Multiobjective Optimal Design of Interior Permanent Magnet Synchronous Motors Considering Improved Core Loss Formula

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Abstract - This paper describes the optimal design of Interior Permanent Magnet Synchronous Motors for which two objective functions regarding motor efficiency and weight are used. Multiobjective optimization technique is applied to finding the optimal solution in this case. An optimal design method that determines both the noninferior set and the best compromise solution employing a modified genetic algorithm is proposed. In order to predict the motor performance more accurately, a core loss formula is derived considering the flux variation due to the armature reaction mmf as well as that due to the magnet.

I. INTRODUCTION

There are many conflicting design objectives in the optimal design of electric machines, so multiobjective optimization technique is required to meet design purposes. Multiobjective optimization problem, in general, has many solutions. The solution set of the multiobjective optimization problem is called as a noninferior solution set. Therefore, in order to apply this method to the optimal design of electric machines, some auxiliary steps are necessary to find the best compromise solution. So, the proposed algorithm consists of two parts of which each finds the noninferior set and the best compromise solution, respectively. In this paper, the algorithm is implemented by a modified genetic algorithm.

The core loss is generally known as no load loss. But, in contrast to the Induction Motor, the air gap flux of the Permanent Magnet Motor is varied according to the armature reaction flux[1,2]. So, the core loss formula considering the flux variation due to the stator currents is required in order to make the more accurate performance prediction possible. In this paper, a formula for the eddy current loss is derived considering the flux variation in the teeth and yoke due to the stator currents as well as the magnet.

The proposed optimal design algorithm is applied to the design of the Interior Permanent Magnet Synchronous Motor for which two objective functions regarding motor efficiency and weight are used. And the dimensions, parameters and characteristics of the optimally designed motor are compared with those of prototype one.

II. DESIGN OBJECTIVES OF THE MOTOR

A. Derivation of Core Loss

The core loss consists of the eddy current and the hysteresis loss and can be calculated from the flux densities and the rate of change of them in the stator teeth and yoke. Analysis of the air gap flux densities due to the magnet, d-axis and q-axis currents of the Interior Permanent Magnet Motor shown in Fig.1 are presented in [3]. Eddy current and hysteresis loss can be decomposed into the term in the stator teeth and that in
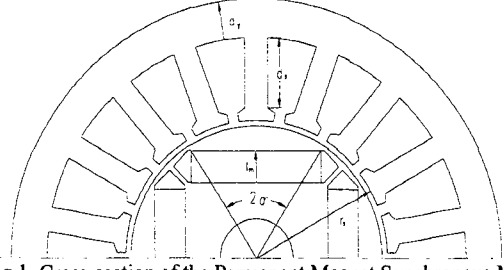

Fig.1. Cross-section of the Permanent Magnet Synchronous Motor

the yoke. Tooth flux density B_t can be given by

$$B_t = \frac{1}{w_t l_r} \int_{\omega_s t/p - \beta/2}^{\omega_s t/p + \beta/2} B_g(p\theta) l_r r_s \, d\theta \qquad (1)$$

where r_s : stator bore radius

     l_r : stator axial length

     p : the number of pole pairs

     w_t : tooth width

     B_g : air gap flux density

     ω_s : source angular frequency

     β : slot pitch in mechanical angle

and the eddy current loss per unit teeth volume P_{elt} is given by

$$
\begin{aligned}
P_{elt} &= \frac{k_{elt}}{T} \int_0^T \left(\frac{dB_t}{dt}\right)^2 dt \\
&= \frac{k_{elt}\omega_s^2}{\pi}\left\{ \frac{2}{p\beta}\left(\left(B_{gm}+B_{dm}-\hat{B}_d \cos\alpha_t\right)^2 + \hat{B}_q^2 \sin^2\alpha_t\right)\right. \\
&\quad \left.+\hat{B}_d^2\left(\frac{\alpha_t - \sin2\alpha_t}{2}\right) + \hat{B}_q^2\left(\frac{\alpha_t + \sin2\alpha_t}{2} + \delta\left(p^2\beta^2 - 1\right)\middle/ p\beta^2\right)\right\}
\end{aligned} \qquad (2)
$$

where k_{elt} : eddy current loss coefficient in teeth

     B_{gm} : air gap flux density due to the magnet

     B_{dm} : induced flux density at the rotor surface

     $\hat{B}_d$: peak flux density due to d-axis current

     $\hat{B}_q$: peak flux density due to q-axis current

     δ : web width in mechanical angle

     α : magnet pole arc angle

     $\alpha_t = p(\alpha - \beta/2)$

Eddy current loss per unit yoke volume P_{ely} can be calculated in the same manner. And the yoke flux density can be calculated as follows:

$$B_y = \frac{1}{d_y l_r} \int_0^{\omega_s t / p} B_g(p\theta) \, l_r \, r_s \, d\theta \qquad (3)$$

where d_y : stator yoke depth

$$P_{ely} = \frac{k_{ely}}{T} \int_0^T \left(\frac{dB_y}{dt}\right)^2 dt$$

$$= \frac{k_{ely}}{\pi} \left(\frac{r_s \omega_s}{p d_y}\right)^2 \left\{ 2p\alpha \left(B_{gm} + B_{dm}\right)^2 + \hat{B}_d^2 \left(p\alpha + \frac{\sin 2p\alpha}{2}\right) \qquad (4)$$

$$+ \hat{B}_q^2 \left(\frac{2p\alpha - \sin 2p\alpha + p\delta + \sin p\delta}{2}\right) - 4\left(B_{gm} + B_{dm}\right)\hat{B}_d \sin p\alpha \right\}$$

where k_{ely} : eddy current loss coefficient in yoke

Hysteresis loss per unit volume can be obtained from the maximum flux density in the teeth and yoke, respectively.

$$P_{hlt} = k_{hlt} \, \omega_s \, \hat{B}_t^2$$
$$P_{hly} = k_{hly} \, \omega_s \, \hat{B}_y^2 \qquad (5)$$

where k_{hlt} : hysteresis loss coefficient in teeth
  k_{hly} : hysteresis loss coefficient in yoke

Stator teeth and yoke volume is given by

$$V_{teeth} = w_t \, d_s \, l_r \, S_n$$
$$V_{yoke} = 2\pi \left(r_s + d_s + d_y/2\right) d_y \, l_r \qquad (6)$$

where d_s : stator slot depth
  S_n: the number of stator slot

Thus the overall core loss is expressed as:

$$P_{cl} = V_{teeth}\left(P_{elt} + P_{hlt}\right) + V_{yoke}\left(P_{ely} + P_{hly}\right) \qquad (7)$$

B. Objective Functions

The motor loss and weight are taken as the objective functions which are to be minimized. The motor loss consists of the stator winding loss and core loss assuming that other losses are negligible. The stator winding loss can be given by

$$P_{sw} = 3 R_s I_s^2$$
$$= \frac{36 \rho_c N_s^2 I_s^2}{\pi f_s d_s (r_s + d_s)} \left(l_r + \frac{\pi r_s k_\sigma}{p}\right) \qquad (8)$$

where ρ_c : resistivity of wire
  I_s : stator phase current
  N_s: the number of stator winding turns
  f_s : stator slot fill factor
  k_σ : overhang coefficient of stator winding

From (7) and (8), the overall motor losses are given as follows:

$$f_{loss} = P_{sw} + P_{cl} + P_{ml} \qquad (9)$$

Mechanical loss P_{ml} is considered as a constant value.

Motor weight is the sum of the stator, rotor, magnet and winding weight. The rotor weight is calculated on the parts

that participate in the energy conversion, i.e.,

$$f_{weight} = \rho_i l_r (\pi r_s d_s + \pi d_y (r_s + d_s))$$
$$+ \rho_i l_r (\pi r_s^2 - l_m (4w_m + 2l_m)) + 4\rho_m w_m l_m l_r \qquad (10)$$
$$+ \rho_w \pi f_s d_s (r_s + d_s)(l_r + k_\sigma \frac{2\pi r_s}{p})$$

where l_m : magnet thickness
  w_m : magnet width
  ρ_i : density of the steel core
  ρ_m : density of the magnet
  ρ_w : density of the wire

C. Decision Variables and Constraints

Since the losses and the weight are represented as a function of design parameters of the motor, several design parameters can be selected as the decision variables. The decision variables are the number of stator winding turns, the stator bore radius, stator axial length, stator yoke depth, stator slot depth, magnet thickness, and the pole arc angle.

The variables are restricted within the range determined by the constraints. The constraints are deduced from the geometry, the electrical and magnetic characteristics of the motor such as the limits of the flux density, current density and magnet protection against the demagnetization. The output power is considered to be same as the rated value of the prototype motor in order to compare the prototype with optimally designed motor.

III. OPTIMIZATION ALGORITHM

A. Formulation of the Problem

Vector optimization problem with p objectives, n decision variables, and m constraints is formulated as

Minimize $f(x)$. $\qquad (11)$

s.t $x_i \geq 0, \quad i = 1, \cdots, n$
  $g_j(x) \leq 0, \quad j = 1, \cdots, m$

where $x = (x_1, \cdots, x_n)$
  $f(x) = (f_1(x), \cdots, f_p(x))$

The solutions must satisfy the noninferiority condition. Noninferiority can be defined in the following way[4];

A feasible solution is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

In general, there exist many solutions satisfying the noninferiority condition. So, it is necessary to find the best compromise solution among the noninferior solutions. If all the objectives have the equal importance, an adequate criterion for the best compromise solution can be given by the min-max optimum of the relative difference from the global optima of the objective functions

$$v(\mathbf{x}^*) = \begin{array}{c} \min \\ \mathbf{x} \in X_f \end{array} \begin{array}{c} \max \\ k \in K \end{array} \left\{ \frac{\left| f_k(\mathbf{x}) - f_k^* \right|}{\left| f_k^* \right|} \right\}. \qquad (12)$$

where X_f : feasible region in decision space

f_k^* : optimal solution of kth objective

B. The Modified Genetic Algorithm

The modified genetic algorithm is a solution method to the vector optimization problem. The algorithm searches both the noninferior solution set and the best compromise solution. Algorithm for searching the noninferior solution set has the same flow as the conventional genetic algorithm, except for the following modifications[5];

a) *Fitness values are high and same ones for all the points satisfying the noninferiority condition, and low ones otherwise.*

b) *Convergence criterion is to be satisfied if no further update of noninferior solution set is done during the predetermined number of iterations.*

The flowchart of the modified genetic algorithm for the vector optimization problem is shown in Fig.2.
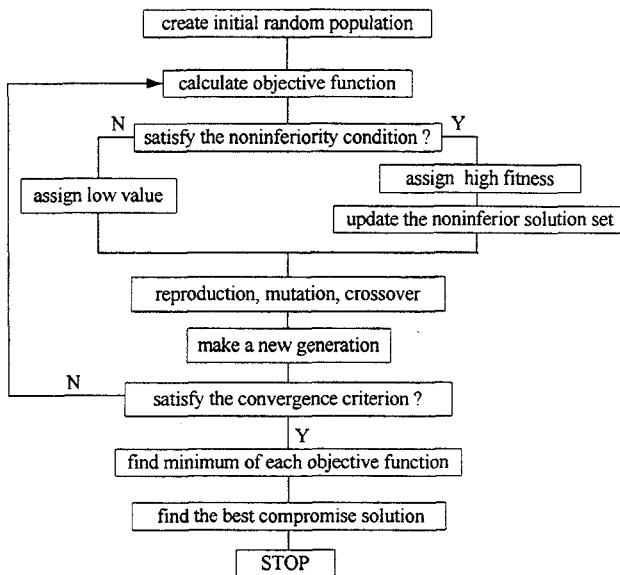

Fig.2. Flowchart of the modified genetic algorithm

IV. RESULTS

The proposed optimization algorithm is applied to the design of the Permanent Magnet Synchronous Motor of 3 phase, 4 pole and 600 W ratings. The optimal design results are given in Table 1 which shows the solution corresponding to the minimum of each objective function. The comparison of the optimally designed motor-the best compromise solution-with the prototype is given in Table 2. The optimally designed motor shows higher efficiency but larger weight than the prototype. But since the rotor dimension of the optimally designed motor is decreased compared with that of the prototype, it can be deduced that the optimally designed motor

could have an enhanced servo performances because of the reduction of the rotor inertia.

Table 1
RESULTS OF THE OPTIMAL DESIGN

	minimum loss solution	minimum weight solution
loss [W]	154.43	198.42
weight [kg]	2.33	1.72
number of winding turns	432	492
stator bore radius [mm]	17.49	19.92
stator axial length [mm]	49.24	42.82
stator slot depth [mm]	14.31	14.17
stator yoke depth [mm]	7.74	5.45
magnet thickness [mm]	3.47	4.15
magnet pole arc angle [deg]	35.62	31.67

Table 2
COMPARISON OF THE OPTIMALLY DESIGNED MOTOR
WITH THE PROTOTYPE

	prototype	optimally designed motor
loss [W]	185.58	167.58
weight [kg]	1.95	2.07
number of winding turns	480	444
stator bore radius [mm]	20.50	18.54
stator axial length [mm]	41.60	48.00
stator slot depth [mm]	13.25	13.30
stator yoke depth [mm]	5.80	7.71
magnet thickness [mm]	4.00	3.34
magnet pole arc angle [deg]	34.35	37.26

V. CONCLUSION

In this paper a multiobjective design algorithm for the interior permanent magnet synchronous motor design which has two conflicting design objectives, i.e., loss and weight, is presented. And the improved core loss formula is used considering the flux variation due to the armature reaction mmf and the magnet. A modified genetic algorithm is presented as search method which finds the noninferior solution set of multiobjective problem. The proposed algorithm has been applied to a sample motor design. It is found that the optimally designed motor shows an improved servo performances comparing to the prototype.

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