MULTIOBJECTIVE OPTIMIZATION OF COMPONENT PLACEMENT ON PLANAR PRINTED WIRING BOARDS

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Abstract

This paper presents a solution methodology for the optimal placement of convectively and conductively air-cooled electronic components on planar printed wiring boards considering thermal and electrical/cost design objectives. The methodology combines the use of a heat transfer solver for the prediction of the temperature distribution among the electronic components and a genetic algorithm for the adaptive search of optimal or near optimal solutions and a multiobjective optimization strategy (Pareto optimization and Multiattribute utility analysis). After proper validation of the elements of the solution methodology (heat transfer solver/genetic algorithm) in isolation, the methodology under consideration is tested using a placement problem (case study) that considers as optimization criteria the minimization of an estimate of the failure rate of the system of components due to thermal overheating (via an Arrhenius relation) and the minimization of the total wiring length (given some interconnectivity requirements. Results corresponding to the case study are presented and discussed for both Pareto optimization and Multiattribute utility analysis.

1 Introduction

The optimal placement of components on printed wiring boards requires satisfying multiple, possibly conflicting, design objectives. As pointed out by Moresco [1], these design objectives may be very different in nature - geometrical, electrical, thermal, mechanical, and cost (manufacturing and maintenance) - which makes finding the "best" design a complicated task.

Two major design objectives are related to the reliability (thermal/mechanical/cost) and the routing (electrical/cost) requirements of the component placement design. Specifically, the minimization of estimates of the *failure rate* of the system and *total wiring length* are design objectives of prominent interest. The former optimization criterion imposes crucial heat transfer requirements on the design because of the combined effects of: i) rapidly increasing packaging density and power dissipation requirements; and, ii) potentially high costs associated with the failure of electronic components, as pointed out by Weiss et al. [2] and Wessely et al. [3] among others. The latter is critical because of electrical performance, speed and transmission line requirements and its impact on the manufacturing costs.

Most optimization studies regarding component placement have modeled the electronic components on printed wiring boards as heat sources distributed along n in-line positions in 2-D channels; see, for example, Dancer et al. [4], Osterman et al. [5], Queipo et al. [8], Queipo et al. [9], and Queipo et al. [11]. The few optimization studies that have taken a more realistic model from a geometrical point of view; that is, to model the electronic components on printed wiring boards as heat sources in 3-D enclosures, i) have not considered multiple design objectives, such as the minimization of the failure rate and total wiring length of the system (Cahlon et al. [12]; Eliasi et al. [6], or, ii) have failed to provide rigorous methods to select the "best" design when multiples objectives are present (Pecht et al. [7]).

This study overcomes the limitations of previous studies in this area and discusses a methodology to select the "best" component placement design when multiple design objectives are present. The methodology is based on the concepts of Pareto optimality (Balachandran et al. [13] and Multi-Attribute Utility Analysis (Keeney et al. [14]). The Pareto optimization provides

a set of alternative component placements from which the "best" design must be selected, and the MAUA assists in the process of articulating the designer's preferences and identifying the "best" component placement (decision problem). As reported by Thurston [15], the MAUA has been successfully applied to a wide variety of decision problems, including trajectory selection for NASA missions, nuclear power plant site selection, telecommunication system architecture design, and many others.

The methodology is illustrated using a model for the problem of finding the optimal placement of air-cooled electronic components on a printed wiring board subject to the minimization of an estimate of the failure rate of the system and of the total inter-component wiring length. The model is formulated and solved with the assistance of adaptive search procedures, loosely based on the Darwinian notion of evolution, called Genetic Algorithms (Holland [22]; Goldberg [23]; Queipo et al. [8]). The model seeks to find the optimal, or near optimal arrangement of $n \times n$ conductively and convectively cooled heat sources distributed among $n \times n$ positions. The heat sources represent electronic components with different heat generation rates and thermal sensitivities. Interconnectivity requirements (specification of functionally related electronic components) are provided through an interconnectivity matrix as illustrated in Figure 3.

Results of the present multiobjective optimization methodology for a case study are presented and discussed. With reference to Figure 1, the results correspond to conductively and convectively air-cooled electronic components with the steady state temperature of the heat sources modeling the aforementioned components calculated numerically using a thermal resistive network model. The maximum temperature of the heat sources are used to estimate the *failure rate* of different component placements (Arrhenius equation). The *total wiring length* of a given arrangement of components is computed as the sum of the length (Manhattan distance) between functionally related components.

The remainder of this paper is structured as follows. Section 2 provides a formal definition of the problem of interest and Section 3 gives a description of the different elements of the present solution methodology and their interaction. In particular, Section 3 describes a heat transfer solver, a genetic algorithm and two different multiobjective optimization strategies (Pareto optimization and Multiattribute utility analysis). A description of a case study designed to validate and evaluate the present solution methodology is the subject of Section 4. The paper ends with the application of the present solution methodology to a multiobjective optimization problem (a variation of the case study) using both Pareto optimization and Multiattribute utility analysis.

2 Problem definition

The problem of interest here corresponds to the optimal placement of conductively and convectively cooled electronic components on printed wiring boards (P-WB) subject to *thermal* and *non-thermal* optimization criteria. Because of its cost effectiveness and mechanical simplicity, forced air cooling is the most frequently used technique for cooling electronic components in personal computers and workstations. These systems comprise a major portion of the market with moderate heat transfer rate requirements. The conductively and convectively cooled electronic components on printed wiring boards are modeled here as equally spaced heat sources placed on a conductive substrate, as illustrated in Figure 2. The printed wiring board is aligned parallel to the coolant flow with each component dissipating a constant amount of heat that may differ among components.

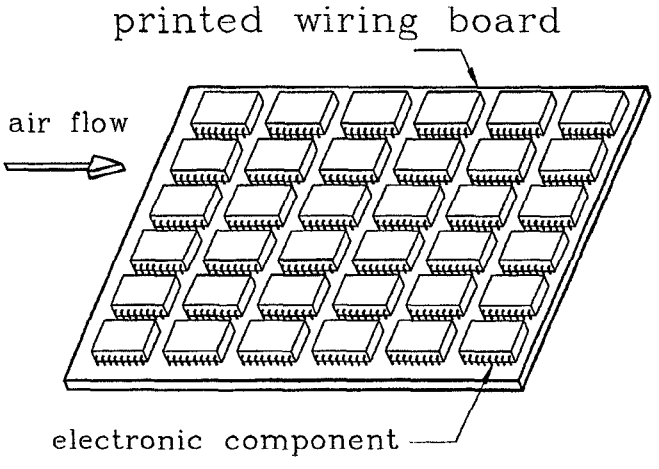

Figure 1: Schematic of the heat transfer configuration of interest.

Regarding thermal optimization, forced air cooling is usually limited by acoustic noise constraints placed on the fan driving the flow, and arrangements of electronic components that maximize reliability and minimize thermo-mechanically induced stresses are highly desirable. Examples of non-thermal optimization criteria include the need to minimize the total wire length on the PWB, clustering functionally related components to conform to speed and transmission line requirements, and keeping analog components and digital

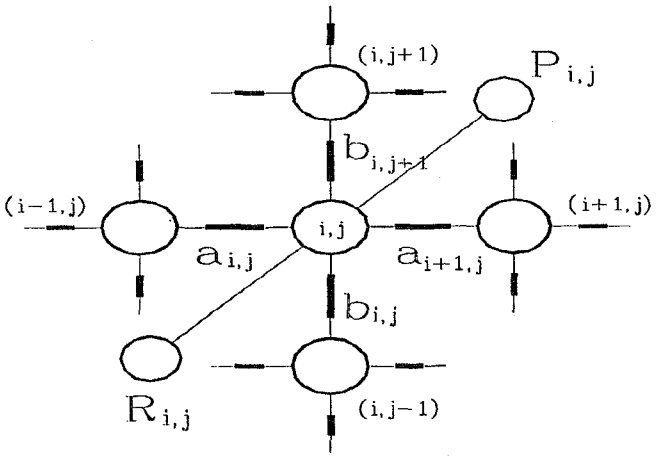

Figure 2: Model of the heat transfer configuration of interest.

components separate to reduce crosstalk.
In this study, the minimization of the failure rate of the electronic components on the printed wiring boards due to thermal overheating, and the minimization of the total wiring length satisfying the requirements specified by an interconnectivity matrix, are selected as *thermal* and *non-thermal* optimization criteria, respectively.
The reliability prediction is the statistical estimate of the value of time over which a device will function. The inverse of the reliability of a device is called its *failure rate* and is measured in failures per megahours (fr Mh^{-1}). As indicated by, for example, Moresco et al. [1], and Wessely et al. [3], the failure rate of an electronic component is a strong function of its temperature.
Even though various functional relationships between failure rate and temperature in electronic components have been suggested (Wong [16]), according to Blanks [17], the Arrhenius relation is the most widespread model among practitioners in the electronic packaging industry. In this study, the failure rate of electronic component "i" is estimated using the Arrhenius relation as:

$$\lambda_i = A_i exp(-B_i/T_i^{max}) \tag{1}$$

Here A_i and B_i are constants associated with the thermal sensitivity of the electronic component, while T_i^{max} is the maximum temperature of component "i". Of interest here is the general case for which the electronic components on the PWB may differ in heat dissipation rate and thermal sensitivity.
One of the objective functions to be minimized in this study is the total failure rate of a system consisting of a number of electronic components equal to N_{com}^2 and given by the sum of the individual component failure rates as shown in Equation 2.

$$\lambda_{total} = \sum_{i}^{N_{com}} \lambda_i \tag{2}$$

The wiring requirements among different components is represented by an interconnectivity matrix (I). An entry I_{ij} in the interconnectivity matrix (see Figure 3) is given the value 1 if component "i" is functionally related to component "j" or the value 0 otherwise.
If we denote the wiring length between components "i" and "j" by the variable L_{ij}, the additional objective function to be satisfied is the minimization of Equation 3.

$$g = \sum_{i=1}^{N_{comp}} \sum_{j=1}^{i} L_{ij}.I_{ij} \tag{3}$$

In summary, the problem of interest may be stated as follows: given N_{com}^2 heat sources to be distributed among $N_{com} \times N_{com}$ equally-spaced locations on a conductive substrate, what are some of the arrangements that minimize both a measure of the failure rate of the system and the total wiring length required to meet the wiring requirements associated with a given interconnectivity matrix?

	1	2	3	4	5
1	1	1	1	0	0
2	1	1	0	1	0
3	1	1	1	0	0
4	0	1	0	1	0
5	0	0	0	0	1

Figure 3: Example of an interconnectivity matrix for five components. A unit entry indicates a pair of components that are "functionally related" while a zero entry indicates a pair of components that are "not functionally related".

3 Solution methodology

The solution methodology, illustrated in Figure 4 has three elements: a heat transfer solver, a genetic algorithm and a multiobjective optimization strategy. The heat transfer solver is responsible for the prediction of the maximum temperature of each heat source used for calculating the individual failure rates. The multiobjective optimization strategy provides the means to convert the original multiobjective optimization problem into a form amenable to be solved by the genetic algorithm. The genetic algorithm is responsible for the adaptive search of optimal or near-optimal solutions. Note that even for the simplified model formulated in this study the thermal optimal placement of electronic components with different heat generation rates and thermal sensitivities would require an exhaustive investigation of the entire solution space which, in this case, is combinatorial. For example, if 36 different components are considered, the number of possible arrangements is $36! = 3.71 \times 10^{41}$ and, as indicated by De Jong [18], non-adaptive search procedures may be computationally prohibitive.

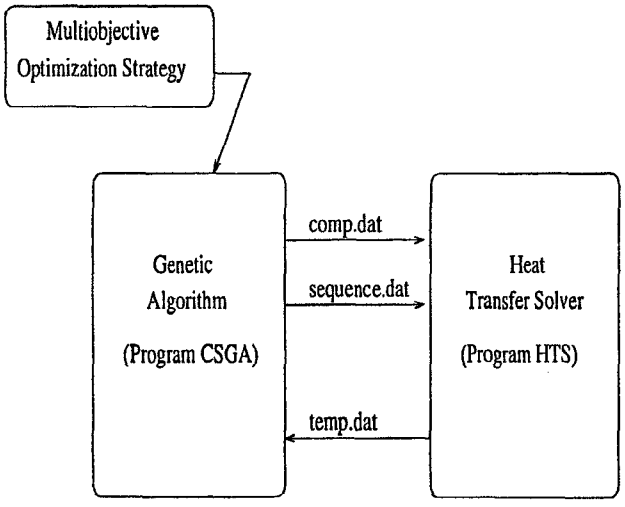

Figure 4: Illustration of the solution methodology.

3.1 Heat transfer solver

For the purpose of estimating the failure rate of a given arrangement of electronic components on a printed wiring board using the model adopted in this investigation, it is necessary to estimate the maximum temperature of each heat source. This temperature, in the resistive network approach, can be obtained by solving the algebraic system of equations that results from performing a heat balance at each node (i,j). The heat sources interact with their neighbors via heat conduction and exchange heat with an air current; Figure 2, despicts an schematic of the node (i,j) under consideration. The discretized form of the heat equation corresponding to node (i,j), can be written in explicit form as:

$$\begin{aligned} T_{ij}^{m+1} = T_{ij}^{m} + h[a_{ij}(T_{i-1,j}^{m} - T_{ij}^{m}) \\ +a_{i+1,j}(T_{i+1,j}^{m} - T_{ij}^{m}) \\ +b_{i,j}(T_{i,j-1}^{m} - T_{ij}^{m}) \\ +b_{i,j+1}(T_{i,j+1}^{m} - T_{ij}^{m})] \\ +hP_{ij} + hR_{i,j}(T_a - T_{ij}^{m})\|T_a - T_{ij}^{m}\|^{\gamma-1} \end{aligned} \tag{4}$$

subject to the following initial and boundary conditions:

$$T_{ij}^{0} = T_{ij}^{*} \; \forall \; i = 1, I; \; j = 1, J$$

$$T_{ij}^{m} = T_{ij}^{b} \; for \; i = 0; i = I + 1 \; and \; j = 0; j = J + 1$$

Note that there are $I \times J$ internal nodes which represent the electronic components while the nodes with $i = 0$; $i = I + 1$ or $j = 0$; $j = J + 1$ specify the boundaries of the computational domain.
Hence, the steady state values associated with maximum temperatures are found through a time marching process where the temperature at time level $t = (m + 1)h$ is computed from the known temperatures at time level $t = mh$. The variables a_{ij} and b_{ij} represent heat conduction coefficients between the node (i,j) and the nodes (i-1,j) and (i,j-1), respectively, T_a denotes the ambient temperature, R_{ij} represents a heat exchange coefficient, P_{ij} expresses a measure of power dissipation, and γ is a coefficient that allows to select between a linear ($\gamma = 1.0$) and non-linear ($\gamma = 1.25$) versions of the problem. The values assigned to a_{ij}, b_{ij}, R_{ij}, P_{ij}, and γ are considered constants throughout the time marching process.
The values for a,b,P and R for predicting the temperature distribution among the heat sources modeling the electronic components, are taken from Steinberg [19] as referenced in Cahlon et al. [12]. Considering a problem with 36 heat sources modeling electronic components, the heat conduction coefficients are:

$$b_1 = b_2 = b_3 = b_4 = b_5 = b_6 = 20/262[sec^{-1}]$$

for the nodes along the bottom wall with temperature T^b;

$$a_1 = a_7 = a_{13} = a_{19} = a_{25} = a_{31} = 25/262[sec^{-1}]$$

for the nodes along the left and right walls;

$$b_{37} = b_{38} = b_{39} = b_{40} = b_{41} = b_{42} = 0[sec^{-1}]$$

for the nodes along the top wall (insulated in this direction);

$$a_k = b_k = 18/262[sec^{-1}],\ otherwise$$

The value for P_{ij} is one of four different values: $0.239/2.62[^oC/sec]$, $0.717/2.62[^oC/sec]$, $1.19/2.62[^oC/sec]$, $1.65/2.62[^oC/sec]$, corresponding to heat dissipation rates of Q_1= 1 watt, Q_2= 2 watts, Q_3=5 watts, and Q_4=7 watts. The heat generation rates p_k assigned to the different heat sources (with $k = 1, 36$) are:

$$p_2 = p_3 = p_4 = p_5 = Q_1$$

$$p_1 = p_6 = p_7 = p_{12} = p_{13} = p_{18} = p_{19} = p_{24} = p_{25} = p_{30} = p_{31} = p_{36} = Q_2$$

$$p_8 = p_9 = p_{10} = p_{11} = p_{14} = p_{17} = p_{20} = p_{23} = p_{26} = p_{29} = p_{32} = p_{35} = Q_3$$

$$p_{15} = p_{16} = p_{21} = p_{22} = p_{27} = p_{28} = p_{33} = p_{34} = Q_4$$

Similarly, the heat exchange coefficients r_k assigned to the different heat sources are selected as $0.02[sec^{-1}]$.

3.2 Genetic algorithm

Genetic algorithms are adaptive search procedures loosely based on the Darwinian notion of evolution that have been employed successfully in a variety of search, optimization and machine learning applications. The genetic algorithm in this study corresponds to the Combinatorial Simple Genetic Algorithm encoded in the program CSGA, documented in Queipo [10]. The CSGA program has the structure of the program GAucsd (v. 1.4) developed by Schraudolph et al. [20], but uses a different representation (integer representation) and different recombination operators (partially matched crossover). In addition, the random number generator in the program CSGA is the routine RAN2 available in Numerical Recipes by Press et al. [21]. For a general introduction to genetic algorithms, see Holland [22], or Goldberg [23]. An introduction to genetic algorithms in the context of thermosciences applications is given by Queipo et al. [8].

The interaction between the Heat Transfer Solver and the Genetic Algorithm is illustrated in Figure 4. There are two key elements to consider in describing the connection between CSGA and HTS: i) the control structure of their coupled execution; and, ii) the information exchange between the two programs.

During the coupled execution of the CSGA and the HTS programs, CSGA is the master process and HTS is the slave process. Each time the program CSGA requires the evaluation of a new candidate solution, a slave process is created and the execution of CSGA is suspended. Within the slave process, the program HTS is invoked and after its successful completion, CSGA resumes its execution. All this is done within a UNIX operating system environment.

The CSGA and the HTS programs exchange information through data files. The program CSGA makes available to HTS two files: i) a file called *comp.dat* describing the geometrical and thermal caracteristics of the heat sources in the candidate solution; and, ii) a file called *sequence.dat* describing the order in which the heat sources specified in *components.dat* are positioned along the bottom wall of the ventilated channel. The program HTS generates the file *temp.dat* after its successful execution. The file *temp.dat* contains the maximum temperature of each of the heat sources in the candidate solution.

3.3 Multiobjective optimization

In contrast to the optimization of a single function where the term optimum value has a unique meaning and geometric interpretation, in the case of multiobjective optimization there is not a general definition of the optimal values. Here, the term optimization means to find a solution that provides acceptable values for the objective functions and that satisfies the preference structure of the person posing the problem; that is, the designer.

Hence, the problem in multiobjective optimization consists in finding a vector of design variables that satisfies a set of constraints and that optimizes a second vector whose elements represent the objective functions. There is no single best approach for solving these problems. Different philosophies and methodologies coexist for addressing optimization problems with multiple objectives. The approaches differ in their view concerning whether or not it is possible (or practical) to capture the preference structure of the designer. The spectrum of methods begins with Pareto optimization where there is no information regarding the preference structure of the designer, and ends with the Multiattribute utility analysis (Keeney et al. [14]) where it is assumed possible to capture the aforementioned preference structure.

3.3.1 Pareto optimization

A vector of decision or design variables belongs to the Pareto optimal set or set of non-dominated solutions if there is no other solution that could improve the value of one of the objective functions without deteriorating at least one of the others objective functions. Examples of Pareto solutions are the solutions obtained by optimizing the objective functions individually.

In the case of Pareto optimization, no information is assumed regarding the designer except for his "preference independence". Preference independence describes the situation where lowering the values of the objective function is always better (assuming the problem is one of minimization). The methods in this category attempt to provide a representative approximation of the Pareto optimal set and some of the criteria to evaluate such methods include: i) how good is the approximation provided by the method of the Pareto optimal set and if it is able to generate a non-convex Pareto set, ii) how fast the computational effort of its use grows with respect to the number of variables, and iii) how easy it is to implement. Some of the methods that belong to this category are: the weighting method, the non-inferior set method and the restriction method (Balachandran et al. [13]).

The Pareto optimization in this work is conducted using the weighting method. The weighting method converts the multiobjective problem to a scalar optimization problem, in which the objective function becomes a weighted sum of the individual objective functions. That is,

$$min \qquad \sum_{i=1}^{n} w_i \, f_i(\bar{x}) \qquad with \qquad 1 \leq i \leq n$$

wherein, the $w_i's$ represent the weights and the $f_i's$ represent the individual objective functions. The above problem is a single-objective optimization problem and it is solved using a genetic algorithm. This is a very simple approach that fits the purpose of this investigation. However, the weighting method is not without its drawbacks: it does not uncover solutions in non-convex regions of the Pareto optimal set; and it finds the Pareto optimal set by solving multiple scalar optimization problems (different set of weights) which may be computational expensive.

Studies of Pareto optimization using genetic algorithms to obtain the set of non-dominated solutions at once have been attempted. The first effort in the use of genetic algorithms in multiobjective optimization problems (Pareto optimization) is due to Shaffer [24]. In his genetic algorithm the population is divided into sub-populations with the fitness of the chromosomes in different sub-populations being evaluated using the different objective functions. Shaffer's approach has the problem that it does not provide a uniform approximation of the pareto set with the solutions obtained concentrated around the extremes of the non-dominated solutions set. A recent genetic algorithm claiming to provide a good approximation of the Pareto optimal set using genetic algorithms is reported by Horn et al. [25].

3.3.2 Multiattribute utility analysis

Pareto optimization is a member of a family of methods based on the measurement of the values of each objective function and on the knowledge of their relative priority. While this approach may be found useful, as pointed out by Thurston [15] it is limited in two respects: i) the direct measurement of the objective functions or attributes of the design, does not necessarily reflect the subsequent value or worth to the designer; and, ii) methods that rely on the concept of relative importance or priority might not accurately quantify attribute tradeoffs. Attribute tradeoffs refer to the designer's willingness to "pay" for improvement in one attribute at the expense of the other. In contrast to Pareto optimization, Multiattribute utility analysis concentrates on finding the overall value of the designs; hence, the design with the highest value to the designer can be identified.

The MUA method becomes practical when the so called *preferential* and *utility* independence assumptions are met. Preferencial independence makes reference to situations where the designer always prefers less to more of an attribute (or more to less depending of the attribute) regardless of the level of the other attributes. Utility independence means that the general shape of the utility functions associated with each attribute (to be discussed later) is not altered by levels of the other attributes. Under this conditions, the overall worth of a design $U(\vec{f})$ can be calculated using Equation 5 (see Keeney et al. [14]),

$$U(\vec{f}) = \frac{1}{K}[[\Phi_{i=1}^{n}(Kk_iU_i(f_i) + 1)] - 1] \qquad (5)$$

wherein,

$U(\vec{f})$ = overall worth of the set of attributes f_i
f_i = level of attribute f_i
$\vec{f}$ = set of attributes levels $(f_1, f_2, ...f_n)$
k_i = assessed single attribute scaling constant
$U_i(f_i)$ = assessed single attribute utility function
K = scaling constant

n = number of attributes
If the more restrictive additive independence condition reported by Thurston [15] is satisfied, that is,

$$\sum_{i=1}^{n} k_i = 1 \tag{6}$$

It can be shown that Equation 5 reduces to,

$$U(\vec{f}) = \sum_{i=1}^{n} k_i U_i(f_i) \tag{7}$$

Equation 7 leaves the designer with two tasks: i) the identification of the worth of the different levels of each attribute in isolation expressed in the single attribute utility function $U_i(f_i)$; and, ii) a measure of the tradeoffs the designer is willing to make, in the form of the attribute's scaling constant k_i. The constants k_i should not be confused with relative importance of attributes or weighting factors.
Points in the single attribute utility functions $U_i(f_i)$ and the attribute's scaling constant k_i can be obtained using the "certainty equivalent" method. An example of the lottery questions used in the certainty equivalent method to determine points in the utility function $U_i(f_i)$ is given in Figure 5.

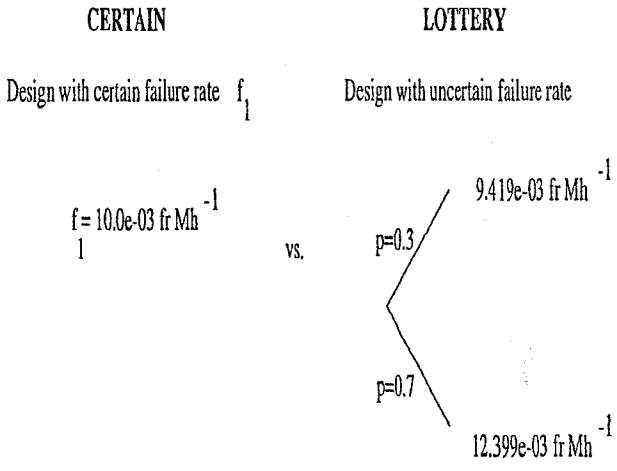

Figure 5: An example of the lottery questions used in the certainty equivalent method to assess the single attribute utility function $U_i(f_i)$ for failure rate.

The designer is asked to imagine two alternative designs: the "certain" alternative is know with certainty to be some value f, while the "lottery" alternative represents a design alternative in which there is uncertainty as to the attribute level. The lottery in Figure 5, shows a probability p of 30% that the failure rate (f_1) will be at the estimated best possible level (f_{1b}) and a probability of $(1-p)$ of 70% that failure rate will be at the estimated worst possible value (f_{1w}). When the indiference point is reached, that is, when the designer is equally likely to take the "lottery" or stay with the "certain" alternative, a point in the single attribute utility function, $U_i(f_i) = p$, is obained. The following equations shows the derivation of this result.

$$U_i(f_i) = p.U_i(f_{ib}) + (1-p).U(f_{iw}) \tag{8}$$

$$U_i(f_i) = p.(1) + (1-p).(0) \tag{9}$$

$$U_i(f_i) = p \tag{10}$$

The value of p at which the designer will be indifferent is obtained by iterating through extreme values of p. The value of k_i is equal to the utility where the attribute f_i is at its best level, f_{ib} and all of the other attributes are at their worst levels; at this point $U(f_{1w}, ...f_{ib}, ...f_{nw}) = k_i$. The "certain" alternative shown in Figure 6 represents a design alternative with attribute levels known with certainty, and the lottery represents a design with uncertain attribute levels. The lottery shows a probability p of 60% that the design has the estimated best attribute levels ($f_1 = 9.419e-03\ fr\ Mh^{-1}; f_2 = 0.4\ m$) and a probability $(1-p)$ and a probability of 40% that the design will exhibit the estimated worst attribute levels ($f_1 = 12.399-03\ fr\ Mh^{-1}; f_2 = 0.8\ m$).

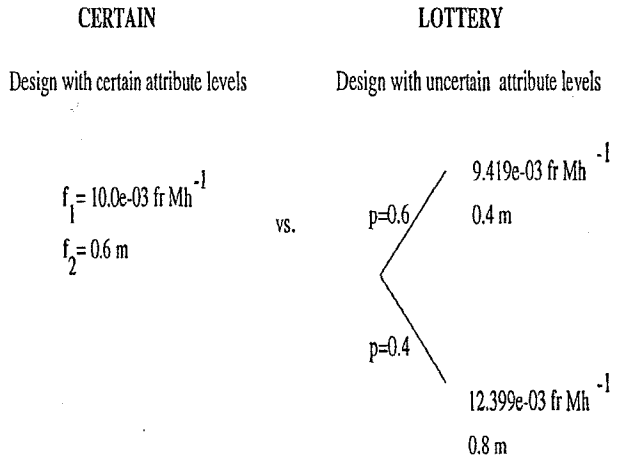

Figure 6: An example of the lottery questions used in the certainty equivalent method to assess the single attribute scaling constant k for failure rate (k_1).

The value of k_i is equal to the value of p corresponding to the indiference point; see the following equations for the derivation of this result.

$$U(f_{1w}...f_{ib}...f_{nw}) = p.U(\vec{f_b}) + (1-p).U(\vec{f_w}) \tag{11}$$

$$U(f_{1w} \cdots f_{ib} \cdots f_{nw}) = p.(1) + (1-p).(0) \quad (12)$$

$$U(f_{1w} \cdots f_{ib} \cdots f_{nw}) = k_i \quad (13)$$

Details of the certainty equivalent method can be found in Keeney et al. [14].

4 Case study

The case study represents the problem of placing a set of thirty six heat sources with heat generation rates and thermal sensitivities as specified in Table 4 with $B_1 = 400\ fr\ Mh^{-1}$,$B_2 = 800\ fr\ Mh^{-1}$, $B_3 = 1200\ fr\ Mh^{-1}$,$B_4 = 1600\ fr\ Mh^{-1}$, using the solution methodology discussed in the previous section. The optimal placement includes both the minimization of the failure rate of the system (Equation 2) and the minimization of the wiring length (Equation 3) using an interconnectivity matrix to be specified later. Before presenting and discussing the results associated with these two multiobjective optimization strategies, the elements of the solution methodology were subject to a validation process. The genetic algorithm has been validated elsewhere (see, Queipo et al. [8], Queipo [10], and Queipo et al. [11]), and the heat transfer solver predicted the same temperature distributions than those reported by Cahlon et al. citecah for selected component placements, such as the one illustrated in Figure 8. See Figure 7 for the interpretation of the figures depicting the temperature distribution among heat sources.

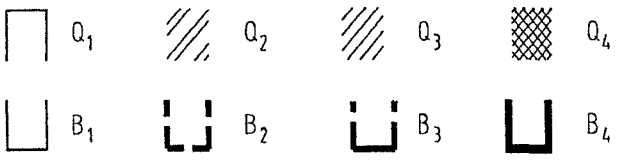

Figure 7: Legend used for graphically identifying the heat generation rates and thermal sensitivities of the heat sources when reporting temperature distributions among them.

4.1 Control parameters for the genetic algorithm

Considering a linear version of the problem ($\gamma = 1.0$) and having the minimization of the failure rate as the sole optimization criterion, numerical simulations of the genetic algorithm were conducted for a range of crossover rates, mutation rates, number of generations and population size. The crossover and mutation rates considered were (0.4,0.6,0.75, and 0.9) and (0.4,0.65 and 0.9), respectively. Furthermore, three different values for the population size (7,10,and 12) and number

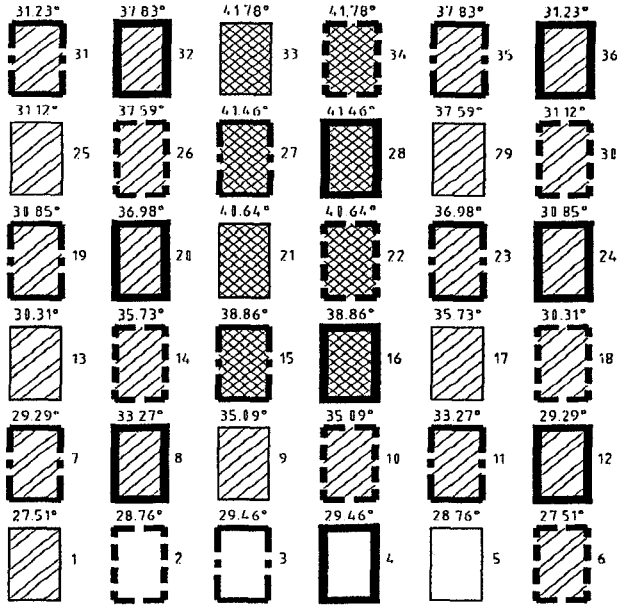

Figure 8: Temperature distribution among the heat sources for a testing configuration.

of generations (8,10,12) were evaluated. The sigma scaling factor was fixed at a value equal to 3. The present genetic algorithm exhibited a robust behavior providing what are considered to be optimal or near optimal solutions for a variety of combinations of crossover rate, mutation rate, population size and number of generations (C,M,P,G); for example, (0.6,0.9,12,10), (0.6,0.65,12,12), (0.75,0.4,7,10), and (0.9,0.65,12,12). The best arragement for the conditions specified in this section exhibited a failure rate of $9.166 \times 10^{-3} fr Mh^{-1}$ and an interconnectivity length of 7.92 m.

All the results reported throughout the study correspond to a crossover rate of 0.6, a mutation rate of 0.9, a population size of 12, and a number of generations equal to 8.

Note that the size of the solution space is given by the expression $36!/(3!^8.2!^4) = 1.38 \times 10^{34}$; if each function evaluation takes 1 second, it would take 4.39×10^{26} years to exhaustively investigate the solution space. As a result, heuristic methods such as genetic algorithms to adaptively search for optimal solutions are mandatory.

5 Results and discussion

This section addresses the situation where the heat sources may differ in their heat generation rates or their thermal sensitivities and the optimization criteria include both *thermal* and *non-thermal* optimization criteria. As previously discussed, in the case of mul-

Heat source ID	Heat generation Watts	Thermal sensitivity (fr Mh^{-1})
5	Q_1	B_1
2	Q_1	B_2
3	Q_1	B_3
4	Q_1	B_4
1,13,25	Q_2	B_1
6,18,30	Q_2	B_2
7,19,31	Q_2	B_3
12,24,36	Q_2	B_4
9,17,29	Q_3	B_1
10,14,26	Q_3	B_2
11,23,35	Q_3	B_3
8,20,32	Q_3	B_4
21,33	Q_4	B_1
22,34	Q_4	B_2
15,27	Q_4	B_3
16,28	Q_4	B_4

Table 1: Thermal characteristics of the heat sources considered in the present case study.

tiobjective optimization there is not a general definition of the optimal values and no single best approach for solving these problems. As a result, different philosophies and methodologies, such as Pareto optimization and Multiattribute utility analysis, co-exist for addressing optimization problems with multiple objectives.

The *thermal* and *non-thermal* optimization criteria correspond to the minimization of the failure rate of the system computed using the Arrhenius relation and of the total wiring length according to an interconnectivity matrix. The present interconnectivity requirement is that each of the heat sources identified with numbers between *1* and *6* must be wired with those identified with numbers between *7* and *12*. Next, each of the heat sources identified with numbers between *13* and *18*, must be wired with those identified with numbers between *19* and *24*. The same pattern continues and ends with the connection of each of the following heat sources (*25,26,27,28,29,30*) with those identified with the numbers between *31* and *36*. The distance between heat sources modeling the electronic components is 0.02m.

The total interconnectivity length and total failure rate of the arrangements of heat sources are denoted by the functions λ (Equation 2) and g (Equation 3), respectively.

5.1 Pareto optimization

Solutions expected to belong to the Pareto optimal set are calculated using the weighting method (Balachandran et al. [13]) which converts the multiobjective problem to a single objective problem, in which the function to be optimized is the weighted sum of the individual objective functions. In this case, the function f to be minimized has the form:

$$f = w_\lambda * \lambda_{total} + w_g * (C_o g) \tag{14}$$

where C_o represents a scaling factor, calculated for each generation in order to render the average contribution of the interconnectivity term in the sum comparable in magnitude to the average contribution due to the total failure rate. The coefficients w_λ and w_g are weighting factors representing the relative importance of the optimization criteria, with $w_\lambda + w_g = 1$. In this work, three sets of weighting coefficients are exercised; the extreme cases and a situation where the optimization criteria are considered to be equally important, that is: (w_λ, w_g): (1.0,0.0), (0.5,0.5) and (0.0,1.0).
The solution corresponding to weighting factors (1.0,0.0) has a failure rate of 9.166×10^{-3} $fr\ Mh^{-1}$ and an interconnectivy length of $7.92\ m$, and a failure rate of 8.525×10^{-3} $fr\ Mh^{-1}$ and an interconnectivy length of $8.44\ m$, for the linear and non-linear versions of the problem respectively. This solution was found for the linear (non-linear) case after 9 (8) generations and 101 (95) objective function evaluations and corresponds to a situation where the minimization of the failure rate is the sole optimization criterion. Figures 9 and 10 show the evolution of the lowest failure rate value found along the search process for the linear and non-linear versions of the problem. Note that better solutions may be found if the search is extended to a greater number of generations.
Figure 11 depicts the temperature distribution for the arrangement with the lowest failure rate for a linear version of the problem ($\gamma = 1.0$).
A solution corresponding to the other extreme of the Pareto optimal set; that is, the situation where the minimization of the wiring length is the only optimization criterion ($w_\lambda = 0$ and $w_g = 1.0$) was selected by inspection of the interconnectivity requirement. The optimal solution selected for this case correspond to the initial configuration depicted in Figure 8 with a failure rate of 10.36×10^{-3} $fr\ Mh^{-1}$ and an optimal wiring length of $6.36\ m$.
The Pareto optimal solutions associated with the situation with equal weighting factors for the linear (non-linear) version of the problem is shown in Table 2 (3), respectively. Figure 14 shows the tempe-

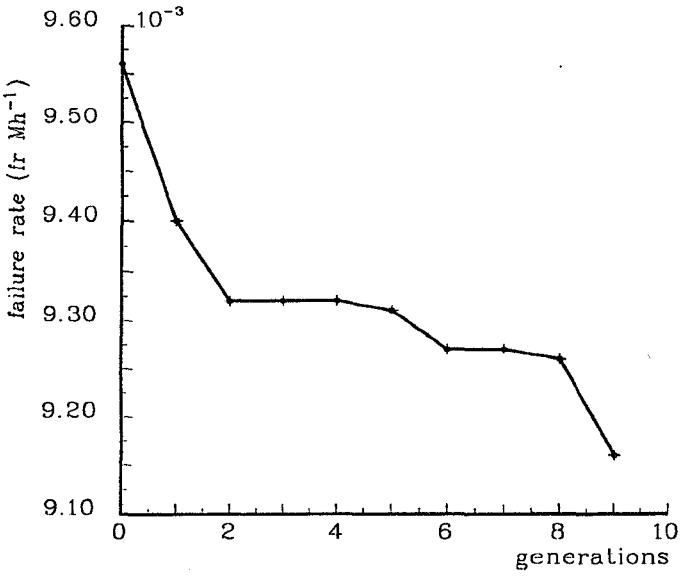

Figure 9: Evolution of the lowest failure rate value along the search process considering $\gamma = 1.0$ and $(w_\lambda = 1.0, w_g = 0.0)$ (Pareto optimization).

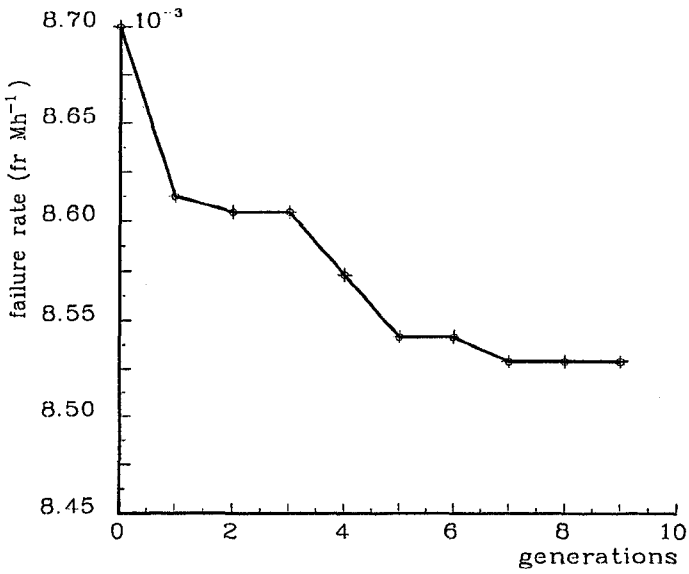

Figure 10: Evolution of the lowest failure rate value along the search process considering $\gamma = 1.25$ and $(w_\lambda = 1.0, w_g = 0.0)$ (Pareto optimization).

Failure rate $frMh^{-1}$	Interconnectivity length (m)	Gen.	No. fun. eval.
9.572×10^{-3}	8.20	9	110
9.578×10^{-3}	7.96	9	117
9.810×10^{-3}	7.80	1	22

Table 2: Pareto optimal solutions (Linear case).

rature distribution among the heat sources for one of the Pareto optimal solutions, the one with $fr = 9.578 \times 10^{-3}\ frMh^{-1}$, and interconnectivity length equal to 7.96 m.

Figures 12 and 13 depict the Pareto optimal solutions for the sets of weighting factors considered in the present situation for the linear ($\gamma = 1.0$) and non-linear cases ($\gamma = 1.25$), respectively.

5.2 Multiattribute utility analysis

This section discusses the solution of the case study using the single attribute utility functions for failure

Failure rate $frMh^{-1}$	Interconnectivity length (m)	Gen.	No. fun. eval.
8.803×10^{-3}	7.68	8	107
8.764×10^{-3}	7.78	9	110
8.661×10^{-3}	8.08	3	42

Table 3: Pareto optimal solutions (Non-linear case).

rate (f_1) and wiring length (f_2) shown in Figures 15 and 16, respectively. Figures 15 and 16 corresponds to quadratic polynomials that interpolate the following three points (f_1,$U_{(f_1)}$): (9.166e-03 fr Mh^{-1},1.0). (9.833e-03 fr Mh^{-1},0.4), (10.500e-03 fr Mh^{-1},0.0); and the points (f_2,$U_{(f_2)}$): (6.36 m,1.0), (8.18 m,0.6), (10.0 m, 0.0), respectively.

The scaling factors reflecting acceptable tradeoffs between attributes, are given as k_1 =0.6 (failure rate) and k_2 =0.4 (wiring length). Both, the utility functions and the scaling factors are assumed to have been obtained with the participation of the designer and the certainty equivalent method discussed in a previous section. The function to be maximized is given by Equation 5 with the aforementioned utility functions and scaling factors.

Considering the linear version of the problem, Table 4 displays the ten best arrangements obtained when using the multiattribute utility analysis approach. The best arrangement exhibits a failure rate and wiring length of $9.179e-03 frMh^{-1}$ and 8.56 m, respectively. The best arrangement was found after 7 generations and 92 function evaluations with the maximum temperatures of the heat sources shown in Figure 17. Note that this approach provides the "best" solution with a single coupled execution of the heat transfer solver and the genetic algorithm provided the utility functions (U_i) and the scalar constants (k_i) are available. In addition, this approach could be used to identify t-

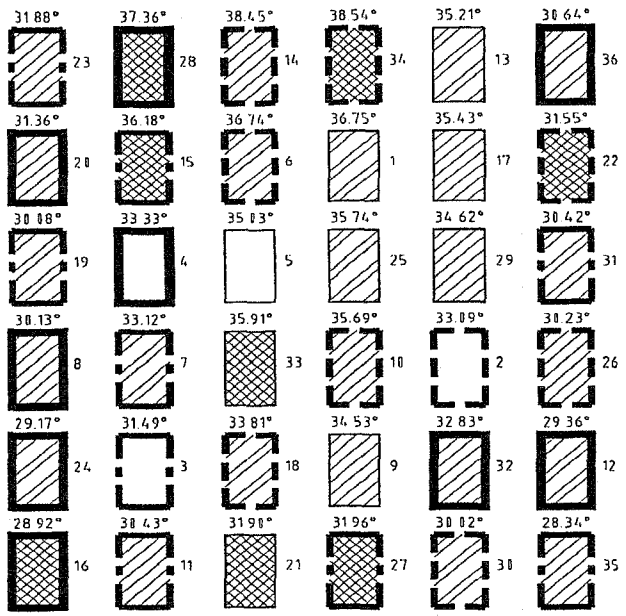

Figure 11: Temperature distribution among the heat sources considering $\gamma = 1.0$ and $(w_\lambda = 1.0, w_g = 0.0)$ (Pareto optimization).

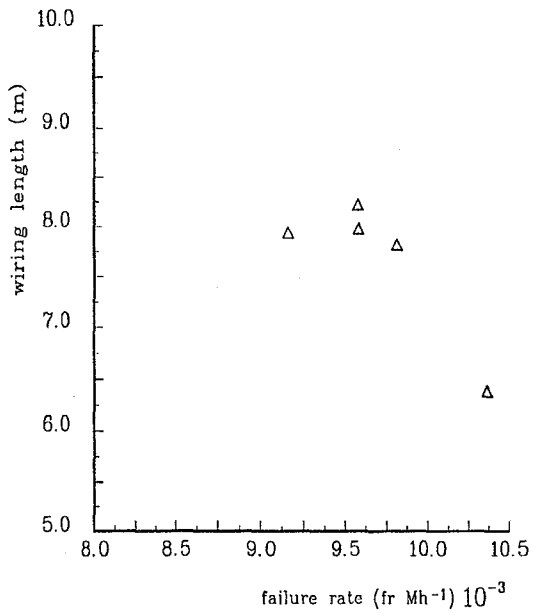

Figure 12: Best arrangements obtained by the CSGA using Pareto optimization for the cases of (w_λ, w_g): (1.0,0.0), (0.5,0.5) and (0.0,1.0) (Linear case).

he "best" solution among the Pareto optimal solutions found (those obtained in the previous section) by computing the utility value of each of the Pareto optimal solutions and selecting the solution with highest utility value.

6 Conclusions

A model for the problem of optimal placement of electronic components on printed wiring boards subject to *thermal* and *non-thermal* optimization criteria has been formulated and solved using a methodology based on three components: i) a heat transfer solver for the prediction of the maximum temperature of the heat sources ; ii) a multiobjective optimization strategy for the scalarization of the vector of design objectives; and, iii) a genetic algorithm for the search of optimal or near optimal solutions.

The inclusion of a more accurate and consequently computationally more expensive heat transfer model, could be handled with the proposed methodology by taking advantage of the fact that the cited methodology can easily be adapted to work in a distributed computing environment.

The multiobjective optimization strategy embedded in the solution methodology is flexible enough to account for two extreme situations (no knowledge/knowledge) regarding the knowledge of the preference structure of the designer by using Pareto optimization and Multiattribute utility analysis.

The solution methodology shows promise as an effective and efficient tool for providing optimal or near-optimal solutions for electronic component placement problems where both thermal and non-thermal optimization criteria are of interest under rather general conditions regarding component geometries, heat generation rates and thermal sensitivities.

Acknowledgments

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References

[1] L. Moresco, "Electronic system packaging: the search for manufacturing the optimum in a sea of constraints," *Seventh IEEE/CHMT International Electronic Manufacturing Technology Symposium,* pp. 149-164, 1989.

[2] J. Weiss, P. Fortner, B. Pearson, K. Watson, and T. Monroe, "Modeling air flow in electronic packages," *Mechanical Engineering,* September, pp. 56-58, 1989.

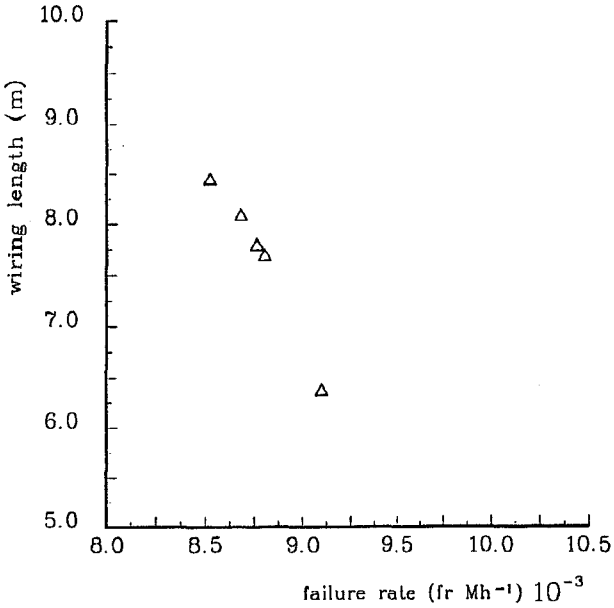

Figure 13: Best arrangements obtained by the CSGA using Pareto optimization for the cases of (w_λ, w_g): (1.0,0.0), (0.5,0.5) and (0.0,1.0) (Non-linear case).

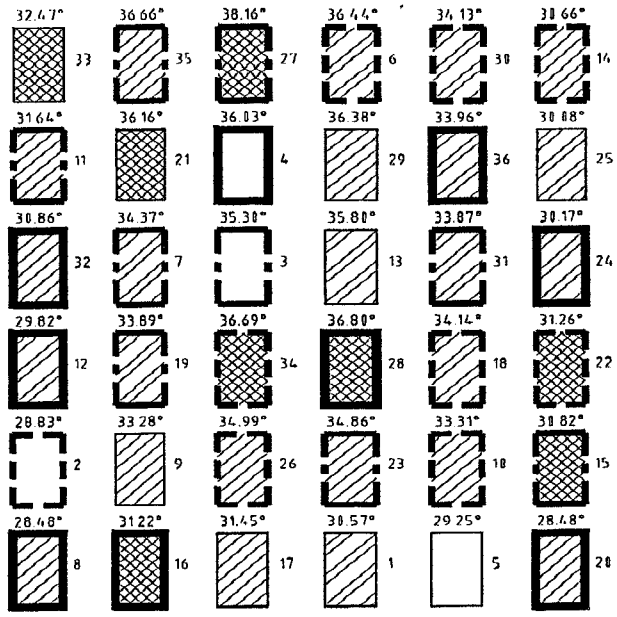

Figure 14: Temperature distribution among the heat sources for a selected Pareto optimal solution; $\gamma = 1.0$ and $(w_\lambda = 0.5, w_g = 0.5)$.

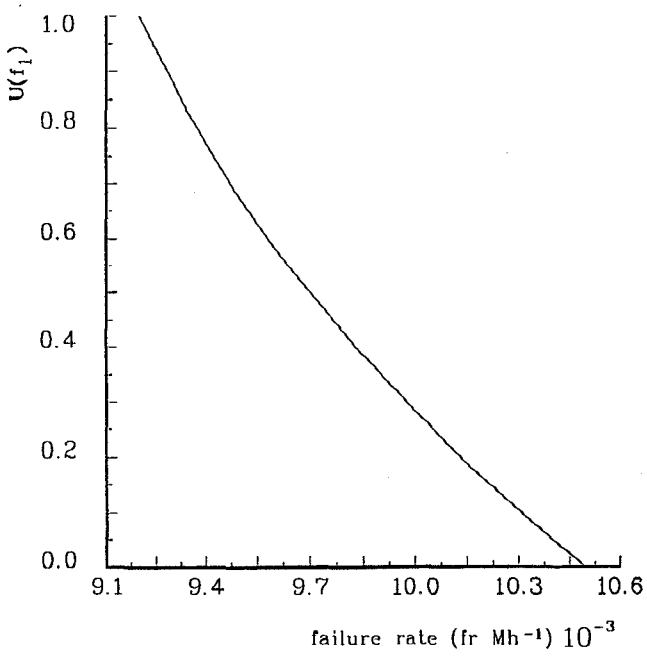

Figure 15: Single attribute utility function for *failure rate.*

[3] H. Wessely, O. Fritz, M. Horn, P. Klimke, W. Koschnick, and K. Schmidt, "Electronic Packaging in the 1990's: The Perspective from Europe," *IEEE Transactions on Components, Hybrids, and Manufacturing Technology,* Vol. 14, No. 2, June, pp. 272-284, 1991.

[4] D. Dancer, and M. Pecht, "Component-Placement Optimization for Convectively Cooled Electronics," *IEEE Transactions on Systems, Man and Cybernetics,* Vol. 38, No. 2, June, pp. 199-205, 1989.

[5] M. Osterman, and M. Pecht, "Component placement for Reliability on Conductively Cooled Printed Wired Boards," *Journal of Electronic Packaging,* Vol. 111, pp. 149-155, 1989.

[6] R. Eliasi, T. Elperin, and A. Bar-Cohen, "Monte Carlo Thermal Optimization of Populated Printed Circuit Board," *IEEE Transactions on Components, Hybrids, and Manufacturing Technology,* Vol. 13, No. 4, December, pp. 953-960, 1990.

[7] M. Pecht, M. Palmer, W. Schenke, and R. Porter, "An investigation into PWB component-placement tradeoffs," *IEEE Trans. on Reliability,* Vol. R-36, No. 5, Dec. 1987.

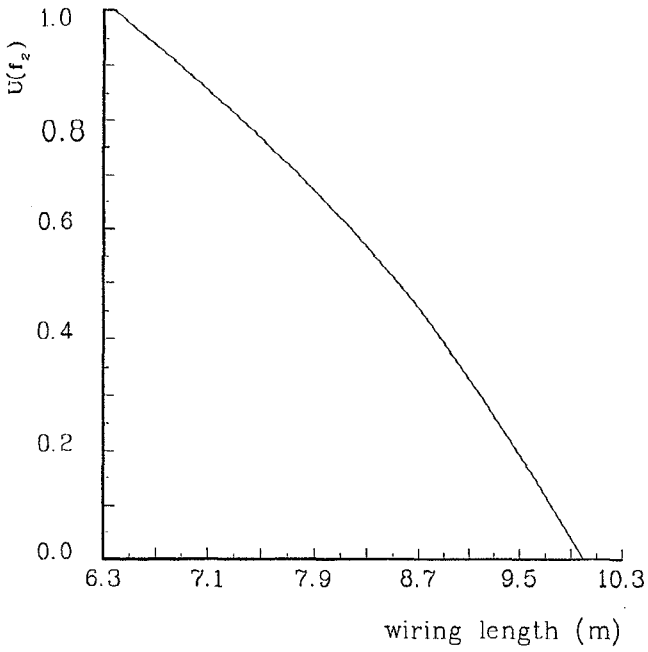

Figure 16: Single attribute utility function for *wiring length.*

Failure rate $frMh^{-1}$	Interconnectivity length (m)	Gen.	No. fun. eval.
9.179×10^{-3}	8.56	7	92
9.372×10^{-3}	8.44	4	59
9.404×10^{-3}	8.28	8	106
9.510×10^{-3}	7.76	8	102
9.404×10^{-3}	8.32	9	119
9.212×10^{-3}	8.84	9	117
9.423×10^{-3}	8.24	7	90
9.304×10^{-3}	8.36	5	61
9.390×10^{-3}	8.36	7	93
9.268×10^{-3}	8.64	9	111

Table 4: Ten best arrangements obtained by the CSGA for the linear version of the case study (Multiattribute utility analysis).

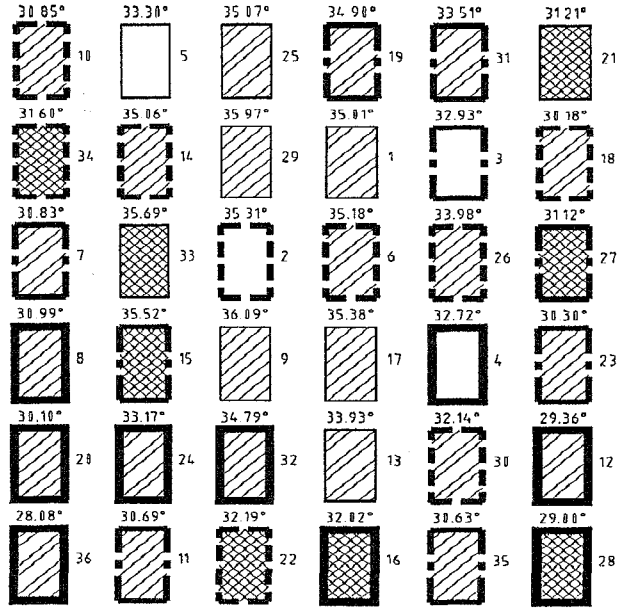

Figure 17: Maximum temperatures of the heat sources associated with the best arrangement uncovered by the CSGA; f_1 = 9.179-03 fr Mh^{-1} and f_2= 8.56 m. (Multiattribute utility analysis).

[8] N. Queipo, R. Devarakonda, and J.A.C. Humphrey, "Genetic Algorithms for Thermosciences Research: Application to the Optimized Cooling of Electronic Components," *International Journal of Heat and Mass Transfer,* Vol. 37, No. 6, pp. 893-908, 1994.

[9] N. Queipo, J.A.C. Humphrey, and A. Ortega, "Optimal placement of convectively and conductively cooled electronic components using genetic algorithms," *Symposium in Honor of Chancellor Chang-Lin Tien,* University of California at Berkeley, pp. 453-464, 1995.

[10] N. Queipo, *On the Optimal Placement of Heat Sources in an Enclosure based on Adaptive Search and Machine Learning,* Ph.D. Thesis, University of California at Berkeley, 1995.

[11] N. Queipo, J.A.C. Humphrey, and A. Ortega, "Multiobjective optimization of component placement on printed wiring boards," *Proceedings of the fifth InterSociety conference on thermal phenomena in electronic systems,* pp. 359-372, 1996.

[12] B. Cahlon, I. Gertsbakh, Schochetman, I., and M. Schillor, "A model for the convective cooling of electronic components with application to optimal placement," *Mathl. Comput. Modeling,* Vol. 15, No. 2, pp. 59-75, 1991.

[13] M. Balanchandran, and J. Gero, "A comparison of three methods for generating the Pareto Optimal Set,", *Engineering Optimization,* Vol. 7, pp. 319-336, 1984.

[14] R. Keeney, and H. Raiffa, *Decision with Multiple Objectives,* Cambridge University Press, 1993.

[15] D. Thurston, "A formal method for subjective design evaluation with multiple attributes," *Research in Engineering Design,* Vol. 3, pp. 105-122, 1991.

[16] K. Wong, "What is wrong with the existing reliability predictions methods?," *Quality and Reliability Engineering International,* Vol. 6, pp. 251-257, 1990.

[17] H. Blanks, "Arrhenius and the temperature dependence of non-constant failure rate," *Quality and Reliability Engineering International,* Vol. 6, pp. 259-265, 1990.

[18] K. De Jong, "Adaptive System Design: A Genetic Approach," *IEEE Transactions on Systems, Man and Cybernetics,* Vol. SMC-10, No. 9, September, pp. 566-574, 1980.

[19] D. Steinberg, *Cooling techniques for electronic equipment,* Wiley, New York, 1982.

[20] N. Schraudolph, and J. Grefenstette, *A User's Guide to GAucsd 1.4,* Available via anonymous ftp from cs.ucsd.edu (132.239.51.3) in the pub-/GAucsd directory, 1992.

[21] W. Press, B. Flannery, S. Tenkolsky, and W. Vetterling, *Numerical Recipes-The Art of Scientific Computing,* Cambridge University Press, p 197, 1986.

[22] J. Holland, *Adaptation in Natural and Artificial Systems,* The MIT Press, Cambridge, Massachusetts, 1975.

[23] D. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning,* Addison Wesley, 1989.

[24] J. Shaffer, "Multiobjective optimization with vector evaluated genetic algorithms," *Proceedings of the first International Conference on Genetic Algorithms,* pp. 93-100, 1985.

[25] J. Horn and N. Nafpliotis, "Multiobjective optimization using niched pareto genetic algorithms," *Illigal Report No. 93005,* University of Illinois at Urbana-Champaign, U.S.A.