

**A Meta-Heuristic Satisficing Tradeoff Method for Solving  
Multiobjective Combinatorial Optimization Problems**  
- With Application to Flowshop Scheduling -

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**ABSTRACT**

In this paper an effective meta-heuristic approach is proposed to realize a satisficing tradeoff method for solving multiobjective *combinatorial* optimization problems. Firstly, Pareto optimal solutions (individuals) are generated by using a genetic algorithm with family elitist concept for a multiobjective combinatorial optimization problem. Then, we try to find a preferred solution of the decision maker based on the satisficing tradeoff method. In this paper a new meta-heuristic satisficing tradeoff method is proposed in which we do not need to solve a complex min-max problem in each iteration, but we try to find a min-max solution in the Pareto optimal solutions (individuals) generated by the genetic algorithm. We further revise the min-max solution by using a local search approach such as a simulated annealing method. As a numerical example a flowshop scheduling problem is included to verify the effectiveness of the method proposed in this paper.

**1. INTRODUCTION**

In advanced production management systems performance evaluation is usually to be done under multiple objectives. Furthermore, in production scheduling, performance evaluation is to be done under combinatorial optimization. Since combinatorial optimization problems are usually NP-hard or sometimes NP-complete (Cook, *et al.*, 1998), an optimal solution or even a suboptimal solution is hard to find even for a single-objective

problem. Therefore, nobody has tried rigorously to solve multiobjective combinatorial optimization problems.

In this paper an effective meta-heuristic approach is proposed to realize a satisficing tradeoff method for solving multiobjective combinatorial optimization problems. Firstly, Pareto optimal solutions (individuals) are generated by using a genetic algorithm (Goldberg, 1989) with family elitist concept (Bedarahally, *et al.*, 1996) for a multiobjective combinatorial optimization problem. Then, we try to find a preferred solution of the decision maker based on the satisficing tradeoff method. A conventional satisficing trade-off method (Nakayama and Sawaragi, 1984) needs to solve a complex min-max problem in each iteration of the algorithm for a given aspiration level of each objective function. In this paper a new meta-heuristic satisficing tradeoff method is proposed in which we do not need to solve a complex min-max problem in each iteration, but we try to find a min-max solution in the Pareto optimal solutions (individuals) generated by the genetic algorithm. We further revise the min-max solution by using a local search approach such as a simulated annealing method. As a numerical example of a multiobjective combinatorial optimization problem a flowshop scheduling problem is included.

**2. SATISFICING TRADEOFF METHOD**

In general, performance evaluation problems in production management systems can be formulated as

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$$\begin{aligned} &\text{minimize } f(x) \equiv (f_1(x), f_2(x), \dots, f_r(x)) \\ &\text{subject to } x \in X \end{aligned}$$

where  $x$  denotes decision vector,  $X$  denotes the feasible region of the decision vector,  $f_i(x)$ ,  $i = 1, 2, \dots, r$  denotes multiple objective functions to be minimized, and  $r$  denotes the number of objective functions.

Basic algorithm of satisficing tradeoff method for solving multiobjective optimization problems can be written as follows (Nakayama and Sawaragi, 1984):

**Step 1.** Specification of the range of each objective function:

Specify the ideal value  $f_i^*$  and the nadir value  $f_i^o$  for each objective function  $f_i(x)$ ,  $i = 1, 2, \dots, r$  by minimizing and maximizing each objective function independently.

**Step 2.** Specification of the aspiration level:

Ask the decision maker the aspiration level  $f_i^a$ ,  $i = 1, 2, \dots, r$  for each objective.

**Step 3.** Solving min-max problem:

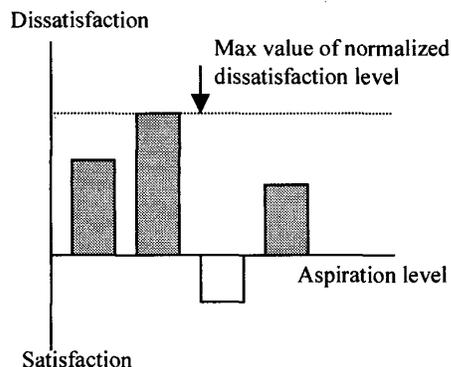
Let normalized weight for each objective be

$$w_i \equiv \frac{1}{f_i^* - f_i^o}$$

Solve min-max problem

$$\begin{aligned} &\text{minimize } \{ \max_{1 \leq i \leq r} w_i (f_i(x) - f_i^a) \} \\ &\text{subject to } x \in X \end{aligned}$$

This min-max problem is interpreted to minimize the maximum value of normalized dissatisfaction level as shown in Figure 1.



**Figure 1** A min-max problem

Instead of solving this min-max problem, an equivalent optimization problem

$$\text{minimize}_{x, z} \{ z + \alpha \sum_{i=1}^r w_i (f_i(x) - f_i^a) \}$$

$$\begin{aligned} &\text{subject to } w_i (f_i(x) - f_i^a) \leq z, \quad i = 1, 2, \dots, r \\ &x \in X \end{aligned}$$

is usually solved for small  $\alpha$ , say  $10^{-6}$ . Let  $x$  be optimal solution to the min-max problem.

**Step 4.** Tradeoff analysis:

Showing the solution  $x$  and the resulting  $f(x)$  to the decision maker we ask him if he would be satisfied with this solution. If he would not be satisfied with this solution, we ask him a new aspiration level for each objective function going back to Step 2.

### 3. GENERATION OF A SET OF PARETO OPTIMAL SOLUTIONS (INDIVIDUALS) BY GENETIC ALGORITHM

In this section we propose a method of generating a set of Pareto optimal solutions (nondominated solutions) of multiobjective optimization problems based on the *family elitist strategy* (Bedarahally, et al., 1996) in Genetic Algorithm (GA).

#### 3.1 Genetic Algorithm and Multiobjective Optimization

GA (Holland, 1975; Goldberg, 1989) is one of the most promising evolutionary computation method in which the process of biological evolution is simulated. GA for a particular problem have the following five components:

- (1) A genetic representation for a solution to the problem,
- (2) A way to create an initial population of individuals which represent potential solutions,
- (3) A function for evaluating fitness of the solutions,
- (4) Genetic operators such as crossover, mutation and inversion that alter the composition of offsprings during the reproduction, and
- (5) Parameter values that the GA uses, e.g. population size, number of generations, crossover rates, probability of mutation.

A significant advantage of GA for applying to multiobjective optimization problems is that GA can generate a set of Pareto optimal solution (individuals) simultaneously (Fonseca and Fleming, 1995; Tamaki and Nishino, 1998), where a Pareto optimal solution is a

nondominated solution such that there exists no feasible solution which improves all the objective functions. Therefore, the decision maker has to tradeoff among multiple objective functions to improve some of them.

### 3.2 Generation of a Set of Pareto Optimal Solutions (Individuals) by GA

For applying GA to multiobjective optimization problems it is necessary to find an effective method for selecting Pareto optimal individuals in the current population. Several approaches have been proposed in this direction. Here, we propose a method to use family elitist strategy (Bedarahally, *et al.*, 1996) in addition to parallel selection (Goldberg, 1989) and Pareto preservation strategy (Tamaki and Nishino, 1998).

#### (1) Parallel Selection:

Individuals of population are divided into sub-populations where the number of sub-populations is equal to  $r$ , the number of objective functions. Sub-populations of the next generation are reproduced from the current population based on the value of each objective function.

#### (2) Pareto Preservation Strategy:

All the Pareto optimal individuals in a population at each generation are preserved in the next generation. If the

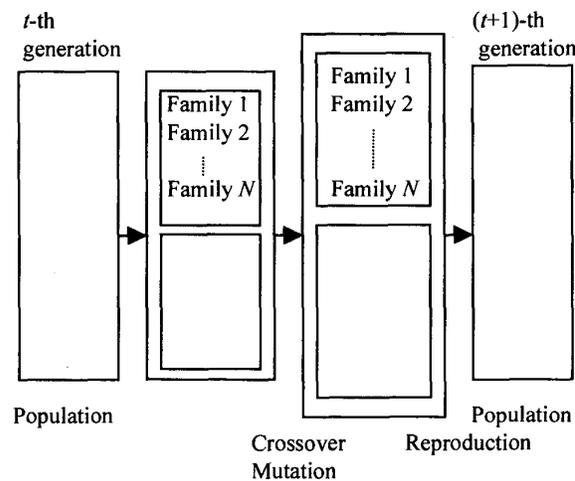
number of Pareto optimal individuals would exceed the size of the population, parallel selection would be performed.

#### (3) Family Elitist Strategy:

Population is divided into several families. Each family contains  $r$  individuals. Pairing, crossover, mutation and reproduction are performed in a family. Parallel selection and Pareto preservation strategy are used for reproduction.

Pareto preservation strategy for multiobjective optimization corresponds to *elitism* (Goldberg, 1989) for single-objective optimization, and non-dominated individuals contained in a population in each generation are all preserved in the next generation. By this strategy compromise solution could be obtained. Parallel selection from the Pareto optimal individuals enables us to improve each objective function further.

Family elitist strategy enables us to avoid the situation that the population is composed of strong individuals only. Therefore, we could expect to get Pareto optimal individuals from wider area of a set of feasible solutions without converging to unbalanced solution at the early stage. **Figure 2** shows a family elitist concept used in GA and **Figure 3** shows genetic operations in the families.



**Figure 2** Introducing a family elitist concept in GA.

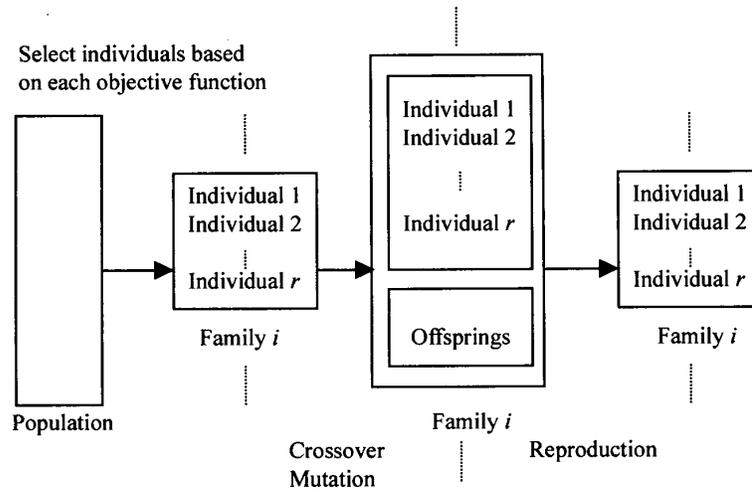


Figure 3 Genetic operations in families.

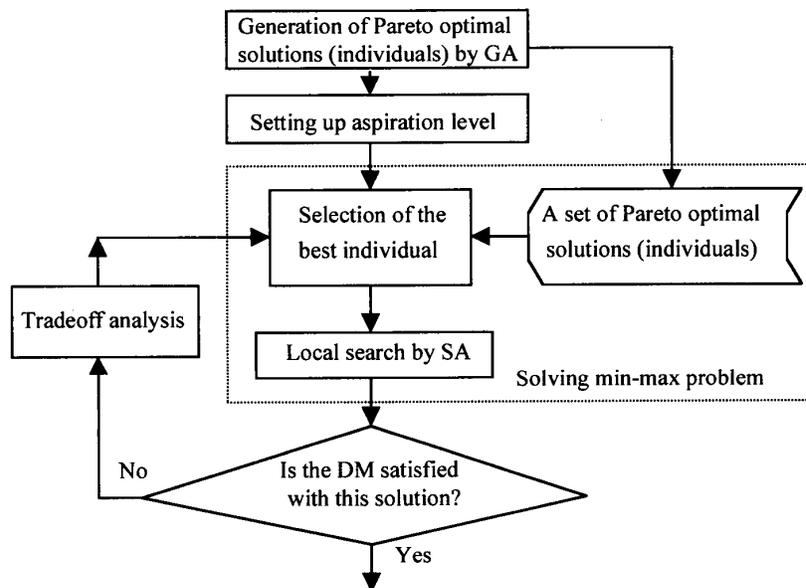


Figure 4 Flow of a meta-heuristic satisfying tradeoff method.

#### 4. META-HEURISTIC SATISFICING TRADEOFF METHOD

Satisficing tradeoff method is applicable for any problems, if we could provide a method to solve min-max problems, but for a complex combinatorial multiobjective problems, it is hard to solve min-max problem. For a larger scale problem it is getting harder to solve min-max

problem in a short time.

In this paper we propose a meta-heuristic satisfying tradeoff method. In this method we generate a set of Pareto optimal solutions (individuals) of a combinatorial multiobjective optimization problem by GA and use them as a candidate of solution of the min-max problem to be solved. That is, we choose a solution of min-max problem

from a set of Pareto optimal solutions (individuals). Starting from this solution we try to pursue local search by a simulated annealing method. Tradeoff analysis among multiple objective functions is performed by interaction with the decision maker. That is, a solution obtained by the local search is shown to the decision maker. If he is satisfied with this solution, iteration is terminated. If not we ask him to revise his aspiration level for each objective function. Then, we try to find another min-max solution from the Pareto optimal solutions (individuals), and so forth. **Figure 4** shows a flow of meta-heuristic satisficing tradeoff method.

In this meta-heuristic satisficing tradeoff method we just try to find a min-max solution from the Pareto optimal solutions (individuals) of the combinatorial multiobjective optimization problem instead of solving complex min-max problem in each iteration. Therefore, we could expect to get a satisfied preferred solution for the decision maker within a short computation time. Furthermore, by using family elitist strategy in GA for obtaining Pareto optimal solutions we could expect to get better compromise solution in a set of Pareto optimal solutions compared with the method without using family elitist strategy.

### 5. FLOWSHOP SCHEDULING PROBLEM

A flowshop scheduling problem in this paper has two cascade processes A and B. Process A has two parallel processing units 1 and 2. There exists no buffer between these two processes. All the products are first processed at the Process A either by the processing unit 1 or 2, and immediately right after this process they are processed at the Process B. There are four objectives to be minimized as follows:

- Minimize  $f_1$  = total processing time
- Minimize  $f_2$  = number of setups at Process A
- Minimize  $f_3$  = the sum of variation rate of the products processed
- Minimize  $f_4$  = the penalty for violating the continuous processing constraints

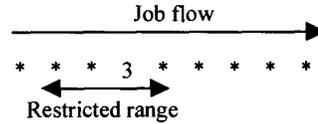
#### 5.1 Generation of Pareto Optimal Solutions (Individuals) by GA

**Figure 5** shows an example of representing individuals in order to generate a set of Pareto optimal solutions (individuals). **Figure 6** shows an example of restriction when we take into account the processing order. **Figure 7** shows an example of crossover operation in GA and **Figure 8** shows an example of mutation operation.

Processing order	4	7	1	2	6	3	5	8
Machine number	0	1	1	0	1	0	1	1

Processing order	7	3	4	5	6	1	2	8
Machine number	0	1	1	0	1	0	1	1

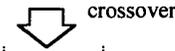
**Figure 5** An example of individuals.



**Figure 6** Generating an individual taking into account the restriction of serial processing order.

Individual 1	4	7	1	2	6	3	5	8
	0	1	1	0	1	0	1	1

Individual 2	7	3	4	5	6	1	2	8
	0	1	1	0	1	0	1	1



Offspring 1	4	7	6	1	2	3	5	8
	0	1	1	0	1	0	1	1

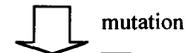
Offspring 2	4	7	2	1	6	3	5	8
	0	1	1	0	1	0	1	1

Offspring 3	7	3	4	5	1	2	6	8
	0	1	1	0	1	0	1	1

Offspring 4	7	3	4	5	6	2	1	8
	0	1	1	0	1	0	1	1

**Figure 7** An example of crossover operation.

Individual	4	7	1	2	6	3	5	8
	0	1	1	0	1	0	1	1



Individual	4	3	1	2	6	7	5	8
	0	0	1	0	1	1	1	1

**Figure 8** An example of mutation operation.

Since there are 4 objective functions to be minimized, number of individuals in a family is set to 4.

## 5.2 Computational Results

We solved a flowshop scheduling problems to process 100 products by the meta-heuristic satisficing tradeoff method. We compared the method having family elitist strategy dealing with 25 families with the method having no family elitist strategy in GA. We obtained better solution by the method having family elitist strategy as shown in **Tables 1 and 2**.

Parameter values used in GA are as follows:

- (a) Number of the individuals: 100
- (b) Number of generations: 1000
- (c) Rate of crossover: 0.6
- (d) Rate of mutation: 0.001

Parameter values used in simulated annealing are as follows:

- (a) Initial temperature: 100
- (b) Freezing temperature: 0.1
- (c) Number of perturbations at each temperature: 1000
- (d) Decreasing rate of temperature: 0.9

**Table 1** Computational result with family elitist strategy (Number of families: 25).

Criteria	$f_1$	$f_2$	$f_3$	$f_4$
1 Aspiration level	416	0	1.56	0
Value	442	27	30.11	1024
2 Aspiration level	442	27	30.11	0
Value	446	28	32.69	52
3 Aspiration level	460.9	32.4	35.82	0
Value	452	30	33.00	24
4 Aspiration level	469.9	38.88	42.67	0
Value	469	35	39.63	0

**Table 2** Computational result without family elitist strategy.

Criteria	$f_1$	$f_2$	$f_3$	$f_4$
1 Aspiration level	416	0	1.56	0
Value	494	49	40.24	1028
2 Aspiration level	494	49.0	40.24	0
Value	471	46	39.41	262
3 Aspiration level	509.6	49.0	40.24	0
Value	508	43	36.54	2
4 Aspiration level	509.6	58.8	47.97	0
Value	502	45	39.33	0

## 6. CONCLUDING REMARKS

In this paper a meta-heuristic satisficing tradeoff method is proposed to solve multiobjective *combinatorial* optimization problems effectively. Since many performance evaluation problems in production management are multiobjective and combinatorial in nature, the method proposed in this paper is expected to be used effectively in many real production management systems.

Further research problems to improve the method are how to set up aspiration levels, how to evaluate Pareto optimality of the individuals, application to many kinds of real problems, and so forth.

## 7. REFERENCES

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