliability (see, for example, Dancer et al. [4], Osterman et al. [5], Eliasi et al. [6]), or routing (see, for example, Kirkpatrick et al. [7], Dunlop et al. [8], and many others reported in Shahookar et al. [9]). The few optimization studies that have addressed multiple objectives have failed to provide rigorous methods to select the "best" design (Pecht et al. [10]) or have made somewhat arbitrary choices regarding the relative importance of the design objectives under consideration (Queipo et al. [11],[12],[13]).

This study overcomes the limitations of previous studies in this area and discusses a methodology to select the "best" component placement design when multiple design objectives are present. The methodology is based on the concepts of Pareto optimality (Balachandran et al. [14]) and Multiattribute utility analysis (Keeney et al. [15]). The Pareto optimization provides a set of alternative component placements from which the "best" design must be selected, and the MUA assists in the process of articulating the designer's preferences and identifying the "best" component placement (decision problem). As reported by Thurston [16], the MUA has been successfully applied to a wide variety of decision problems, including trajectory

selection for NASA missions, nuclear power plant site selection, telecommunication system architecture design, and many others.

The remainder of this paper is structured as follows. Section 2 provides a formal definition of the problem of interest and Section 3 gives a description of the different elements of the present solution methodology and their interaction. In particular, Section 3 describes a flow and heat transfer solver, a genetic algorithm and two different multiobjective optimization strategies (Pareto optimization and Multiattribute utility analysis). A description of a case study designed to validate and evaluate the present solution methodology is the subject of Section 4. The paper ends with the application of the present solution methodology to a multiobjective optimization problem (a variation of the case study) using both Pareto optimization and Multiattribute utility analysis.

2 Problem definition

The problem of interest here corresponds to the optimal placement of convectively cooled electronic components on printed wiring boards (PWB) subject to *thermal* and *non-thermal* optimization criteria. Because of its cost effectiveness and mechanical simplicity, forced air cooling is the most frequently used technique for cooling electronic components in personal computers and workstations. These systems comprise a major portion of the market with moderate heat transfer rate requirements. The convectively cooled electronic components on printed wiring boards are modeled here as equally spaced heated elements placed on the bottom wall of a ventilated two dimensional channel, as illustrated in Figure 1. The printed wiring board is aligned parallel to the coolant flow which is assumed laminar and two-dimensional. Each component is assumed to dissipate a constant heat flux and the heat fluxes may differ among components.

Regarding thermal optimization, forced air cooling is usually limited by acoustic noise constraints placed on the fan driving the flow, and arrangements of electronic components that maximize reliability and minimize thermo-mechanically induced stresses are highly desirable. Examples of non-thermal optimization criteria include the need to minimize the total wire length on the PWB, clustering functionally related components to conform to speed and transmission line requirements, and keeping analog components and digital components separate to reduce crosstalk.

In this study, the minimization of the failure rate of the electronic components on the printed wiring boards due to thermal overheating, and the minimization of

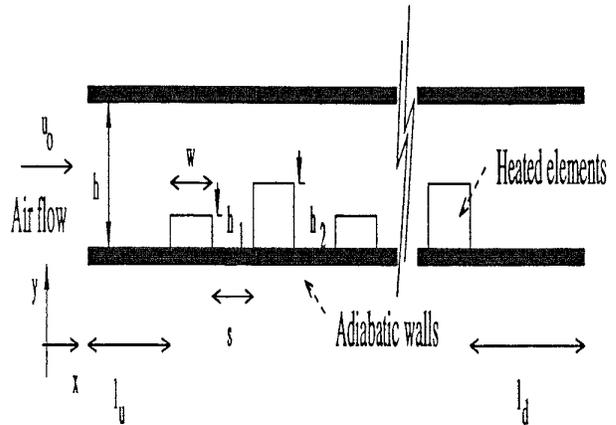


Figure 1: Schematic of the heat transfer configuration of interest.

the total wiring length satisfying the requirements specified by an interconnectivity matrix, are selected as *thermal* and *non-thermal* optimization criteria, respectively.

The reliability prediction is the statistical estimate of the value of time over which a device will function. The inverse of the reliability of a device is called its *failure rate* and is measured in failures per megahours (fr Mh^{-1}). As indicated by, for example, Moresco et al. [1], and Wessely et al. [3], the failure rate of an electronic component is a strong function of its temperature.

Even though various functional relationships between failure rate and temperature in electronic components have been suggested (Wong [17]), according to Blanks [18], the Arrhenius relation is the most widespread model among practitioners in the electronic packaging industry. In this study, the failure rate of electronic component "i" is estimated using the Arrhenius relation as:

$$\lambda_i = A_i \exp(-B_i/T_i^{max}) \quad (1)$$

Here A_i and B_i are constants associated with the thermal sensitivity of the electronic component, while T_i^{max} is the maximum temperature of component "i". Of interest here is the general case for which the electronic components on the PWB may differ in heat dissipation rate and thermal sensitivity. Since the failure rate of a component depends strongly on temperature as specified by Equation 1, the maximum temperature of each component is calculated by solving numerically the conservation equations for continuity, momentum and energy.

One of the objective functions to be minimized in this study is the total failure rate of a system consisting of a number of electronic components equal to N_{com} and given by the sum of the individual component failure rates as shown in Equation 2.

$$\lambda_{total} = \sum_i^{N_{com}} \lambda_i \quad (2)$$

The wiring requirements among different components is represented by an interconnectivity matrix (I). An entry I_{ij} in the interconnectivity matrix (see Figure 2) is given the value 1 if component "i" is functionally related to component "j" or the value 0 otherwise.

If we denote the wiring length between components "i" and "j" by the variable L_{ij} , the additional objective function to be satisfied is the minimization of Equation 3.

$$g = \sum_{i=1}^{N_{comp}} \sum_{j=1}^i L_{ij} \cdot I_{ij} \quad (3)$$

In summary, the problem of interest may be stated as follows: given N_{com} heated elements to be distributed among N_{com} equally-spaced locations on the bottom wall of a two-dimensional ventilated channel, what are some of the arrangements that minimize both a measure of the failure rate of the system and the total wiring length required to meet the wiring requirements associated with a given interconnectivity matrix?

	1	2	3	4	5
1	1	1	1	0	0
2	1	1	0	1	0
3	1	1	1	0	0
4	0	1	0	1	0
5	0	0	0	0	1

Figure 2: Example of an interconnectivity matrix for five components. A unit entry indicates a pair of components that are "functionally related" while a zero entry indicates a pair of components that are "not functionally related".

3 Solution methodology

The solution methodology, illustrated in Figure 3 has three elements: a flow and heat transfer solver, a genetic algorithm and a multiobjective optimization strategy. The flow and heat transfer solver is responsible for the accurate prediction of the maximum temperature of each heated element used for calculating the individual failure rates. The multiobjective optimization strategy provides the means to convert the original multiobjective optimization problem into a form amenable to be solved by the genetic algorithm. The genetic algorithm is responsible for the adaptive search of optimal or near-optimal solutions. Note that even for the simplified model formulated in this study the thermal optimal placement of electronic components with different heat generation rates and thermal sensitivities would require an exhaustive investigation of the entire solution space which, in this case, is combinatorial. For example, if 8 different components are considered, the number of possible arrangements is 40320 (8!) and, as indicated by De Jong [19], non-adaptive search procedures may be computationally prohibitive.

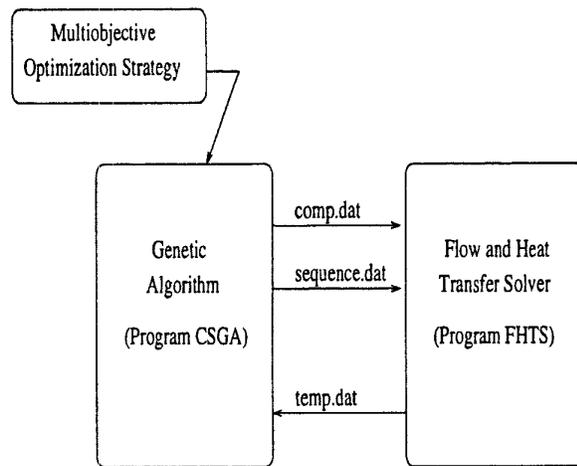


Figure 3: Illustration of the solution methodology.

3.1 Flow and heat transfer solver

For the purpose of estimating the failure rate of a given arrangement of electronic components on a printed wiring board using the model adopted in this investigation, it is necessary to estimate the maximum temperature on the surface of each heated element. This temperature is a function of the air velocity field, the thermal boundary conditions exhibited by the heated elements, the specific geometry (size and height of the heated elements), distance between heated elements,

substrate conduction characteristics, and the distance from the inflow boundary to the first heated element. If the substrate is assumed to be adiabatic, and the heat fluxes on the element surfaces are specified, the maximum surface temperature of each heated element can be obtained by solving the conservation equations of mass, momentum and energy in the fluid subject to appropriate boundary conditions.

Steady state results for the velocity and temperature fields can be obtained by either solving the unsteady form of the conservation equations and marching in time or by solving the steady state form of the conservation equations in an iterative framework using under-relaxation. Jang et al. [20] show that the latter approach may be more efficient in terms of CPU time required to achieve steady state results and is the approach adopted in this investigation.

Assuming steady, two-dimensional laminar fluid flow with constant physical properties, and neglecting natural convection (this assumption is justified later) and viscous dissipation effects, the non-dimensional conservation equations for mass, momentum and energy in the fluid phase are given by Equations 4, 5, 6 and 7, respectively.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (4)$$

$$\frac{\partial UU}{\partial X} + \frac{\partial VU}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left\{ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right\} \quad (5)$$

$$\frac{\partial UV}{\partial X} + \frac{\partial VV}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left\{ \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right\} \quad (6)$$

$$\frac{\partial U\theta}{\partial X} + \frac{\partial V\theta}{\partial Y} = \frac{1}{Re Pr} \left\{ \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right\} \quad (7)$$

The dimensionless quantities appearing in these equations are:

$$\begin{aligned} X &= \frac{x}{h}, & Y &= \frac{y}{h} \\ U &= \frac{u}{u_o}, & V &= \frac{v}{u_o}, & P &= \frac{p}{\rho u_o^2} \\ \theta &= \frac{T - T_o}{q_r'' h/k}, & Re &= \frac{u_o h}{\nu}, & Pr &= \frac{\nu}{\alpha} \end{aligned}$$

In these expressions, the symbols α and ν denote thermal diffusivity and kinematic viscosity, respectively. The symbol q_r'' represents the total heat flux provided by any one of the heated elements taken as a reference value.

For the channel flow considered in this investigation (see Figure 1) uniform velocity and temperature distributions are imposed at the inlet. Except for the inflow boundary, all exposed channel surfaces are taken

as adiabatic. In general, selecting accurate boundary conditions at open boundaries has been shown to be difficult (Sani et al. [21]). In this study, the streamwise velocity and temperature gradients are assumed to be zero at the outlet. Blosch et al. [22] have shown that the specification of velocity gradients equal to zero is a good boundary condition for the open boundary of channel flow configurations, provided the velocity at the outlet is corrected so that global conservation of mass is enforced. The temperature gradient equal to zero (thermally fully developed flow) is imposed at a distance sufficiently removed from the last heated element so that the computed maximum temperature of each heated element is unaffected.

The boundary conditions are:

- Inlet plane: $U = 1.0, V = 0, \theta = 0$
- Outlet plane: $\partial U/\partial X = 0, \partial V/\partial X = 0, \partial \theta/\partial X = 0$, and $\int_{0.0}^{1.0} U dY = 1$
- Top and bottom walls: $U = V = 0$
- Top and bottom walls: $\partial \theta/\partial Y = 0$
- Heated elements: $U = V = 0, \partial \theta/\partial n|_s = q''/q_r''$

The initial condition imposed on the flow field calculations corresponded to developed flow in a channel at every streamwise location except within the solid heated elements where velocities were set to zero. The initial temperature field in the fluid was set equal to the inlet temperature.

The configuration geometry is specified by the number of heated elements, the channel height (h), the heated element width (w), heated elements height (h_1, h_2), the inter-element spacing (s), the distance from the inlet plane to the first element (l_u) and the distance from the last element to the exit plane (l_d). See Figure 1.

The non-dimensional values adopted for these quantities are summarized in Table 1 and are close to the values reported by Kim [23] as typical of models of electronic components on printed wiring boards. The value of the geometrical parameter l_d/h is selected such that the location of the exit plane does not significantly affect the maximum temperatures calculated on the surfaces of the heated elements. As in Queipo et al. [11], the number of heated elements considered is eight.

In this study, the cooling fluid is air and all physical properties are evaluated for air at 300 K. The channel height was assumed to be 0.02 m and the inlet velocity $u_o = 0.59$ m/s corresponding to a Reynolds number of $Re = 750$. The Prandtl number of air at the reference temperature was taken $Pr = 0.7$.

Parameter	Re	Pr	h_1/h	h_2/h	s/h	w/h	L_w/h
Value	750.0	0.7	0.1	0.2	0.5	0.5	3.0

Table 1: Non-dimensional and geometric parameter values used in this work.

The program FHTS (Flow and Heat Transfer Solver) was used to perform the numerical calculations of flow and heat transfer. The program finds its origins in the ROTFLO2 program developed by Hayase et al. [24] and allows the direct numerical simulation of unsteady, three-dimensional, non-isothermal, constant property laminar flow in cartesian or cylindrical coordinates. The numerical procedure solves for the primitive variables (velocity and pressure) and is based on the finite difference equations derived using the staggered grid control-volume formulation presented by Patankar [25], but with the convective coefficients discretized using the QUICK scheme as suggested by Hayase et al. [26]. FHTS includes the codification of a variety of velocity-pressure coupling algorithms such as the SIMPLE procedure of Patankar and Spalding [27], the SIMPLER procedure of Patankar [25] and the SIMPLEC procedure of VanDoormaal et al. [28]. The program FHTS has been described and successfully tested in Queipo [13] using a variety of standard flow and heat transfer benchmark test cases. These include the cavity driven flow of Ghia et al. [29], and the backward facing step flow and heat transfer of Gartling [30] and Runchal [31], among others.

The velocity field for each configuration is calculated using the SIMPLE algorithm with an under-relaxation factor for the velocities α equal to 0.7. The under-relaxation factor for pressure was taken as $1-\alpha$. According to Peric [33] (see Dainese [32]) this relation between the velocity and pressure under-relaxation factors is optimal. Convergence was achieved when a variable (ϵ) representing the maximum mass, U-momentum and V momentum residual fell below a given predefined value. These residuals are computed as the sum of the absolute values of the corresponding mass or momentum imbalances over each of the control volumes in the domain. A convergence criteria of $\epsilon \leq 10^{-4}$ was used for computing the velocity field.

The temperature field is obtained using an under-relaxation parameter equal to 0.9. Convergence in the numerical calculation of temperature was achieved when a variable ϵ representing the energy residual fell below a given predefined value. This residual is computed as the sum of the absolute value of the energy imbalance of each of the control volumes in the domain. A convergence criteria of $\epsilon \leq 10^{-5}$ was used to compute the temperature field.

3.2 Genetic algorithm

Genetic algorithms are adaptive search procedures loosely based on the Darwinian notion of evolution that have been employed successfully in a variety of search, optimization and machine learning applications. The genetic algorithm in this study corresponds to the Combinatorial Simple Genetic Algorithm encoded in the program CSGA, documented in Queipo [13]. The CSGA program has the structure of the program GAUCSD (v. 1.4) developed by Schraudolph et al. [34], but uses a different representation (integer representation) and different recombination operators (partially matched crossover). In addition, the random number generator in the program CSGA is the routine RAN2 available in Numerical Recipes by Press et al. [35]. For a general introduction to genetic algorithms, see Holland [36], or Goldberg [37]. An introduction to genetic algorithms in the context of thermosciences applications is given by Queipo et al. [12].

The interaction between the Flow and Heat Transfer Solver and the Genetic Algorithm is illustrated in Figure 3. There are two key elements to consider in describing the connection between CSGA and FHTS: i) the control structure of their coupled execution; and, ii) the information exchange between the two programs. During the coupled execution of the CSGA and the FHTS programs, CSGA is the master process and FHTS is the slave process. Each time the program CSGA requires the evaluation of a new candidate solution, a slave process is created and the execution of CSGA is suspended. Within the slave process, the program FHTS is invoked and after its successful completion, CSGA resumes its execution. All this is done within a UNIX operating system environment.

The CSGA and the FHTS programs exchange information through data files. The program CSGA makes available to FHTS two files: i) a file called *comp.dat* describing the geometrical and thermal characteristics of the heated elements in the candidate solution; and, ii) a file called *sequence.dat* describing the order in which the heated elements specified in *components.dat* are positioned along the bottom wall of the ventilated channel. The program FHTS generates the file *temp.dat* after its successful execution. The file *temp.dat* contains the maximum temperature on the surface of each of the heated elements in the candidate solution.

3.3 Multiobjective optimization

In contrast to the optimization of a single function where the term optimum value has a unique meaning and geometric interpretation, in the case of multiobjective optimization there is not a general definition of the

optimal values. Here, the term optimization means to find a solution that provides acceptable values for the objective functions and that satisfies the preference structure of the person posing the problem; that is, the designer.

Hence, the problem in multiobjective optimization consists in finding a vector of design variables that satisfies a set of constraints and that optimizes a second vector whose elements represent the objective functions. There is no single best approach for solving these problems. Different philosophies and methodologies co-exist for addressing optimization problems with multiple objectives. The approaches differ in their view concerning whether or not it is possible (or practical) to capture the preference structure of the designer. The spectrum of methods begins with Pareto optimization where there is no information regarding the preference structure of the designer, and ends with the Multiattribute utility analysis (Keeney et al. [15]) where it is assumed possible to capture the aforementioned preference structure.

3.3.1 Pareto optimization

A vector of decision or design variables belongs to the Pareto optimal set or set of non-dominated solutions if there is no other solution that could improve the value of one of the objective functions without deteriorating at least one of the others objective functions. Examples of Pareto solutions are the solutions obtained by optimizing the objective functions individually.

In the case of Pareto optimization, no information is assumed regarding the designer except for his "preference independence". Preference independence describes the situation where lowering the values of the objective function is always better (assuming the problem is one of minimization). The methods in this category attempt to provide a representative approximation of the Pareto optimal set and some of the criteria to evaluate such methods include: i) how good is the approximation provided by the method of the Pareto optimal set and if it is able to generate a non-convex Pareto set, ii) how fast the computational effort of its use grows with respect to the number of variables, and iii) how easy it is to implement. Some of the methods that belong to this category are: the weighting method, the non-inferior set method and the restriction method (Balachandran et al. [14]).

The Pareto optimization in this work is conducted using the weighting method. The weighting method converts the multiobjective problem to a scalar optimization problem, in which the objective function becomes a weighted sum of the individual objective func-

tions. That is,

$$\min \sum_{i=1}^n w_i f_i(\bar{x}) \quad \text{with} \quad 1 \leq i \leq n$$

wherein, the w_i 's represent the weights and the f_i 's represent the individual objective functions. The above problem is a single-objective optimization problem and it is solved using a genetic algorithm. This is a very simple approach that fits the purpose of this investigation. However, the weighting method is not without its drawbacks: it does not uncover solutions in non-convex regions of the Pareto optimal set; and it finds the Pareto optimal set by solving multiple scalar optimization problems (different set of weights) which may be computational expensive.

Studies of Pareto optimization using genetic algorithms to obtain the set of non-dominated solutions at once have been attempted. The first effort in the use of genetic algorithms in multiobjective optimization problems (Pareto optimization) is due to Shaffer [38]. In his genetic algorithm the population is divided into sub-populations with the fitness of the chromosomes in different sub-populations being evaluated using the different objective functions. Shaffer's approach has the problem that it does not provide a uniform approximation of the Pareto set with the solutions obtained concentrated around the extremes of the non-dominated solutions set. A recent genetic algorithm claiming to provide a good approximation of the Pareto optimal set using genetic algorithms is reported by Horn et al. [39].

3.3.2 Multiattribute utility analysis

Pareto optimization is a member of a family of methods based on the measurement of the values of each objective function and on the knowledge of their relative priority. While this approach may be found useful, as pointed out by Thurston [16] it is limited in two respects: i) the direct measurement of the objective functions or attributes of the design, does not necessarily reflect the subsequent value or worth to the designer; and, ii) methods that rely on the concept of relative importance or priority might not accurately quantify attribute tradeoffs. Attribute tradeoffs refer to the designer's willingness to "pay" for improvement in one attribute at the expense of the other. In contrast to Pareto optimization, Multiattribute utility analysis concentrates on finding the overall value of the designs; hence, the design with the highest value to the designer can be identified.

The MUA method becomes practical when the so called *preferential* and *utility* independence assumptions are met. Preferential independence makes reference to situations where the designer always prefers less to more of an attribute (or more to less depending of the attribute) regardless of the level of the other attributes. Utility independence means that the general shape of the utility functions associated with each attribute (to be discussed later) is not altered by levels of the other attributes. Under this conditions, the overall worth of a design $U(\vec{f})$ can be calculated using Equation 8 (see Keeney et al. [15]),

$$U(\vec{f}) = \frac{1}{K} [[\Phi_{i=1}^n (K k_i U_i(f_i) + 1)] - 1] \quad (8)$$

wherein,

$U(\vec{f})$ = overall worth of the set of attributes f_i

f_i = level of attribute f_i

\vec{f} = set of attributes levels (f_1, f_2, \dots, f_n)

k_i = assessed single attribute scaling constant

$U_i(f_i)$ = assessed single attribute utility function

K = scaling constant

n = number of attributes

If the more restrictive additive independence condition reported by Thurston [16] is satisfied, that is,

$$\sum_{i=1}^n k_i = 1 \quad (9)$$

It can be shown that Equation 8 reduces to,

$$U(\vec{f}) = \sum_{i=1}^n k_i U_i(f_i) \quad (10)$$

Equation 10 leaves the designer with two tasks: i) the identification of the worth of the different levels of each attribute in isolation expressed in the single attribute utility function $U_i(f_i)$; and, ii) a measure of the tradeoffs the designer is willing to make, in the form of the attribute's scaling constant k_i . The constants k_i should not be confused with relative importance of attributes or weighting factors.

Points in the single attribute utility functions $U_i(f_i)$ and the attribute's scaling constant k_i can be obtained using the "certainty equivalent" method. An example of the lottery questions used in the certainty equivalent method to determine points in the utility function $U_i(f_i)$ is given in Figure 4.

The designer is asked to imagine two alternative designs: the "certain" alternative is known with certainty to be some value f , while the "lottery" alternative represents a design alternative in which there is uncertainty as to the attribute level. The lottery in Figure 4,

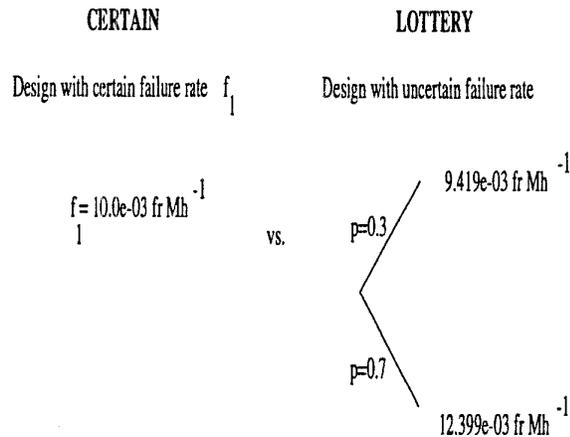


Figure 4: An example of the lottery questions used in the certainty equivalent method to assess the single attribute utility function $U_i(f_i)$ for failure rate.

shows a probability p of 30% that the failure rate (f_1) will be at the estimated best possible level (f_{1b}) and a probability of $(1 - p)$ of 70% that failure rate will be at the estimated worst possible value (f_{1w}). When the indifference point is reached, that is, when the designer is equally likely to take the "lottery" or stay with the "certain" alternative, a point in the single attribute utility function, $U_i(f_i) = p$, is obtained. The following equations show the derivation of this result.

$$U_i(f_i) = p.U_i(f_{ib}) + (1 - p).U_i(f_{iw}) \quad (11)$$

$$U_i(f_i) = p.(1) + (1 - p).(0) \quad (12)$$

$$U_i(f_i) = p \quad (13)$$

The value of p at which the designer will be indifferent is obtained by iterating through extreme values of p . The value of k_i is equal to the utility where the attribute f_i is at its best level, f_{ib} and all of the other attributes are at their worst levels; at this point $U(f_{1w}, \dots, f_{ib}, \dots, f_{nw}) = k_i$. The "certain" alternative shown in Figure 5 represents a design alternative with attribute levels known with certainty, and the lottery represents a design with uncertain attribute levels. The lottery shows a probability p of 60% that the design has the estimated best attribute levels ($f_1 = 9.419e - 03 \text{ fr Mh}^{-1}$; $f_2 = 0.4 \text{ m}$) and a probability $(1 - p)$ and a probability of 40% that the design will exhibit the estimated worst attribute levels ($f_1 = 12.399 - 03 \text{ fr Mh}^{-1}$; $f_2 = 0.8 \text{ m}$).

The value of k_i is equal to the value of p corresponding to the indifference point; see the following equations for

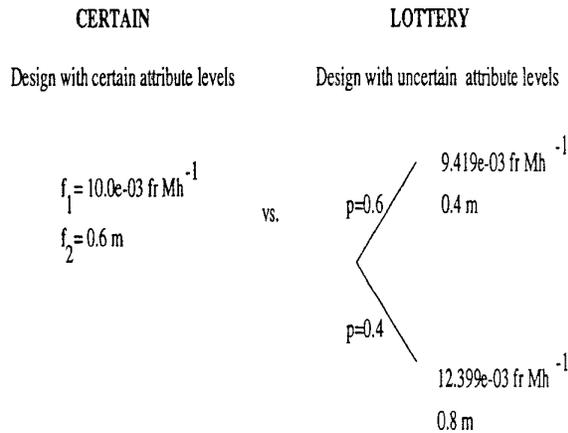


Figure 5: An example of the lottery questions used in the certainty equivalent method to assess the single attribute scaling constant k for failure rate (k_1).

the derivation of this result.

$$U(f_{1w} \dots f_{ib} \dots f_{nw}) = p.U(\vec{f}_b) + (1-p).U(\vec{f}_w) \quad (14)$$

$$U(f_{1w} \dots f_{ib} \dots f_{nw}) = p.(1) + (1-p).(0) \quad (15)$$

$$U(f_{1w} \dots f_{ib} \dots f_{nw}) = k_i \quad (16)$$

Details of the certainty equivalent method can be found in Keeney et al. [15].

4 Case study

For the purpose of illustrating and evaluating the present solution methodology, a case study is introduced. With reference to Figure 1, the case study represents the problem of optimally placing a set of eight heated elements with heat flux and thermal sensitivities as specified in Table 2 using the solution methodology discussed in the previous section. The optimal placement includes both the minimization of the failure rate of the system (Equation 2) and the minimization of the wiring length (Equation 3) using an interconnectivity matrix to be specified later.

Using an arrangement considered representative of the set of possible configurations (15263748), a grid refinement study was first conducted. The grid refinement study included three different non-uniform grids: 110×48 , 150×56 , and 186×68 , and $l_d/h = 4.5$ and 6.0 . The description of the grids corresponds to number of nodes in the streamwise and transverse direction, respectively. In all cases, the difference in the prediction of the maximum temperature on each of the heated elements were not significantly affected. The parameter

Element	Height (cm)	q_i ($W m^{-2}$)	B_i ($fr Mh^{-1}$)
1	0.2	200	400
2	0.2	300	400
3	0.2	200	1600
4	0.2	300	1600
5	0.4	200	400
6	0.4	300	400
7	0.4	200	1600
8	0.4	300	1600

Table 2: Thermal characteristics of the heated elements considered in the present case study.

Gr/Re^2 was $\ll 1$, where $Gr = g\beta(T^{max} - T_o)w^3/\nu^2$ is the Grashof number. Therefore, the assumption of negligible natural convection effects is justified.

Throughout the rest of the study (except for the validation run, to be discussed later) the maximum temperatures on the heated elements along the channel were computed using the 110×48 grid with $l_d/h = 4.5$. The selected grid has, in the streamwise direction, expansion factors in the interval [1.04,1.43] and a minimum grid spacing of 0.08. In the transverse direction, the expansion factors are in the interval [1.11,1.51] and a minimum grid spacing of 0.05. This grid allows the prediction of the maximum temperature of the heated elements within 1% of those obtained using the most refined grid with 75% less CPU time. For the case study, the results of using the two multiobjective optimization strategies under consideration, that is, Pareto optimization and Multiattribute utility analysis, are presented.

Before presenting and discussing the results associated with these two multiobjective optimization strategies, a thermal placement problem with *known* optimal solutions is first addressed (Validation run).

4.1 Validation run

Consider the placement of the heated elements listed in Table 3 so that the total failure rate of the system (Equation 2) is minimized. Observe that the heated elements generate the same heat flux and that the maximum temperature of the heated elements is only a function of their position along the channel. Under these conditions, it can be shown (Queipo et al. [13]) that the optimal arrangement requires placing the heated elements in descending order of thermal sensitivity. Hence, optimal sequences are, for example, 34726815, 74386251, 43762815, etc. The total number of possible arrangements is 40320 (8!), and there are 72 (3! \times 3! \times 2!) optimal solutions representing 0.18 %

Element	Height (cm)	q_i ($W m^{-2}$)	B_i (fr Mh^{-1})
1	0.2	400	200
2	0.2	400	800
3	0.2	400	2000
4	0.2	400	2000
5	0.2	400	200
6	0.2	400	800
7	0.2	400	2000
8	0.2	400	800

Table 3: Thermal characteristics of the heated elements considered in the validation run.

of the total solution space.

4.2 Control parameters for the genetic algorithm

The population size was taken as 7 (as in Queipo et al. [11]) and the number of generations was specified as 9. Numerical simulations of the genetic algorithm were conducted for a range of crossover rates, mutation rates and scaling factors. The crossover rate and mutation rates considered were between 0.1 and 0.9 with increments of 0.1 (with the restriction of mutation rates lower than crossover rates). The scaling factors were taken between 1.0 and 3.0 with increments of 1.0. The present genetic algorithm exhibited a robust behavior. At the end of 9 generations, a significant number of combinations of crossover rate, mutation rate and scaling factor (C,M,S) generated optimal solutions. For example (0.9,0.4,1.0), (0.9,0.2,1.0), (0.8,0.4,1.0), (0.6,0.3,1.0), (0.4,0.1,1.5), (0.8,0.4,1.5), (0.4,0.1,2.0), and many others. All the results reported in this section and throughout the study correspond to a crossover rate of 0.9, a mutation rate of 0.4 and a sigma scaling factor equal to 1.0.

Figure 6 shows temperature isocontours corresponding to the Validation run. As expected, the maximum temperatures of the heated elements increase with positions farther downstream of the inflow boundary and the maximum temperature gradients are located near the walls of the heated elements.

Table 4 shows the ten best arrangements uncovered by the genetic algorithm. Each entry in the table shows a given arrangement of components, its failure rate, the number of the generation in which it appeared and the corresponding number of objective function evaluations. The best elements correspond to the sequences 47368215 (ninth generation - fifty seven objective function evaluations) and 37468215 (eighth generation - fifty four objective function evaluations) with failure rate of 5.643×10^{-3} fr Mh^{-1} . These sequences have

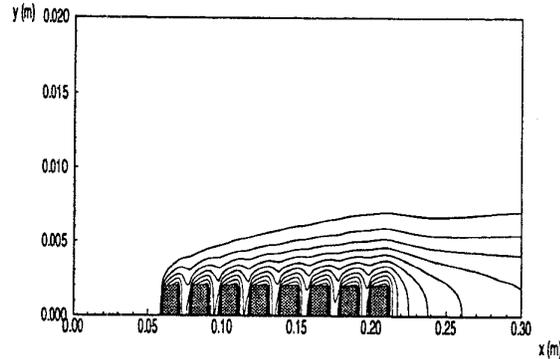


Figure 6: Temperature isocontours corresponding to the validation run. The figure shows fifteen equally spaced contours between 302 K and 330 K.

Arrangement	f (fr Mh^{-1})	Gen.	Fn. Evals
4 7 3 6 8 2 1 5	5.643e-03	9	57
4 7 3 2 5 6 1 8	5.700e-03	2	19
4 3 7 6 5 2 1 8	5.700e-03	8	53
3 7 4 8 5 6 1 2	5.700e-03	9	58
7 3 4 6 5 2 1 8	5.700e-03	7	47
4 3 7 2 5 6 1 8	5.700e-03	7	51
7 3 4 2 5 6 1 8	5.700e-03	5	37
4 7 3 6 5 8 1 2	5.700e-03	8	56
3 7 4 6 8 2 1 5	5.643e-03	8	54
4 7 3 6 5 2 1 8	5.700e-03	4	30

Table 4: Ten best arrangements uncovered by the CS-GA (validation run).

the heated elements positioned in decreasing order of thermal sensitivity and are optimal solution. Note that the expected number of objective function evaluations to randomly find an optimal solution is given by the number of possible arrangements ($8!$) divided by the number of optimal solutions ($3! \times 3! \times 2!$) and equal to 560. The genetic algorithm found an optimal solution using an order of magnitude fewer objective function evaluations.

5 Results and discussion

This section addresses the more complex situation where all the heated elements are different in their heat generation rates or their thermal sensitivities and the optimization criteria include both *thermal* and *non-thermal* optimization criteria. As previously discussed, in the case of multiobjective optimization there is not a general definition of the optimal values and no single

best approach for solving these problems. As a result, different philosophies and methodologies, such as Pareto optimization and Multiattribute utility analysis, co-exist for addressing optimization problems with multiple objectives.

The *thermal* and *non-thermal* optimization criteria correspond to the minimization of the failure rate of the system computed using the Arrhenius relation and of the total wiring length according to an interconnectivity matrix. The present interconnectivity requirement is that the heated elements identified with numbers between 1 and 4 inclusive and those identified with numbers between 5 and 8 must be wired among themselves, respectively. The total interconnectivity length and total failure rate of the arrangements of heated elements are denoted by the functions λ (Equation 2) and g (Equation 3), respectively.

Table 2 presents a description of the thermal characteristics of the heated elements under consideration. The control parameters of the genetic algorithm adopted were exactly those selected in the Validation run.

5.1 Pareto optimization

Solutions expected to belong to the Pareto optimal set are calculated using the weighting method (Balachandran et al. [14]) which converts the multi-objective problem to a single objective problem, in which the function to be optimized is the weighted sum of the individual objective functions. In this case, the function f to be minimized has the form:

$$f = w_\lambda * \lambda_{total} + w_g * (C_o g) \quad (17)$$

where C_o represents a scaling factor, calculated for each generation in order to render the average contribution of the interconnectivity term in the sum comparable in magnitude to the average contribution due to the total failure rate. The coefficients w_λ and w_g are weighting factors representing the relative importance of the optimization criteria, with $w_\lambda + w_g = 1$. In this work, three points in the Pareto optimal set (including the extremes of the set and a situation where the optimization criteria are considered to be equally important) are sought (w_λ, w_g): (1.0,0.0), (0.5,0.5) and (0.0,1.0).

The failure rate and interconnectivity length associated with the three Pareto optimal solutions are plotted in Figure 7. The solution corresponding to weighting factors (1.0,0.0) has a failure rate of $9.419 \text{ fr } Mh^{-1}$ and an interconnectivity length of 0.64 m . This solution was found after five generations (thirty one objective function evaluations) and corresponds to a situation where the minimization of the failure rate is the sole optimization criterion. A solution corresponding

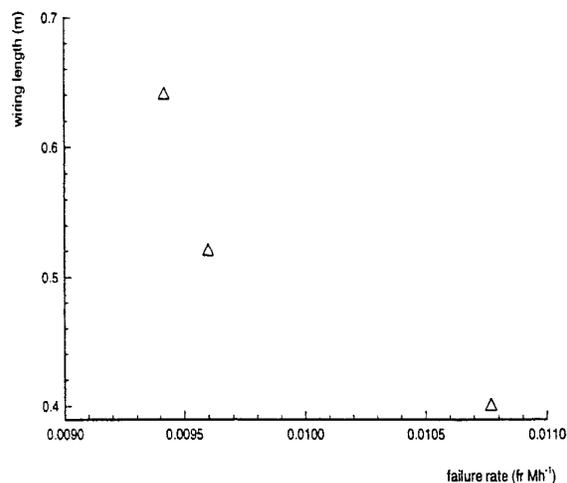


Figure 7: Solutions uncovered by the genetic algorithm (Pareto optimization).

to the other extreme of the Pareto optimal set; that is, the situation where the minimization of the wiring length is the only optimization criterion ($w_\lambda = 0$ and $w_g = 1.0$) was selected by inspection of the interconnectivity requirement. The optimal solution selected for this case was the sequence *12345678* with a failure rate of $10.770 \text{ fr } Mh^{-1}$ and an optimal wiring length of 0.4 m . The Pareto optimal solution associated with the situation with equal weighting factors was obtained after nine generations (sixty six objective function evaluations) and corresponds to the sequence *42371658*. The aforementioned sequence has a failure rate of $9.600 \text{ fr } Mh^{-1}$ and an interconnectivity length of 0.52 m . Note that the Pareto optimal solutions under consideration are in fact non-dominated solutions (see Figure 8), with their failure rate and interconnectivity length varying by up to 14 % and 60 %, respectively.

5.2 Multiattribute utility analysis

This section discusses the solution of the case study using the single attribute utility functions for failure rate (f_1) and wiring length (f_2) shown in Figures 9 and 10, respectively. Figures 9 and 10 corresponds to quadratic polynomials that interpolate the following three points ($f_1, U(f_1)$): ($9.4e-03 \text{ fr } Mh^{-1}, 1.0$), ($10.0e-03 \text{ fr } Mh^{-1}, 0.7$), ($12.4e-03 \text{ fr } Mh^{-1}, 0.0$); and respectively. The scaling factors reflecting acceptable tradeoffs between attributes, are given as $k_1 = 0.6$ (failure rate) and $k_2 = 0.4$ (wiring length). Both, the utility functions and

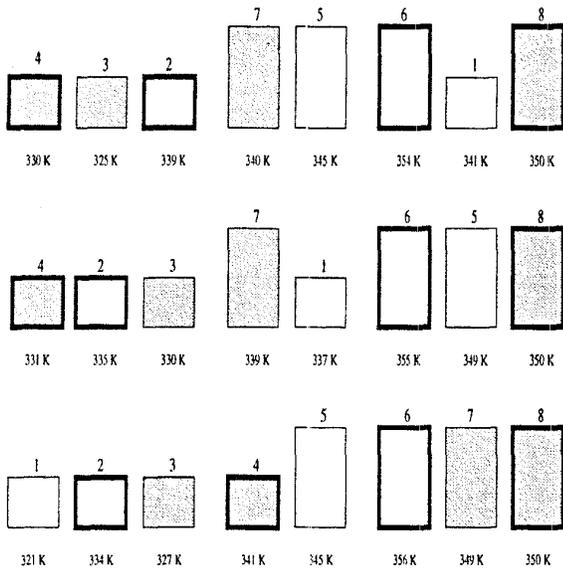


Figure 8: Best arrangements obtained by the CSGA using Pareto optimization for the cases of (w_λ, w_g) : (1.0,0.0), (0.5,0.5) and (0.0,1.0). Top (1.0,0.0); 43275618, $f_1 = 9.419e-03$ fr Mh^{-1} and $f_2 = 0.64$ m. Middle (0.5,0.5); 42371658, $f_1 = 9.419e-03$ fr Mh^{-1} and $f_2 = 0.52$ m. Bottom (0.0,1.0); 12345678, $f_1 = 10.770e-03$ fr Mh^{-1} and $f_2 = 0.4$ m. Maximum temperatures of the heated elements are also shown.

the scaling factors are assumed to have been obtained with the participation of the designer and the certainty equivalent method discussed in a previous section. The function to be maximized is given by Equation 8 with the aforementioned utility functions and scaling factors.

The ten best arrangements obtained when using the multiattribute utility analysis approach are shown in Table 5. The best arrangement corresponds to the sequence 42371568 with a failure rate and wiring length of $9.725e-03$ fr Mh^{-1} and 0.52 m, respectively. The best arrangement was found after 9 generations and 65 function evaluations.

The temperature isocontours as well the maximum temperature on each of the heated elements corresponding to the sequence 42371568 are shown in Figures 11 and 12, respectively.

Note that this approach provides the "best" solution with a single coupled execution of the fluid and heat transfer solver and the genetic algorithm provided the utility functions (U_i) and the scalar constants (k_i) are available. In addition, this approach could be used to identify the "best" solution among the Pareto op-

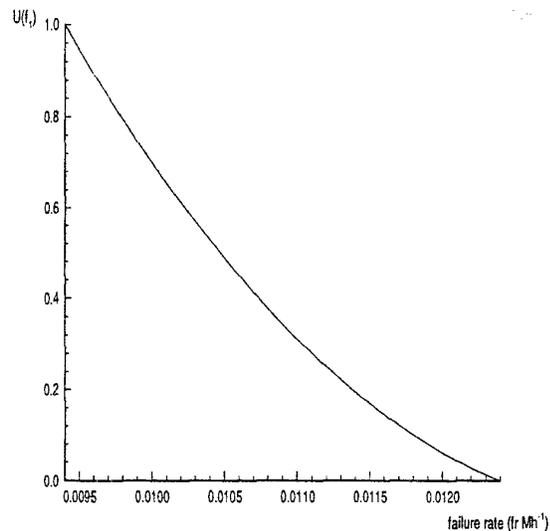


Figure 9: Single attribute utility function for failure rate.

timal solutions found (those obtained in the previous section) by computing the utility value of each of the Pareto optimal solutions and selecting the solution with highest utility value.

6 Conclusions

A model for the problem of optimal placement of electronic components on printed wiring boards subject to thermal and non-thermal optimization criteria has been formulated and solved using a methodology based on three components: i) a fluid and heat transfer sol-

Arrangement	Utility	Gen.	Fn. Evals
4 2 3 7 5 6 1 8	7.8583e-01	1	11
4 2 3 7 6 5 1 8	7.1768e-01	4	35
3 2 4 7 5 6 1 8	7.2541e-01	6	48
2 3 4 7 5 6 1 8	7.0141e-01	0	7
4 2 3 5 7 6 1 8	6.8448e-01	5	39
4 2 3 6 1 7 5 8	8.1152e-01	8	63
4 2 3 1 5 6 7 8	7.6526e-01	6	47
3 4 2 8 5 6 1 7	6.9352e-01	1	13
4 2 3 6 5 7 1 8	7.5229e-01	4	32
4 2 3 7 1 5 6 8	8.4586e-01	9	65

Table 5: Ten best arrangements uncovered by the CSGA (Multiattribute utility analysis).

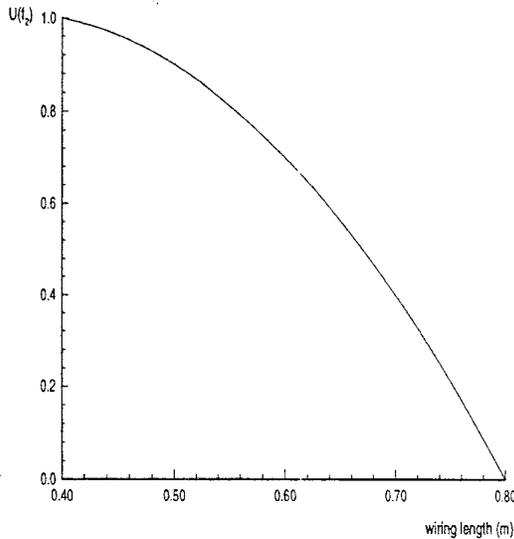


Figure 10: Single attribute utility function for *wiring length*.

ver for the prediction of the maximum temperature of the heated elements; ii) a multiobjective optimization strategy for the scalarization of the vector of design objectives; and, iii) a genetic algorithm for the search of optimal or near optimal solutions.

The multiobjective optimization strategy embedded in the solution methodology is flexible enough to account for two extreme situations (no knowledge/knowledge) regarding the knowledge of the preference structure of the designer by using Pareto optimization and Multiattribute utility analysis.

The solution methodology shows promise as an effective and efficient tool for providing optimal or near-optimal solutions for electronic component placement problems where both thermal and non-thermal optimization criteria are of interest under rather general conditions regarding component geometries, heat generation rates and thermal sensitivities.

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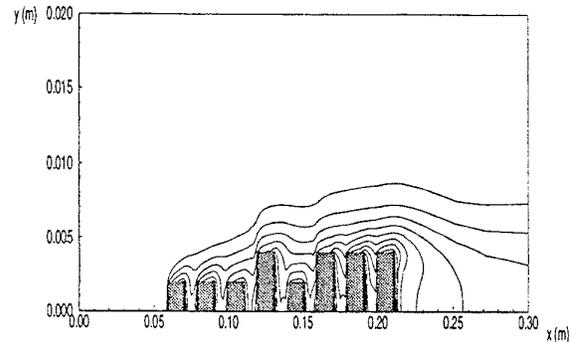


Figure 11: Temperature isocontours associated with the best arrangement (42371568) uncovered by the CSGA. The figure shows fifteen equally spaced contours between 302 K and 352 K. (Multiattribute utility analysis).

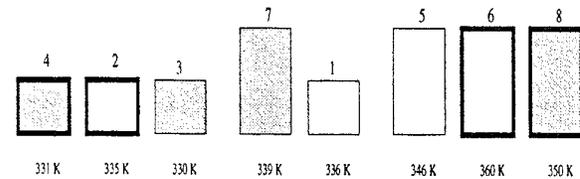


Figure 12: Maximum temperatures of the heated elements associated with the best arrangement uncovered by the CSGA; $f_1 = 9.725 \cdot 10^{-3} \text{ fr } Mh^{-1}$ and $f_2 = 0.52 \text{ m}$. (Multiattribute utility analysis).

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